UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4/M DIGITAL (ENG4042/ENG5022) [RESULTS]

XX December 2019
xx:xx – xx:xx

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 …

Q2 (a) Consider a signal whose z-transform has two complex conjugate poles. Sketch and discuss the time sequences associated with various positions of the poles in the complex plane. [6]

(b) Given a discrete transfer function $H(z) = U(z)/E(z)$, demonstrate that, in the time domain, $u_k = \sum_{j=-\infty}^{+\infty} e_j h_{k-j}$. What is this formula commonly known as? [6]

(c) Consider a first order continuous transfer function $H(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$. Demonstrate that the forward rectangular numerical integration rule can be implemented through the substitution $s \Leftrightarrow \frac{z-1}{T}$ [6]

(d) What is aliasing? [2]

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SECTION B

Q3 (a) … []

Q4 (a) ... []

SECTION C

Q5 Consider the following transfer function:

$$H(s) = 10\frac{10s+1}{100s+1}$$

(a) Find the gain (in dB) and phase (in deg) at $\omega_1 = 0.5$ rad/s . Assuming a sample time T = 2 s, calculate the Nyquist frequency ω_n. [0.1686 dB; -10.1642 deg; 1.5708 rad] [3]

(b) Design the discrete equivalent of $H(s)$ using the forward rectangular rule. [$\frac{z-0.8}{z-0.98}$] [4]

(c) Compute the discrete equivalent of $H(s)$ using the pole-zero matching technique (match the steady-state gain). [$1.0921\frac{z-0.8187}{z-0.9802}$] [9]

(d) Find the gain (in dB) phase (in deg) at ω_1 of the two discrete equivalents, and compare with that of the continuous $H(s)$. Which one is closest? [-0.6533 dB; -10.4372 deg; 0.1693 dB; -9.2907 deg; PZ map] [4]

Q6 Given the following digital feedback control loop (sample time T = 0.1 s):

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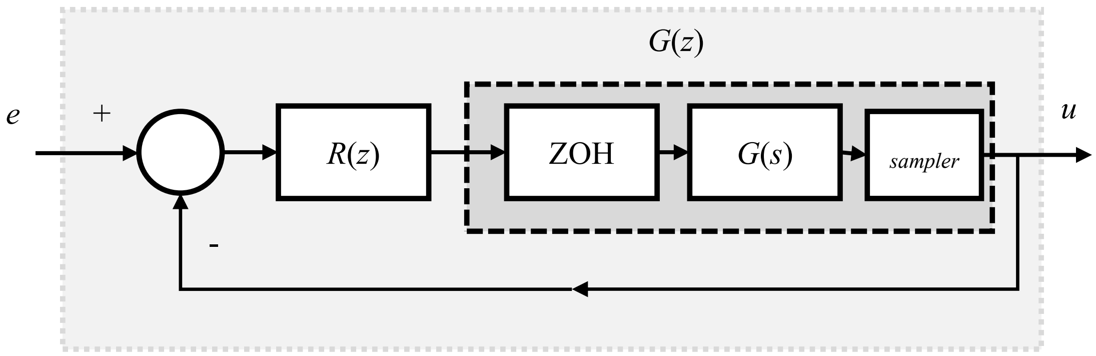

$C_L(z)$

$$G(s) = \frac{20s-10}{2s+1} \qquad R(z) = \frac{z-0.9512}{10z-1}$$

(a) Find the discrete equivalent G(z) of the plant $G(s)$. [$\frac{10z-10.49}{z-0.9512}$] [8]

(b) Find the closed loop transfer function of the system $C_L(z)$. (Simplify poles and zeros if possible) [$0.5\frac{z-1.0490}{z-0.5745}$] [2]

(c) Find the difference equation corresponding to $C_L(z)$. [$u_k = 0.5745\, u_{k-1} + 0.5\, e_k - 0.5245\, e_{k-1}$] [4]

(d) Estimate the steady-state output of the closed-loop system $C_L(z)$ for the following input:

$$e_k = 10\sin\left(0.1\, kT\right)$$

[$0.585\sin\left(0.1kT + 2.9161\right)$] [6]

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Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^{\,k}}{\partial \alpha^{\,k}} \left(\dfrac{z}{z - e^{-\alpha T}} \right)$
$e^{-a t}$	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-a t}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-a t} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-a t} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper