UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4/M (ENG4042/ENG5022)
[RESULTS]

XX December 2017
xx:xx – xx:xx

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 …

Q2 (a) Discuss the features of a digital signal, as opposed to an analogue one. [4]

(b) With the help of sketches as necessary, highlight the main differences of a digital feedback controller, with respect to an analogue one. [6]

(c) Consider a first order continuous transfer function $H(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$.

Demonstrate that the trapezoid numerical integration rule can be implemented through the substitution $s \Leftrightarrow \frac{2}{T}\frac{z-1}{z+1}$ [6]

(d) Derive how the stability region of a continuous transfer function maps into the z-plane, using that rule, and use sketches to explain your results. What are the consequences of your result, when applying this rule to a real system. [4]

Continued overleaf

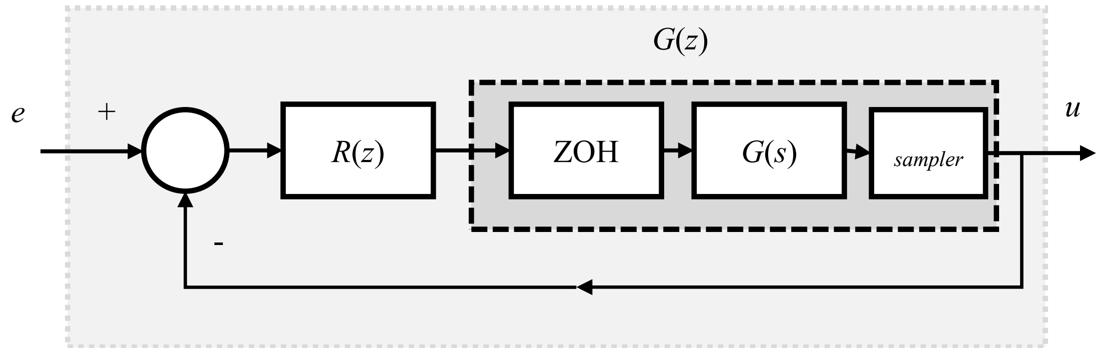
SECTION B

Q3 (a) … []

Q4 (a) ... []

SECTION C

Q5 Given the following digital feedback control loop $C_L(z)$ (sample time $T = 0.1$ s):



$C_L(z)$

$$G(s) = \frac{s+1}{20s+1} \qquad R(z) = \frac{z-0.995}{z-1}$$

(a) Find the discrete equivalent G(z) of the plant $G(s)$. [$G(z) = 0.05\frac{z-0.9002}{z-0.995}$] [8]

(b) Find the transfer function of the closed loop system $C_L(z)$. (Simplify poles and zeros if possible) [$C_L = 0.047619\frac{z-0.9002}{z-0.9952}$] [2]

(c) Find the difference equation corresponding to $C_L(z)$. [$u_k = 0.9952u_{k-1} + 0.047619e_k - 0.042866e_{k-1}$] [4]

(d) By verifying a suitable condition, demonstrate whether the difference equation is BIBO stable. [yes because…] [6]

Q6 Consider a continuous transfer function of a first-order low pass filter with cutoff frequency (-3 dB) of 15 rad/s and steady-state gain of 0 dB.

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(a) Design the discrete equivalent of it, using the Tustin rule, considering a sampling time of 0.1 s. Compute the gain (in dB) of the digital filter at the cutoff frequency, and compare it with the analogue version. [$G(z)=\dfrac{0.4286z+0.4286}{z-0.1429}$; $-4.0533\,\text{dB} <$ -3 dB] [8]

(b) Re-design the same, but this time apply a pre-warping such that the gain is preserved at the original cutoff frequency. Once designed, verify the gain numerically. [$G(z)=0.4823\dfrac{z+1}{z-0.03541}$; $-3.0103\,\text{dB}$] [8]

(c) Finally, re-design the discrete equivalent using the backward rectangular rule, and compare the gain at the cutoff frequency. [$G(z)=\dfrac{z}{1.66667z-0.66667}$; $-4.8644\,\text{dB}$] [4]

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Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit}\ \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper