UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL () [RESULTS]

December 2015
xx:xx – xx:xx

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Continued overleaf

SECTION A

Q1 (a) Sketch two block diagrams, describing the main components and signals of a typical analogue and digital feedback control loops. Discuss the main differences (blocks, types of signals) between the two. [5]

(b) Discuss advantages and disadvantages of using a digital computer as a controller as opposed to an analogue circuit. [5]

(c) Define the BIBO stability of a discrete system H(z). State a sufficient condition for it. Starting from the discrete convolution, $u_k = \sum_{j=-\infty}^{+\infty} e_j h_{k-j}$, derive that condition for a transfer function H(z) = U(z)/E(z) [10]

Q2 (a) … []

SECTION B

Q3 Consider the following continuous transfer function:

$$G(s) = \frac{s+1}{s^2+s+1}$$

(a) Design the discrete equivalent of it, using the pole-zero mapping and a sample time $T = 0.1$ seconds. Make sure the discrete equivalent preserves the gain for a constant signal. [$H(z) = 0.0515\frac{z^2+0.0952z-0.9048}{z^2-1.895z+0.9048}$] [10]

(b) Design another discrete equivalent, this time using the forward integration rule. Calculate the poles in the discrete domain, and assess the stability of the discrete TF. [$G(z) = \frac{0.1z-0.09}{z^2-1.9z+0.91}$; 0.9500 +/– 0.0866i; stable] [10]

Q4 (a) Calculate the discrete equivalent of the following continuous plant:

$$H(s) = \frac{2}{s+2}$$

When preceded by a zero-order hold (ZOH). Sample time is $T = 0.2$ seconds. [$H(z) = \frac{0.3297}{s-0.6703}$] [12]

Continued overleaf

(b) Remembering the identity $\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$, find the z-transform of the discrete sinusoid:

$$e_k = r^k \cos(k\theta)\,\mathrm{step}(k)$$

Compute its poles, and discuss the stability of the signal. [$r\cos\theta \pm j\sin\theta$; stable if $|r|<1$] [8]

Section C

Q5 (a) … []

Q6 (a) ... []

Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *(t=kT, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t - kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{T\cdot z}{(z-1)^2}$
$\dfrac{t^k}{k!}$	$\left(\dfrac{1}{s}\right)^{k+1}$	$\lim\limits_{\alpha\to 0} \dfrac{(-1)^k}{k!}\cdot\dfrac{\partial^k}{\partial\alpha^k}\left(\dfrac{z}{z-e^{-\alpha T}}\right)$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
$t\cdot e^{-at}$	$\dfrac{1}{(s+a)^2}$	$\dfrac{T\cdot z\cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at}\cdot\cos(bt)$	$\dfrac{(s+a)}{(s+a)^2+b^2}$	$\dfrac{z\cdot(z-e^{-aT}\cdot\cos(bt))}{z^2 - z\cdot(2\cdot e^{-aT}\cdot\cos(bt)) + e^{-2aT}}$
$e^{-at}\cdot\sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{z\cdot e^{-aT}\cdot\sin(bt)}{z^2 - z\cdot(2\cdot e^{-aT}\cdot\cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper