UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4 (ENG4042)

Monday 16th December 2019
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

Please see below for instructions on how to answer:

- Script books will NOT be used in this exam.
- You will be issued with four A3 double sided answer sheets.
- Answer only one question on each A3 answer sheet.
- Ensure you CLEARLY add your GUID and date of birth on each answer sheet. GUID goes on front and back.
- Continuation answer sheets (pink) are available on request. Please ensure you CLEARLY add your GUID, date of birth and question number to all continuation sheets. GUID goes on front and back.

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SECTION A

Q1 (a) Draw the structure of an open-loop control system and of a two-degree-of-freedom feedback control system. Show the different signals, including disturbances, and explain their physical meaning. Explain which additional components are required in the feedback control structure. [5]

(b) Show that for an open-loop structure the ideal compensator should cancel the plant dynamics, and state the drawbacks of this approach. Demonstrate that a feedback control structure can provide approximate plant inversion and show therefore that high-gain closed-loop control implicitly inverts plant dynamics.[5]

(c) What is observability in the context of state feedback control? Describe a test for observability of a state space system. [5]

(d) What is meant by state estimator feedback control? Use a block diagram to illustrate your explanations and mark the elements which form the compensator. Explain what is meant by the Separation Theorem in the context. [5]

Q2 (a) Consider a signal whose z-transform has two complex conjugate poles. Sketch and discuss the time sequences associated with various positions of the poles in the complex plane. [6]

(b) Given a discrete transfer function $H(z) = U(z)/E(z)$, demonstrate that, in the time domain, $u_k = \sum_{j=-\infty}^{+\infty} e_j h_{k-j}$. What is this formula commonly known as? [6]

(c) Consider a first order continuous transfer function $H(s) = \dfrac{U(s)}{E(s)} = \dfrac{a}{s+a}$.

Demonstrate that the forward rectangular numerical integration rule can be implemented through the substitution $s \Leftrightarrow \dfrac{z-1}{T}$ where T is the sample time.[6]

(d) What is aliasing in the context of sampled system? [2]

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SECTION B

Q3 (a) Explain why 'peaking' in the sensitivity function S_0 and the complementary sensitivity function T_0 should be avoided. The explanation should be based on performing the following analysis.

i. Derive the closed-loop equations of a feedback control system in terms of S_o and T_o, considering the output responses to the reference, disturbance and noise. Discuss how peaking would affect the system's response to these signals. [4]

ii. Show that the complementary vector margin for the inverse loop gain is equal to the inverse of the peak value of $|T_0|$. By a symmetry argument, briefly describe the effect of $|S_0|$ on the vector margin for the loop gain. Based on this, explain what a strongly oscillatory response of a closed-loop system means for stability-robustness. [5]

iii. Show that T_0 is equal to the sensitivity of S_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of S_0 on the sensitivity of T_0 to plant changes. [5]

(b) Refer to a closed loop control structure. Assume that the compensator $C(s)$ is chosen in such a way that the nominal loop gain $L_o(s)$ gives a stable closed-loop system. Assume also that the nominal loop gain $L_o(s)$ is perturbed to the actual loop gain $L(s)$, i.e. $L_o(s) \to L(s)$ (or, equivalently, that the nominal inverse loop gain is perturbed as $1/L_o(s) \to 1/L(s)$). Derive a sufficient condition for closed-loop stability which combines two tests, one involving $|S_o|$ and the other involving $|T_o|$. What is the name of this criterion? [6]

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Q4 (a) Consider the linear state space system

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

with $x(0) = x_0$.

(i) Show how a linear transfer function *G(s)* can be derived. [5]

(ii) What are the requirements in terms of the properties of the matrices / vectors A, B, C and D for which the system is stable? What are the corresponding requirements for the transfer function *G(s)*? [4]

(b) Consider a state space system. What is controller design by pole assignment and how can it be used to design a state feedback controller? What is the relevance of the *controller canonical form* and of *controllability* in this context? [6]

(c) Design a state feedback controller *K* for the state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

using the pole placement method in such a way that the closed system has an overshoot of $M_p = 1\%$ and a rise time of $t_r = 0.1$ seconds. You can use the following equations to derive the natural frequency and the damping of the desired closed loop system:

$$\omega_n \cong \frac{1.8}{t_r} \quad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + (\ln M_p)^2}}; \; 0 \leq \xi < 1$$

[5]

Continued overleaf

Section C

Q5 Consider the following transfer function:

$$H(s) = 10\frac{10s+1}{100s+1}$$

(a) Find the gain (in dB) and phase (in deg) at $\omega_1 = 0.5\,\text{rad/s}$. Assuming a sample time $T = 2$ s, calculate the Nyquist frequency ω_n in rad/s. [3]

(b) Design the discrete equivalent of $H(s)$ for the sample time given in (a), using the forward rectangular rule. [4]

(c) Compute the discrete equivalent of $H(s)$ using the pole-zero matching technique (match the steady-state gain), for the sample time given in (a). [9]

(d) Find the gain (in dB) phase (in deg) at ω_1 of the two discrete equivalents derived in (b) and (c), and compare with that of the continuous $H(s)$. Which one is closest? [4]

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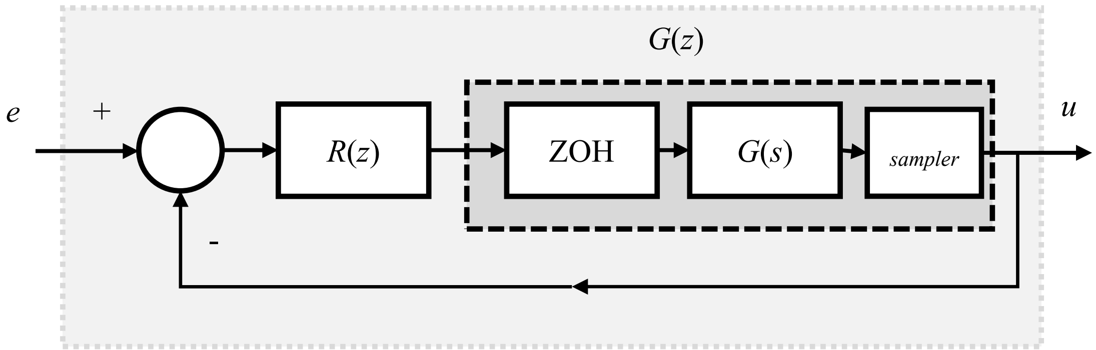
Q6 Consider the following digital feedback control loop (sample time $T = 0.1$ s):



$C_L(z)$

$$G(s) = \frac{20s - 10}{2s + 1} \qquad R(z) = \frac{z - 0.9512}{10z - 1}$$

(a) Find the discrete equivalent $G(z)$ of the plant $G(s)$. [8]

(b) Find the closed loop transfer function $C_L(z)$ of the system from e to u. (Simplify poles and zeros if possible.) [2]

(c) Find the difference equation corresponding to $C_L(z)$ from (b). [4]

(d) Estimate the steady-state output of the closed-loop system $C_L(z)$ from (b) for the following input:

$$e_k = 10\sin(0.1\,kT)$$

[6]

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Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ *(causal)*	$F(s)$	$F(z)$ *($t=kT$, T = Sample Time, k = Index)*
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\,(t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit}\ \frac{(-1)^k}{k!} \cdot \frac{\partial^{\,k}}{\partial \alpha^{\,k}}\left(\frac{z}{z-e^{-\alpha T}}\right)$
$e^{-a t}$	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$t \cdot e^{-a t}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-a t} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2+b^2}$	$\frac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-a t} \cdot \sin(bt)$	$\frac{b}{(s+a)^2+b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

End of question paper