UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4/M DIGITAL (ENG4042/ENG5022)
[OPEN BOOK EXAM] [RESULTS]

XX December 2020
xx:xx – xx:xx

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 (a) … []

(b) … []

Q2 (a) The formula:

$$r(t) = \sum_{k=-\infty}^{+\infty} r(kT)\,\mathrm{sinc}\left(\frac{\pi(t-kT)}{T}\right)$$

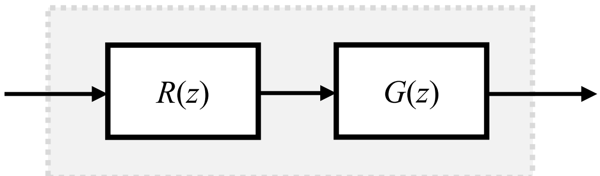
can be used for reconstructing a continuous signal $r(t)$ from its samples $r(kT)$

(i) State under which condition(s) exact reconstruction is theoretically possible. [3]

(ii) Explain why this formula cannot be implemented in a realistic scenario, and state a realizable, approximated version of the formula. [4]

(iii) Explain what can be done to improve the reconstruction of the signal in quasi-real-time, and what is the potential impact on system control and signal broadcasting. Use sketches of time-signal plots as required. [8]

(b) Given the following system of two transfer functions in series:



$$R(z) = \frac{z-1}{(z+0.1)(z+2)}; \qquad G(z) = \frac{(z+2)(z+0.1)}{(z-0.5)}$$

Discuss the system's asymptotic and BIBO stabilities, after zero-pole cancellations, and justify your answers [5]

Continued overleaf

SECTION B

Q3 (a) … []

Q4 (a) ... []

SECTION C

Q5 Consider the following digital plant with sample time $T = 0.1\,s$:

$$G(z) = \frac{1}{(z-2)(z+0.5)}$$

(a) Design a digital PD controller that cancels the unstable pole of the plant, and find the open-loop transfer function $L(z)$. [$\frac{K}{z(z+0.5)}$] [4]

(b) Select the open-loop gain K such that the time constant of the closed-loop system is approximately $\tau = 0.2\,s$. [0.3679] [6]

(c) Find the values of the damping and natural frequency of the closed-loop system. [0.243; 20.5733 rad/s] [5]

(d) Estimate the steady-state error (of the closed-loop system) in response to a unit step input. [0.803] [5]

Q6 Consider the following low-pass filter:

$$G(s) = \frac{20}{(s-1)(s-2)}$$

(a) Find the sampling time that corresponds to 10 times the bandwidth of the filter (frequency at 0 dB). [0.15 s] [4]

(b) Now set a sample time $T = 0.1\,s$. Calculate the Nyquist frequency. [31.4159 rad/s] [2]

(c) Find the discrete equivalent of the filter, using the pole-zero matching rule; calculate the gain at $\omega_c = 4.19\,rad/s$, and compare it to that of the continuous filter at the same frequency. [$\frac{0.058213(z+1)^2}{(z-1.105)(z-1.221)}$; -0.2569 dB; Less] [8]

Continued overleaf

(d) Now design another discrete equivalent, using the Tustin rule with pre-warping at frequency $\omega_c = 4.19\,rad/s$. Show that the gain is now preserved. [$\dfrac{0.060381(z+1)^2}{(z-1.107)(z-1.226)}$; 0 dB; Same] [6]

Continued overleaf

Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t - kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \frac{(-1)^k}{k!} \cdot \frac{\partial^k}{\partial \alpha^k}\left(\frac{z}{z - e^{-\alpha T}}\right)$
$e^{-a t}$	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$
$t \cdot e^{-a t}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z - e^{-aT})^2}$
$e^{-a t} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2 + b^2}$	$\frac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-a t} \cdot \sin(bt)$	$\frac{b}{(s+a)^2 + b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper