Frequency response design

- involves **loop-shaping**, i.e. the modification of the shape of the loop gain to achieve desired properties.
- Design targets:
  - **Performance** (disturbance attenuation and good command response): This often involves increasing the bandwidth of the closed loop system which is related to increasing the *cross-over frequency,* ω_c, of the loop gain, $L(j\omega_c)$.
  - **Robustness** with respect to **stability** and **performance**.

These targets can be translated into requirements for the complementary sensitivity $T(j\omega) = \frac{L(j\omega)}{1+L(j\omega)}$ and the sensitivity $S(j\omega) = \frac{1}{1+L(j\omega)}$ and ultimately the **loop gain** $\boldsymbol{L(j\omega)}$

Example

$L(s) = C(s)P_o(s)$ with

plant $P_o(s) = \frac{3.75}{(s+36.5)(s+1)}$

and simple integral controller $C(s) = KD(s)$ where K is a **constant** and

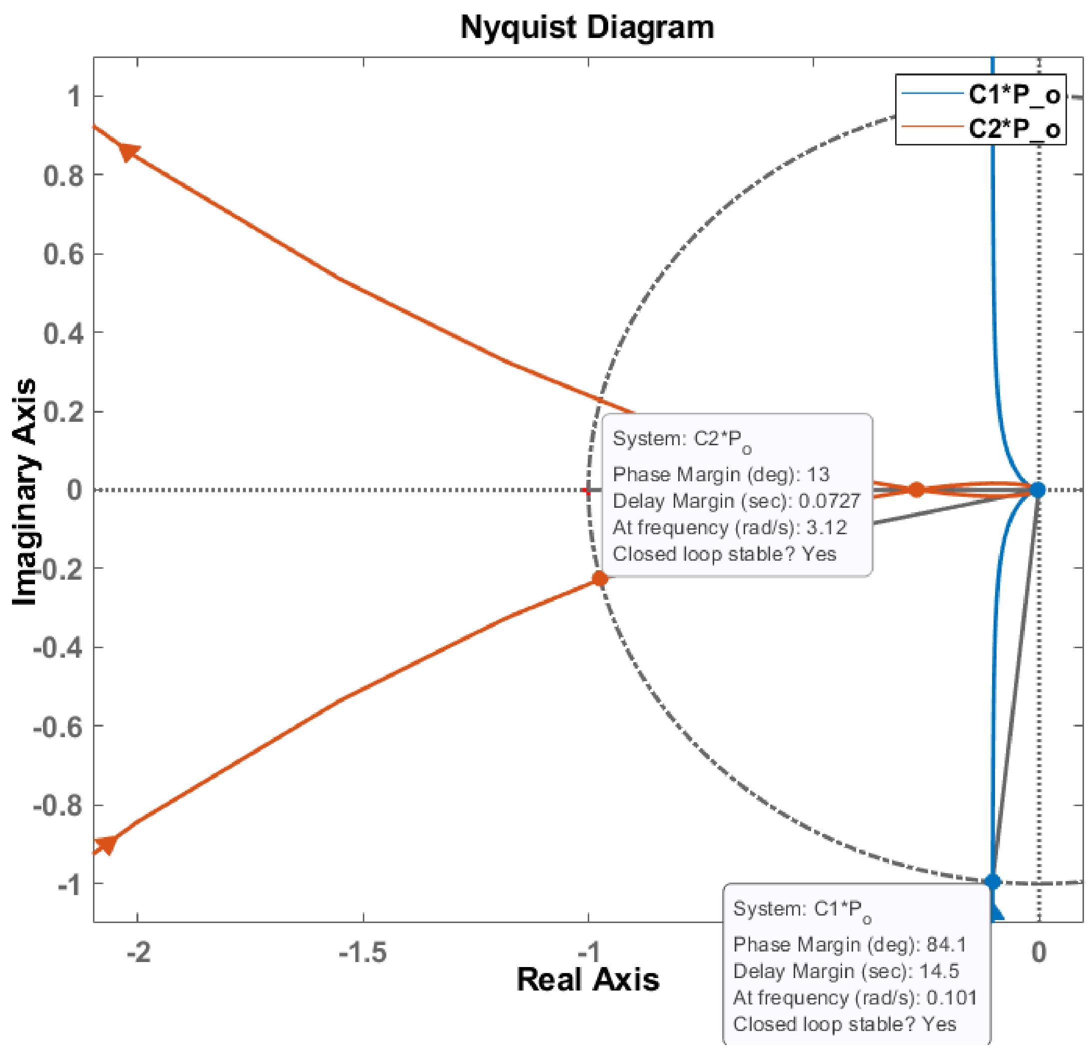
$D(s) = \frac{1}{s}$ is an **integrator**

Consider:

- $K_1 = 1$; i.e. $C_1(s) = K_1 D(s) = \frac{1}{s}$,
- $K_2 = 100$; i.e. $C_2(s) = K_2 D(s) = \frac{100}{s}$

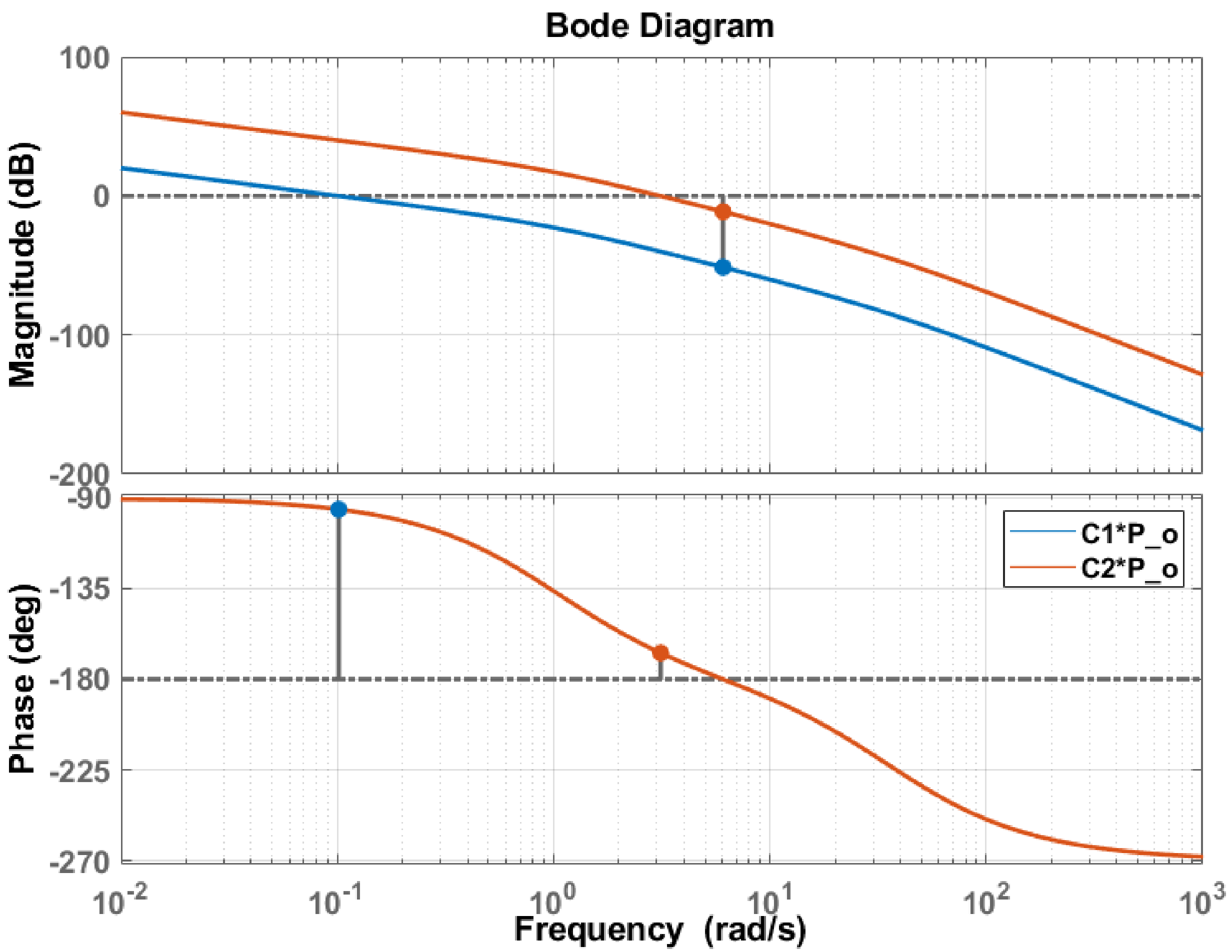
Example: Nyquist plot

$P(s) = \frac{3.75}{(s+36.5)(s+1)}$ with controllers: $C_1(s) = \frac{1}{s}$ and $C_2(s) = \frac{100}{s}$



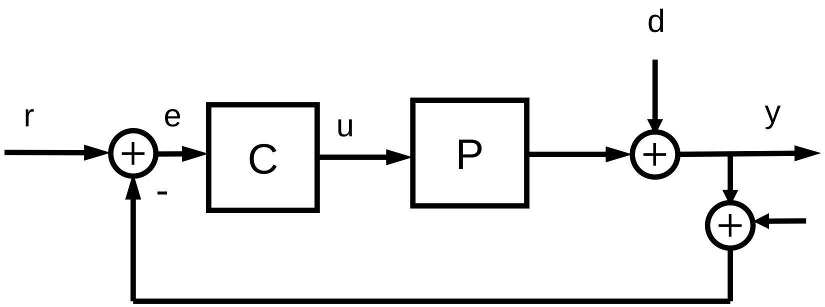
Example: Bode plot

$P(s) = \frac{3.75}{(s+36.5)(s+1)}$ with controllers: $C_1(s) = \frac{1}{s}$ and $C_2(s) = \frac{100}{s}$



PID Controller

Proportional+Integral+Derivative (PID)



In the continuous time domain:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \dot{e}(t)$$
$$= u_p(t) + u_i(t) + u_d(t)$$

In the Laplace domain:

$$\mathrm{U(s)} = \left(\mathrm{k}_p + \frac{k_i}{s} + k_d s \right) E(s)$$

- **C** is a PID controller:
  - Proportional gain k_p
  - Integral gain k_i
  - Differential gain k_d

$$\mathrm{C(s)} = \left(\mathrm{k}_p + \frac{k_i}{s} + k_d s \right)$$

- Proportional control: $u_p(t) = K e(t) \quad \Rightarrow \quad C_P(s) = K$
- Integral control: $u_I(t) = \dfrac{K}{T_I}\int_0^t e(\tau)d\tau \quad \Rightarrow \quad C_I(s) = \dfrac{K}{T_I s}$
- Derivative control:

$$u_D(t) = KT_D\dot{e}(t) \quad \Rightarrow \quad C_D(s) = KT_D s$$

for proper transfer function: $C_D(s) = \dfrac{KT_D s}{\tau_D s + 1}$

Proportional control

$$u_p(t) = Ke(t) \quad \Rightarrow \quad C_P(s) = K$$

- provides a contribution which depends on the **instantaneous value of the control error**.
- can control any stable plant,
- but provides limited performance and nonzero steady state errors. This latter limitation is due to the fact that its frequency response is bounded for all frequencies.

Integral control

$$u_I(t) = \frac{K}{T_I}\int_0^t e(\tau)\,d\tau \quad \Rightarrow \quad C_I(s) = \frac{K}{T_I\,s}$$

- gives a controller output that is **proportional to the accumulated error**, it is a **slow reaction control mode** (low pass frequency response).
- plays a fundamental role in achieving perfect plant inversion at $\omega = 0$ (steady state error = zero for constant reference or disturbance).
- major shortcoming: its pole at the origin is detrimental to loop stability.
- Undesirable effect in the presence of actuator saturation known as **wind-up**.

Derivative control

$$u_D(t) = KT_D\dot{e}(t) \quad \Rightarrow \quad C_D(s) = KT_D s$$

- acts on the **rate of change of the control error**.
- is a **fast mode** which ultimately disappears in the presence of constant errors.
- main limitation: tendency to yield **large control signals** in response to high frequency control errors, such as errors induced by setpoint changes or measurement noise.
- Since implementation requires that the transfer functions are proper, a pole is typically added to the derivative.

PID controller

$$u(t) = K\left(1 + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\dot{e(t)}\right)$$

$$C(s) = K\left(1 + \frac{1}{T_I s} + T_D s\right) = \frac{K}{s}\left(T_D s^2 + s + \frac{1}{T_I}\right)$$

with $T_D \ll T_I$

$$C(s) \approx \frac{K}{s}\left(T_D s^2 + \left(1 + \frac{T_D}{T_I}\right)s + \frac{1}{T_I}\right) = \frac{K}{s}(T_D s + 1)\left(s + \frac{1}{T_I}\right)$$

