UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022) [RESULTS]

Monday 11 August 2014
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

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SECTION A

Q1 (a) Sketch two block diagrams, describing the main components and signals of an analogue and a digital feedback control loop. Describe the main differences (blocks, types of signals) between the two. [5]

(b) Define the BIBO stability of a discrete system and state a necessary condition for it, given a discrete transfer function H(z). [4]

(c) Describe what is a zero-order hold (ZOH) and how a works, by splitting it in its two ideal building blocks, and sketching the time history of a sinusoidal signal at each stage in the process. (No need to use electronic diagrams) [5]

(d) State the formula that allows to map poles from the continuous domain to the discrete one, defining its terms. Explain why this is not a one-to-one correspondence. Plot two complex planes, one for the continuous domain and one for the discrete domain. Show how the negative real axis, the imaginary axis and the II/IV quadrant bisector in the continuous domain are mapped into the discrete domain. [6]

Q2 (a) For a closed loop system with a nominal plant P_0, a phase margin of 70deg at a frequency of 10Hz, and a gain margin of 6dB was derived from a Nyquist diagram of the loop gain.

i. By how much can the plant gain increase before the closed loop becomes unstable? [2]

ii. What additional delay can be added to the closed loop before stability is lost? [3]

(b) Show that closed-loop control implicitly performs an approximate inversion of the plant dynamics, at least for those frequencies where the loop gain is high. [5]

(c) Consider a state space system. What is controller design by pole assignment and how can it be used to design a state feedback controller? [5]

(d) What is controllability in the context of state feedback control and what is meant by stabilisability? [3]

(e) What are the components of a PID controller? Give the control law in the time domain. [2]

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SECTION B

Q3 (a) Derive the z-transform of the discrete exponential signal below, find its poles, and derive a condition for its convergence for $t \to +\infty$. [...; $|r| < 1$] [6]

$$e(k) = \begin{cases} r^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

(b) A sampled (not held) signal $r^*(t)$ can be represented in the time domain as a train of impulses modulated by the original continuous signal $r(t)$.

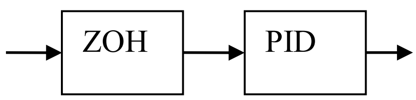
(i) State the general expression of $r^*(t)$, by using the Dirac δ function. [$r^*(t) = \sum_{k=-\infty}^{\infty} r(t)\delta(t - kT)$]

(ii) Derive its Laplace transform. [$R^*(s) = \sum_{k=-\infty}^{\infty} r(kT)e^{-skT}$] [6]

(c) Consider a PID controller, with proportional gain K, integral gain K_I and derivative transfer function:

$$D(s) = K_D \frac{s}{s+1}$$

Find the discrete equivalent of the PID preceded by a zero-order hold with sample time *T*, as shown in Figure 1 below. [8]



[$G(z) = K + \frac{K_I T}{z-1} + \frac{K_D(z-1)}{z-e^{-T}}$]

FIGURE 1

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Q4 (a) Derive the difference equation and the transfer function of the forward and backward integration rules. [$u_k = u_{k-1} + Te_{k-1}$; $H(z) = \frac{T}{z-1}$; $u_k = u_{k-1} + Te_k$; $H(z) = \frac{zT}{z-1}$] [6]

(b) Using the zero-pole matching rules, find the discrete equivalent of the following transfer function (TF). Select the appropriate parameters such that the discrete TF preserves the gain for a constant signal. Consider a sample time *T*. [8]

$$H(s) = \frac{1}{s^2 + 3s + 2}$$

[$H(z) = \frac{1}{8}\left(1 - e^{-T}\right)\left(1 - e^{-2T}\right)\frac{(z+1)^2}{\left(z - e^{-T}\right)\left(z - e^{-2T}\right)}$]

(c) Demonstrate whether the Tustin integration rule is BIBO stable. Explain your answer through a simple example. [… not BIBO stable; Integrating a constant signal causes integral grow linearly to infinity] [6]

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Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t - kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z - e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z - e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z - e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2 + b^2}$	$\dfrac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2 + b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

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SECTION C

Q5 Refer to the closed-loop system shown in Figure 2.

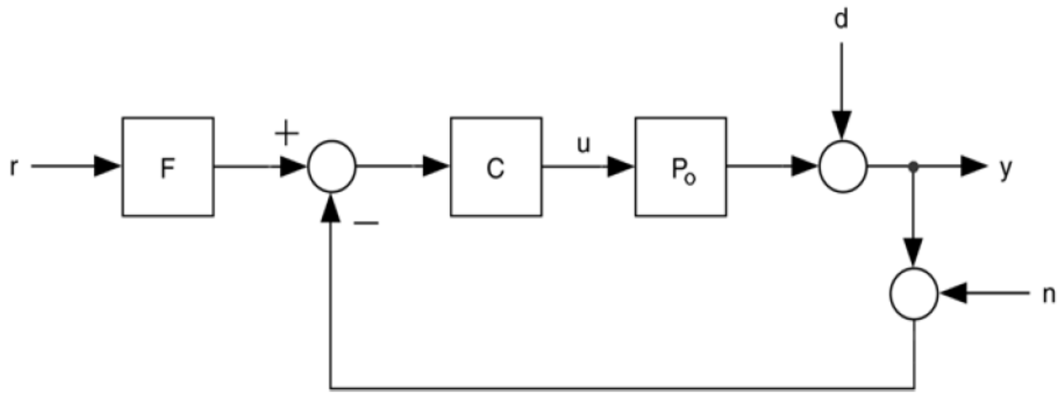

FIGURE 2

(a) By means of a sketch of a typical Nyquist diagram, indicate the vector margin for the loop gain, and the alternative vector margin for the inverse loop gain, and explain how these can be used to characterise the relative stability of the closed-loop. [5]

(b) Explain why 'peaking' in the sensitivity function S_0 and the complementary sensitivity function T_0 should be avoided. The explanation should be based on performing the following analysis.

i. Derive the closed-loop equations of the system, and discuss how peaking would affect the system's response to *r*, *d* and *n*, which are defined in Figure Q4. [4]

ii. Show that the vector margin for the inverse loop gain is equal to the inverse of the peak value of $|T_0|$. By a symmetry argument, briefly describe the effect of $|S_0|$ on the vector margin for the loop gain. [6]

iii. Show that T_0 is equal to the sensitivity of S_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of S_0 on the sensitivity of T_0. [5]

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Q6 (a) The control system in Figure 2 has a plant $P_0(s) = \dfrac{B_0(s)}{A_0(s)}$ and a controller $C(s) = \dfrac{G(s)}{H(s)}$. Using the definitions of the sensitivity function and the complementary sensitivity function, derive the expression for the characteristic polynomial of the closed loop and discuss its significance. Based on this, explain what is meant by feedback controller design using pole assignment. [7]

Consider the plant $P_0(s) = \dfrac{5}{s+5}$

(b) Sketch the frequency response of this plant in form of a Bode plot. What is the bandwidth of the open loop system? [2]

(c) Describe how a proportional controller can be used to increase the bandwidth of the closed loop system by considering how it affects the loop gain. Mention two limitations associated with using only a proportional component in the controller and how the other terms of the PID controller could be used to overcome these limitations. [5]

(d) Given the plant $P_0(s) = \dfrac{5}{s+5}$, a strictly proper 1^{st} order controller C(s) should be designed using the pole placement design method. The closed loop rise time t_r should be 0.9sec and the overshoot M_p should be 0.1.

(i) Using the following formulae to relate time-domain specifications to parameters of the closed loop polynomial, specify the desired characteristic polynomial A_{cl}.

$$\omega_n \cong \frac{1.8}{t_r},\ \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + \left(\ln M_p\right)^2}};\ M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right),\ 0 \le \xi < 1.$$ [2]

(ii) Calculate the parameters of the controller by solving the Diophantine equation. [4]

End of question paper