UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4 (ENG4042)

Monday 10 December 2018
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

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SECTION A

Q1 (a) Compare and contrast the properties of open- and closed-loop control structures. In your analysis, focus on the characteristics of each structure with respect to plant disturbances, changes in the plant gain, and stabilisation. [5]

(b) Refer to the closed-loop system shown in Figure Q1. Define the sensitivity function S_o and the complementary sensitivity function T_o and show explicitly how these transfer functions determine the properties of the system with respect to reference, disturbance and measurement noise signals. [5]

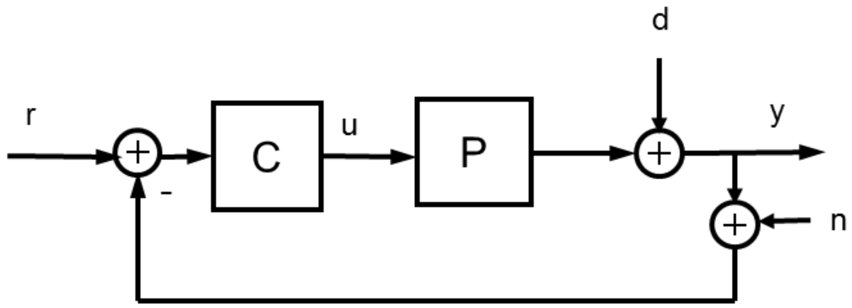

Figure Q1

(c) Consider the linear state space system

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with $x(0) = x_0$. Show how a linear transfer function *G(s)* can be derived. [5]

(d) Given is the equation of motion of a pendulum as

$$I\ddot{\theta}(t) + c\dot{\theta}(t) + \frac{mgl}{2}\sin\theta(t) = \tau(t)$$

Linearise this equation and present it in the standard state-space form of a differential equation,

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with the angle θ as the output, and the external torque τ as the input. [5]

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Q2 (a) With the help of sketches as necessary, highlight the main differences of a digital feedback controller, with respect to an analogue one. [6]

(b) Under which condition(s) the difference equation $u_k = a_1 u_{k-1} + a_2 u_{k-2} + \ldots + b_0 e_k + b_{-1} e_{k-1} + b_{-2} e_{k-2} + \ldots + b_1 e_{k+1} + b_2 e_{k+2} + \ldots$ is causal? What is the physical meaning of causality? [4]

(c) Consider a first order continuous transfer function $H(s) = \dfrac{U(s)}{E(s)} = \dfrac{a}{s+a}$.

Demonstrate that the backward rectangular numerical integration rule can be implemented through the substitution $s \Leftrightarrow \dfrac{z-1}{Tz}$ [6]

(d) Derive how the stability region of a continuous transfer function maps into the z-plane, using that rule, and use sketches to explain your results. What are the consequences of your result, when applying this rule to a real system? [4]

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SECTION B

Q3 (a) Describe design goals which one aims to achieve when using feedback control. Relate these to requirements for the complementary sensitivity function T_o and the sensitivity function S_o, and explain how these can be translated into requirements of the magnitude and phase of the loop gain L. [6]

(b) Describe what is meant by controller design using loop-shaping. [4]

(c) Consider a PID controller in the context of frequency response design using loop-shaping.

(i) What are the three components of the PID controller? Give their responses in the Laplace domain. [3]

(ii) Derive the transfer function of a PID controller $C(s)$, and express the result in terms of a gain K, a time constant relating to the integral term, T_i, and a time constant relating to the derivative term, T_d. What are the poles and zeros of $C(s)$. [4]

(iii) Based on the transfer function derived in (ii), sketch the Bode frequency response plot of an ideal PID controller. [3]

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Q4 (a) What is meant by state estimator feedback control. Use a block diagram to illustrate your explanations and mark the elements which form the compensator. What are the advantages of using a state estimator? [5]

(b) For the structure described in (a), describe in detail the structure of the state estimator (observer) and discuss the behaviour of the state estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$. [5]

(c) Consider the plant

$$\dot{x}(t) = \begin{bmatrix} -10 & 1 \\ -20 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Calculate the observer gain vector L such that the closed loop observer poles are located at -80 and -90. [5]

(d) Describe a test for observability of a state space system. Show whether the following system is observable:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 10 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[5]

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Section C

Q5 The following transfer function is a lead network:

$$H(s) = \frac{s+1}{0.2s+1}$$

(a) Find the discrete equivalent of it, when preceded by a zero-order hold (ZOH), for sample time T = 0.5 s. Use 4 significant digits for all numbers in the solution. [8]

(b) Using the inverse z-transform, find the corresponding difference equation. [4]

(c) State a necessary and sufficient condition for BIBO stability and determine whether the difference equation is BIBO stable. [8]

Q6 Consider a continuous transfer function of a first-order low pass filter with steady-state gain of 20 dB and cutoff frequency (gain decays by -3 dB) at 10 rad/s.

(a) Design the discrete equivalent of it, using the Tustin rule, considering a sampling time of 0.2 s. Compute the gain (in dB) of the digital filter at the cutoff frequency, and compare it with the analogue version. [8]

(b) Re-design the same, but this time apply a pre-warping such that the gain is preserved at the original cutoff frequency. Once designed, verify the gain numerically. [8]

(c) Finally, re-design the discrete equivalent using the forward rectangular rule, and compare the gain at the cutoff frequency. [4]

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Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T\cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha\to 0}{Limit}\ \dfrac{(-1)^k}{k!}\cdot\dfrac{\partial^k}{\partial\alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t\cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T\cdot z\cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at}\cdot\cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z\cdot(z-e^{-aT}\cdot\cos(bt))}{z^2-z\cdot(2\cdot e^{-aT}\cdot\cos(bt))+e^{-2aT}}$
$e^{-at}\cdot\sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z\cdot e^{-aT}\cdot\sin(bt)}{z^2-z\cdot(2\cdot e^{-aT}\cdot\cos(bt))+e^{-2aT}}$

TABLE 1

End of question paper