UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022)

15th December 2017
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

DATA SHEET INCLUDED AT END OF PAPER

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Continued overleaf

SECTION A

Q1 (a) Draw a Nyquist plot for a stable closed loop system and for an unstable system. For the stable system, mark the gain margin, phase margin and the vector margin in the plot. Explain the meaning of gain margin, phase margin and vector margin. [5]

(b) For a control system with a nominal plant P_0, the Nyquist plot of the loop-gain L shows a phase margin of 60deg at a frequency of 4rad/sec. The plot crosses the negative real axis at -0.8.

(i) What additional delay can be added to the closed loop before stability is lost? [3]

(ii) By how much can the plant gain increase before the closed loop becomes unstable? [2]

(c) Given the transfer function

$$G(s) = \frac{25}{(s+2)(s+5)}$$

derive the state space description in observer canonical form. [5]

(d) What is meant by the Separation Theorem in the context of state-estimator feedback control? [5]

Q2 (a) Discuss the features of a digital signal, as opposed to an analogue one. [4]

(b) With the help of sketches as necessary, highlight the main differences of a digital feedback controller, compared to an analogue one. [6]

(c) Consider a first order continuous transfer function $H(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$. Show that the trapezoid numerical integration rule can be implemented through the substitution $s \Leftrightarrow \frac{2}{T}\frac{z-1}{z+1}$ [6]

(d) Derive how the stability region of a continuous transfer function maps into the z-plane, using the trapezoid rule from (c), and use sketches to explain your results. What are the consequences of your result, when applying this rule to a real system? [4]

Continued overleaf

SECTION B

Q3 (a) Describe potential benefits and drawbacks of a closed-loop control strategy and contrast these to an open-loop strategy. In your analysis, focus on the characteristics of each structure with respect to plant disturbances, changes in the plant gain, and stabilisation. [5]

(b) Consider the closed loop system shown in Figure Q3.

(i) Derive the closed-loop equation of this system, and define the loop gain L, the sensitivity function S and the complementary sensitivity function T. [5]

(ii) Sketch the typical shapes of the frequency responses of the magnitudes of L, S, and T. Explain how this is related to characteristics of the closed loop system, in particular how the closed loop behaviour at low frequencies and at high frequencies is defined by this. [5]

(c) Show that the sensitivity function S_0 is equal to the sensitivity of the complementary sensitivity T_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of T_0 on the sensitivity of S_0 to plant changes. [5]

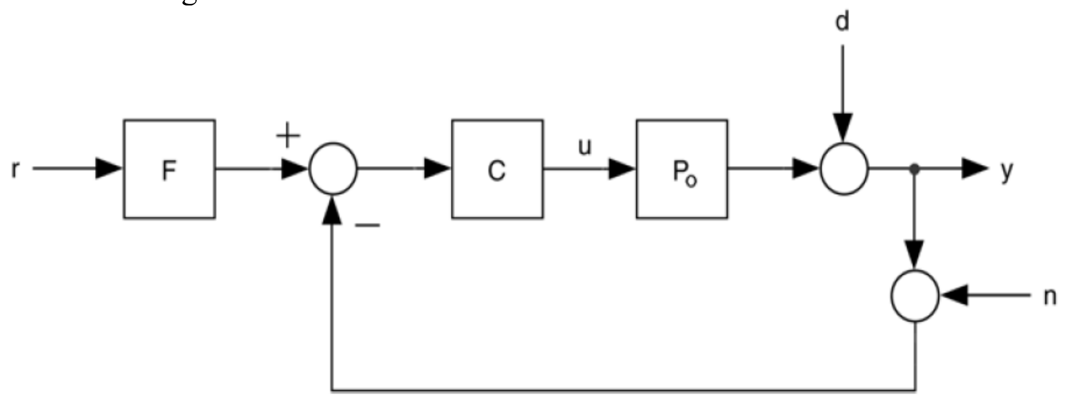

Figure Q3

Continued overleaf

Q4 (a) Describe the concept of state feedback control using a block diagram and derive the differential equation of the closed loop system. [5]

(b) For the state feedback system from (a), what is controller design by pole assignment and how can it be used to design a state feedback controller? [5]

(c) What is controllability in the context of state feedback control and what is meant by stabilisability? [3]

(d) Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & -10 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Show whether this system is controllable or not. [2]

(e) Design a state feedback controller K for the state space system given in (d) using the pole placement method in such a way that the closed system has an overshoot of Mp=1% and a rise time of tr=0.1 seconds. You can use the following equations to derive the natural frequency and the damping of the desired closed loop system:

$$\omega_n \cong \frac{1.8}{t_r} \quad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + \left(\ln M_p\right)^2}}; \; 0 \le \xi < 1$$

[5]

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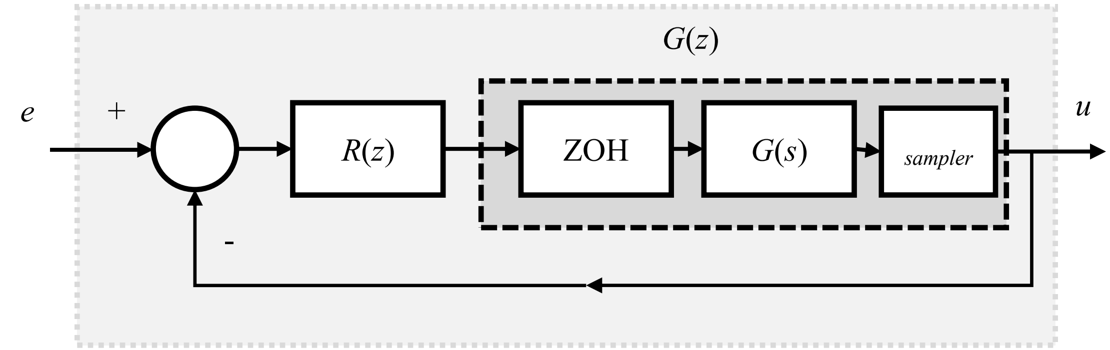
Section C

Q5 Given the following digital feedback control loop $C_L(z)$ (sample time $T = 0.01$ s):



$C_L(z)$

$$G(s) = \frac{10s - 100}{5s + 1} \qquad R(z) = \frac{z - 0.998}{10z - 1}$$

(a) Find the discrete equivalent $G(z)$ of the plant $G(s)$. [8]

(b) Find the transfer function of the closed loop system $C_L(z)$. (Simplify poles and zeros if possible) [2]

(c) Find the difference equation corresponding to $C_L(z)$. [4]

(d) By verifying a suitable condition, demonstrate whether the difference equation is BIBO stable. [6]

Q6 Consider a continuous transfer function of a first-order low pass filter with cutoff frequency (-3 dB) of 15 rad/s and steady-state gain of 0 dB.

(a) Design the discrete equivalent of it, using the Tustin rule, considering a sampling time of 0.1 s. Compute the gain (in dB) of the digital filter at the cutoff frequency, and compare it with the analogue version. [8]

(b) Re-design the system in (a), but this time apply a pre-warping such that the gain is preserved at the original cutoff frequency. Once designed, verify the gain numerically. [8]

(c) Finally, re-design the discrete equivalent using the backward rectangular rule, and compare the gain at the cutoff frequency. [4]

Continued overleaf

Data sheet

Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit}\ \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper

UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022)

Monday 10 December 2018
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 (a) Compare and contrast the properties of open- and closed-loop control structures. In your analysis, focus on the characteristics of each structure with respect to plant disturbances, changes in the plant gain, and stabilisation. [5]

(b) Refer to the closed-loop system shown in Figure Q1. Define the sensitivity function S_o and the complementary sensitivity function T_o and show explicitly how these transfer functions determine the properties of the system with respect to reference, disturbance and measurement noise signals. [5]

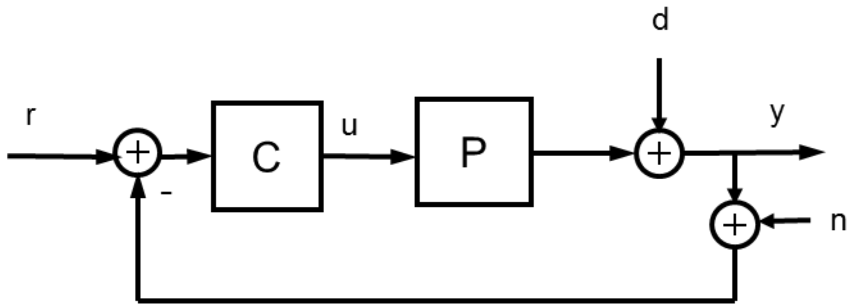


Figure Q1

(c) Consider the linear state space system

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with $x(0) = x_0$. Show how a linear transfer function *G(s)* can be derived. [5]

(d) Given is the equation of motion of a pendulum as

$$I\ddot{\theta}(t) + c\dot{\theta}(t) + \frac{mgl}{2}\sin\theta(t) = \tau(t)$$

Linearise this equation and present it in the standard state-space form of a differential equation,

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with the angle θ as the output, and the external torque τ as the input. [5]

Continued overleaf

Q2 (a) With the help of sketches as necessary, highlight the main differences of a digital feedback controller, with respect to an analogue one. [6]

(b) Under which condition(s) the difference equation $u_k = a_1 u_{k-1} + a_2 u_{k-2} + \ldots + b_0 e_k + b_{-1} e_{k-1} + b_{-2} e_{k-2} + \ldots + b_1 e_{k+1} + b_2 e_{k+2} + \ldots$ is causal? What is the physical meaning of causality? [4]

(c) Consider a first order continuous transfer function $H(s) = \dfrac{U(s)}{E(s)} = \dfrac{a}{s+a}$.

Demonstrate that the backward rectangular numerical integration rule can be implemented through the substitution $s \Leftrightarrow \dfrac{z-1}{Tz}$ [6]

(d) Derive how the stability region of a continuous transfer function maps into the z-plane, using that rule, and use sketches to explain your results. What are the consequences of your result, when applying this rule to a real system? [4]

Continued overleaf

SECTION B

Q3 (a) Describe design goals which one aims to achieve when using feedback control. Relate these to requirements for the complementary sensitivity function T_o and the sensitivity function S_o, and explain how these can be translated into requirements of the magnitude and phase of the loop gain L. [6]

(b) Describe what is meant by controller design using loop-shaping. [4]

(c) Consider a PID controller in the context of frequency response design using loop-shaping.

(i) What are the three components of the PID controller? Give their responses in the Laplace domain. [3]

(ii) Derive the transfer function of a PID controller $C(s)$, and express the result in terms of a gain K, a time constant relating to the integral term, T_i, and a time constant relating to the derivative term, T_d. What are the poles and zeros of $C(s)$. [4]

(iii) Based on the transfer function derived in (ii), sketch the Bode frequency response plot of an ideal PID controller. [3]

Continued overleaf

Q4 (a) What is meant by state estimator feedback control. Use a block diagram to illustrate your explanations and mark the elements which form the compensator. What are the advantages of using a state estimator? [5]

(b) For the structure described in (a), describe in detail the structure of the state estimator (observer) and discuss the behaviour of the state estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$. [5]

(c) Consider the plant

$$\dot{x}(t) = \begin{bmatrix} -10 & 1 \\ -20 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Calculate the observer gain vector L such that the closed loop observer poles are located at -80 and -90. [5]

(d) Describe a test for observability of a state space system. Show whether the following system is observable:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 10 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[5]

Continued overleaf

Section C

Q5 The following transfer function is a lead network:

$$H(s) = \frac{s+1}{0.2s+1}$$

(a) Find the discrete equivalent of it, when preceded by a zero-order hold (ZOH), for sample time T = 0.5 s. Use 4 significant digits for all numbers in the solution. [8]

(b) Using the inverse z-transform, find the corresponding difference equation. [4]

(c) State a necessary and sufficient condition for BIBO stability and determine whether the difference equation is BIBO stable. [8]

Q6 Consider a continuous transfer function of a first-order low pass filter with steady-state gain of 20 dB and cutoff frequency (gain decays by -3 dB) at 10 rad/s.

(a) Design the discrete equivalent of it, using the Tustin rule, considering a sampling time of 0.2 s. Compute the gain (in dB) of the digital filter at the cutoff frequency, and compare it with the analogue version. [8]

(b) Re-design the same, but this time apply a pre-warping such that the gain is preserved at the original cutoff frequency. Once designed, verify the gain numerically. [8]

(c) Finally, re-design the discrete equivalent using the forward rectangular rule, and compare the gain at the cutoff frequency. [4]

Continued overleaf

Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper

UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022)

Monday 16th December 2019
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 (a) Draw the structure of an open-loop control system and of a two-degree-of-freedom feedback control system. Show the different signals, including disturbances, and explain their physical meaning. Explain which additional components are required in the feedback control structure. [5]

(b) Show that for an open-loop structure the ideal compensator should cancel the plant dynamics, and state the drawbacks of this approach. Demonstrate that a feedback control structure can provide approximate plant inversion and show therefore that high-gain closed-loop control implicitly inverts plant dynamics. [5]

(c) What is observability in the context of state feedback control? Describe a test for observability of a state space system. [5]

(d) What is meant by state estimator feedback control? Use a block diagram to illustrate your explanations and mark the elements which form the compensator. Explain what is meant by the Separation Theorem in the context. [5]

Q2 (a) Consider a signal whose z-transform has two complex conjugate poles. Sketch and discuss the time sequences associated with various positions of the poles in the complex plane. [6]

(b) Given a discrete transfer function $H(z) = U(z)/E(z)$, demonstrate that, in the time domain, $u_k = \sum_{j=-\infty}^{+\infty} e_j h_{k-j}$. What is this formula commonly known as? [6]

(c) Consider a first order continuous transfer function $H(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$.
Demonstrate that the forward rectangular numerical integration rule can be implemented through the substitution $s \Leftrightarrow \frac{z-1}{T}$ where T is the sample time. [6]

(d) What is aliasing in the context of sampled system? [2]

Continued overleaf

SECTION B

Q3 (a) Explain why 'peaking' in the sensitivity function S_0 and the complementary sensitivity function T_0 should be avoided. The explanation should be based on performing the following analysis.

i. Derive the closed-loop equations of a feedback control system in terms of S_o and T_o, considering the output responses to the reference, disturbance and noise. Discuss how peaking would affect the system's response to these signals. [4]

ii. Show that the complementary vector margin for the inverse loop gain is equal to the inverse of the peak value of $|T_0|$. By a symmetry argument, briefly describe the effect of $|S_0|$ on the vector margin for the loop gain. Based on this, explain what a strongly oscillatory response of a closed-loop system means for stability-robustness. [5]

iii. Show that T_0 is equal to the sensitivity of S_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of S_0 on the sensitivity of T_0 to plant changes. [5]

(b) Refer to a closed loop control structure. Assume that the compensator $C(s)$ is chosen in such a way that the nominal loop gain $L_o(s)$ gives a stable closed-loop system. Assume also that the nominal loop gain $L_o(s)$ is perturbed to the actual loop gain $L(s)$, i.e. $L_o(s) \to L(s)$ (or, equivalently, that the nominal inverse loop gain is perturbed as $1/L_o(s) \to 1/L(s)$). Derive a sufficient condition for closed-loop stability which combines two tests, one involving $|S_o|$ and the other involving $|T_o|$. What is the name of this criterion? [6]

Continued overleaf

Q4 (a) Consider the linear state space system

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with $x(0) = x_0$.

(i) Show how a linear transfer function *G(s)* can be derived. [5]

(ii) What are the requirements in terms of the properties of the matrices / vectors A, B, C and D for which the system is stable? What are the corresponding requirements for the transfer function *G(s)*? [4]

(b) Consider a state space system. What is controller design by pole assignment and how can it be used to design a state feedback controller? What is the relevance of the *controller canonical form* and of *controllability* in this context? [6]

(c) Design a state feedback controller K for the state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

using the pole placement method in such a way that the closed system has an overshoot of $M_p = 1\%$ and a rise time of $t_r = 0.1$ seconds. You can use the following equations to derive the natural frequency and the damping of the desired closed loop system:

$$\omega_n \cong \frac{1.8}{t_r} \quad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + \left(\ln M_p\right)^2}}; \; 0 \le \xi < 1$$

[5]

Continued overleaf

Section C

Q5 Consider the following transfer function:

$$H(s) = 10\frac{10s+1}{100s+1}$$

(a) Find the gain (in dB) and phase (in deg) at $\omega_1 = 0.5\ \text{rad/s}$. Assuming a sample time $T = 2$ s, calculate the Nyquist frequency ω_n in rad/s. [3]

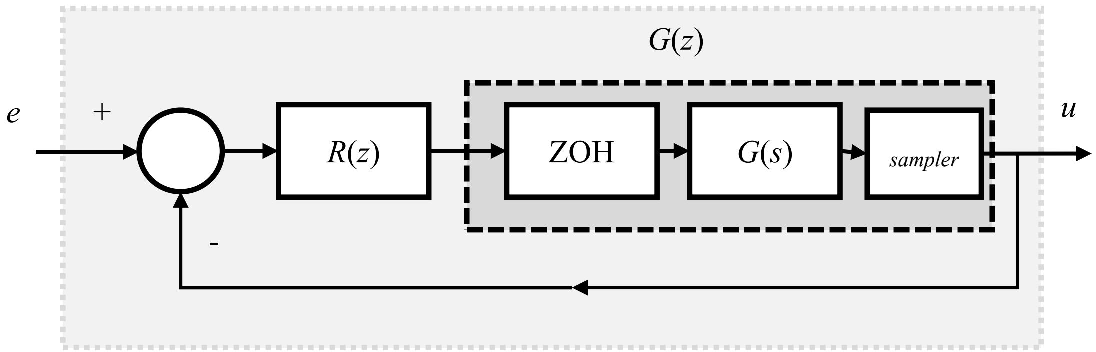
(b) Design the discrete equivalent of $H(s)$ for the sample time given in (a), using the forward rectangular rule. [4]

(c) Compute the discrete equivalent of $H(s)$ using the pole-zero matching technique (match the steady-state gain), for the sample time given in (a). [9]

(d) Find the gain (in dB) phase (in deg) at ω_1 of the two discrete equivalents derived in (b) and (c), and compare with that of the continuous $H(s)$. Which one is closest? [4]

Continued overleaf

Q6 Consider the following digital feedback control loop (sample time $T = 0.1$ s):



$C_L(z)$

$$G(s) = \frac{20s - 10}{2s + 1} \qquad R(z) = \frac{z - 0.9512}{10z - 1}$$

(a) Find the discrete equivalent $G(z)$ of the plant $G(s)$. [8]

(b) Find the closed loop transfer function $C_L(z)$ of the system from e to u. (Simplify poles and zeros if possible.) [2]

(c) Find the difference equation corresponding to $C_L(z)$ from (b). [4]

(d) Estimate the steady-state output of the closed-loop system $C_L(z)$ from (b) for the following input:

$$e_k = 10\sin(0.1\,kT)$$

[6]

Continued overleaf

Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \frac{(-1)^k}{k!} \cdot \frac{\partial^k}{\partial \alpha^k}\left(\frac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2+b^2}$	$\frac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\frac{b}{(s+a)^2+b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper

UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022)

Monday 14th December 2020
Start time: 09:15 – 11:15

This exam should take you: 2 hours to complete
However, you have a 4-hour window to download/complete/upload your submission.

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

A FORMULA SHEET IS PROVIDED AT THE END OF PAPER

A calculator may be used. Show intermediate steps in calculations.

Continued overleaf

SECTION A
Attempt BOTH questions

Q1 (a) Refer to the closed-loop system shown in Figure Q1(a)

Derive the closed-loop equations relating the plant output *y* to the signals *r*, *d*, and *n*. Discuss how should the feedback system be designed in order to respond appropriately to each of these signals and what the associated limitations are. [5]

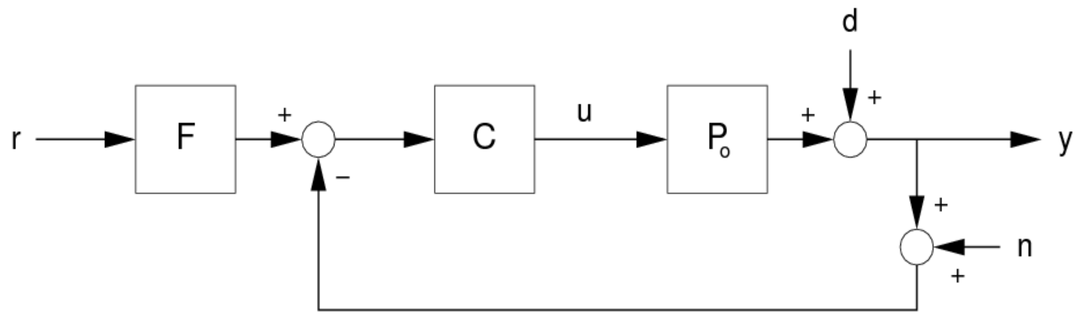


Figure Q1(a)

(b) Derive an expression linking the vector margin s_m of the closed-loop system to the peak magnitude of the sensitivity function S_o. Derive an equivalent expression linking the complementary vector margin r_m of the closed-loop system to the peak magnitude of the complementary sensitivity function T_o. Briefly discuss the relevance of these expressions in relation to stability robustness of the feedback system. [5]

(c) Consider the linear state space system

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \quad x(0)=x_0$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Show how a linear transfer function $G(s)$ can be derived. Explain which element of the linear state space representation cannot be represented in the linear transfer function $G(s)$. [5]

(d) In your own words, explain what is meant by *controllability* in the context of state feedback control. Describe a test for controllability of a state space system. [5]

Continued overleaf

Q2 (a) The formula:

$$r(t)=\sum_{k=-\infty}^{+\infty} r(kT)\,\mathrm{sinc}\left(\frac{\pi(t-kT)}{T}\right)$$

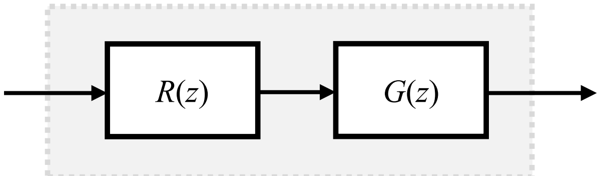
can be used for reconstructing a continuous signal $r(t)$ from its samples $r(kT)$

(i) State under which condition(s) exact reconstruction is theoretically possible. [3]

(ii) Explain why this formula cannot be implemented in a realistic scenario, and state a realizable, approximated version of the formula. [4]

(iii) Explain what can be done to improve the reconstruction of the signal in quasi-real-time, and what is the potential impact on system control and signal broadcasting. Use sketches of time-signal plots as required. [8]

(b) Given the following system of two transfer functions in series:



$$R(z)=\frac{z-1}{(z+0.1)(z+2)}; \qquad G(z)=\frac{(z+2)(z+0.1)}{(z-0.5)}$$

Discuss the system's asymptotic and BIBO stabilities, after zero-pole cancellations, and justify your answers. [5]

Continued overleaf

SECTION B
Attempt ONE question

Q3 (a) Consider a feedback control system with loop gain $L_o(s)$. Discuss design targets of the closed loop system in terms of the sensitivity function $S_o(s)$ and the complementary sensitivity function $T_o(s)$. How can these design targets be translated into requirements for the frequency response of L_o? [6]

(b) Consider a PID controller.

(i) State the control law in the time domain and in the Laplace domain. [3]

(ii) Derive the transfer function of the PID controller in terms an overall controller gain K, a time-constant associated with the integral term, T_I, and a time constant associated with the derivative term, T_D. What are the poles and zeros of $C(s)$. [4]

(iii) Sketch the Bode frequency response of a PID controller with $K = 100$, $T_I = 1$ and $T_D = 0.05$. Clearly marks the corner frequencies and the corresponding asymptotes of the magnitude and phase components of the frequency response. [4]

(iv) Describe how the PID controller can be extended to make it realisable. Based in the numerical values in Q3(b)(iii), choose a suitable value for the extra component and explain your choice. Amend the Bode plot of the PID controller accordingly. [3]

Continued overleaf

Q4 (a) Explain in your own words what is meant by state estimator feedback control. Use a block diagram to illustrate your explanations and mark the elements which form the compensator. Discuss the reasons for using a state estimator. [5]

(b) For the structure described in (a), describe in detail the state estimator (observer). Derive the equations for the state estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$ and discuss its behaviour. [5]

(c) Consider the plant

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -11 & 1 \\ -15 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 23 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Derive the observer gain vector L such that the closed loop observer poles are located at -100 and -110. [5]

(d) Describe a test for observability of a state space system. Show whether the following system is observable:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[5]

Continued overleaf

SECTION C
Attempt ONE question

Q5 Consider the following digital plant with sample time $T = 0.1\,s$:

$$G(z) = \frac{z}{(z-2)(z+0.5)}$$

(a) Design a digital PD controller that cancels the unstable pole of the plant, and find the open-loop transfer function $L(z)$. [4]

(b) Select the open-loop gain K such that the time constant of the closed-loop system is approximately $\tau = 0.2\,s$. [6]

(c) Find the values of the damping and natural frequency of the closed-loop system. [5]

(d) Estimate the steady-state error (of the closed-loop system) in response to a unit step input. [5]

Continued overleaf

Q6 Consider the following low-pass filter:

$$G(s) = \frac{20}{(s-1)(s-2)}$$

(a) Find the sampling time that corresponds to 10 times the bandwidth of the filter (frequency at 0 dB). [4]

(b) Now set a sample time $T = 0.1\,s$. Calculate the Nyquist frequency. [2]

(c) Find the discrete equivalent of the filter, using the pole-zero matching rule; calculate the gain at $\omega_c = 4.19\,rad/s$, and compare it to that of the continuous filter at the same frequency. [8]

(d) Now design another discrete equivalent, using the Tustin rule with pre-warping at frequency $\omega_c = 4.19\,rad/s$. Show that the gain is now preserved. [6]

Continued overleaf

Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper