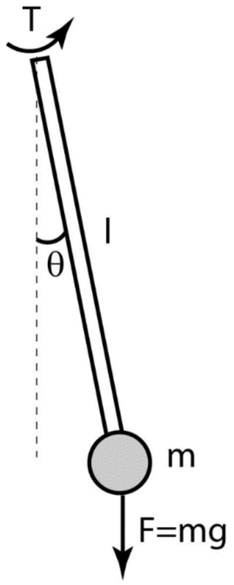
Frequency Domain Analysis and Design

Tutorial questions

Differential Equations

Q1 Lecture note Part I, section 1.2 & 1.4, page 5ff. Ignore equilibrium solutions



Torque balance leads to

$$I\ddot{\theta}(t) + c\dot{\theta}(t) + mg\frac{l}{2}\sin\theta(t) = \tau(t)$$

Linear differential equation (valid for small θ, such that $\sin\theta \approx \theta$): [1]

$$\ddot{\theta}(t) + \frac{c}{I}\dot{\theta}(t) + mg\frac{l}{2I}\theta(t) = \frac{1}{I}\tau(t)$$

Q2 Pendulum model handouts, page 3.

Laplace transform: [3]

$$\Theta(s)\left(s^2 + \frac{c}{I}s + mg\frac{l}{2I}\right) = \frac{1}{I}\mathrm{T}(s)$$

$$P(s) = \frac{\Theta(s)}{\mathrm{T}(s)} = \frac{1/I}{s^2 + \frac{c}{I}s + mgl/(2I)}$$

Define $\omega_n = \sqrt{mgl/(2I)}$, $\xi = c/(2I\omega_n)$ [1]

$$P(s) = \frac{1/I}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Q3 substitute the general values of the parameters in Q2 with the specific values given here.

Q4 Pendulum model handouts, page 5

$$\begin{bmatrix} \ddot{\theta}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -2\xi\omega_n & -\omega_n^2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 1/I \\ 0 \end{bmatrix} \tau(t)$$

$$\theta(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \end{bmatrix} + 0\tau(t)$$

Define A, B, C, D and x, u, y

Q5 (i) steady state gain is given by magnitude at 0 frequency = -20dB (=0.1)

peak of magnitude at approx. 7 rad/sec indicates overshoot (damped oscillations) with that frequency.

(ii) steady state gain is unchanged.

no peak in magnitude means that there is no overshoot in the time domain response (overdamped response).

Open-loop and Closed-loop Control

Q1

See figure 3 below.

Explain meaning of the signals!

Q2 Lecture Notes Part II ("Feedback Control"), page 5 & page 7f.

Q3 Lecture Notes Part II ("Feedback Control"), page 5-7.

Q4 Lecture Notes Part II ("Feedback Control"), page 8ff

Dynamic Closed-loop Control: sensitivity functions

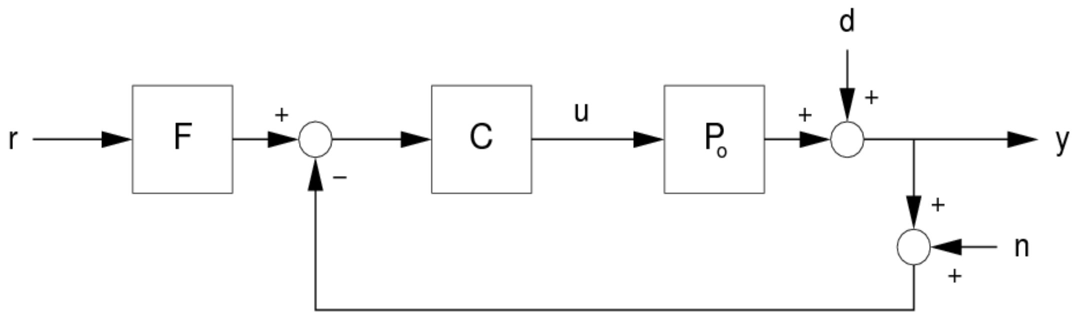


Figure 3

Refer to the closed-loop system shown in Figure 3.

Q1 Lecture Notes Part II (“Feedback Control”), page 11ff

Q2 *r* = reference signal, *d*=disturbance, *n*=measurement noise

output should follow the reference, especially in steady state (i.e. at low frequencies).

disturbance should be attenuated, especially at low frequencies (eliminating constant disturbances)

measurement noise should be attenuated, especially at higher frequencies (where this noise is more likely to occur)

Q3 Definition and sketch: see lecture notes Part II (page 12) and handouts (“Sensitivity functions”)

tradeoffs: (i) disturbances can only be attenuated at lower frequencies, (ii) output will only follow reference at lower frequencies, (iii) measurement noise can only be attenuated at higher frequencies.

(lower frequencies = below bandwidth, higher frequencies = above bandwidth)

Q4 see lecture notes Part II, page 11ff

Q5.1/2 Lecture notes Part II, page 12-13

Closed-loop Stability and Stability Robustness

Q1 Lecture notes Part II, page 14f

Q2 Lecture notes Part II page 15f, and handouts (“Nyquist Stability Robustness”)

Q3.1/2 Lecture notes Part II, Figure 10 and page 16f, and in handouts. Unstable plot encircles (“goes to the left of”) -1.

