Introduction

20 September 2021 18:04

Note created by Zak, more information https://github.com/Zak3225/Glasgow_ENG5022_Control-M

需要使用matlab/Simulink
周一control， 周五digital control

- Postgraduate (MSc) students
  - 90% exam
  - 10% lab assignment

Matlab

11 October 2021 22:51

You can use logical indexing to reassign values in an array. For example, if you wish to replace all values in the array `x` that are equal to `999` with the value `1`, use the following syntax.

```
x(x==999) = 1
```

TASK
Modify `v1` so that any value less than `4` is replaced with the value `0`.

Go to task

C2d

https://blog.csdn.net/hjhjhx26364/article/details/84107150

类型转换

https://www.cnblogs.com/daxiaoyuyu/archive/2013/01/31/2887199.html

Simulink

14 October 2021 11:26

PI

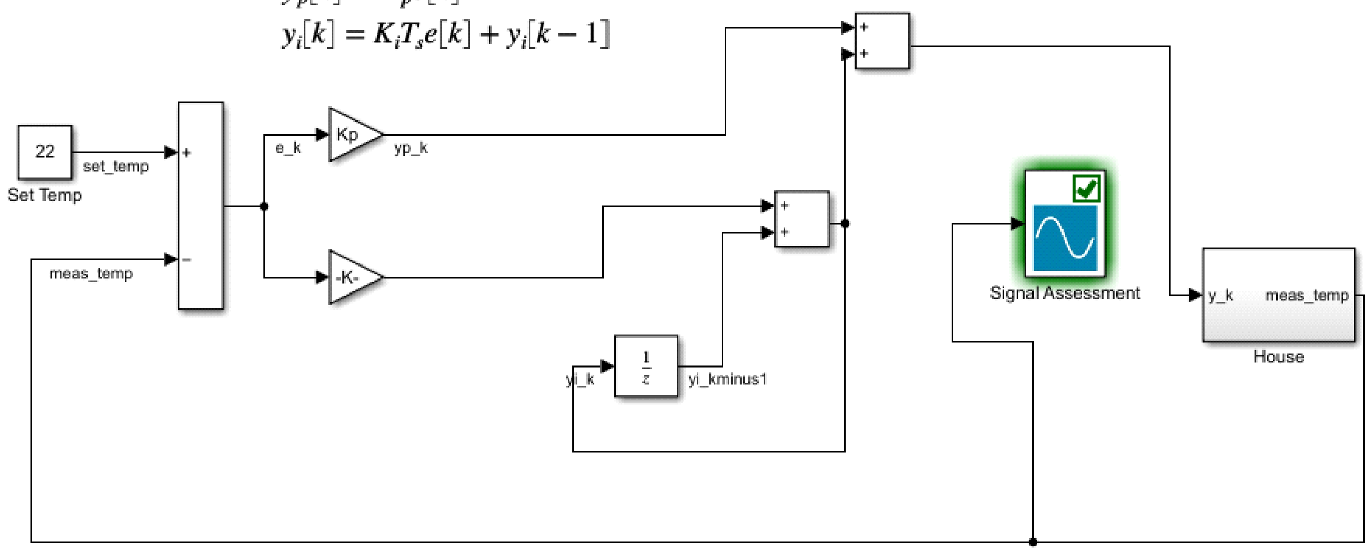
Proportional-integral control:

$y[k] = y_p[k] + y_i[k]$,

where:

$y_p[k] = K_p e[k]$

$y_i[k] = K_i T_s e[k] + y_i[k-1]$



(Time given in units of hours)

Control System Analysis and Design

03 October 2021 15:52

Classic Control: Frequency domain analysis of feedback system

11 December 2021 17:37

Pendulum Model

03 October 2021 15:50

See folder

System Modelling

03 October 2021 15:52

Introduction

1. Time domain(state space representation)
   a. Systems are modeled by differential equations
   b. Systems are characterized by the evolution of their variables in time
   c. The evolution of variables in time is computed by solving differential equations
2. Frequency domain(LaPlace transform)
   a. Systems are modelled by transfer functions, which capture this impact as a function of frequency

Linearization

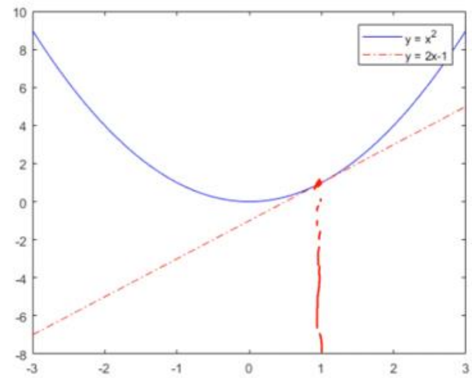
1. Linearising two-variable equation
   a. $$f_{\text{lin}}(x,y) = f(x_0,y_0) + \frac{\partial f(x,y)}{\partial x}\bigg]_{\substack{x=x_0\\ y=y_0}}(x-x_0) + \frac{\partial f(x,y)}{\partial y}\bigg]_{\substack{x=x_0\\ y=y_0}}(y-y_0)$$
2. Example1:

   Linearize the nonlinear function $y = x^2$ about $x_0 = 1$

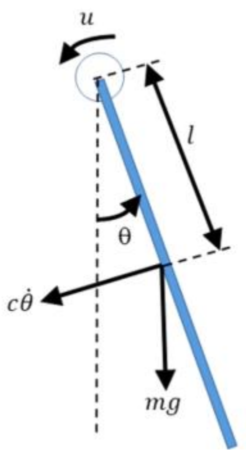
   a. $$\mathrm{f}_{\text{lin}}(x) = \mathrm{f}(x_0) + \frac{\partial f(x)}{\partial x}\bigg]_{x=x_0}(x-x_0)$$

   x = x0带入导数中

   $$\mathrm{f}_{\text{lin}}(x) = 1 + 2(x-1) = 2x-1$$



3. Example2:



   a. $$f(\theta,\mathrm{u}) \coloneqq \ddot{\theta} = -\frac{c}{ml^2}\dot{\theta} - \frac{g}{l}\sin\theta + \frac{1}{ml^2}u$$

   $$\mathrm{f}_{\text{lin}}(\theta,\mathrm{u}) = \mathrm{f}(\theta_0,\mathrm{u}_0) + \frac{\partial f(\theta,u)}{\partial \theta}\bigg]_{\substack{\theta=\theta_0\\ u=u_0}}(\theta-\theta_0) + \frac{\partial f(\theta,u)}{\partial u}\bigg]_{\substack{\theta=\theta_0\\ u=u_0}}(u-u_0)$$

   Given $\mathrm{f}(\theta_0,\mathrm{u}_0)$=0

   $$\mathrm{f}_{\text{lin}}(\theta,\mathrm{u}) = -\frac{c}{ml^2}\ddot{\theta} - \frac{g}{l}\cos\theta\bigg]_{\substack{\theta=0\\ u=0}}(\theta-\theta_0) + \frac{1}{ml^2}\bigg]_{\substack{\theta=0\\ u=0}}(u-u_0)$$

   Get the linear EOM for the pendulum:

   $$\boxed{\mathrm{f}_{\text{lin}}(\theta,\mathrm{u}) = -\frac{g}{l}\theta + \frac{1}{ml^2}u}$$
4. Linear Function is only valid for a certain region about the operating point

Laplace transform

1. Converts a time-domain based equation into the frequency domain equation

   Laplace transform of $f(t)$ is defined as:
2. $$F(s) = \int_0^\infty f(t)e^{-st}dt = \mathcal{L}[f(t)]$$

3. Inverse Laplace transform of $F(s)$ is defined as:

$$f(t) = \mathcal{L}^{-1}[F(s)] = \int_{c-j\infty}^{c+j\infty} F(s)e^{st}\,ds$$

4. $s = \sigma + j\omega$

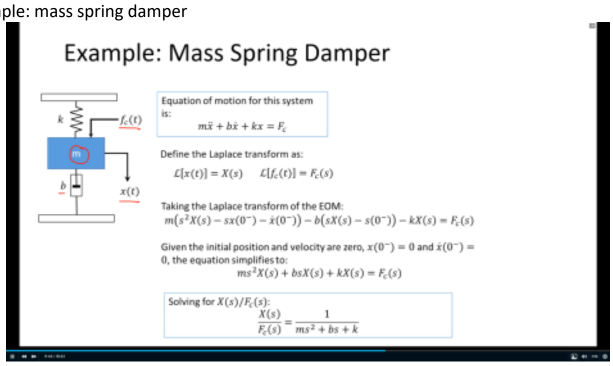
5.

f(t)	F(s)
δ(impulse)	1
$u(t)$ (step)	$1/s$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$

6.

	F(s)
Multiplication by a constant	$\mathcal{L}[kf(t)] = kF(s)$
Sum and difference	$\mathcal{L}[f(t) \pm g(t)] = F(s) \pm G(s)$
Differentiation	$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-)$ $\mathcal{L}\left[\frac{d^2f(t)}{dt}\right] = s^2F(s) - s\,f(0^-) - \dot{f}(0^-)$ $\mathcal{L}\left[\frac{d^nf(t)}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0^-)$
Integration	$\mathcal{L}\left[\int_0^t f(\tau)d\tau\right] = \frac{1}{s}F(s)$
Final-value theorem	$\mathcal{L}\left[\lim_{t\to\infty} f(t)\right] = \lim_{s\to 0} sF(s)$ Only applies if $sF(s)$ has no poles on or to right of the imaginary axis in the s-plane.

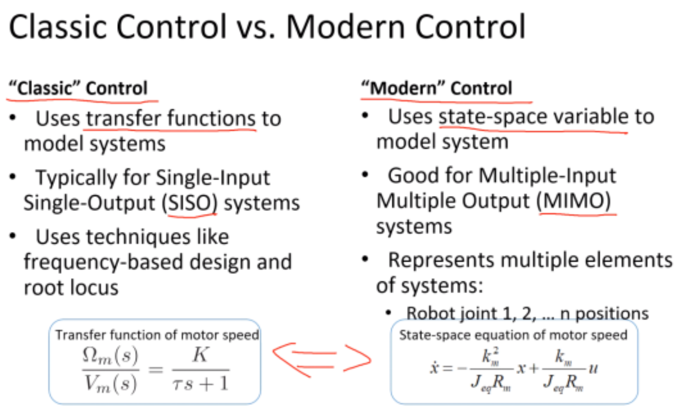
7. Example: mass spring damper

a. 

8. Cannot be used for nonlinear systems

State space model

1. Easier for more complex systems with multiple degrees of freedom, and MIMO systems

2. 

State-Space Representation

3. **General form**:

$$\dot{x} = f(x, u)$$
$$y = h(x, u)$$

Where:
x = vector of state variables ($n \times 1$)
u = the control input vector ($r \times 1$)
y = is the output vector ($m \times 1$)

This is for a **n[th] order system with *r* inputs and *m* outputs.**

This can describe both *linear* and *nonlinear* systems.

Linear system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

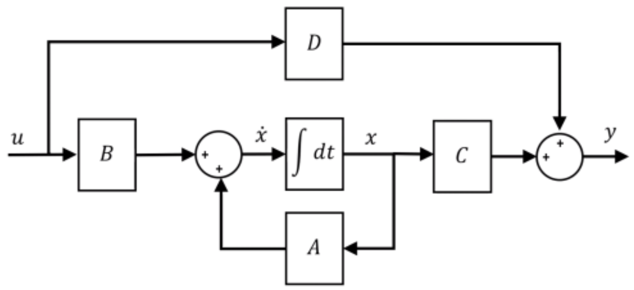
Where:
A = system matrix ($n \times n$)
B = input matrix ($n \times r$)
C = output matrix ($m \times n$)
D = feedforward matrix ($m \times r$)

The system state

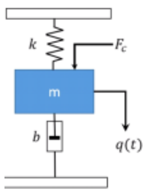
$$x = [x_1 \quad x_2 \quad x_3 \ldots x_{n-1} \quad x_n]^T$$

4. State: captures effects of the past
5. Inputs: external excitation
6. Dynamics: describes how the state changes
7. Outputs: describe measured quantities

State-Space Block Diagram

8. 

Example: Mass Spring Damper

9. 

Equation of motion for this system is:

$$m\ddot{q} + b\dot{q} + kq = F_c$$

Define the **states** as:

$$x_1 = q$$
$$x_2 = \dot{q}$$

Input is the force:

$$u = F_c$$

Output (i.e. measured variable) is position:

$$y = q$$

Rewrite as 1[st] order equations:

$$\dot{x}_1 = \dot{q}$$
$$\dot{x}_2 = \frac{1}{m}(-kq - b\dot{q}) + \frac{1}{m}F_c$$

Define w.r.t the system state

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m}(-kx_1 - b\dot{x}_2) + \frac{1}{m}u$$

The output equation is:

$$y = x_1$$

To get the matrix form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

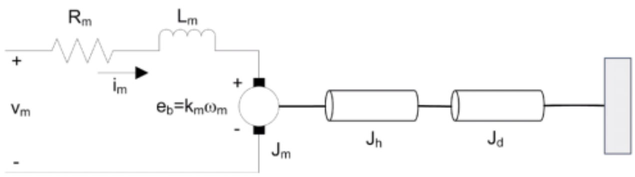
We have

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

And

$$y = [1 \quad 0]x$$

Example: DC Motor Modelling

10. 

The equation of motion are:

$$R_m i_m(t) + L_m \frac{di_m(t)}{dt} = v_m(t) - k_m \dot{\theta}_m$$

$$J_{eq}\ddot{\theta}_m(t) + b\dot{\theta}_m(t) = k_t i_m(t)$$

Defining the state variable as

$$x = [\theta_m \quad \dot{\theta}_m \quad i_m]^T$$

Input is the voltage

$$u = [v_m]$$

Output is the motor position

$$y = [\theta_m]$$

11. Write the system as first-order equations based on the state and output vector:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{J_m}(-bx_2 + k_t x_3)$$

$$\dot{x}_3 = \frac{1}{L_m}(-k_m x_2 - R_m x_3 + u)$$

$$y = x_1$$

Expressing this is terms of the matrix form:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

We have

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/J_{eq} & k_t/J_m \\ 0 & -k_t/L_m & -R_m/L_m \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/L_m \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0]$$

$$D = [0]$$

Block diagrams

1.

Static control

03 October 2021 22:25

Open loop control

Open loop control

P - plant

C - controller

y - output

u - control signal

r - reference

1. d - disturbance

Control aim:

minimise control error $\boxed{e = r - y}$

$$y = P\cdot u + d = C\cdot P r + d$$

$$e = r - (C\cdot P r + d)$$

best controller $C = \frac{1}{P} \Rightarrow C\cdot P = 1$

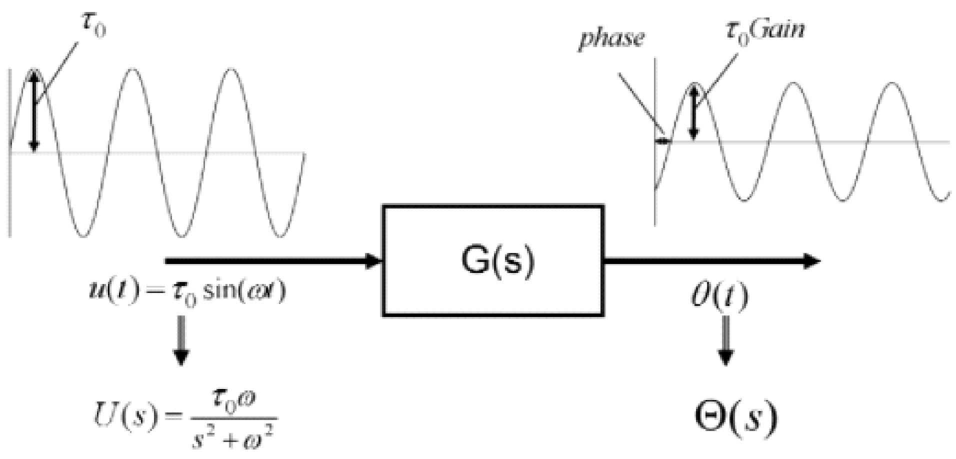
$$\Rightarrow e = r - \left(\frac{1}{P} P r + d\right) = -d$$

no disturbance attenuation

Frequency response

18 October 2021 17:33

Frequency response



$$\Theta(s) = G(s)\frac{\tau_0 \omega}{s^2 + \omega^2}$$

$$\Downarrow$$

$$\theta_{ss}(t) = \tau_0 \left|G(j\omega)\right| \sin\left(\omega t + \arg\left(G\left(j\omega\right)\right)\right)$$

滑动 2

Frequency response

Transfer function $G(s)$

$\Downarrow$

replace $s \rightarrow j\omega$

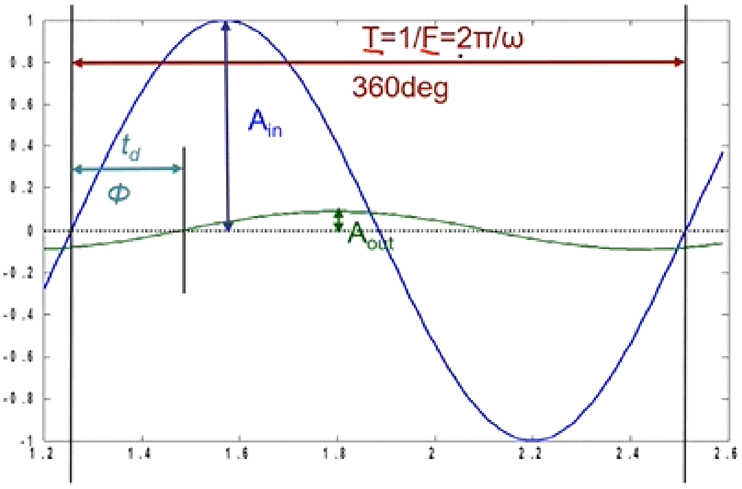
$\Downarrow$

Frequency response $G(j\omega)$

complex number which can be represented as

- Gain $|G(j\omega)|$ and phase $\arg G(j\omega)$
- Real and imaginary parts $\Re(G(j\omega)) + j\Im(G(j\omega))$

Gain and phase



T = period of oscillation [s]
F = frequency [Hz]
ω = angular frequency [rad/s]
t_d = delay [s]
Φ = phase [deg]
A_{in} = input amplitude
A_{out} = output amplitude

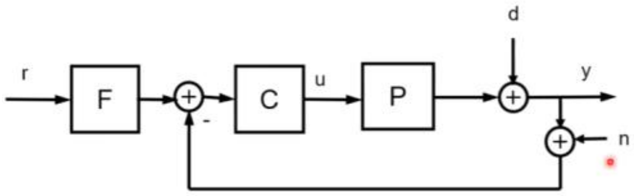
$\frac{t_d}{T} = \frac{\varphi}{360^o} \rightarrow$ **time delay** $t_d = \frac{\varphi}{360^o}\frac{2\pi}{\omega}$

Phase: $\varphi(j\omega) = \arg(G(j\omega)) = 360^o\frac{t_d}{T} = 360^o\frac{\omega}{2\pi}t_d$

Gain: $|G(j\omega)| = \frac{A_{out}}{A_{in}}$ $\quad |G(j\omega)|_{dB} = 20\,log_{10}\,|G(j\omega)|$

Sensitivity functions and frequency response

26 October 2021 14:22



- d is an output disturbance
- n is measurement noise

$$y=\underbrace{\frac{1}{1+PC}}_{S}d+\underbrace{\frac{PC}{1+PC}}_{T}Fr-\underbrace{\frac{PC}{1+PC}}_{T}n$$

- S is the sensitivity function
- T is the complementary sensitivity function

$$\text{Loop gain}: L = PC$$

$$S=\frac{1}{1+L}$$

$$T=\frac{L}{1+L}$$

$$\Rightarrow S+T=1$$

- Shape of loop gain L determined by typical low-pass character of physical plant

Loop gain L

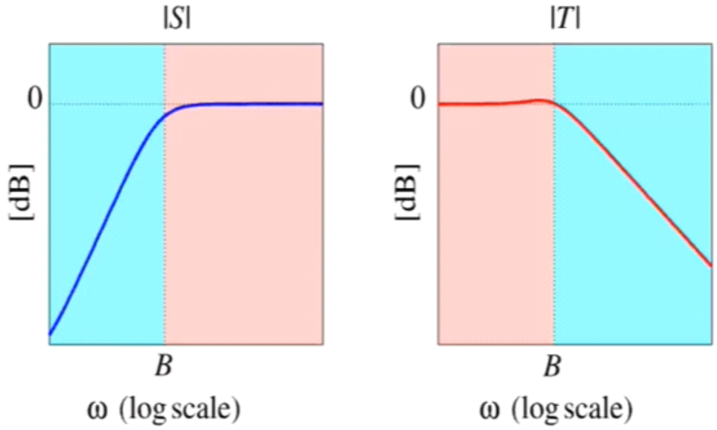
Large at low frequencies: $|L(j\omega)| >> 1$

Small at high frequencies: $|L(j\omega)| << 1$

Crossover region: $|L(j\omega)| \approx 1$

- at low frequency S is small and T is close to 1
- at high frequency T is small while S is close to 1

Typical shapes
for S and T

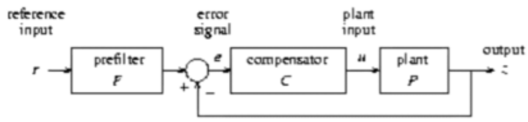


	S	T
low frequency	small	near 1
high frequency	near 1	small

Nyquist stability robustness

26 October 2021 17:15

Stability

1. 

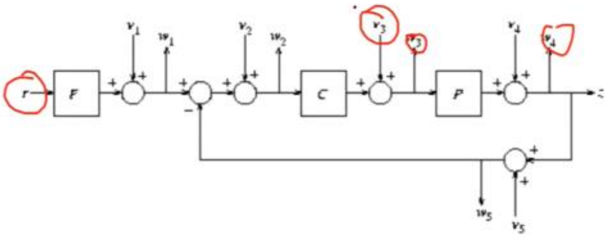
The control system is BIBO stable if every bounded input signal r results in bounded output signals e, u and z

BIBO = “bounded input bounded output”

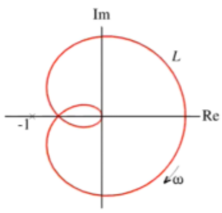
Asymptotic stability ⇒ BIBO stability

Inject “internal” signals into each “exposed interconnection” of the system, and define additional “internal” output signals after each injection point

Then the system is internally stable if it is BIBO stable with respect to all inputs (external and internal) and all (external and internal) outputs

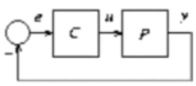
2. 

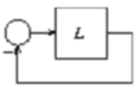
Nyquist plot

1. 

- Nyquist plot is of the loop gain is the curve traced in the complex plane by L
- Nyquist plot is symmetric with respect to the real axis

$$L(j\omega), \omega \in \Re$$





2. L = CP, 画在图上之后可以得到L的轨迹图
3. 如果轨迹不包括-1点则系统稳定，否则系统不稳定

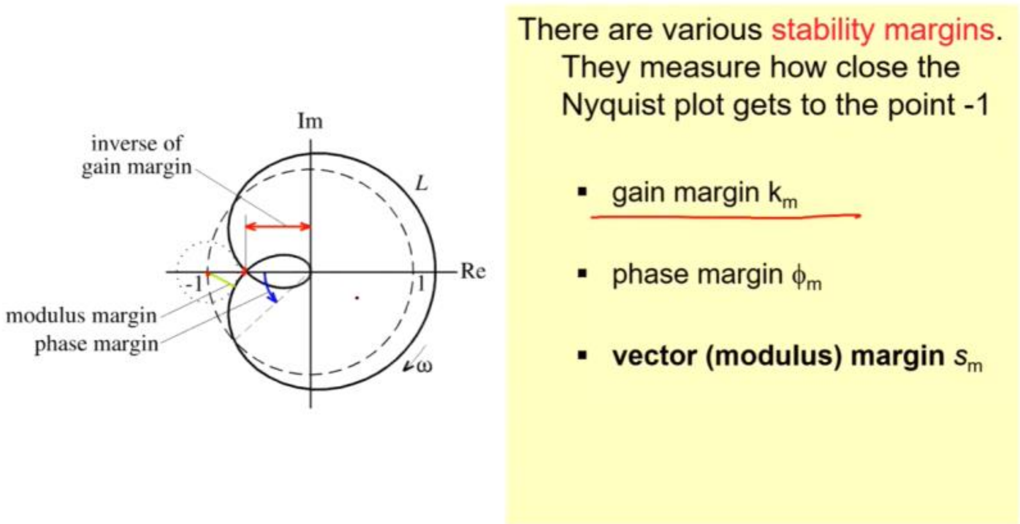
4. Stability margin

   a. If the closed-loop system is stable but the Nyquist plot of L passes close to -1 then

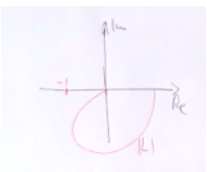
      ▪ the system is **near-unstable**, i.e. has an oscillatory response

      ▪ the system **may become unstable** by small perturbations of the plant, i.e. the system is **not robust**

   b. 轨迹靠近-1的时候偶称作stability margin

   c. 

   d. Gain margin: 红点标记处的值到点-1的距离为最大值，即红点到-1的连线。如果超过这个值，系统就不稳定。Gain margin的值为红线的倒数

      i. 

      ii. 曲线与实轴无交点，直接过原点的时候，gain margin为正无穷（可以理解为交点为0，取倒数之后为正无穷）

   e. Phase margin： 蓝色标记的角度，曲线与单位圆交点和原点的连线与实轴加成的角度。相当于给系统添加一个phase margin代表添加时滞

   f. Modulus margin: 点-1到曲线的最短距离

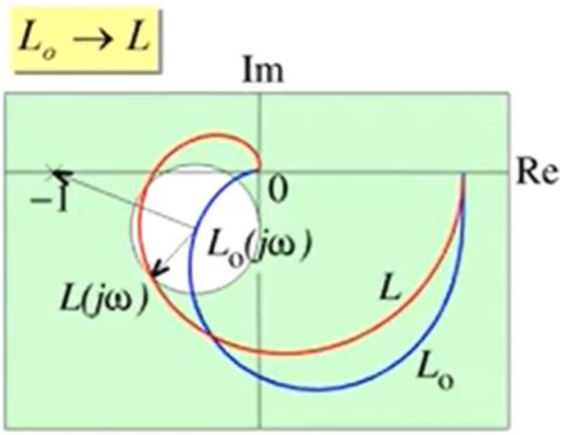
5. Find these margins by bode plot

   a. Gain margin: 在伯德图phase图像作出-180°的直线，与图像的交点为对应的频率，通过这个频率在gain图像找到对应的点。这个点到x轴的距离为gain margin。 如果这个点在x轴下方，则表示奈奎斯特图像上点在-1右侧，系统稳定

   b. Phase margin：在gain图像上找到与x轴交点，这一点gain=1，正好对应奈奎斯特图曲线与单位圆的交点。通过gain图像找到的x轴交点对应到phase上找到交点，这一点与-180°的差距为phase margin

1. 判断稳定性条件

a.



- **Sufficient condition** for stability under perturbation:

$$\left|L(j\omega)-L_o(j\omega)\right| < \left|1+L_o(j\omega)\right|, \quad \omega \in \Re$$

b.

Equivalently,

$$\left|\frac{L(j\omega)-L_o(j\omega)}{L_o(j\omega)}\right| < \left|\frac{1+L_o(j\omega)}{L_o(j\omega)}\right|, \omega \in \Re$$

or

$$\left|\frac{L(j\omega)-L_o(j\omega)}{L_o(j\omega)}\right| < \frac{1}{\left|T_o(j\omega)\right|}, \omega \in \Re$$

- The left side of the inequality is the relative loop gain perturbation
- This is Doyle's stability robustness criterion
- It is a sufficient condition

c.

Equivalently,

$$\left|\frac{\dfrac{1}{L(j\omega)}-\dfrac{1}{L_o(j\omega)}}{\dfrac{1}{L_o(j\omega)}}\right| < \frac{1}{\left|S_o(j\omega)\right|}, \omega \in \Re$$

d. 充分条件，满足的一定稳定，稳定的不一定满足

e. 核心是L的轨迹不能过-1点，标示为上述距离的式子

Feedback summary

27 October 2021 11:08

1. https://www.zhihu.com/question/27347401?sort=created

2. **Benefits**
   - disturbance rejection
   - robustness
   - linearity improvement
   - bandwidth improvement

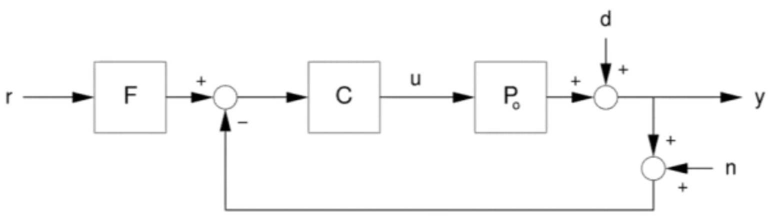
   Closed-loop Stability Required

   Good tracking and disturbance attenuation are retained as long as
   - closed-loop system remains stable
   - the $\boxed{\text{gain remains high}}$

   Under these conditions high-gain feedback implies robustness with respect to loop uncertainty

3. ## Potential problems
   - naively $\boxed{\text{making the gain large}}$ can easily result in an unstable feedback system
   - even if feedback system is stable, $\boxed{\text{overly large plant inputs}}$ may arise that exceed the plant capacity
   - measurement noise causes loss of performance

4. System functions: L and S



Loop gain L

$$L = PC$$

- **Sensitivity function** S

$$y = \underbrace{\frac{1}{1+L}}_{S} d$$

5.

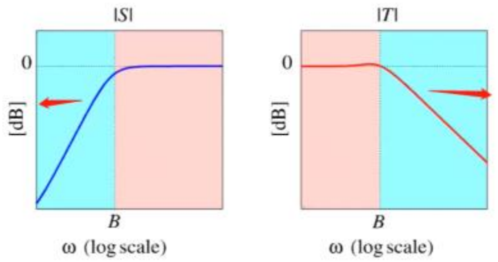
Disturbance Attenuation

- The smaller |S(jω)| is the more disturbances are attenuated at frequency ω
- |S| is small if the magnitude of the loop gain L is large
- L needs to be made large for frequencies where disturbance attenuation is needed
- However, this is limited by plant capacity

Bandwidth

- L can only be made large over a limited frequency band
- The size of this band is called the bandwidth B

6.



- Typical shape of magnitude of the sensitivity function:
  - low at frequencies up to the bandwidth
  - near 1 at frequencies above the bandwidth

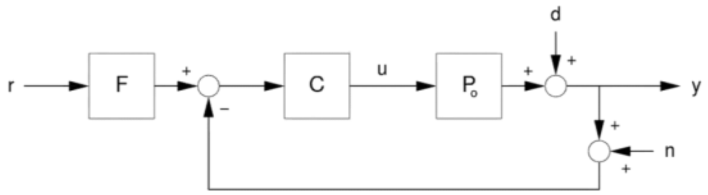
bandwidth取两边

- The frequency range around B is the crossover region
  - "peaking" of S should be avoided
  - otherwise disturbances are amplified

crossover region不应该取

7.

System functions: *TF* and *T*

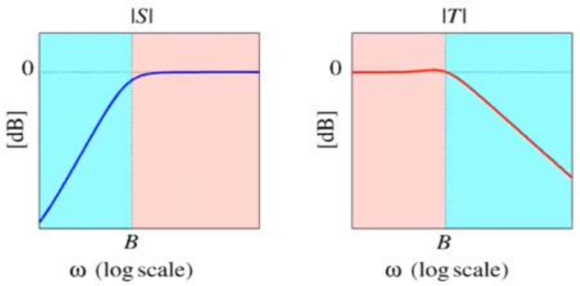


- ***Closed-loop transfer function*** TF

$$y=\underbrace{\frac{L}{1+L}F}_{TF}\,r$$

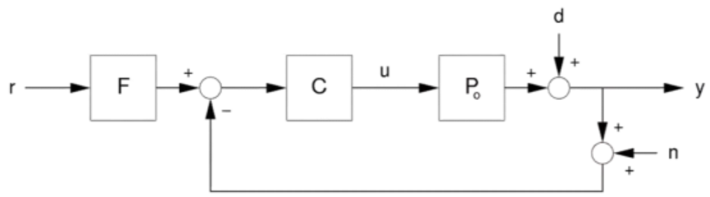
Complementary sensitivity function T

$$TF=\underbrace{\frac{L}{1+L}}_{T}F$$

8. 

- Recall that T = 1 - S
- T determines the command response - it is close to 1 up to B
- When F = 1, closed-loop transfer function *TF* is low pass with the same bandwidth as the band for disturbance attenuation F=1的时候带宽与单自由度的相同
- If a different command response is required, F can compensate for this 如果需要其他的特征，可以通过F调整

Measurement noise (T)

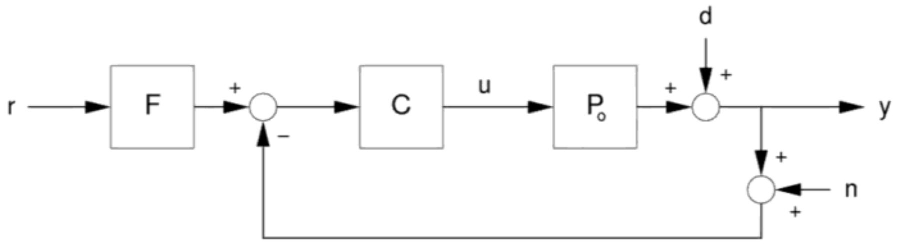
9. 

$$y = \underbrace{\frac{1}{1+PC}}_{S} d + \underbrace{\frac{PC}{1+PC}}_{T} Fr - \underbrace{\frac{PC}{1+PC}}_{T} n$$

noise高频迅速衰减
低频无法衰减

- T determines measurement noise sensitivity
- high frequencies: T should decrease as quickly as possible
- low frequencies: T is close to 1 - this emphasises the need for good low-noise sensors

System functions: S_u

10. 

- ***Control sensitivity function*** S_u

$$u = \underbrace{\frac{C}{1+CP}}_{S_u}(Fr - n - d)$$

Plant Capacity (S_u)

11. Note that $T = S_u P$

Thus, requirements on S_u can be translated into requirements on T

高频迅速衰减
低频 gain较大

- To prevent overly large inputs S_u (T) should not be too large
- At low frequencies the loop gain should be high for low sensitivity, and the magnitude of T is close to 1
- This can lead to plant capacity being exceeded
- At high frequencies S_u should decrease as fast as possible, otherwise measurement noise affects the input - this is consistent with the robustness requirement that T decrease fast

Plant Capacity (S_u) - r.h.p. zeros

When $L = CP >> 1$ then $S_u \approx \dfrac{1}{P}$

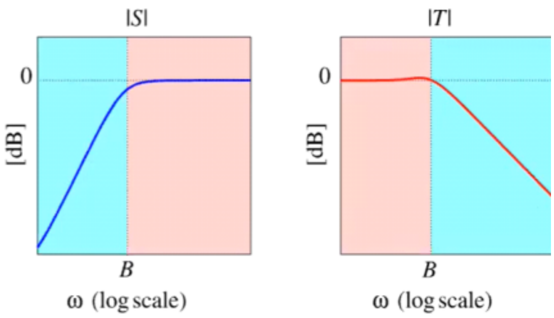
12. If the plant P has zeros in the right half plane, $1/P$ is unstable

- Unstable open loop plant zeros limit the closed-loop bandwidth
- S_u may only be made equal to *1/P* up to the frequency which equals the magnitude of the r.h.p. zero with the smallest magnitude

Stability Robustness

13.

- Robustness
  - For loop gain perturbations T needs to be small
  - For inverse loop gain perturbations S needs to be small
- Performance
  - At high frequencies T needs to be small
  - At low frequencies S needs to be small
- Perturbations
  - High frequency uncertainty (parasitics) causes significant loop gain perturbations
  - low frequency uncertainty (load changes etc.) causes significant inverse loop gain perturbations
- Crossover
  - Neither S nor T can be small, they must therefore be prevented from peaking. Good stability margins help to ensure this



	S	T
low frequencies	small	$\cong 1$
high frequencies	$\cong 1$	small

14.

Performance Robustness

Henrik Gollee

- Performance is determined by S, T, S_u and TF
- Since S_u and TF depend on S and T, we need only consider the effect of perturbations on S and T
- For robust S we need T_o small
- For robust T we need S_o small
- Normally, S is small at low frequencies, making T robust at low frequencies - this is the region where T's values are significant
- Conversely, T is normally small at high frequencies, making S robust at high frequencies - the region where S is significant

Denote nominal quantities by S_o etc.

Relative changes in $1/S$ and $1/T$

$$\frac{\frac{1}{S}-\frac{1}{S_o}}{\frac{1}{S_o}} = \frac{S_o - S}{S} = T_o\frac{L-L_o}{L_o} = T_o\frac{\frac{1}{L_o}-\frac{1}{L}}{\frac{1}{L}}$$

$$\frac{\frac{1}{T}-\frac{1}{T_o}}{\frac{1}{T_o}} = \frac{T-T_o}{T} = S_o\frac{L-L_o}{L} = S_o\frac{\frac{1}{L}-\frac{1}{L_o}}{\frac{1}{L_o}}$$

15.

Sensitivity S small at low frequency to achieve

- disturbance attenuation
- good command response
- robustness at low frequencies

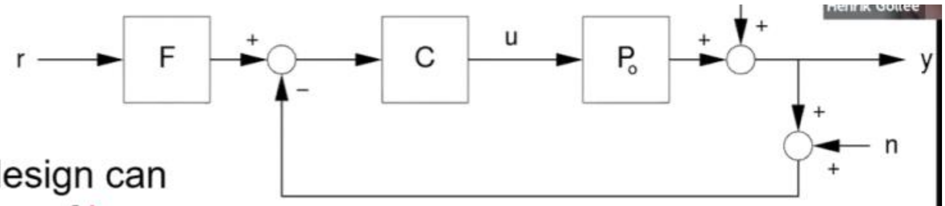
Complementary sensitivity T small at high frequencies to prevent

- exceeding plant capacity
- adverse effects of measurement noise
- loss of robustness at high frequencies

In the Crossover Region peaking of both S and T should be avoided to prevent

- overly large disturbance sensitivity
- excessive influence of measurement noise
- loss of robustness

Loop Gain L



- Feedback system design can be seen as a process of loop shaping

16. - Performance and robustness requirements result in specifications on |S| in the low frequency region and on |T| in the high frequency region
- This results in bounds on the loop gain L

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad L = CP$$

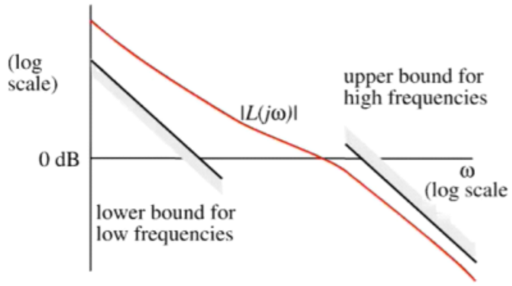
Low frequencies:

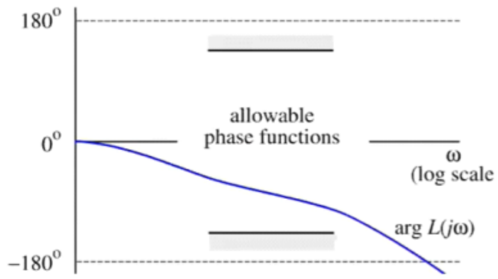
require $S \ll 1, T \approx 1 \;\Leftrightarrow\; |L(j\omega)| \gg 1$

High frequencies:

require $T \ll 1, \; S \approx 1 \;\Leftrightarrow\; |L(j\omega)| \ll 1$

Loop shaping

17. 



Low frequencies: large loop gain

High frequencies: small loop gain

- In the crossover region the phase is constrained because of stability

18.

Crossover Region

- The more closely the Nyquist plot of L approaches -1 the more S peaks
- If the plot of L approaches -1 so does the plot of 1/L. Hence the more closely the plot of L approaches -1 the more T peaks
- Thus to avoid peaking we need good stability margins
- But gain and phase are not independent

$$S = \frac{1}{1+L},$$
$$T = \frac{L}{1+L} = \frac{1}{1+\frac{1}{L}}$$

Modern Control - State Space Control

22 November 2021 13:41

State space, state feedback control and controllabilityExternal tool

11 December 2021 17:37

Derive the transfer function from a state space model

1. $\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \quad x(0)=x_0$
   $y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$

$$X(s) = (s\mathbf{I}-\mathbf{A})^{-1}x(0) + (s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}U(s)$$

$$Y(s) = [\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}]U(s) + \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}x(0)$$

2. $G(s)$ is the system transfer function.

$$Y(s) = G(s)U(s)$$

$$G(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}$$

滑动1

3. From state space to transfer function

$\dot{x}(t) = Ax(t) + Bu(t)$ with $x(0)=x_0$

$y(t) = Cx(t) + Du(t)$

take laplace transform

$x(0)=0$

$\rightarrow sX(s) - \cancel{x(0)} = AX(s) + Bu(s)$

$\rightarrow Y(s) = CX(s) + DU(s)$

$\rightarrow sX(s) - AX(s) = (sI-A)X(s) = Bu(s)$

identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\cancel{(sI-A)^{-1}(sI-A)}X(s) = (sI-A)^{-1}Bu(s)$

$\rightarrow Y(s) = C\cdot(sI-A)^{-1}Bu(s) + DU(s)$

$$\hookrightarrow Y(s) = C\cdot(sI-A)^{-1}BU(s) + DU(s)$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D}$$

Transfer functions describe the input-output properties of linear systems in algebraic form.

