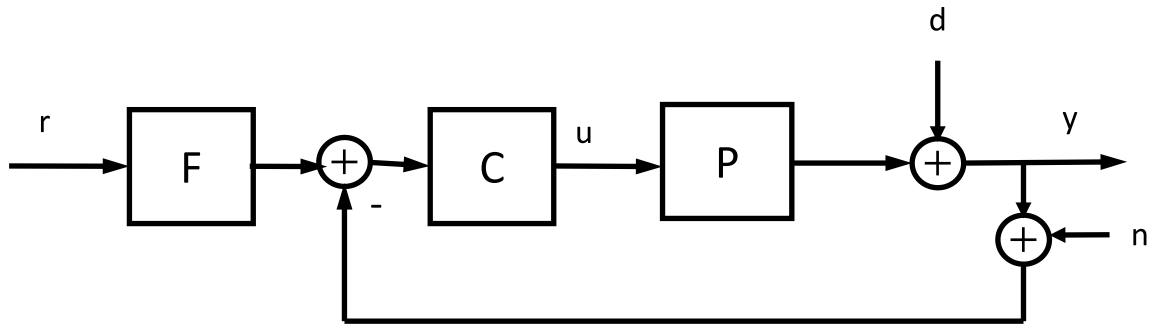
# Summary

## Themes

- Feedback
- Stability
- Robustness
- Performance
  - Frequency response
  - Loop shaping



# Summary of Feedback

## Benefits

- disturbance rejection
- robustness
- linearity improvement
- bandwidth improvement

## Closed-loop Stability Required

Good tracking and disturbance attenuation are retained as long as

- closed-loop system remains stable
- the gain remains high

Under these conditions high-gain feedback implies robustness with respect to loop uncertainty

# Pitfalls of High Gain Feedback

## Potential problems

- naively making the gain large can easily result in an unstable feedback system
- even if feedback system is stable, overly large plant inputs may arise that exceed the plant capacity
- measurement noise causes loss of performance

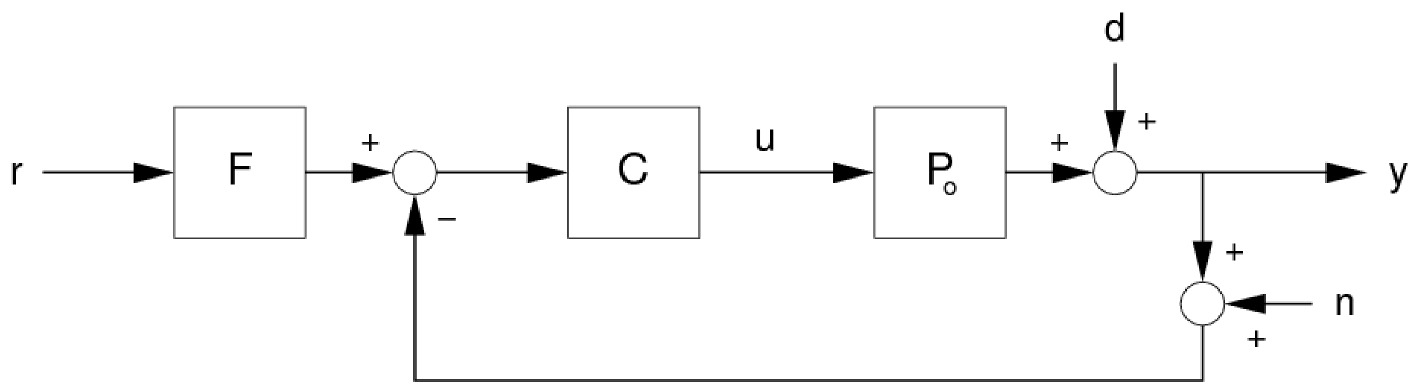
# Design Issues (recap)

## Targets

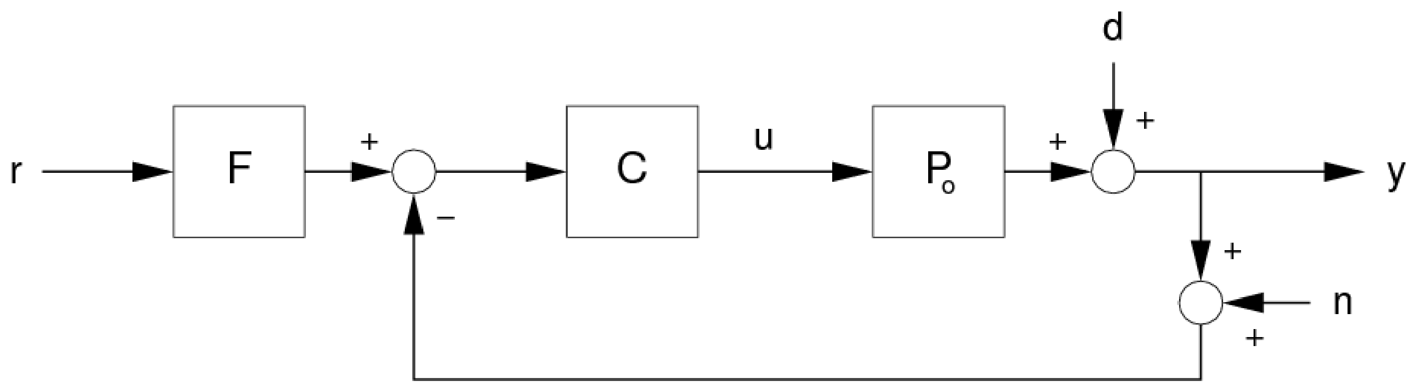
- Closed-loop stability
- Disturbance attenuation
- Good command response
- Robustness
  - stability
  - performance

## Limitations

- Plant capacity
- Measurement noise



# System functions: *L* and *S*




**Loop gain** $L$

$$L = PC$$

- **Sensitivity function** $S$

$$y = \underbrace{\frac{1}{1+L}}_{S} d$$

# Disturbance Attenuation and Bandwidth

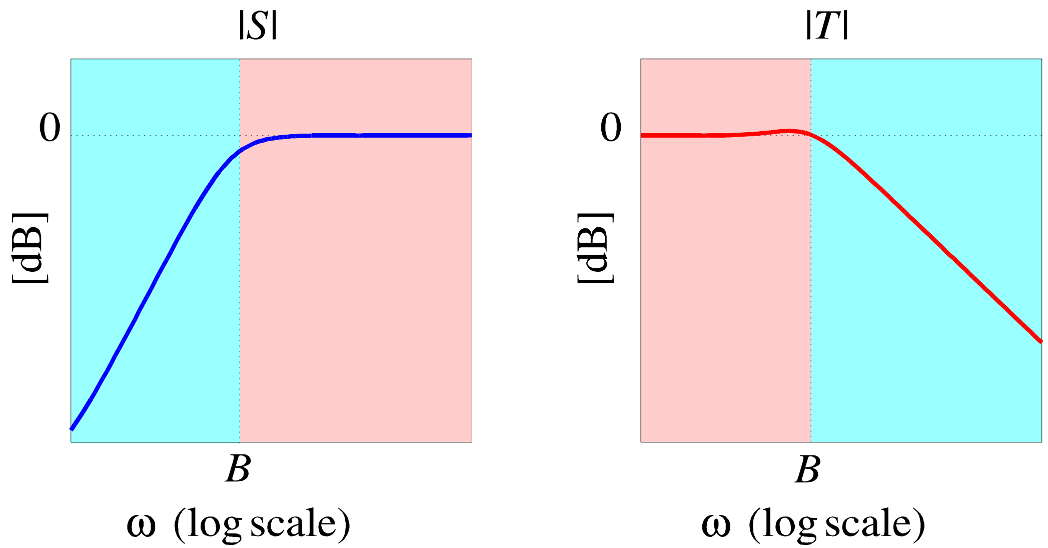
## Disturbance Attenuation

- The smaller $|S(j\omega)|$ is the more disturbances are attenuated at frequency $\omega$
- $|S|$ is small if the magnitude of the loop gain L is large
- L needs to be made large for frequencies where disturbance attenuation is needed
- However, this is limited by plant capacity

## Bandwidth

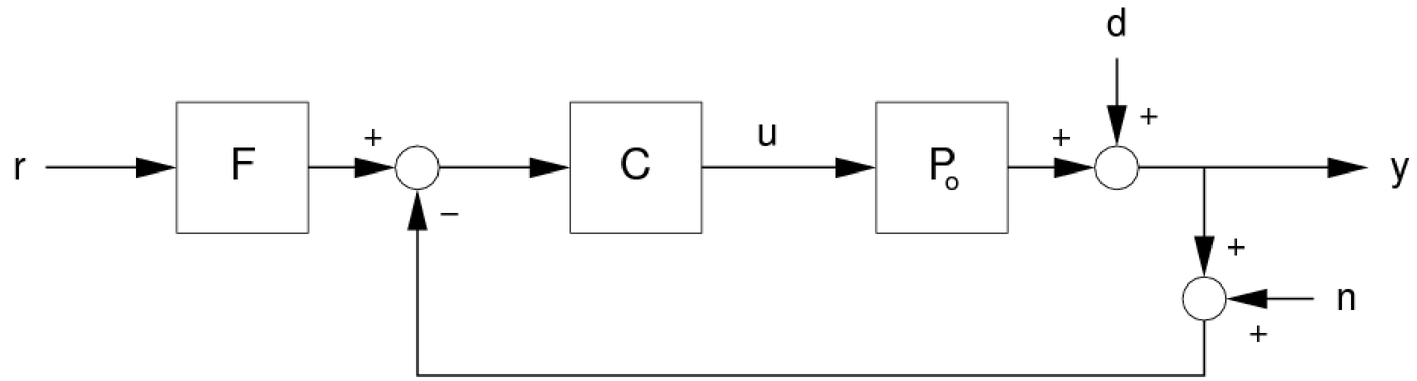
- L can only be made large over a limited frequency band
- The size of this band is called the bandwidth B

# Bandwidth and Crossover Region (S)




- Typical shape of magnitude of the sensitivity function:
  - low at frequencies up to the bandwidth
  - near 1 at frequencies above the bandwidth
- The frequency range around B is the crossover region
  - "peaking" of S should be avoided
  - otherwise disturbances are amplified

# System functions: *TF* and *T*



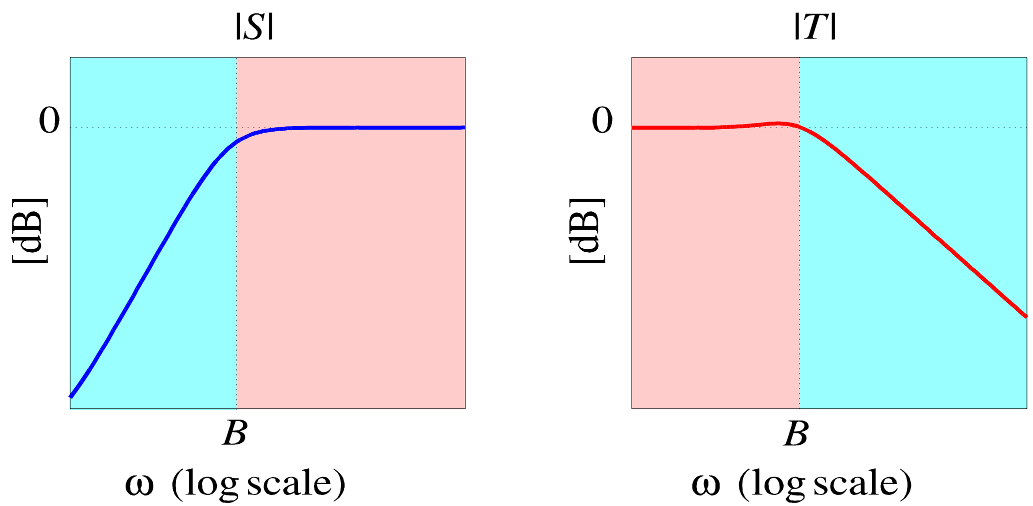

- ***Closed-loop transfer function*** $TF$

$$y = \underbrace{\frac{L}{1+L}F}_{TF}\, r$$

***Complementary sensitivity function*** $T$

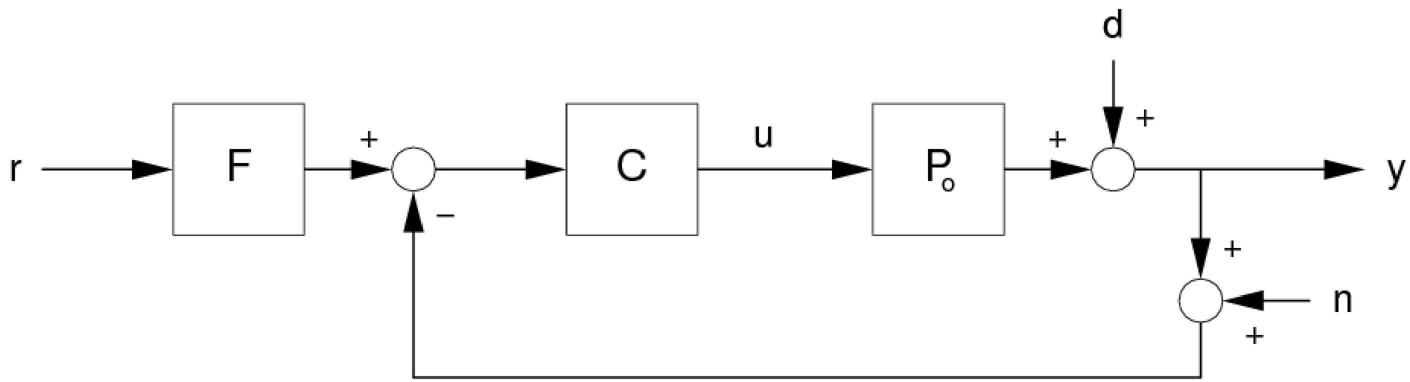
$$TF = \underbrace{\frac{L}{1+L}}_{T} F$$

# Command Response (T,F)




- Recall that T = 1 - S
- T determines the command response - it is close to 1 up to B
- When F = 1, closed-loop transfer function *TF* is low pass with the same bandwidth as the band for disturbance attenuation
- If a different command response is required, F can compensate for this

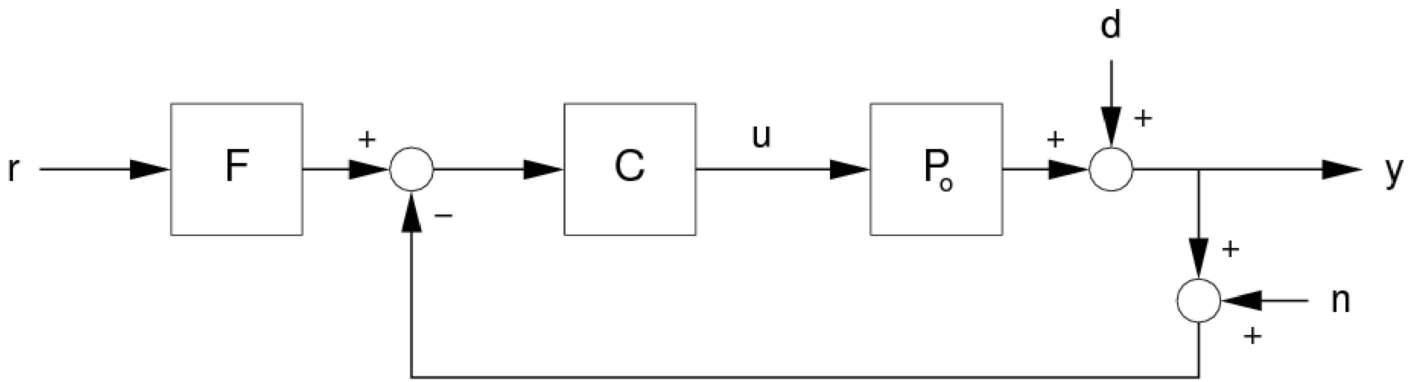
# Measurement noise (T)




$$y = \underbrace{\frac{1}{1+PC}}_{S} d + \underbrace{\frac{PC}{1+PC}}_{T} Fr - \underbrace{\frac{PC}{1+PC}}_{T} n$$

- T determines measurement noise sensitivity
- high frequencies: T should decrease as quickly as possible
- low frequencies: T is close to 1 - this emphasises the need for good low-noise sensors

# System functions: $S_u$




- ***Control sensitivity function*** $S_u$

$$u = \underbrace{\frac{C}{1+CP}}_{S_u}(Fr - n - d)$$

# Plant Capacity ($S_u$)

Note that $T = S_u P$

Thus, requirements on $S_u$ can be translated into requirements on $T$

- To prevent overly large inputs $S_u$ ($T$) should not be too large
- At low frequencies the loop gain should be high for low sensitivity, and the magnitude of T is close to 1
- This can lead to plant capacity being exceeded
- At high frequencies $S_u$ should decrease as fast as possible, otherwise measurement noise affects the input - this is consistent with the robustness requirement that T decrease fast

# Plant Capacity ($S_u$) - r.h.p. zeros

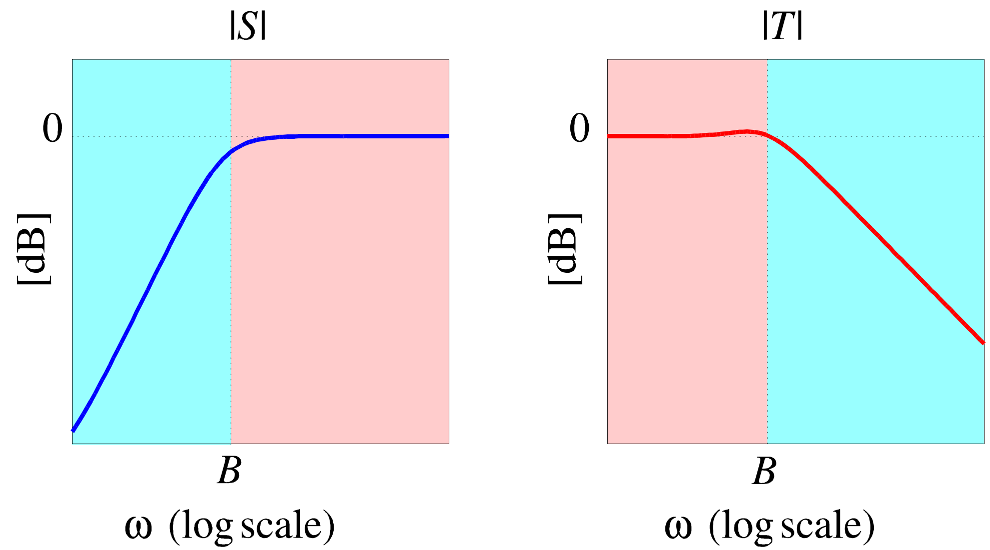
When $L = CP >> 1$ then $S_u \approx \frac{1}{P}$

If the plant $P$ has zeros in the right half plane, $1/P$ is unstable

- Unstable open loop plant zeros limit the closed-loop bandwidth
- $S_u$ may only be made equal to *1/P* up to the frequency which equals the magnitude of the r.h.p. zero with the smallest magnitude

# Stability Robustness

- Robustness
  - For loop gain perturbations T needs to be small
  - For inverse loop gain perturbations S needs to be small
- Performance
  - At high frequencies T needs to be small
  - At low frequencies S needs to be small
- Perturbations
  - High frequency uncertainty (parasitics) causes significant loop gain perturbations
  - low frequency uncertainty (load changes etc.) causes significant inverse loop gain perturbations
- Crossover
  - Neither S nor T can be small, they must therefore be prevented from peaking. Good stability margins help to ensure this




| | $S$ | $T$ |
|---|---|---|
| low frequencies | small | $\cong 1$ |
| high frequencies | $\cong 1$ | small |

# Performance Robustness

- Performance is determined by S, T, $S_u$ and TF
- Since $S_u$ and TF depend on S and T, we need only consider the effect of perturbations on S and T

- For robust S we need $T_o$ small
- For robust T we need $S_o$ small

- Normally, S is small at low frequencies, making T robust at low frequencies - this is the region where T's values are significant
- Conversely, T is normally small at high frequencies, making S robust at high frequencies - the region where S is significant

Denote nominal quantities by $S_o$ etc.

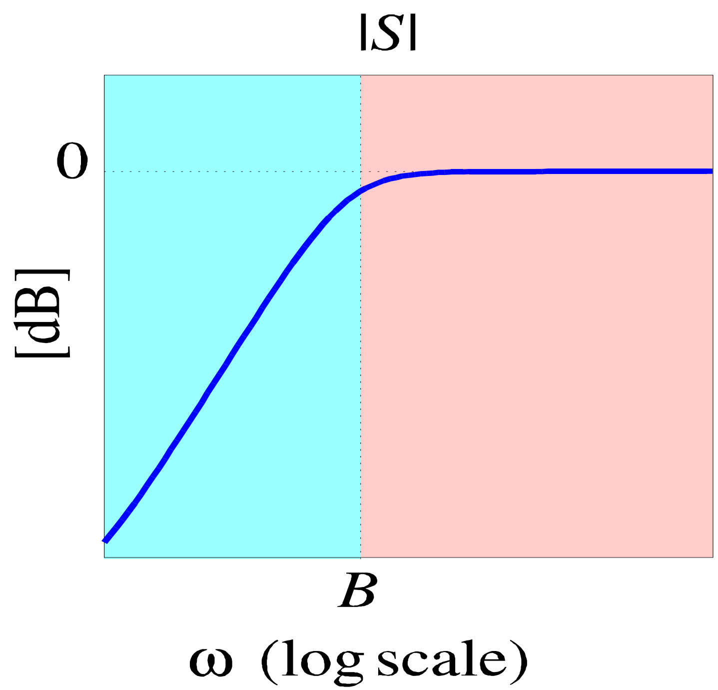
Relative changes in $1/S$ and $1/T$

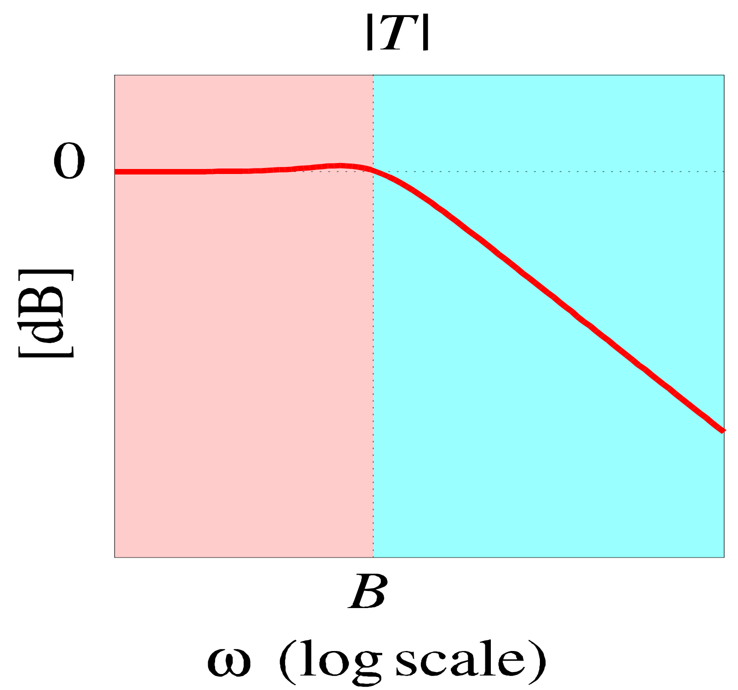
$$\frac{\frac{1}{S}-\frac{1}{S_o}}{\frac{1}{S_o}} = \frac{S_o - S}{S} = T_o \frac{L - L_o}{L_o} = T_o \frac{\frac{1}{L_o}-\frac{1}{L}}{\frac{1}{L}}$$

$$\frac{\frac{1}{T}-\frac{1}{T_o}}{\frac{1}{T_o}} = \frac{T - T_o}{T} = S_o \frac{L - L_o}{L} = S_o \frac{\frac{1}{L}-\frac{1}{L_o}}{\frac{1}{L_o}}$$

Vector margin related to peak S

Complementary vector margin related to peak T




$$s_m = \frac{1}{\max_{\omega} |S_o(j\omega)|}$$

$$r_m = \frac{1}{\max_{\omega} |T_o(j\omega)|}$$

# Review of Design Requirements

Sensitivity S small at low frequency to achieve

- disturbance attenuation
- good command response
- robustness at low frequencies

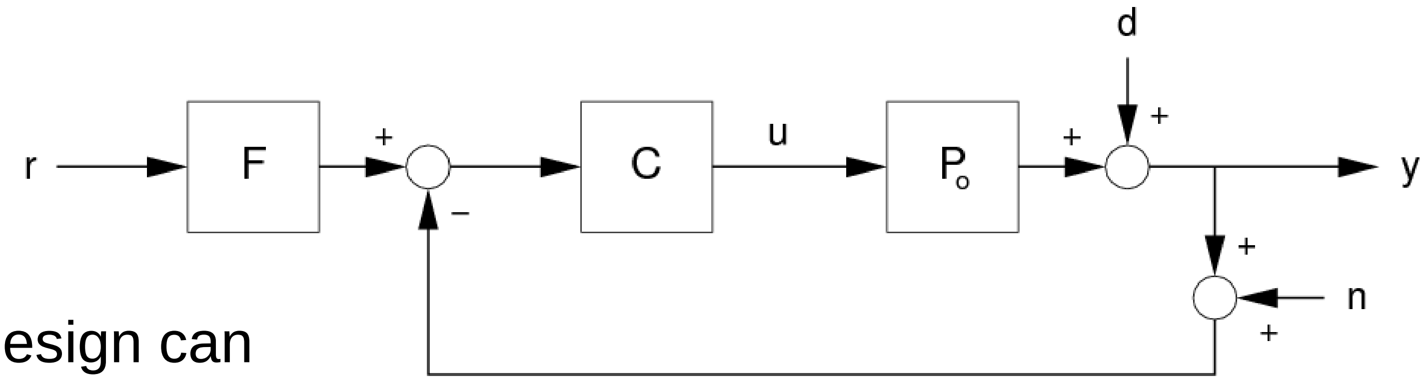
Complementary sensitivity T small at high frequencies to prevent

- exceeding plant capacity
- adverse effects of measurement noise
- loss of robustness at high frequencies

In the Crossover Region peaking of both S and T should be avoided to prevent

- overly large disturbance sensitivity
- excessive influence of measurement noise
- loss of robustness

# Loop Gain L




- Feedback system design can be seen as a process of loop shaping
- Performance and robustness requirements result in specifications on |S| in the low frequency region and on |T| in the high frequency region
- This results in bounds on the loop gain L

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad L = CP$$

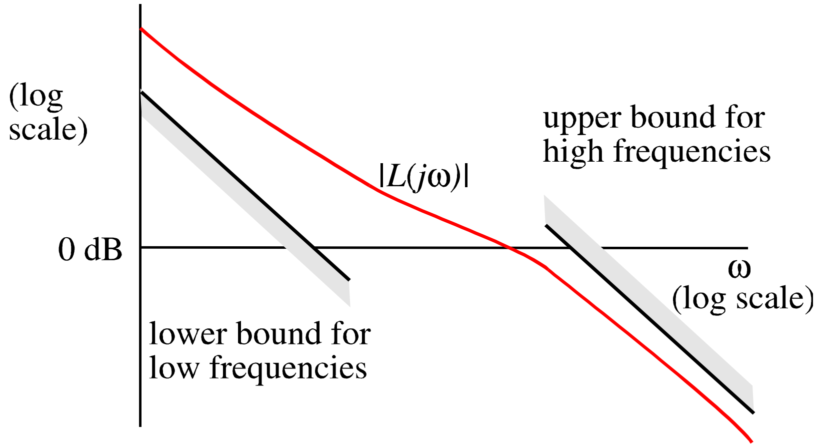
Low frequencies :

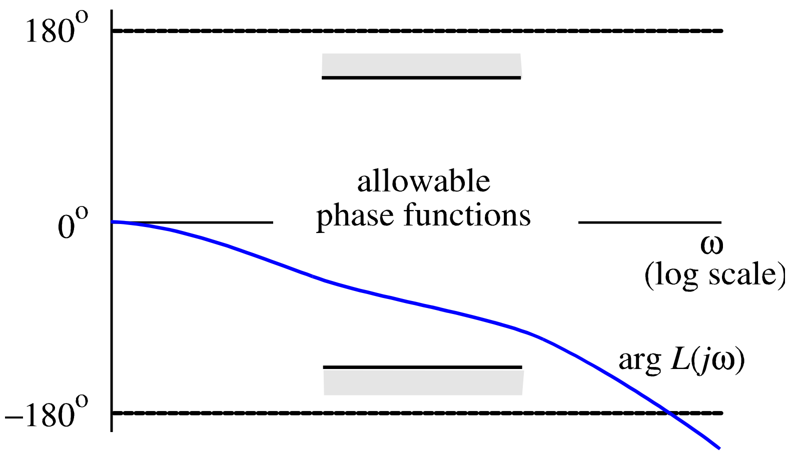
require $S << 1, T \approx 1 \iff |L(j\omega)| >> 1$

High frequencies :

require $T << 1, \; S \approx 1 \iff |L(j\omega)| << 1$

# Loop shaping




Low frequencies: large loop gain

High frequencies: small loop gain

- In the crossover region the phase is constrained because of stability

# Crossover Region

- The more closely the Nyquist plot of L approaches -1 the more S peaks
- If the plot of L approaches -1 so does the plot of 1/L. Hence the more closely the plot of L approaches -1 the more T peaks
- Thus to avoid peaking we need good stability margins
- But gain and phase are not independent

$$S = \frac{1}{1+L},$$

$$T = \frac{L}{1+L} = \frac{1}{1+\frac{1}{L}}$$