Q4 (i) 11.8dB=10^(11.8/20)=3.8, Gain can increase by 3.8

(ii) 120deg at 3 Hz => $\frac{120\text{deg}}{360\text{deg}} \times \frac{1}{3\text{Hz}} = 0.11\text{sec}$. Maximal delay is 0.11sec

Q5 Handout “Nyquist Stability Robustness”, slide 13ff

Q6.1/2 Lecture notes Part II, page 17f

Design Goals

Q1 Lecture Notes Part II, page 18ff., in particular section 4.6

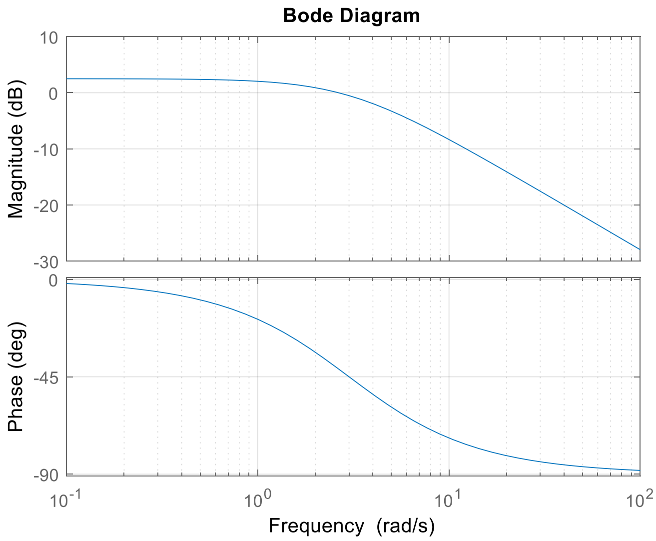
Q2 Lecture Notes Part II, page 18ff, in particular section 4.8 (Summary) and handout “Design summary”

Frequency Domain Controller Design – Loop Shaping

Q1 PID handout, page 3

Q2 PID handout, page 4.

Q3



Steady state gain is close to slightly >0dB. The bandwidth is thus approx. 3 rad/sec (i.e. the frequency of the pole).

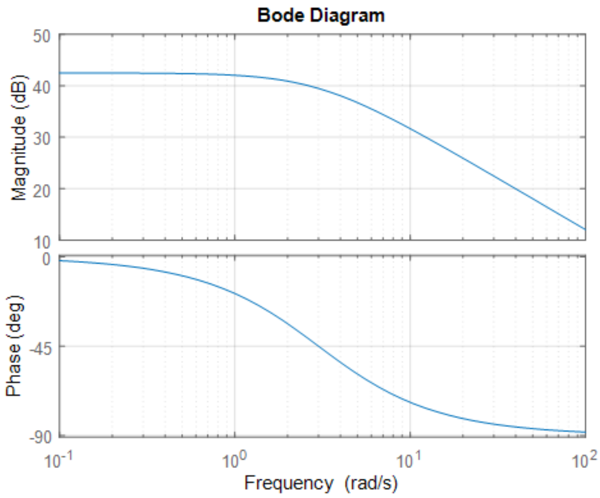
Q4

A proportional controller will affect the magnitude of the frequency response and therefore shift the crossover frequency to higher frequencies.

Limitations: (i) no change of phase, therefore the phase margin will decrease with higher bandwidth. (ii) low frequency gain is limited.

(i) can be addressed by adding a derivative term, (ii) by adding an integral term which gives infinite low frequency gain.

Q5

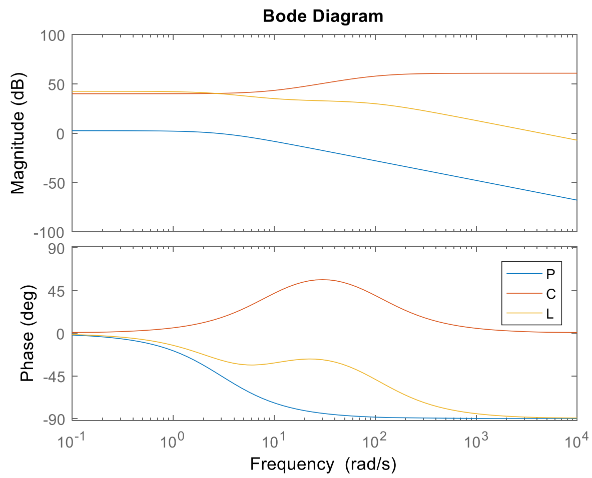


Magnitude response shifted up by 100 (=40dB). No change in phase.

L(0)=400/3, so steady state of closed loop is L(0)/[L(0)+1]=0.99

Phase does not approach -180deg, so phase margin remains large.

Q6



No change of steady state gain (L(0) remains unchanged).

Cross-over frequency becomes larger, and phase margin increases.

Probably increased control signal due to integral action.