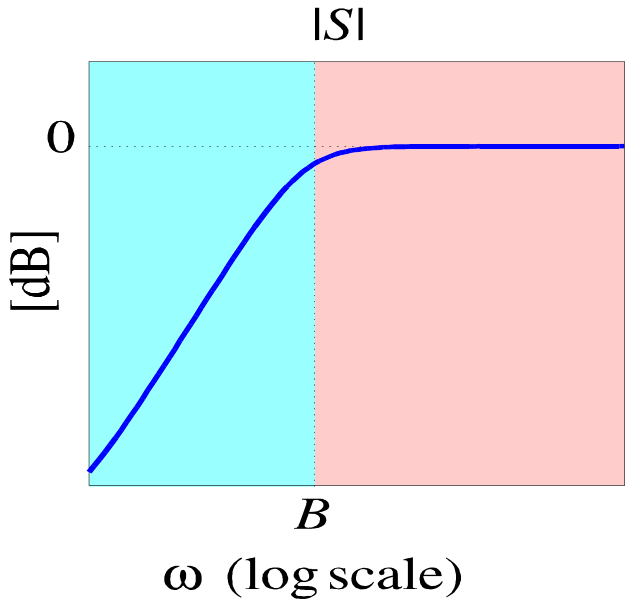
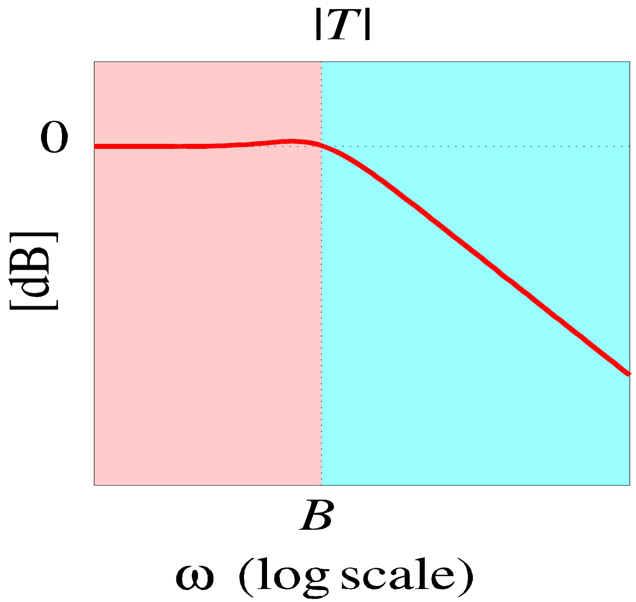
Vector margin – S & T

- Vector margin s_m related to peak sensitivity function *S*
- Complementary vector margin r_m related to peak in complimentary sensitivity function *T*





Vector margin

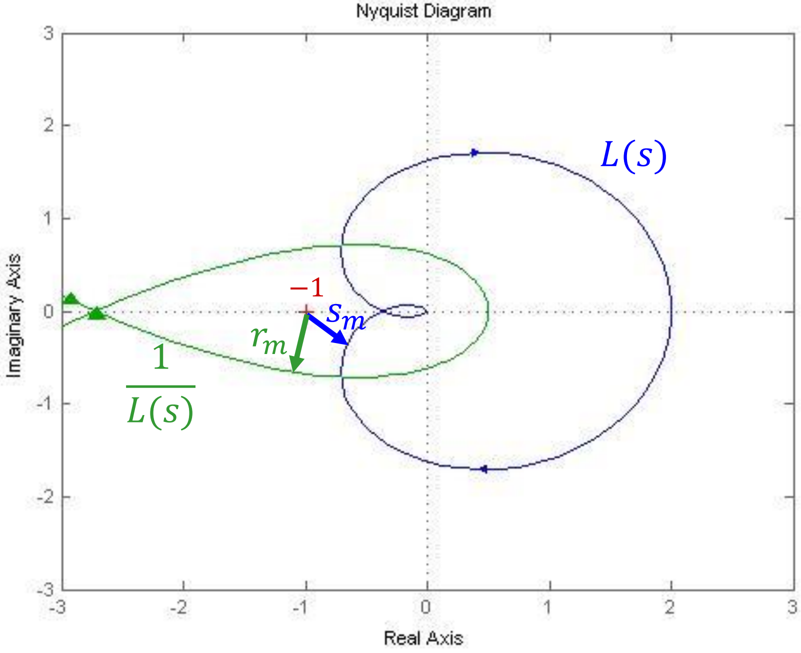
$$s_m = \min_{\omega} |1 + L(j\omega)| = \frac{1}{\max_{\omega} |S_o(j\omega)|}$$

Complementary vector margin

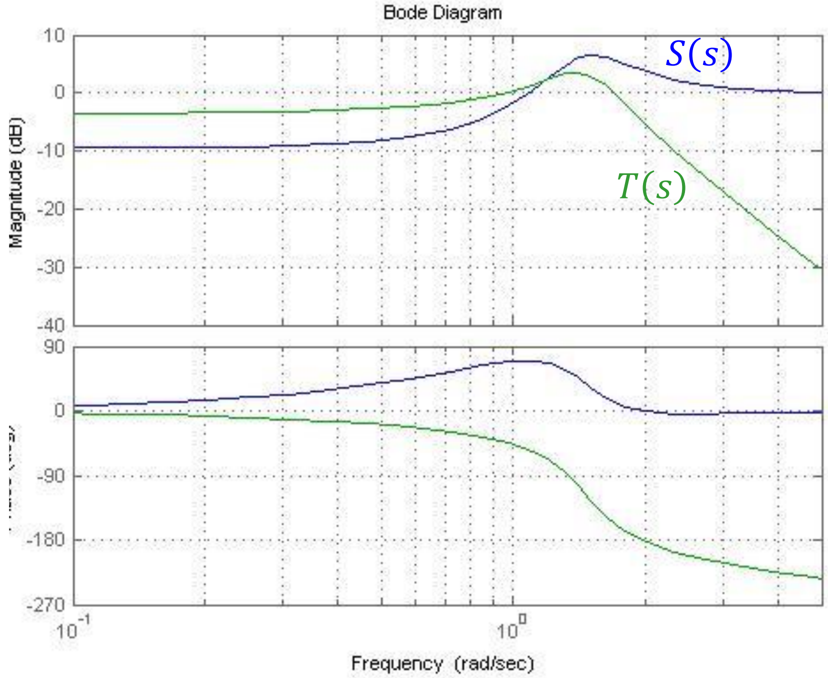
$$r_m = \min_{\omega} |1 + \frac{1}{L(j\omega)}| \frac{1}{\max_{\omega} |T_o(j\omega)|}$$

Vector margin – S & T

Vector margin and **complementary *vector* margin**: relatively large

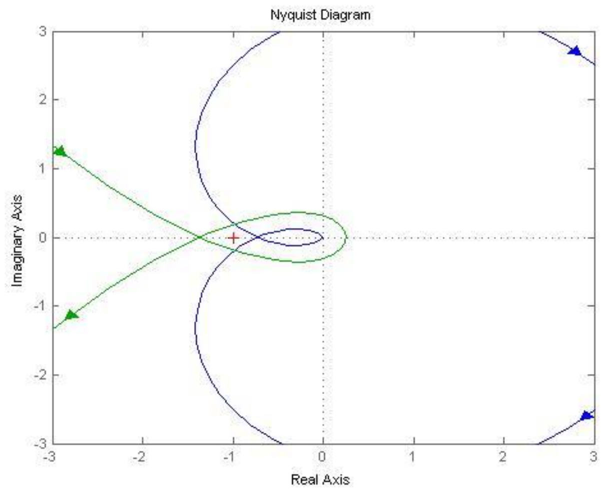


Peaks of S and T: relatively small

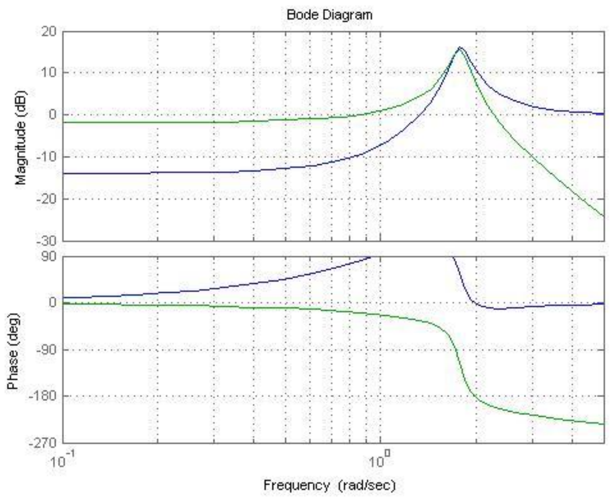


Vector margin – S & T

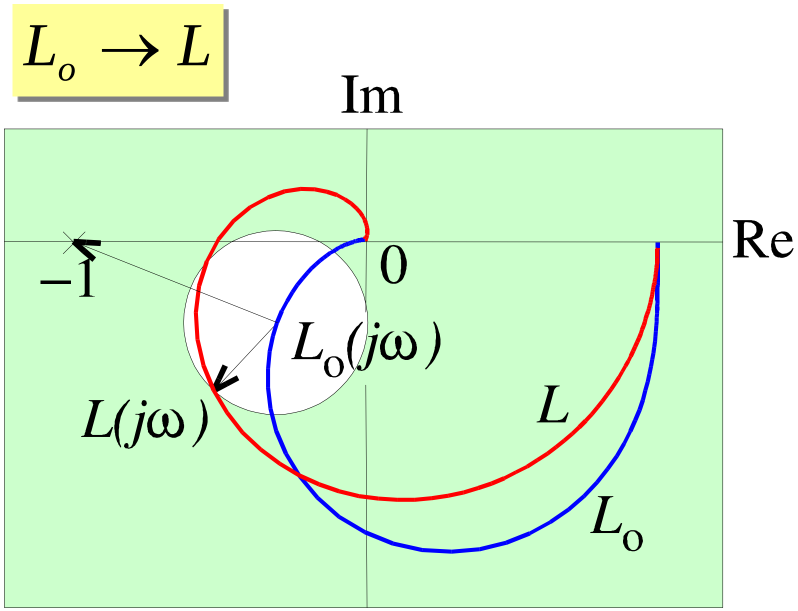
Vector margin and **complementary *vector* margin**: relatively small



Peaks of S and T: relatively large



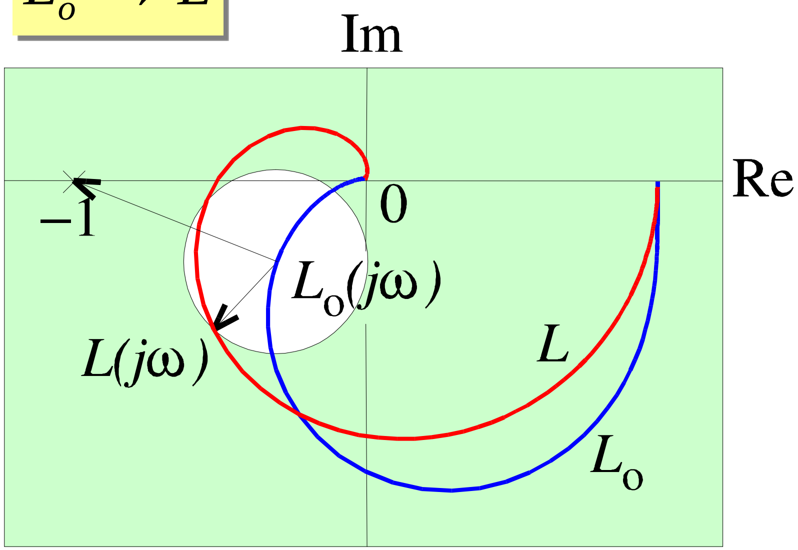
Robustness



- Nominal loop gain L_o is designed so that the closed-loop is stable
- We investigate whether the feedback system remains stable when the nominal loop gain L_o is perturbed to the actual loop gain L
- By the Nyquist criterion the plot of L_o does not encircle -1
- The actual closed-loop is stable if also L does not encircle -1

Robustness Functions

$L_o \rightarrow L$



- **Sufficient condition** for stability under perturbation:

$$\left|L(j\omega) - L_o(j\omega)\right| < \left|1 + L_o(j\omega)\right|,$$
$$\omega \in \Re$$

Robustness Functions
Doyle's stability criterion (1)

Equivalently,

$$\left|\frac{L(j\omega)-L_o(j\omega)}{L_o(j\omega)}\right| < \left|\frac{1+L_o(j\omega)}{L_o(j\omega)}\right|, \omega \in \Re$$

or

$$\left|\frac{L(j\omega)-L_o(j\omega)}{L_o(j\omega)}\right| < \frac{1}{|T_o(j\omega)|}, \omega \in \Re$$

- The left side of the inequality is the relative loop gain perturbation
- This is Doyle's stability robustness criterion
- It is a sufficient condition

Robustness Functions
Doyle's stability criterion (2)

Equivalently,

$$\left| \frac{\frac{1}{L(j\omega)} - \frac{1}{L_o(j\omega)}}{\frac{1}{L_o(j\omega)}} \right| < \frac{1}{|S_o(j\omega)|}, \omega \in \Re$$

- The left side of the inequality is the relative inverse loop gain perturbation
- This is the inverse loop gain stability robustness criterion
- It is a sufficient condition

- Notice that with respect to the loop gain criterion, the role of T is now taken by S