4. The dynamic behaviour and stability of the system are determined by the location of the **poles of the transfer function**.
   For the state space representation, these correspond to the **eigenvalues of the matrix A**$_+$

Controllability

1. Canonical forms
   a.

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & & \cdots & \cdots & 0 \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ \vdots & & \ddots & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & -a_2 & -a_1 \end{bmatrix} \quad \mathbf{B}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$$

$$\mathbf{C}_c = \begin{bmatrix} b_n & b_{n-1} & \cdots & \cdots & b_1 \end{bmatrix} \quad \mathbf{D}_c = 0$$

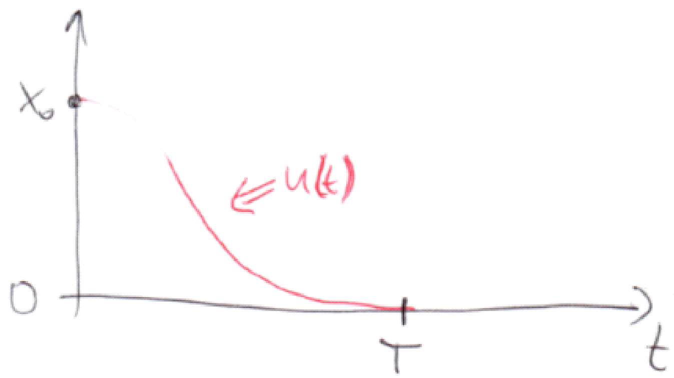
The equivalent transfer function form is

$$G(s) = \frac{b_1 s^{n-1} + \ldots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n} = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$$

   b. Any completely controllable system can be expressed in this way
2. Controllability

a. **Definition:** A state x is said to be **controllable** if, given any $x_0 \neq 0$ and any finite time interval $[0, T]$, there exists an input $\{u(t), t \in [0, T]\}$ such that $x(T) = 0$. If all states are controllable, then the system is said to be **completely controllable**.

b. Controllability



c. Controllability matrix

   i. The **controllability matrix** is defined as

$$\Gamma_c[\mathbf{A},\mathbf{B}] = [\mathbf{B}\ \mathbf{AB}\ \mathbf{A}^2\mathbf{B}\ \ldots\ \mathbf{A}^{n-1}\mathbf{B}]$$

   The model is **completely controllable** if and only if $\mathbf{\Gamma}_c[\mathbf{A}, \mathbf{B}]$ has full row rank (ie is non-singular).

d. If a system is not completely controllable, it can be decomposed into a controllable and a completely uncontrollable subsystem, as explained below.

   i. If the uncontrollable subsystem is stable, then the whole system is stable. But if the uncontrollable subsystem is unstable, the whole system can never be stable.

e. Controllable decomposition

i.

滑动 1

Similarity transformations

T nonsingular matrix (i.e. T^{-1} exists)

for example 0 is singular because $1/0 = \infty$

$$\bar{x}(t) = T^{-1}x(t) \quad \text{or} \quad T\bar{x}(t) = x(t)$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$T^{-1} \rightarrow T^{-1}\dot{x} = T^{-1}Ax + T^{-1}Bu$$

with $x = T\bar{x}$ and $\dot{x} = T\dot{\bar{x}}$

$$T^{-1}T\dot{\bar{x}} = \underbrace{T^{-1}AT}\bar{x} + \underbrace{T^{-1}B}u$$

$$\boxed{\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u} \; *$$

where $\bar{A} = T^{-1}AT$, $\bar{B} = T^{-1}B$

$$y = CT\bar{x} + Du$$

$$\boxed{y = \bar{C}\bar{x} + Du} \; *$$

where $\bar{C} = CT$

3. Reachability

a. **Definition:** A state $\bar{x} \neq 0$ is said to be **reachable** (*from the origin*) if, given $x(0) = 0$, there exist a finite time interval $[0, T]$ and an input $\{u(t), \; t \in [0, T]\}$ such that $x(T) = \bar{x}$. If all states are reachable, the system is said to be **completely reachable**.

Observability

12 December 2021 16:04

Observability

Observability

Observability is concerned with the issue of what can be said about the state when one is given measurements of the plant output.

A formal definition is as follows:

Definition: The state $x_0 \neq 0$ is said to be unobservable if, given $x(0) = x_0$, and $u(T) = 0$ for $T \geq 0$, then $y(T) = 0$ for $T \geq 0$. The system is said to be completely observable if there exists no nonzero initial state that it is unobservable.

1. Given output y, whether we can reconstruct the initial state x. if yes, the system is observable

Test of observability

Test for Observability (cont.)

The set of all unobservable states is equal to the null space of the observability matrix $\Gamma_0[\mathbf{A}, \mathbf{C}]$, where

$$\Gamma_o[\mathbf{A}, \mathbf{C}] = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{\mathbf{n}-1} \end{bmatrix}$$

The system is completely observable if and only if $\Gamma_0[\mathbf{A}, \mathbf{C}]$, has full column rank n (ie. it is non-singular).

Duality

Duality

We see a remarkable similarity between the Theorems for controllability and for observability. We can formalize this as follows:

Theorem (*Duality*). Consider a state space model described by the 4-tuple $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$. Then the system is completely controllable if and only if the dual system $(\mathbf{A}^T, \mathbf{C}^T, \mathbf{B}^T, \mathbf{D}^T)$ is completely observable.

Note: $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$.

1. (ABCD) is controllable --- (AtBtCtDt) is observable.

Observer Canonical form

Observer Canonical Form

Consider a **completely observable** SISO system given by

$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t); \quad x(0) = x_0$$

$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t)$$

Then there exists a similarity transformation that converts the model to the **observer-canonical form**

$$\mathbf{A}_o = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & 0 & \cdots & \vdots \\ \vdots & & \ddots & & & 1 \\ -a_n & 0 & & 0 & & 0 \end{bmatrix}; \quad \mathbf{B}_o = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix};$$

$$\mathbf{C}_o = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Pole placement

12 December 2021 16:27

Control law

Control law:

$$u(t) = -K x(t)$$

with $K = [k_1 \quad k_2]$

$$u(t) = -K x(t) = -[k_1 \quad k_2]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\dot{x} = Ax + Bu = Ax - BKx$$

$$= (A - BK)x$$

$$\dot{x} = \left(\begin{bmatrix} -37.5 & -36.8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}[k_1, k_2]\right)x$$

$$\dot{x} = \left(\begin{bmatrix} -37.5 & -36.8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \\ 0 & 0 \end{bmatrix}\right)x$$

$$\dot{x} = \begin{bmatrix} -37.5-k_1 & -36.8-k_2 \\ 1 & 0 \end{bmatrix}x$$

1. By changing the value of k1 and k2, we can change the value of the first row in A
2. The martix A contains the element of the denominator polynomial of the TF, they define the location of the poles
3. We can change k so that the location of poles are changed, thus the dynamic performance of the system is changed
4. Generally

$$\dot{x} = (\mathbf{A} - \mathbf{BK})x \quad y = \mathbf{C}x$$

with

a. $$(\mathbf{A} - \mathbf{BK}) = \begin{bmatrix} -(a_1 + k_1) & -(a_2 + k_2) & \cdots & \cdots & -(a_n + k_n) \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ \vdots & & \ddots & 1 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} b_1 & b_2 & \cdots & \cdots & b_n \end{bmatrix}$$

Ackermann's formula

Example: Pole assignment (cont.)

- For our example, the closed loop polynomial is

$$A_{cl}(s) = s^2 + 30s + 200$$

- It then follows that:

$$\alpha_c(\mathbf{A}) = \mathbf{A}^2 + 30\mathbf{A} + 200\mathbf{I} = \begin{bmatrix} 27 & -276 \\ 23 & 188 \end{bmatrix}$$

$$\mathbf{\Gamma}_c = [\mathbf{B} \quad \mathbf{AB}] = \begin{bmatrix} 1 & -7 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{\Gamma}_c^{-1} = \begin{bmatrix} 1 & 7 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{K} = [0 \quad 1]\mathbf{\Gamma}_c^{-1}\alpha_c(\mathbf{A}) = [0 \quad 1]\begin{bmatrix} 1 & 7 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 27 & -276 \\ 23 & 188 \end{bmatrix} = [23 \quad 188]$$

Section of poles

1. Why placing poles? To meet the control target(rise time, overshoot, settling time, etc.)
2. 离远点越远的pole, nature freq越大, 影响越小
3. Dominate poles: 离原点最近的pole

4. Rise time: tr = 1.8/wn

Introducing the reference input

1.

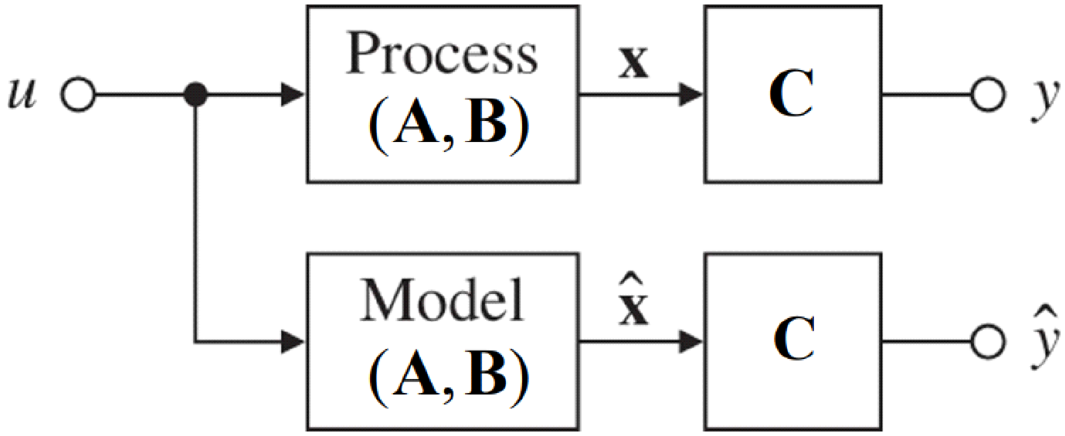
Observer design

12 December 2021 17:21

Motivation

1. To imply state feedback control, we have to know all states.
2. In most cases, this is unrealistic requirement.
3. We design an observer to estimating the state from the available measurements

Open loop observer



- Model and process outputs are only identical if **model and process** are identical, **initial conditions** are know, and **disturbances** are zero!

Estimation error

Estimation error

$$\tilde{x} = x - \hat{x}$$

$$\dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu}$$

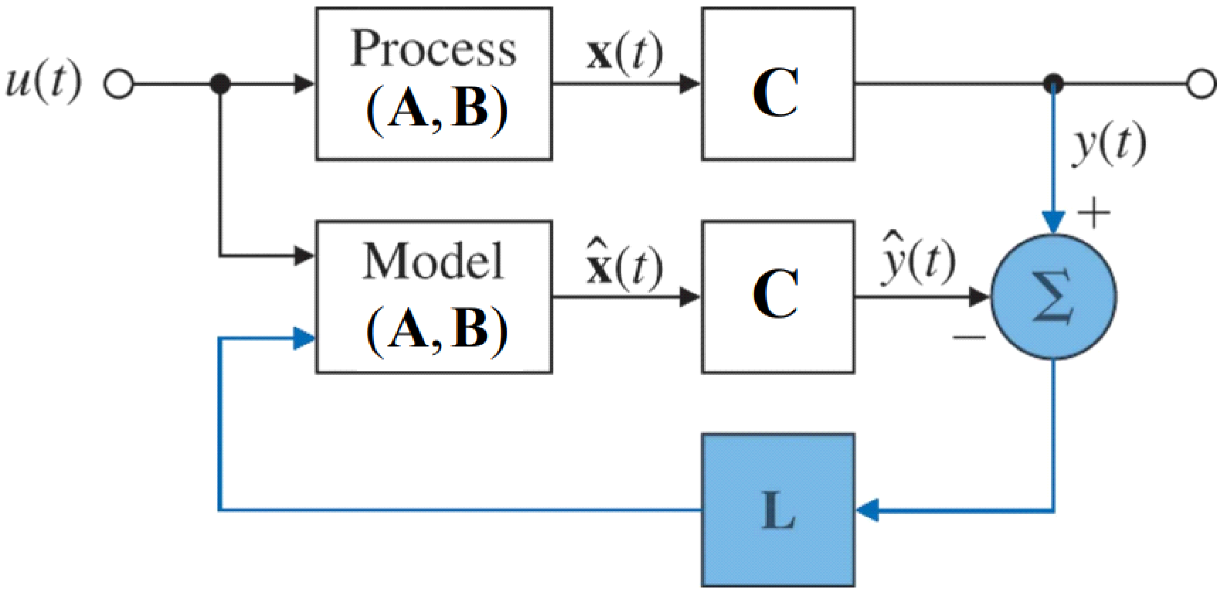
$$\dot{x} - \dot{\hat{x}} = A(x - \hat{x})$$

$$\underline{\underline{\dot{\tilde{x}} = A\tilde{x}}} \qquad \tilde{x}(0) = x(0) - \hat{x}(0) = x_0$$

- The error converges to zero for stable systems (**A** stable), but we have no ability to influence the rate at which the state estimate converges to the true state.
- Furthermore, the error is converging to zero at the same rate as the natural dynamics **A**, which may not be satisfactory.
- We can now invoke the Golden Rule:

When in trouble, use feedback

Closed loop observer



- We can feed back the difference between the measured and the estimated output

output error $$\boxed{v(t)} = y(t) - C\hat{x}(t)$$

using a feedback vector **L**

$$\mathbf{L} = [l_1 \quad l_2 \quad \cdots \quad l_n]^T$$

- The equation for this scheme is

$$\dot{\hat{x}}(t) = \mathbf{A}\hat{x}(t) + \mathbf{B}u(t) + \mathbf{L}\big(y(t) - \mathbf{C}\hat{x}(t)\big)$$

Lemma: The estimation error $\tilde{x}(t)$ satisfies

$$\dot{\tilde{x}}(t) = \left(\mathbf{A} - \mathbf{LC}\right)\tilde{x}(t)$$

Moreover, provided the model is **completely observable**, then the eigenvalues of (**A** - **LC**) can be arbitrarily assigned by choice of **L**.

The characteristic equation of the error is now given by

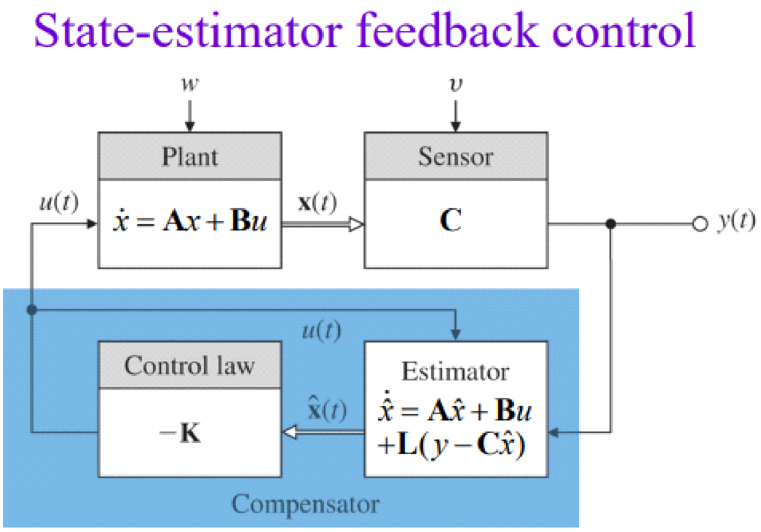
$$\det\left[s\mathbf{I} - \left(\mathbf{A} - \mathbf{LC}\right)\right] = 0$$

1. We can modify the error by choosing L

State estimator feedback

12 December 2021 13:15

State-estimator feedback control



1. It can present the unmeasurable state by estimator
2. It separate the task of observer design from the controller design
3. It is not clear how the observer poles and the state feedback interact
4. The resultant closed-loop poles are the combination of observer and the state-feedback poles.

Separation Theorem

1. **Theorem:** (*Separation theorem*). Consider the state space model and assume that it is completely controllable and completely observable. Consider also an associated observer and state-variable feedback, where the state estimates are used in lieu of the true states:

$$u(t) = \bar{r}(t) - \mathbf{K}\hat{x}(t)$$

$$\mathbf{K} \triangleq \begin{bmatrix} k_0 & k_1 & \dots & k_{n-1} \end{bmatrix}$$

Then

(i) the closed-loop poles are the combination of the poles from the observer and the poles that would have resulted from using the same feedback on the true states - specifically, the closed-loop polynomial $A_{cl}(s)$ is given
2. by

$$A_{cl}(s) = \det\left(sI - \left(\boxed{\mathbf{A} - \mathbf{BK}}\right)\right)\det\left(s\mathbf{I} - \left(\boxed{\mathbf{A} - \mathbf{LC}}\right)\right)$$

independent

(ii) The state-estimation error $\tilde{x}(t)$ cannot be controlled from the external signal $\bar{r}(t)$.

System types

12 December 2021 17:55

SISO

Digital Control

03 October 2021 16:09

Vocabulary

绝对值|a|	The magnitude of a
等比数列	Geometric sequence /series
带	Strip
径向	Radial
对称	Symmetry
渐近稳定性	Asymptotic stability
受到	Subject to
有界的	Bounded
零极点相消	pole/zero cancellation

https://zhuanlan.zhihu.com/p/266041991?utm_source=qq

Review of continuous systems

03 October 2021 22:23

6-stability

11 December 2021 22:04

Pole-zero matching

1. https://ctms.engin.umich.edu/CTMS/index.php?aux=Extras_PZ
2. 让plant的分母与controller的分子相互匹配, 从而消除整体系统的不稳定极点, 这种方法为p-z matching

Lab

09 November 2021 22:22

Q1.14

1. Pole-zero matching
   a. Discrete control #4: Discretize with the matched method

Q1.8

1. underdamped and overdamped
   a. https://courses.lumenlearning.com/physics/chapter/16-7-damped-harmonic-motion/
   b. https://zhuanlan.zhihu.com/p/134176474
   c. https://www.zhihu.com/question/41318067/answer/152614792?iam=f6142c76fb408f626961cd3a3bff0bd6

Q1.17

1. 临界稳定
   a. https://www.zhihu.com/question/300650404

出现的分析

离散奈奎斯特图	DC-习题-5	26:55

Exam

23 November 2021 18:16

General

1. Section B
   a. Frequency domain or state space aspect

2020

1. Q1
   a. Derive the closed-loop equation
   b. Vector margin, complementary vector margin, sensitivity function
   c. State space form to G(s)
   d. Controllability
2. Q2
   a. Sampling theorem (Nyquist)
   b. 解释公式为何不可实现, 改的可实现
   c. 如何在qua si-real-time改进, 对system control and signal broadcasting的影响
   d. asymptotic and BIBO stabilities
3. Q3
   a. Discussed in the end of design section
      i. design target

1) **Targets**
   - Closed-loop stability
   - Disturbance attenuation
   - Good command response
   - Robustness
     - stability
     - performance

2) Review of Design Requirements

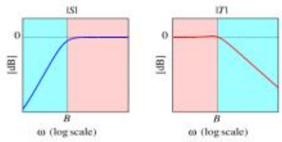
   Sensitivity S small at low frequency to achieve
   - disturbance attenuation
   - good command response
   - robustness at low frequencies

   Complementary sensitivity T small at high frequencies to prevent
   - exceeding plant capacity
   - adverse effects of measurement noise
   - loss of robustness at high frequencies

   In the Crossover Region peaking of both S and T should be avoided to prevent
   - overly large disturbance sensitivity
   - excessive influence of measurement noise
   - loss of robustness

3) Disturbance attenuation
   a) S(jw) the smaller, L the larger, attenuation the better. In the crossover region, peaking of S should be avoided, otherwise disturbances are amplified.
4) T determines the command response

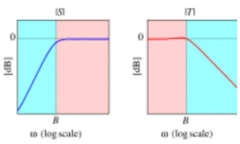
   Command Response (T,F)

   a) 

   - Recall that T = 1 - S
   - T determines the command response - it is close to 1 up to B
   - When F = 1, closed-loop transfer function *TF* is low pass with the same bandwidth as the band for disturbance attenuation
   - If a different command response is required, F can compensate for this
5) T determines measurement noise sensitivity, so for high freq, T should decrease as fast as possible
6) Stability robustness

   Stability Robustness

   a)
   - Robustness
     - For loop gain perturbations T needs to be small
     - For inverse loop gain perturbations S needs to be small
   - Performance
     - At high frequencies T needs to be small
     - At low frequencies S needs to be small
   - Perturbations
     - High frequency uncertainty (parasitics) causes significant loop gain perturbations
     - low frequency uncertainty (load changes etc.) causes significant inverse loop gain perturbations
   - Crossover
     - Neither S nor T can be small, they must therefore be prevented from peaking. Good stability margins help to ensure this

	S	T
low frequencies	small	≈ 1
high frequencies	≈ 1	small

7) Performance robustness

a)

Performance Robustness

- Performance is determined by S, T, S_u and TF
- Since S_u and TF depend on S and T, we need only consider the effect of perturbations on S and T
- For robust S we need T_o small
- For robust T we need S_o small
- Normally, S is small at low frequencies, making T robust at low frequencies - this is the region where T's values are significant
- Conversely, T is normally small at high frequencies, making S robust at high frequencies - the region where S is significant

Denote nominal quantities by S_o etc.

Relative changes in $1/S$ and $1/T$

$$\frac{\frac{1}{S}-\frac{1}{S_o}}{\frac{1}{S_o}}=\frac{S_o-S}{S}=T_o\frac{L-L_o}{L_o}=T_o\frac{\frac{1}{L_o}-\frac{1}{L}}{\frac{1}{L}}$$

$$\frac{\frac{1}{T}-\frac{1}{T_o}}{\frac{1}{T_o}}=\frac{T-T_o}{T}=S_o\frac{L-L_o}{L}=S_o\frac{\frac{1}{L}-\frac{1}{L_o}}{\frac{1}{L_o}}$$

b)

ii. S should be small so that disturbance can be attenuated
iii. T should be close to 1 so that the reference can be followed and noise can be attenuated
iv. Peaking of S and T should be avoided so that the system is robust (relate to the vector margin)
v. ? Drop off low freq and high freq, the L is close to one in the crossover region

b. PID

i. PID的公式, 时域和频域

1)

$$u(t)=K\left(e(t)+\frac{1}{T_I}\int_0^t e(\tau)d\tau+T_D\dot{e}(t)\right) \quad (7)$$

$$C(s)=K\left(1+\frac{1}{T_I s}+T_D s\right) \quad (8)$$

ii. PID的TF, 零极点

1) Combining proportional, integral and derivative terms leads to the following control law

$$C(s)=K\left(1+\frac{1}{T_I s}+T_D s\right)=\frac{K}{s}\left(T_D s^2+s+\frac{1}{T_I}\right) \quad (9)$$

with $T_D \ll T_I$, i.e. $\frac{T_D}{T_I}\ll 1$

$$C(s)\approx\frac{K}{s}\left(T_D s^2+\left(1+\frac{T_D}{T_I}\right)s+\frac{1}{T_I}\right)=\frac{K}{s}(T_D s+1)\left(s+\frac{1}{T_I}\right) \quad (10)$$

$C(s)$ in equation (10) has two zeros (at $\frac{1}{T_D}$ and $\frac{1}{T_I}$) and one pole (at zero). Since the number of zeros is larger than the number of poles, this transfer function is not realisable[1]. To obtain a realisable version $C_r(s)$ of this controller, an additional pole needs to be implemented, as shown in equation (6), leading to

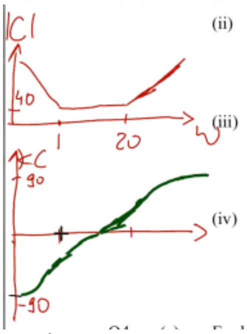
$$C_r(s)\approx K\frac{(T_D s+1)\left(s+\frac{1}{T_I}\right)}{s(\tau_D s+1)} \quad (11)$$

Figure 4 shows example Bode plots of $C(s)$ and $C_r(s)$.

For frequency domain design, the parameters K, T_I and T_D are to be chosen in such a way that the frequency response of the controller combined with the plant results in a satisfactory shape of the loop gain.

iii. 画伯德图, 标出转折频率, 幅值渐近线, 相位

1)



2) Corner freq is important

iv. 对上述系统校正, 画出校正后的伯德图

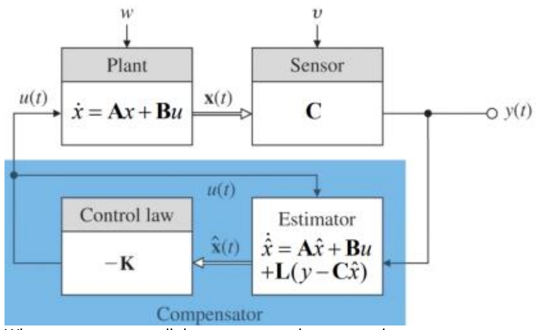
1) It is unrealizable as the magnitude goes to infinity
2) It is unrealizable as the num of zeros is larger than poles
3) Add a pole so that the final magnitude is flat
4) 0.1Td<Td_new<0.2Td, lecture note, Td_new = 0.005

4. Q4

a. 解释state estimator feedback control

State-estimator feedback control

i.



ii. Why: cannot measure all the states, can only measure the output which is a subset of all states. but for states feedback , we have to know all the states, thus we have state estimator to estimate all the states based on output

b. 用state estimator的原因

i. $\dot{\tilde{x}}=(A-LC)\tilde{x}$

ii. Donot use the open loop estimator use close loop
iii. Lecture, do not copy

c. 求observer gain vector L

i. Close loop polynomial: (s+100)(s+110)

$A_{obs} = (s+100)(s+110) = s^2+210s+11000$

ii. $\begin{bmatrix}-210 & 1\\ -11000 & 0\end{bmatrix} = \begin{bmatrix}-11 & 1\\ -15 & 0\end{bmatrix} - \begin{bmatrix}L_1 & 0\\ L_2 & 0\end{bmatrix}$

Page 4 of 5 Continued overleaf

$210 = 11 + L_1$ $\quad L_1 = 199$ $\quad L = \begin{bmatrix}199\\ 10985\end{bmatrix}$

$11000 = 15 + L_2$ $\quad L_2 = 10985$

d. 证明系统观测性
   i. lecture notes

   n a state space system. Show whether the

   ii. $\Gamma_o = \begin{bmatrix}C\\ CA\end{bmatrix} = \begin{bmatrix}-2 & 1\\ 6 & -1\end{bmatrix}$

   iii. Rank of the matrix is 2, thus it is fully observable

5. Q5
   a. 设计PD校正, 消去不稳定极点
   b. 已知τ求K
   c. 求damping和Wn
   d. 求稳态误差
6. Q6-low pass filter
   a. 求滤波器带宽及sampling time
   b. 采样定理
   c. G(s)求G(z), 用零极点匹配
   d. 已知Wc求K
   e. Tustin rule

2019

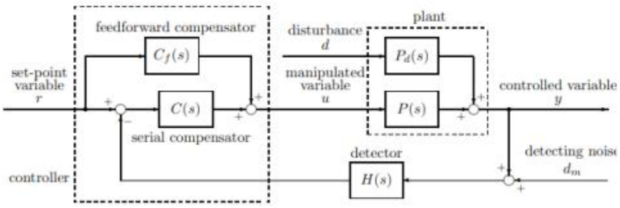
1. Q1
   a. 画2自由度框图
      i. https://www.mathworks.com/help/control/ug/two-degree-of-freedom-2-dof-pid-controllers.html
      ii. 

Fig. 1. Two-degree-of-freedom (2DOF) control system.

   b. ideal compensator should cancel the plant dynamics
      i. Feedback control 1.4

      ii. Consider the open-loop control structure shown in figure 5. Here, the plant dynamics are

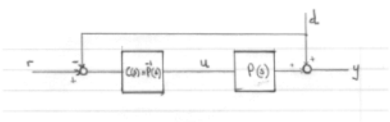


Figure 5: Open-loop control via explicit plant inversion.

given by $P(s)$ and the dynamic controller is given by the plant inverse $P^{-1}(s)$. The input to the controller includes the reference signal r and a measurement of the disturbance d, i.e. the signal $r - d$ is the controller input. With this structure it is clear that exact reference tracking, $y = r$, is achieved. However, the limitations of this approach are:

- the plant dynamics $P(s)$ must be known exactly;
- the plant dynamics must be stable;

8

- the disturbance d must be exactly measurable;
- the plant inverse dynamics $P^{-1}(s)$ must be stable and realisable (proper).

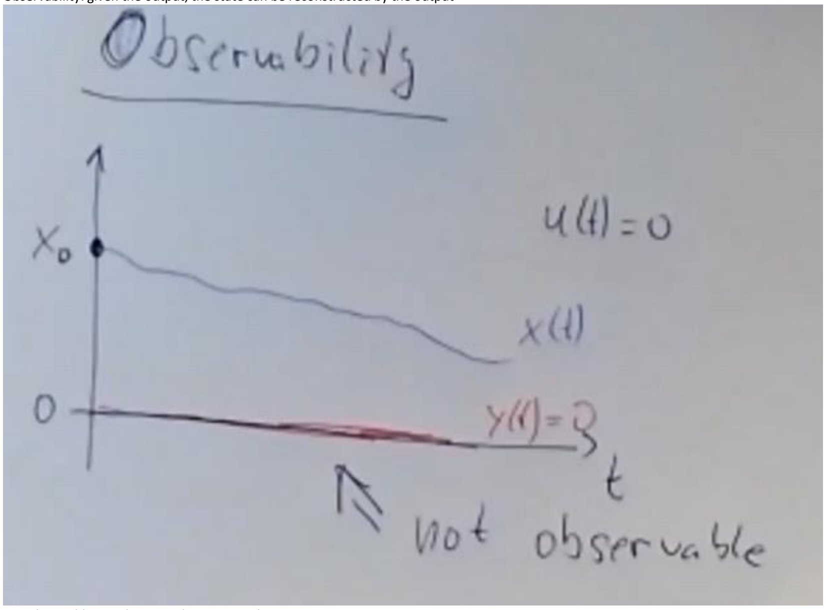
   c. Observability, state space描述

Observability is concerned with the issue of what can be said about the state when one is given measurements of the plant output.

A formal definition is as follows:

i. **Definition:** The state $x_0 \neq 0$ is said to be unobservable if, given $x(0) = x_0$, and $u(T) = 0$ for $T \geq 0$, then $y(T) = 0$ for $T \geq 0$. The system is said to be completely observable if there exists no nonzero initial state that it is unobservable.

ii. Observability: given the output, the state can be reconstructed by the output

iii. 

iv. Not observable as y does not change according x

v. Consider the following state space model:

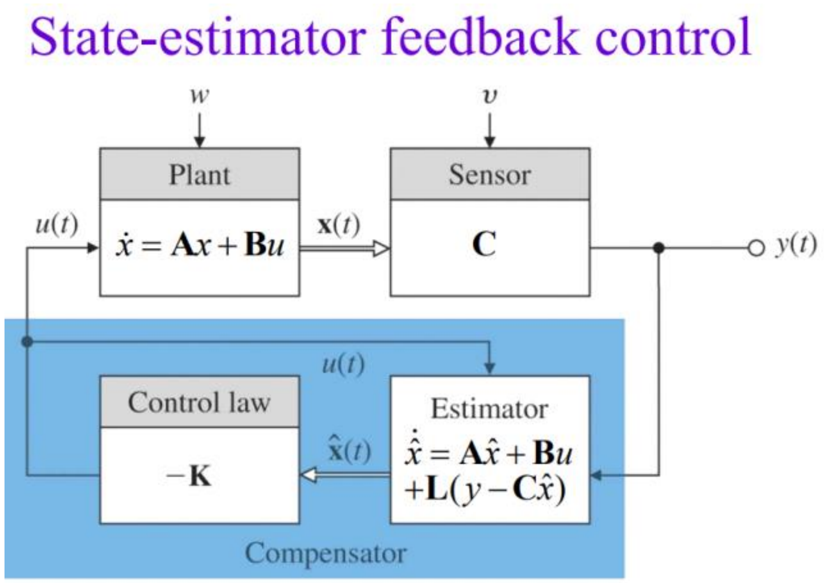
$$\mathbf{A} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Then

$$\mathbf{\Gamma}_o[\mathbf{A}, \mathbf{C}] = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -4 & -2 \end{bmatrix}$$

Hence, rank $\Gamma_0[\mathbf{A}, \mathbf{C}] = 2$, and the system is completely observable.

d. 解释state estimator feedback control

i. 

e. Separation Theorem

Separation Theorem

Theorem: (*Separation theorem*). Consider the state space model and assume that it is completely controllable and completely observable. Consider also an associated observer and state-variable feedback, where the state estimates are used in lieu of the true states:

$$u(t) = r(t) - \mathbf{K}\hat{x}(t)$$
$$\mathbf{K} \triangleq [k_0 \quad k_1 \quad \dots \quad k_{n-1}]$$

i.

Then

(*i*) the closed-loop poles are the combination of the poles from the observer and the poles that would have resulted from using the same feedback on the true states - specifically, the closed-loop polynomial $A_{cl}(s)$ is given by

$$A_{cl}(s) = \det\left(sI - (\mathbf{A} - \mathbf{BK})\right)\det\left(s\mathbf{I} - (\mathbf{A} - \mathbf{LC})\right)$$

(*ii*) The state-estimation error $\tilde{x}(t)$ cannot be controlled from the external signal $\bar{r}(t)$.

2. Q2
   a. 根据极点写出G(z)并画图, 解释time sequences
   b. H(z)与时域转化
   c. forward rectangular numerical integration rule, dc-ex37
   d. aliasing解释 dc-8.3
3. Q3
   a. sensitivity function求解, 讨论peaking
      i. **(a)**
         i)
         $y = \frac{CP}{1+CP}r + \frac{CP}{1+CP}r - \frac{CP}{1+CP}n$ [1]
         $y = Tr + Sd - Tn$ [1]
         Peaking T = increased noise and reference response [1]
         Peaking S = increased disturbance sensitivity [1]
      ii. **(ii)**
         $s_m = \min_\omega|1 + L_o(j\omega)| = \min_\omega \frac{1}{|S(j\omega)|} = \frac{1}{\max_\omega|S(j\omega)|}$ [2]
         $r_m = \min_\omega|1 + 1/L_o(j\omega)| = \min_\omega \frac{1}{|T(j\omega)|} = \frac{1}{\max_\omega|T(j\omega)|}$ [2]
         Strongly oscillatory response implies small (complementary) vector margins. [1]

(iii)

$\frac{\partial T}{T_o} = S_{P_o}^{T_o} \frac{\partial P}{P_o}$ [1]

Con

Page 8 of 17

iii.

$$S_{P_o}^{T_o} = \frac{\partial T}{\partial P} = \frac{P_o}{T_o}$$

with T_o from (i) and differentiating, this results in

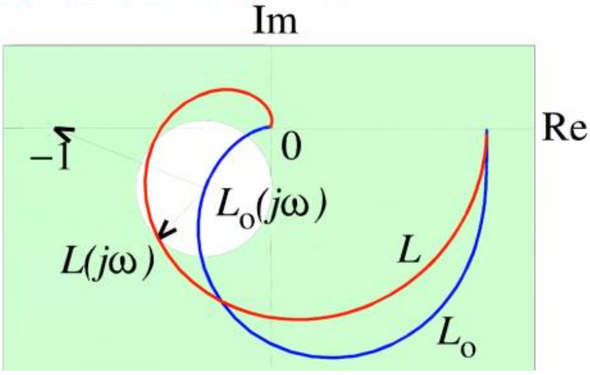
$S_{P_o}^{T_o} = \frac{1}{1+CP_o}$ [2]

Replace S with T gives the equivalent result for the effect of S on T. [2]

(b)

Doyle's stability robustness criterion: [1]

iv.



$$|L - L_o| < |1 + L_o|$$

$$\left|\frac{L - L_o}{L_o}\right| < \left|\frac{1 + L_o}{L_o}\right| = \frac{1}{|T_o|}$$

[2]

v. Equivalently for 1/L:

+

$$|1/L - 1/L_o| < |1 + 1/L_o|$$

$$\left|\frac{\frac{1}{L} - \frac{1}{L_o}}{\frac{1}{L_o}}\right| < \left|\frac{1 + \frac{1}{L_o}}{\frac{1}{L_o}}\right| = \left|\frac{1 + L_o}{1}\right| = \frac{1}{|S_o|}$$

[2]

b. complementary vector margin 与peak的关系
c. T0推导过程, S0和T0的作用
d. closed-loop stability的标准
   i. Perturbed loop gain and nominal loop gain
   ii. Doyle's criterion

4. Q4
   a.
      i. State space form to G(s)
      ii. 稳定系统ABCD的条件
         1) Eigen value of A is the poles of the TF, they must all have real negative parts to be stable
         2) B,C,D are irrelevant
         3) G(s): poles(roots of denominator) all should be on the left half of the plane,
   b. controller design by pole assignment
      i. If a system is completely controllable, it can be transformed into the canonical form

      ii. $\dot{x} = \mathbf{A}x + \mathbf{B}u \quad y = \mathbf{C}x \quad u = -\mathbf{K}x$

      with

$$\mathbf{A} = \begin{bmatrix} -a_1 & -a_2 & \cdots & \cdots & -a_n \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ \vdots & & \ddots & 1 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} b_1 & b_2 & \cdots & \cdots & b_n \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & \cdots & \cdots & k_n \end{bmatrix}$$

      iii. By changing the value of k
   c. 解释controller canonical form and of controllability
   d. pole placement method 求参数

5. Q5

    a. 给出H(s)求gain, phase, Nyquist frequency求解
    b. G(s)求G(z), forward rectangular rule dc-ex37
    c. G(s)求G(z), zoh
    d. 对比前项矩形和zoh的参数, 看哪个最好
6. Q6
    a. G(s)求G(z)
    b. 求闭环
    c. 求DE dc-ex4
    d. 求稳态误差

2018

1. Q1
    a. 对比开环和闭环, 三个指标(plant disturbances, changes in the plant gain, and stabilisation.)
    b. sensitivity function So and the complementary sensitivity function T�
    c. State space form to G(s)
    d. 微分方程求state space form
2. Q2
    a. 对比digital 与 analogue 的差别
    b. 解释causality
    c. Backword rectangular numerical integration rule dc-ex38
    d. Stability region map dc-7.4.2
3. Q3
    a. sensitivity function, complementary sensitivity function
    b. loop-shaping
    c. PID公式及responses
    d. 同2020-Q3-b-ii
    e. 画伯德图
4. Q4-同2020-Q4
    a. state estimator解释及优势
    b. 同2020-Q4
    c. 同2020-Q4
    d. 同2020-Q4
5. Q5
    a. G(s)求G(z)
    b. G(z)求微分方程
    c. BIBO stability 条件
6. Q6-low pass filter
    a. 给条件求滤波器, Tustin rule, 对比digital 与analogue, 同2020-Q6-d
    b. Pre-warping preserve the gain, 同2020-Q6-d
    c. forward rectangular rule

2017

1. Q1
    a. 奈奎斯特曲线画图, 标注(gain ,phase, vector)margin并解释
    b. 给出图的特征求delay
    c. 求gain临界值
    d. G(s)求state space
    e. state-estimator feedback control, Separation Theorem
2. Q2
    a. 同2018 Q2
3. Q3
    a. 同2018-Q1-a
    b. 给出tf求sensitivity function, complementary sensitivity function
    c. 对于上一题的frequency responses
    d. T0和S0对P0的关系
4. Q4
    a. state feedback control, 求微分方程
    b. pole assignment
    c. 解释controllability
    d. 给出TF求controllable
    e. 同2018-Q4-c
5. Q5
    a. 同2019 Q6
6. Q6
    a. 同2018 Q6

一阶高通滤波器的输入到输出行为可以通过以下标准化传递函数来描述：

$$T(s)_{HP} = \frac{a_1 s}{s + \omega_O}$$

让我们将它与相应的低通表达式进行比较：

$$T(s)_{LP} = \frac{a_O}{s + \omega_O}$$

如您所见，两种情况下分母是相同的。在这两种情况下，我们在s =-ωO处都有一个极点，这意味着低通滤波器和高通滤波器都具有以下特性：

1、ωO处的幅度响应将比最大幅度响应低3 dB;使用无源滤波器时，当最大幅度响应为单位值时，在这种情况下，ωO处的幅度值为-3 dB。

2、电路在ωO的相移绝对值为45°。

因此，这两个电路中ωO的响应非常相似。然而，在ωO以上和以下频率的响应受到T(s)的分子的影响，并且两个分子之间的差异使得低通滤波器与高通滤波器的频率响应非常不同。

2016

1. Q2
   a. B
      i. portional, Integral and Derivative components. [2]

$$u(t) = K\left(1+\frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\dot{e}(t)\right)$$

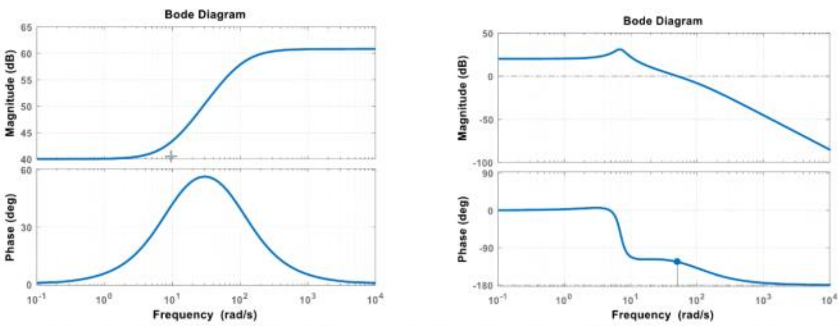
$$C(s) = K\left(1+\frac{1}{T_I s}+T_D s\right)$$

[2]

   b. C: 不考

2. Q5
   a. C
      i. Open loop plant
         1) -20db: k = 0.1
         2) under damped: diagram peaks
         3) Take the freq of the peak
      ii. Gain goes up by 40db as only a P controller is applied
         1) T(0) = 10/(1+10), get error
         2) Bandwidth: freq that cross the 0db line
         3) Robustness: phase is unaffected, gain margin is infinite, phase margin is very small(180-phase(wc))
         4) Damping: cannot derive this from bode plot, but phase margin is small, indicating vector margin is small as well, indicating an underdamped close-loop margin
      iii. PD controller
         1)

Controller C(s) Loop gain L(s)



Loop gain at zero freq unchanged, i.e. steady state characterstics unchanged

         2) Bandwidth is increased as wc move to the right
         3) Stead characteristic remain the same as the gain does not change
         4) ocializations are decreased
         5) Still underdamped

总计

采样定理解释及应用, 求Nyquist frequency	3
state estimator feedback control解释	5
sensitivity function, complementary sensitivity function	5
State space form, G(s) 相互转化	4
controllability解释	3
G(s)求G(z), forward, backward, 另一种	5
G(z) 与微分方程的转化	2
BIBO stability	2
对比digital 与analogue	2
Pre-warping preserve the gain	2

Template

13 December 2021 13:36

CA

1. Open and closed loop
2. S and T
3. Controllability and test
4. Observability and test
5. State estimator feedback control
6. Derive linear tf

DC

1. Signal broadcasting
2. p-z matching
3. 性能指标整理
4. Steady state error 整理
5. Poles diagram
6. Convolution
7. S = a/s+a
8. Aliasing