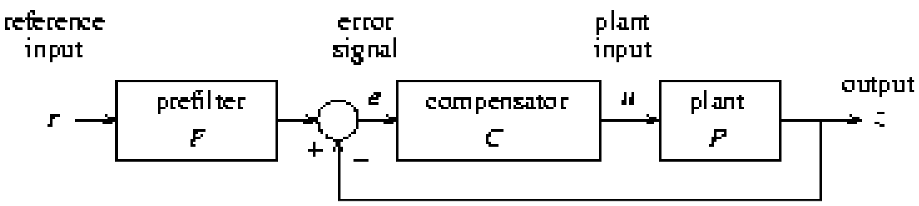
Stability



The control system is BIBO stable if every bounded input signal r results in bounded output signals e, u and z

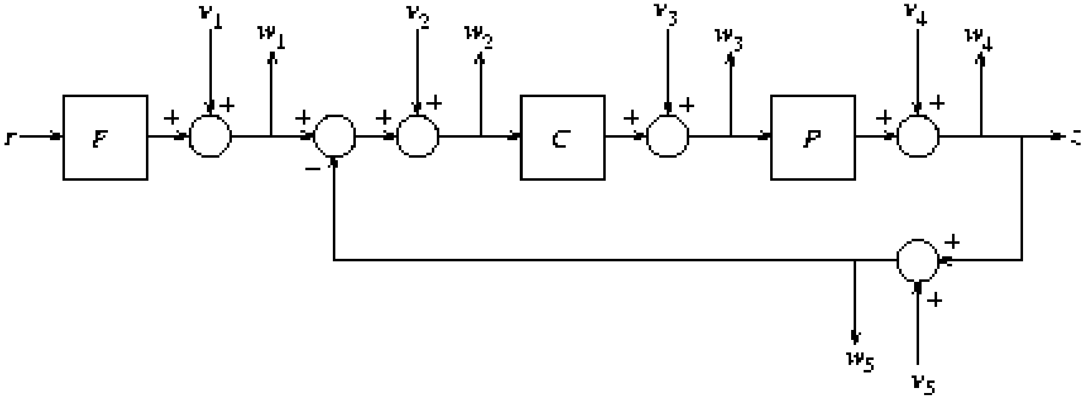
BIBO = “bounded input bounded output”

Asymptotic stability $\Rightarrow$ BIBO stability

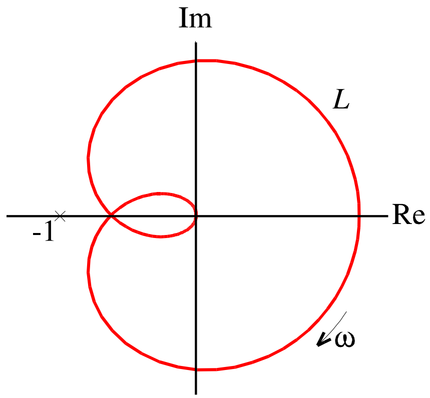
Stability – Internal stability

Inject "internal" signals into each "exposed interconnection" of the system, and define additional "internal" output signals after each injection point

Then the system is internally stable if it is BIBO stable with respect to all inputs (external and internal) and all (external and internal) outputs

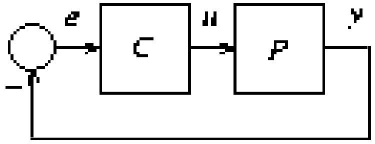


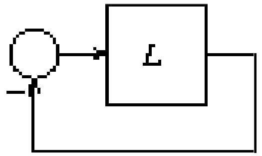
Nyquist Plot



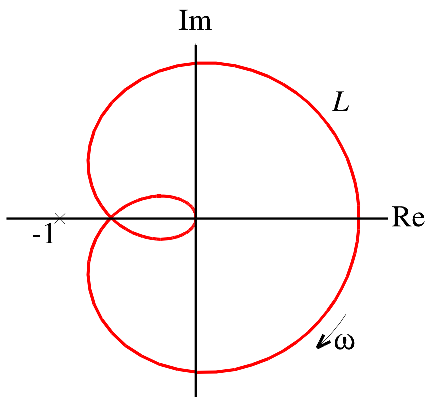
- Nyquist plot is of the loop gain is the curve traced in the complex plane by L
- Nyquist plot is symmetric with respect to the real axis

$$L(j\omega), \omega \in \Re$$



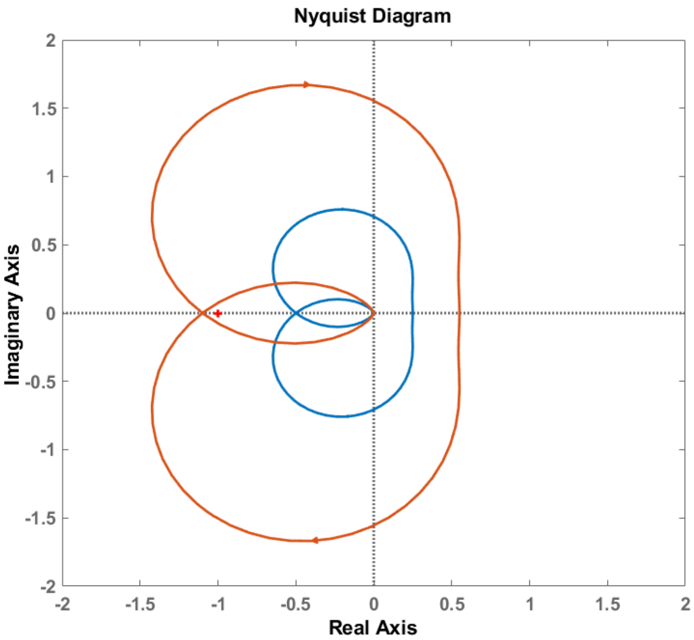


Nyquist Stability
Criterion for Open-loop Stable Systems

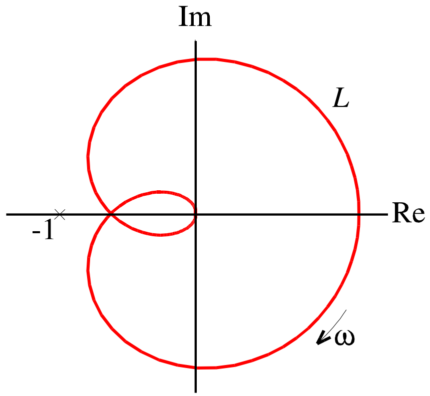


$$L(j\omega), \omega \in \Re$$

- Assume the SISO system L is open-loop stable
- Then the closed-loop system is stable if and only if the Nyquist plot of L does not encircle the point -1



Generalised Nyquist Stability Criterion



$$L(j\omega), \omega \in \Re$$

- SISO system L can have r.h.p. poles (unstable)
- The following holds:

  the number of **unstable closed-loop poles**

  =

  the number of times the Nyquist plot **encircles the point -1**

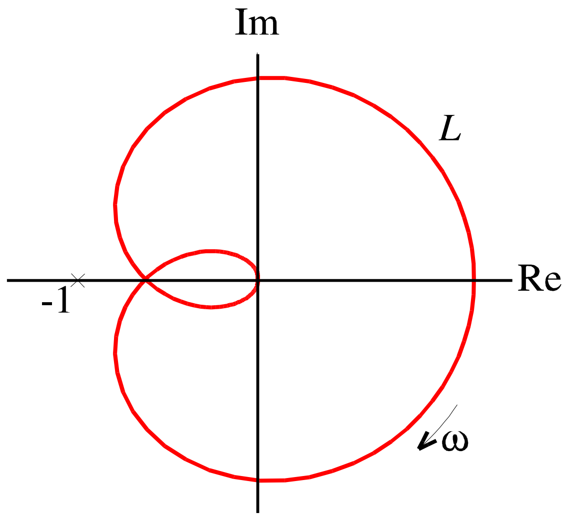
  -

  the number of **unstable open-loop poles**

- It follows that the closed-loop system is stable if and only if the number of encirclements of the point -1 equals the number of unstable open-loop poles.

Stability Margins

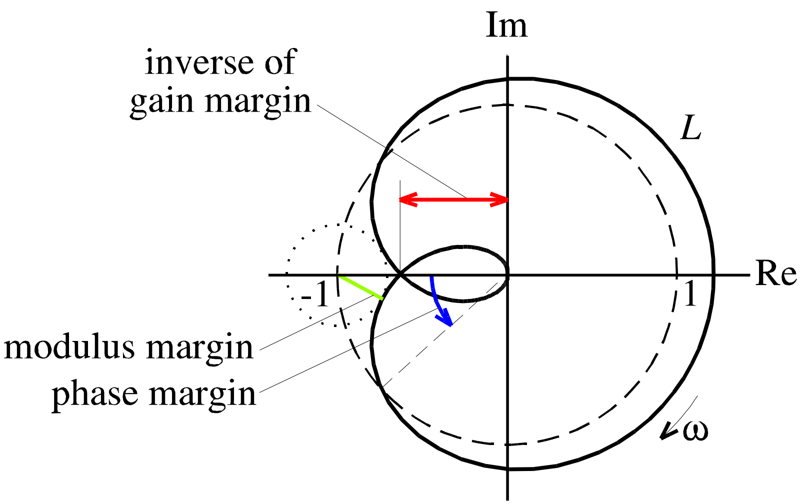
In the SISO case the **point -1** is a critical point for the Nyquist plot of the system. If the Nyquist plot is changed ("perturbed") so that it crosses the point -1 then the system becomes unstable



If the closed-loop system is stable but the Nyquist plot of L passes close to -1 then

- the system is **near-unstable**, i.e. has an oscillatory response
- the system **may become unstable** by small perturbations of the plant, i.e. the system is **not robust**

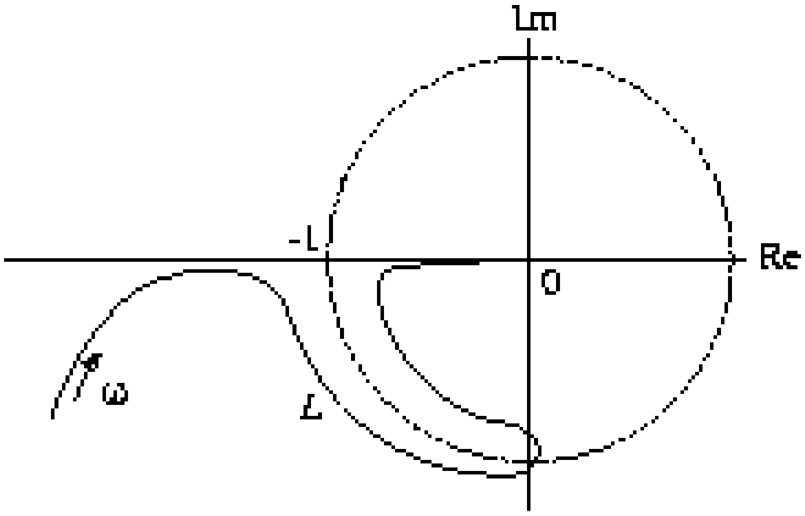
Stability Margin



There are various stability margins. They measure how close the Nyquist plot gets to the point -1

- gain margin k_m
- phase margin ϕ_m
- **vector (modulus) margin** s_m

Stability Margins



Gain and phase margin do not always adequately characterise robustness

- in the example small joint perturbations of gain and phase can destabilise the system
- modulus margin is better in this case

Note that good margins are important also for performance

- with poor margins the -1 point is approached closely
- this gives closed-loop poles close to the imaginary axis, leading to an oscillatory response