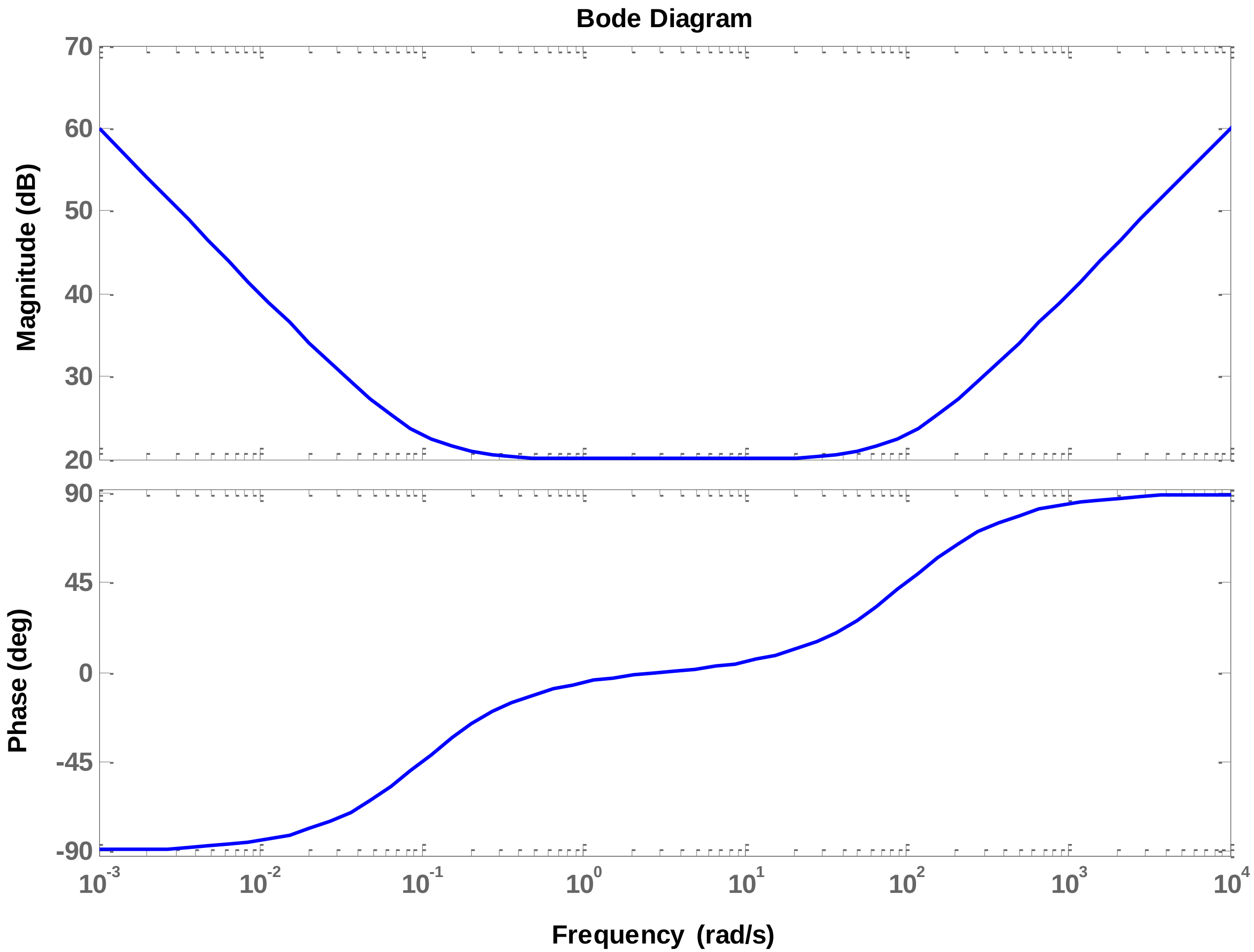
Thus, the zeros of C(s) are located at $-\frac{1}{T_D}$ and $-\frac{1}{T_I}$

This also means that the corner frequencies of the bode plot of C(s) are at $\frac{1}{T_D}$ and $\frac{1}{T_I}$.

We can *shape* of the frequency response of C(s) by choosing T_D and T_I, and hence *design* the shape of the loop gain $L(j\omega)$.

Frequency response of PID controller

- $K = 10, T_I = 10sec, T_D = 0.01sec$



Frequency response (K=100, Ti=1, Td=0.01)

$$P(s) = \frac{3.75}{(s+36.5)(s+1)} \qquad C(s) = \frac{100}{s}(0.01s+1)(s+1)$$