UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL M (ENG5022)

Monday 13th December 2021
Release time: 09:00AM (GMT) for 2.5 hours

Exam duration: 2 hours to complete exam plus 30 mins for download/upload of submission

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

A FORMULA SHEET IS PROVIDED AT THE END OF PAPER

A calculator may be used. Show intermediate steps in calculations.

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SECTION A
Attempt BOTH questions

Q1 (a) Draw the block diagram of an open-loop control system and that of a two-degree-of-freedom feedback control system. Show the different signals, including disturbances, and explain their physical meaning. Explain which additional components are required in the feedback control structure. [5]

(b) Describe potential benefits and drawbacks of a closed-loop control strategy and contrast these to an open-loop strategy. In your analysis, focus on the characteristics of each structure with respect to plant disturbances, changes in the plant gain, and stabilisation. [5]

(c) In your own words, explain what is meant by observability in the context of state feedback control. Describe a test for observability of a state space system. [5]

(d) Explain in your own words what is meant by state estimator feedback control. Use a block diagram to illustrate your explanations and mark the elements which form the compensator. [5]

Q2 (a) Consider a continuous and a discrete sinusoidal signal, both with the same frequency.

(i) Derive an expression relating the continuous and discrete poles. [4]

(ii) Show why this correspondence is one-to-many, and how the frequencies relate. [3]

(iii) With the aid of a sketch of the complex s-plane, show and justify a possible way to remove the one-to-many ambiguity. [4]

(b) Given a generic continuous signal $R(s)$ in the frequency domain:

(i) State an expression for the spectrum (also in the frequency domain) of its sampled counterpart. [3]

(ii) Sketch the spectrum of a generic signal after it has been sampled, highlighting its components and graphically showing the phenomenon of aliasing. [3]

(iii) Draw a similar sketch for the same signal, this time using an anti-aliasing filter before sampling. [3]

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SECTION B
Attempt ONE question

Q3 (a) Explain why 'peaking' in the sensitivity function S_0 and the complementary sensitivity function T_0 should be avoided. The explanation should be based on performing the following analysis.

(i) Derive the closed-loop equations of a feedback control system in terms of S_o and T_o, considering the output responses to the reference, disturbance and noise. Discuss how peaking would affect the system's response to these signals. [4]

(ii) Show that the complementary vector margin for the inverse loop gain is equal to the inverse of the peak value of $|T_0|$. By a symmetry argument, briefly describe the effect of $|S_0|$ on the vector margin for the loop gain. Based on this, explain what a strongly oscillatory response of a closed-loop system means for stability-robustness. [5]

(iii) Show that T_0 is equal to the sensitivity of S_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of S_0 on the sensitivity of T_0 to plant changes. [5]

(b) Explain in your own words what is meant by controller design using loop-shaping. [3]

(c) Consider a simple proportional controller in a feedback control configuration. Explain how varying the controller gain affects the frequency response of the loop gain and discuss related limitations of using a proportional controller. [3]

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Q4 (a) Describe the concept of state feedback control using a block diagram and derive the differential equation of the closed loop system. Explain the role of the different components in your block diagram. [5]

(b) How can the structure in Q4(a) be extended to include a reference input? Use a block diagram to illustrate your answer and derive the corresponding equations. [5]

(c) Explain in your own words what is meant by *controller design by pole assignment* in the context of state feedback control. [3]

(d) A pendulum can be represented by the state following state space system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -9.375 & -18.39 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0.9375 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Design a state feedback controller K for this system using the pole placement method in such a way that the closed system has an overshoot of $M_p = 1\%$ and a rise time of $t_r = 0.1$ seconds. You can use the following equations to derive the natural frequency and the damping of the desired closed loop system:

$$\omega_n \cong \frac{1.8}{t_r} \qquad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + (\ln M_p)^2}}; 0 \le \xi < 1$$

[4]

(e) For the system in Q4(d), calculate the corresponding reference input gain. [3]

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SECTION C
Attempt ONE question

Q5 Consider the following digital plant, with sample time $T = 0.1\,s$:

$$G_{ZAS}(z) = 0.0043361\frac{z+0.861}{(z-1.051)(z-0.6065)}$$

(a) Select a controller (PI or PD) with proportional gain K, that cancels the unstable pole of the plant, such that the closed-loop system has zero steady-state error due to a step input. Justify your choice. Find the open-loop transfer function. [4]

(b) Describe the procedure and equations to find the gain K of the controller chosen in (a), such that the percentage overshoot of the closed loop is 10%. [7]

(c) Now set the proportional gain $K = 12.84$, and find the natural frequency of the closed-loop system. [3]

(d) Estimate settling time and the steady-state error (of the closed-loop system) in response to a unit ramp input. [6]

Q6 Consider the following low-pass filter:

$$G(s) = \frac{20}{(s+2)(s+5)}$$

(a) Calculate its discrete equivalent when preceded by a zero-order hold (ZOH) and followed by a sampler ($T = 0.1\,s$). [6]

(b) Re-design another discrete equivalent, using the Tustin rule with pre-warping such that the gain is preserved at $\omega_c = 3.14$ rad/s . [6]

(c) Compute gain (in dB) and phase (in degrees) of the Tustin-equivalent at ω_c, and verify that the gain is preserved by comparing it to the analogue filter. [4]

(d) Estimate the steady-state response of the digital filter (Tustin) to the following input:

$$u_k = 2\cos(0.314k) \qquad [4]$$

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Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T \cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \dfrac{(-1)^k}{k!} \cdot \dfrac{\partial^k}{\partial \alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t \cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper