UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4/M (ENG4042/ENG5022) DIGITAL [RESULTS]

XX December 2016
xx:xx – xx:xx

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

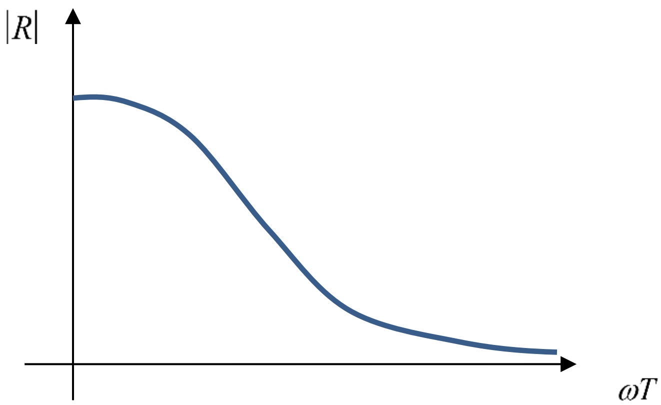
Data sheet included within paper.

Continued overleaf

SECTION A

Q1. (a) State the Nyquist sampling theorem. Given a sampling time T, what is the shift in frequency between two aliased harmonics ω_1 and ω_2? [5]

(b) Consider a continuous signal $R(z)$ with the following qualitative spectral content:



Sketch the magnitude of the same signal after sampling, $\left|TR^*\right|$, highlighting aliasing. [5]

(c) Describe a workaround to prevent aliasing to occur, assuming that you cannot alter the sampling frequency ω_s. [4]

(d) Sketch the spectral content of the sampled signal again, when the workaround is in place, showing and explaining how aliasing is prevented. [6]

Q2 (a) … []

Continued overleaf

SECTION B

Q3 The following transfer function is a lag network designed to increase the steady-state gain by a factor of 10 and have negligible phase lag at $\omega_1 = 3$ rad/s:

$$H(s) = 10\frac{10s+1}{100s+1}$$

(a) Find the gain (in dB) at ω_1. Assuming a sample time T = 0.25 s, calculate the Nyquist frequency ω_n. [0.0048 dB; 12.5664 rad/s] [2]

(b) Design the discrete equivalent of $H(s)$ using the backward rectangular rule. [$H(z) = 1.0224\frac{z-0.9756}{z-0.9975}$] [4]

(c) Compute the discrete equivalent of $H(s)$ using the pole-zero matching technique (match the steady-state gain). [$H(z) = 1.011\frac{z-0.9756}{z-0.9975}$] [10]

(d) Find the gain (in dB) at ω_1 of the discrete equivalents, and compare with that of $H(s)$. [0.1013 dB; 0.0048 dB] [4]

Q4 The following transfer function is a lead network:

$$H(s) = \frac{s+1}{0.1s+1}$$

(a) Find the discrete equivalent of it, when preceded by a zero-order hold (ZOH), for sample time T = 0.25 s. Use 4 significant digits for all numbers in the solution. [$H(z) = 10\frac{z-0.9082}{z-0.08208}$] [8]

(b) Using the inverse z-transform, find the corresponding difference equation. [$u_k = 0.08208u_{k-1} + 10e_k - 9.082e_{k-1}$] [4]

(c) State a necessary and sufficient condition for BIBO stability and determine whether the difference equation is BIBO stable. […, BIBO stable] [8]

Section C

Q5 (a) … []

Q6 (a) ... []

Continued overleaf

Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	**$F(s)$**	**$F(z)$** *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper