UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4 (ENG4042)

Monday 16 December 2013
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

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SECTION A

Q1 (a) Discuss advantages and disadvantages of using a digital computer as a controller as opposed to an analogue circuit. [4]

(b) State the Nyquist's sampling theorem. Describe in which way the continuous signal should be conditioned before being sampled. [4]

(c) Discuss the phenomenon of aliasing, showing the implications on the spectral content of a sampled signal with respect to its continuous counterpart. Explain its meaning in the time domain. [6]

(d) Consider a signal whose z-Transform has two complex conjugate poles. Sketch and discuss the time sequences associated with various positions of the poles in the complex plane. [6]

Q2 (a) Refer to the closed loop system shown in Figure 1. Typically, what are the signals *r*, *d* and *n*, and how should the feedback system be designed in order to respond appropriately to each of these signals? [5]

(b) Compare and contrast the properties of open- and closed-loop control structures. In your analysis, focus on the characteristics of each structure with respect to plant disturbances, changes in the plant gain, and stabilisation. [5]

(c) Describe the concept of state feedback control using a block diagram and derive the differential equation of the closed loop system. [5]

(d) Briefly analyse the advantages and disadvantages of each term in the standard PID control scheme. [5]

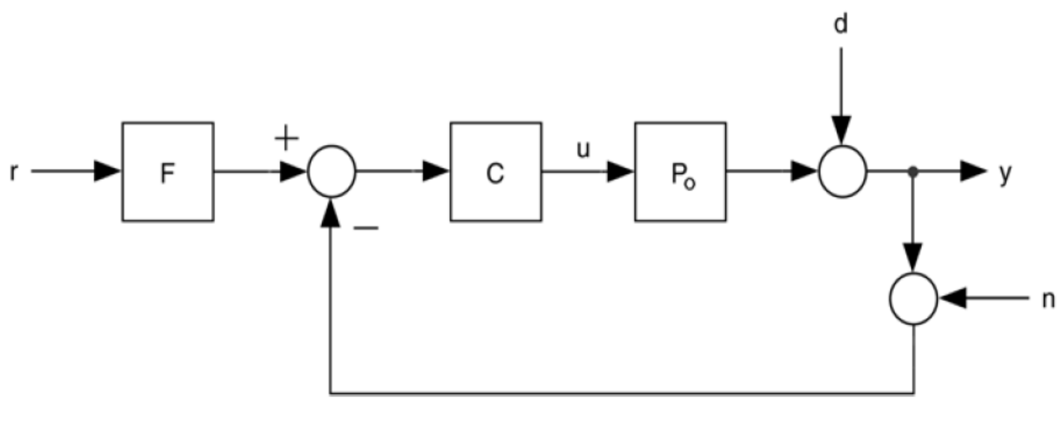

Figure 1

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SECTION B

Q3 (a) Show that the discrete transfer function can be defined as the z-transform of the response to a unit pulse input. [6]

(b) Derive the expression to find the discrete transfer function of a generic continuous transfer function G(s) preceded by a zero-order hold. [7]

(c) Find the discrete equivalent transfer function of a proportional-integral controller with gains K_P, K_I respectively, preceded by a zero-order hold with sample time T, as shown in Figure 2. [$G(z)=\frac{K_P z+(K_I T-K_P)}{z-1}$] [7]

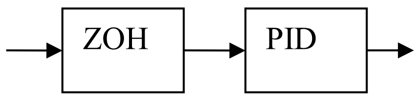

Figure 2

Q4 (a) Derive the difference equation and the transfer function of the trapezoid integration rule. [$u_k = u_{k-1}+\frac{T}{2}(e_k+e_{k-1})$; $H(z)=\frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}}$] [5]

(b) Design a discrete equivalent transfer function via numerical integration using the Tustin's method of the following continuous transfer function: [6]

$$G(s)=\frac{1}{s^2+s+1}$$

[$G(z)=\frac{T^2(z+1)^2}{(T^2+2T+4)z^2+(2T^2-8)z+(T^2-2T+4)}$]

(c) Show how the poles in the s-plane are mapped in the z-plane (using appropriate sketches) and describe what the consequences are on the stability of the discrete system. Compare with the forward and backward integration rules. [Sketches of the mapping of the stable region from s-plane to z-plane with the three rules, discussion] [9]

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Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \frac{(-1)^k}{k!} \cdot \frac{\partial^k}{\partial \alpha^k}\left(\frac{z}{z-e^{-\alpha T}}\right)$
$e^{-a t}$	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$t \cdot e^{-a t}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-a t} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2+b^2}$	$\frac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-a t} \cdot \sin(bt)$	$\frac{b}{(s+a)^2+b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

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SECTION C

Q5 (a) Describe the design goals which one attempts to achieve in the design of closed-loop feedback systems, and describe factors which limit the extent to which these goals can be achieved. [5]

(b) Given is the feedback control loop shown in Figure 1. Formulate the Nyquist stability criterion. Under which conditions does this criterion apply? [3]

(c) Draw a Nyquist plot for a stable closed loop system and for an unstable system. For the stable system, mark the gain margin, phase margin and the vector margin in the plot. Explain the meaning of gain margin, phase margin and vector margin. [5]

(d) Refer to the closed-loop system shown in Figure 1. Assume that the compensator C is chosen such that the nominal loop gain L_0 gives a stable closed-loop system. Assume also that the nominal loop gain L_0 is perturbed to the actual loop gain L, i.e. $L_0 \to L$ (or, equivalently, that the nominal inverse loop gain is perturbed as $1/L_0 \to 1/L$). Derive a sufficient condition for closed-loop stability which combines two tests, one involving the magnitude of the nominal sensitivity function, $|S_0|$, and the other involving the magnitude of the complementary sensitivity, $|T_0|$. [7]

Q6 (a) What is controller design by pole assignment and how can this method be used to design a controller in transfer function form (as shown in Figure 1)? Discuss the use of the polynomial Diophantine equation in this design method. What is the characteristic polynomial? [7]

(b) Describe how pole assignment can be used to design a state feedback controller. [4]

(c) Consider the state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -10 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Show if this system is controllable. [4]

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(d) For the system given in (c), design a state feedback controller K using the pole placement method in such a way that the closed system has an overshoot of M_p=10% and a rise time of t_r=0.1 seconds. You can use the following equations to derive the natural frequency and the damping ratio of the desired closed loop system:

$$\omega_n \cong \frac{1.8}{t_r} \quad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + \left(\ln M_p\right)^2}}; \; 0 \le \xi < 1 \qquad [5]$$

End of question paper