Control M Lab Report

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Lab Work Description

In order to regulates the position of the output shaft of a servo motor, both continuous and discrete controller were designed and evaluated. All the code is applied in MATLAB and models are applied in Simulink.

Firstly, the transfer function of QUBE Servo 2 system(P) is derived by 'tf' according to the given parameters. The poles and zeros are computed by 'pole' and 'zero'. Then P is simulated by Simulink.

Secondly, a PD controller(D) is designed by cancelling the high frequency pole of D. Parameters and margins of D is evaluated by bode diagram and root-locus plot using 'bode' and 'rlocus'. Then the Nyquist plot is applied through 'nyqust' command to analyse the stability of the system. With the help of Simulink, the system is pushed to limit in order to test the margins.

Thirdly, the system is implemented using a digital computer. The discrete equivalent of the plant(P_zoh) is computed by zero-order hold using command 'c2d'. After that the controller(D) is converted into digital by pole-zero matching, which maps the s-plane poles and zeros into z-plane equivalent. Both digital and continuous compensator are compared by bode plot. The performance of digital feedback system is simulated in Simulink. Then the sample time is changed to assess its effect of the system.

Finally, the simulated plant is replaced by a realistic model. An open-loop system is implemented to find the differences between them. Then the responses of both digital and continuous compensator are assessed by Simulink. The effect of disk is assessed as well.

Results

Note that the results of Q1.3, Q1.6, Q1.7 and Q1.9 are shown in the quiz on Moodle.

Part 1

Q1.1

$$P(s) = \frac{0.042}{0.000176s^2 + 0.001764s}$$

Q1.2

$$P(s) = \frac{238.6}{s^2 + 10.02s}$$

Q1.4

According to Figure 1, with the increase of input frequencies, the time shift decreases from 17.34 to 0.42 and the amplitude decreases from 23.68 to 4.21. These correspond to the prediction as phase and gain are functions of input frequencies.

$$t_d = \frac{2\pi}{\omega}\frac{\varphi}{360}$$

Q1.5

According to Figure 2, the trend can be explained by the poles and zeros. Generally, the curve of L(s) equals to the sum of P(s) and C(s). Note that the magnitude does not change immediately at the corner frequency, it takes a 3 dB compensation.

The shape of bode diagram is determined by transfer function. For example, $P(s) = \frac{238.6}{s^2+10s} = G * \frac{1}{D(s)} * \frac{1}{E(s)} = (238.6) * \frac{1}{s} * \frac{1}{s+10}$. The gain is 238.6, for low frequency and 0 for high frequency, thus the magnitude begins with 47.5 dB and goes down to munis-infinity. The corner frequencies are determined by poles of A(s) and B(s), which are 0 and -10. When frequency is less than 10, A(s) works mainly, thus the slope of magnitude is -20 dB and the phase is -90 degree. When frequency is greater than 10, B(s) works mainly, thus the slope of magnitude decrease by -20 dB and the phase decrease by -180 degrees.

Q1.8

According to figure3, the response with K_A is critically damped while K_B is under damped, which means K_A moves to equilibrium at once while K_B oscillate before it moves to equilibrium.

Q1.10

According to figure5, the phase margin from bode plot is exactly the same with the one from Nyquist plot.

Q1.11

According to figure6, with the increase of time delay(t), the system becomes more and more unstable, but it remains stable only if t < 0.0238.

Q1.12

$$P_{zo}\ (z) = \frac{0.01154z + 0.01116}{z^2 - 1.905z + 0.9046}$$

Q1.13

$$D_d(z) = \frac{z - 0.9048}{z + 0.3679}, K_d = 14.61$$

The details of pole-zero matching are shown in figure7.

Q1.14

According to figure8, the magnitude of digital and continuous compensators is the same. The phase of them remains the same at low frequency but the digital one reach zero faster than continuous. The phase(deg) at 200rad/s of continuous and digital are about 23.8 and 15.7.

Q1.15

According to figure9, digital system has a higher overshoot about 20%, while continuous only have 5%. The settle time is about the same.

Q1.16

$$P_{zoh}(z) = \frac{0.1678z + 0.1468}{z^2 - 1.67z + 0.6697}$$

$$D_d(z) = \frac{z - 0.6703}{z - 0.01832}, K_d = 6.5506$$

Q1.17

According to figure10, the digital system at T = 0.04 is marginally stable as it oscillates too long to become stable.

Part 2

Q2.1

According to figure11, the performances of real servo and simulated plant are quiet the same. The only difference is that the magnitude of real servo is a little smaller than simulated plant, due to the effect of unmodelled friction.

Q2.2

According to figure12, the real servo has a little offset at the steady state. This is because the simulated plant ignored the friction. Mathematically:

$$J_{eq}R_m\ddot{\theta}(t) + k_t k_m \dot{\theta}(t) + k_f \theta(t) = k_t v_m(t)$$

Here k_f means the friction coefficient.

Q2.3

According to figure13, with the increase of time delay, the system goes from stable to unstable. At time delay at 0.3, the simulated plant is marginally stable, while the real one is unstable. It can be inferred that the delay margin is smaller for the real servo.

Q2.4

According to figure 14, the magnitude is smaller when the disk is removed, but the system remains stable. The load moment of inertia has changed when the disk is removed.

Q2.5

According to figure15, the overshooting and setter time are higher for the digital controller.

Q2.6

According to figure 16, the system is marginally stable. The real plant affects the stability by friction and its disk.

Discussion

In order to improve the results, the following method can be used:

1. Take friction into consideration
2. Use the fastest sample time
3. Implement the real servo instead of the real servo model

Appendix

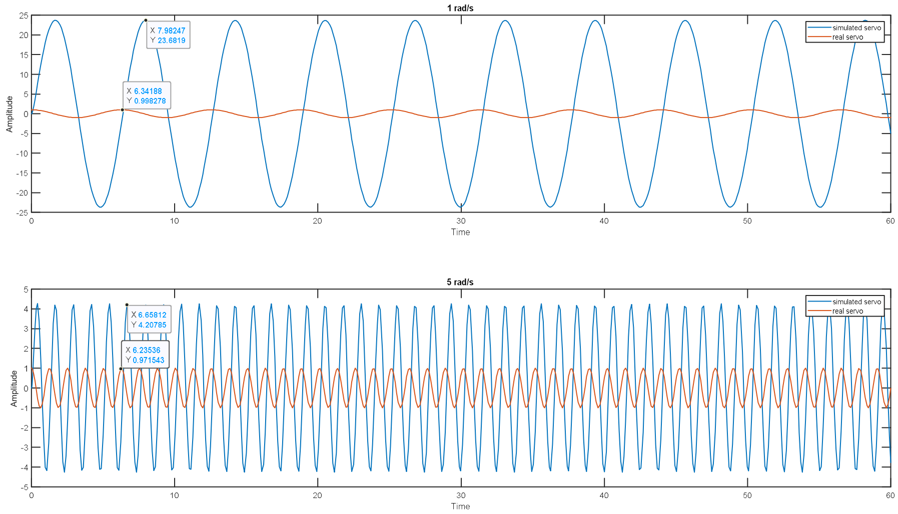

Figure 1: Q1.4

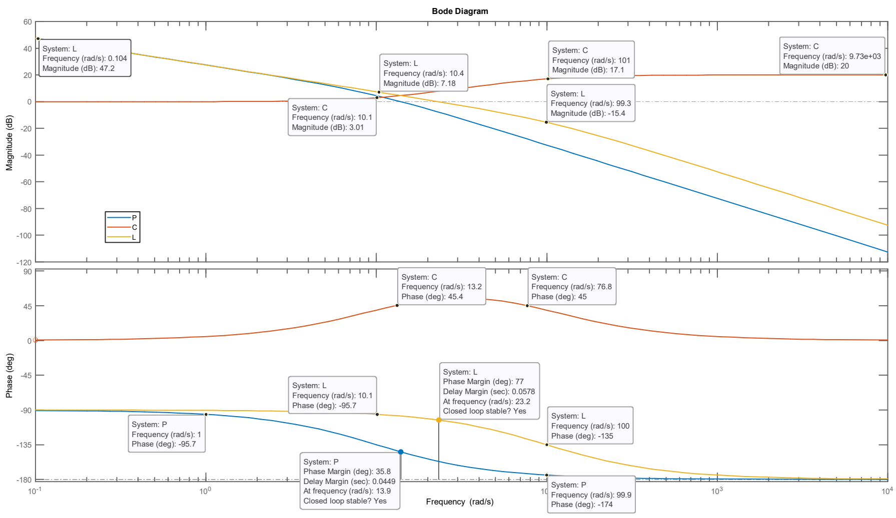

Figure 2: Q1.11

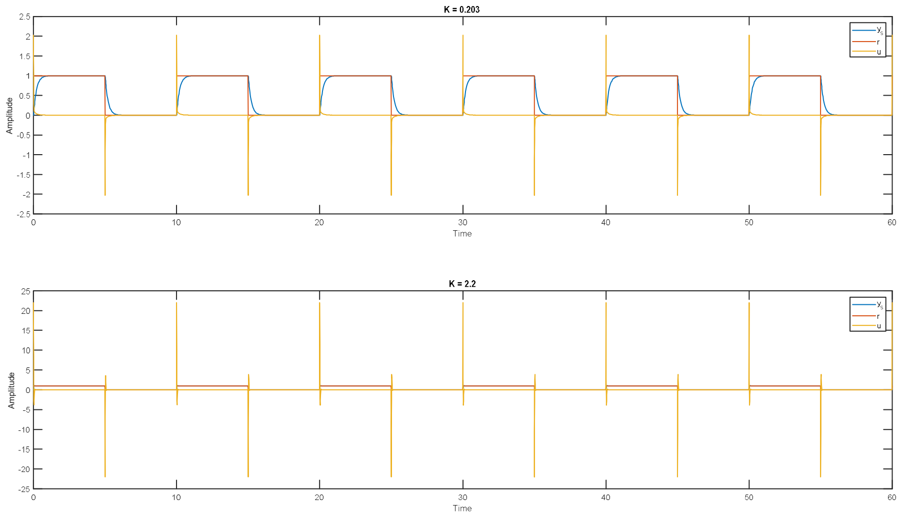

Figure 3: Q1.8

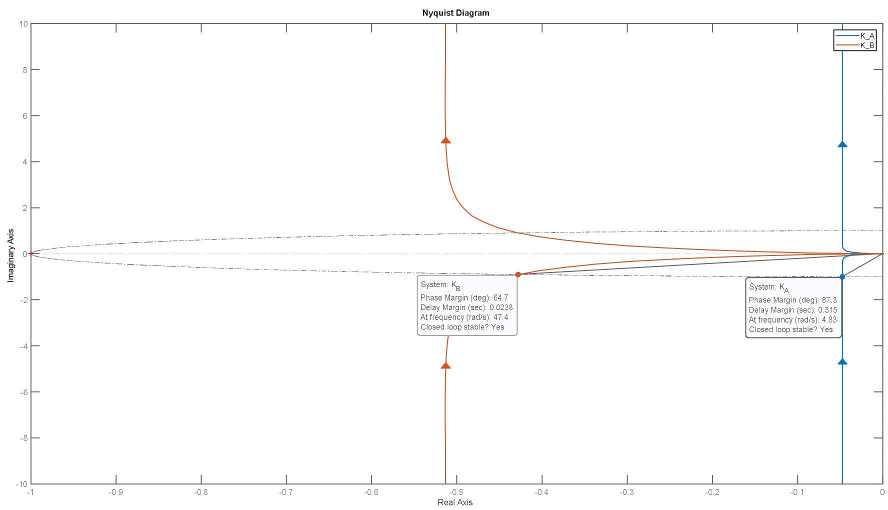

Figure 4: Q1.9

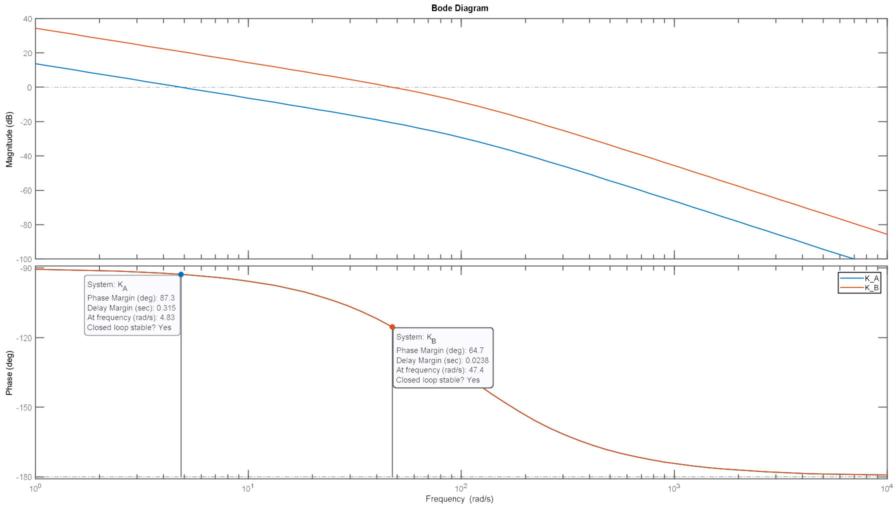

Figure 5: Q1.10

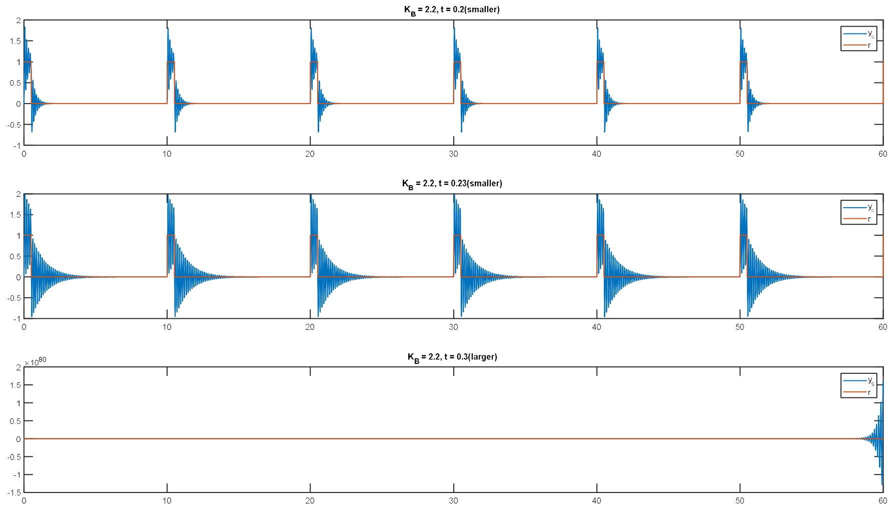

Figure 6: Q1.11

Q1.13: For the controller gain from B (`K_B`), find the zeros and poles of the discrete transfer function of the compensator, Dd(z), and its gain (`K_d`) analytically:

$K_d =$

$D_d(z) = \frac{z+a}{z+b} =$

$D = \frac{10s+100}{s+100}$, $\begin{cases} S_z = -10 \\ S_p = -100 \end{cases}$

1) $Z_1 = e^{S_z T} = e^{-0.1} = 0.9048$

$Z_2 = e^{S_p T} = e^{-1} = 0.3679$

2) $D(Z) = k_d \cdot \frac{(Z-0.9048)}{(Z-0.3679)}$

3) $2.2 \cdot \frac{100}{100} = Kd \cdot \frac{Z-0.9048}{Z-0.3679}$

$Kd = 14.61$

Figure 7: Q1.13

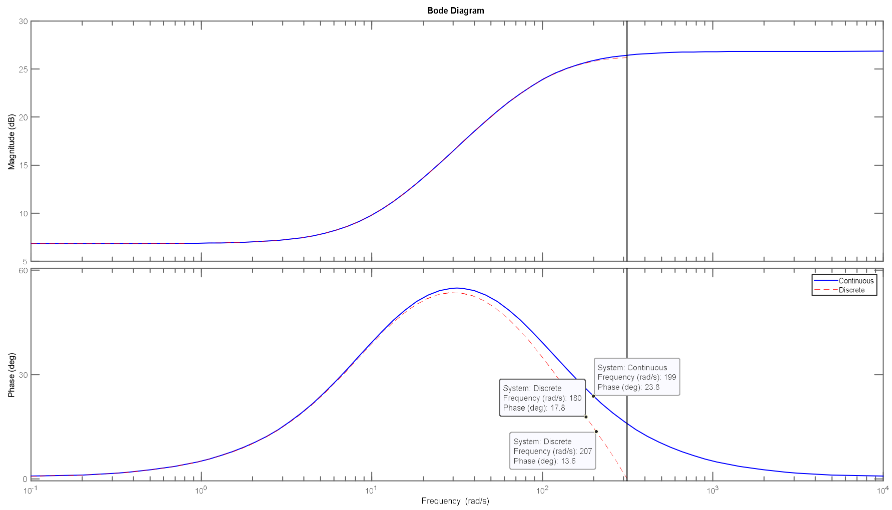

Figure 8: Q1.14

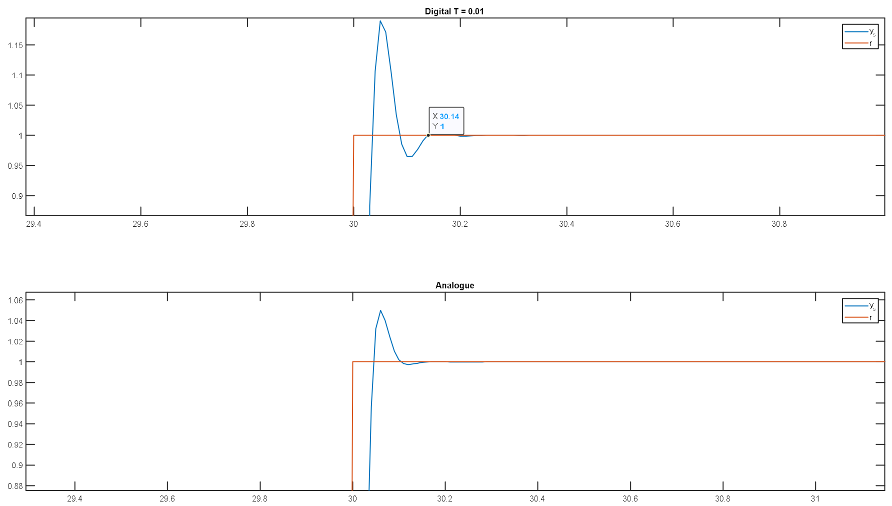

Figure 9: Q1.15

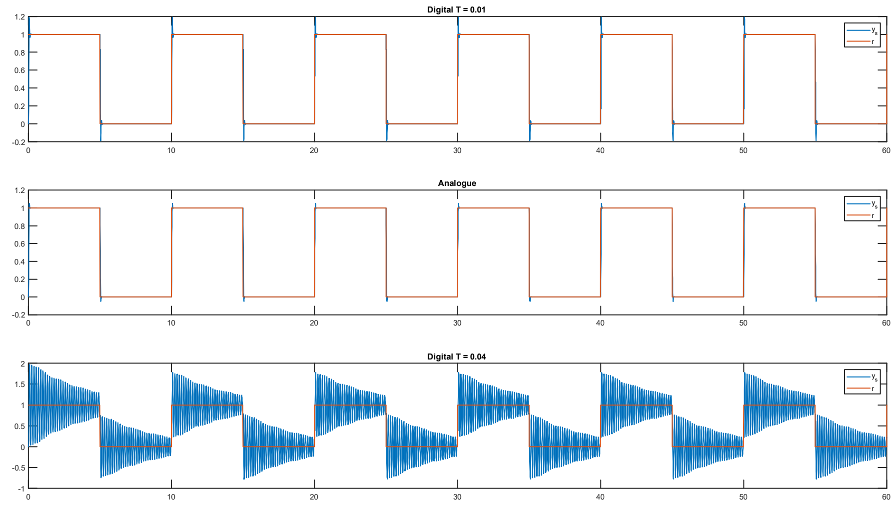

Figure 10: Q1.17

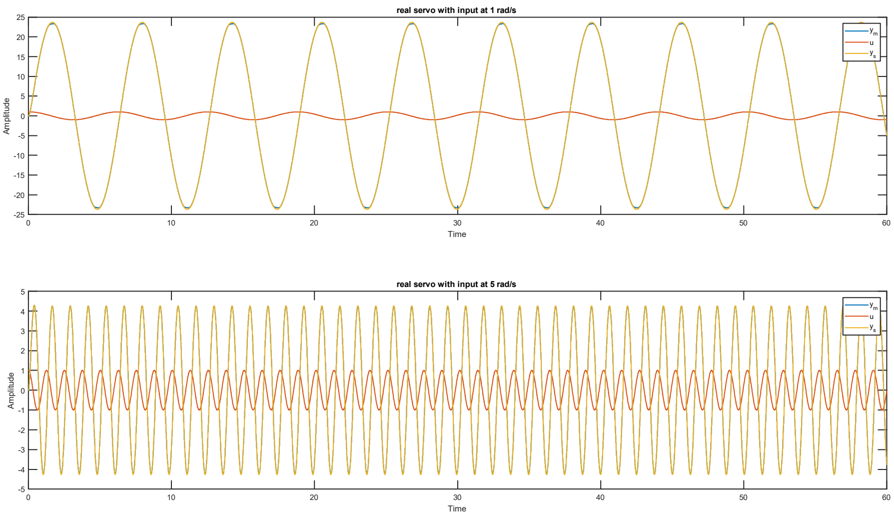

Figure 11: Q2.1

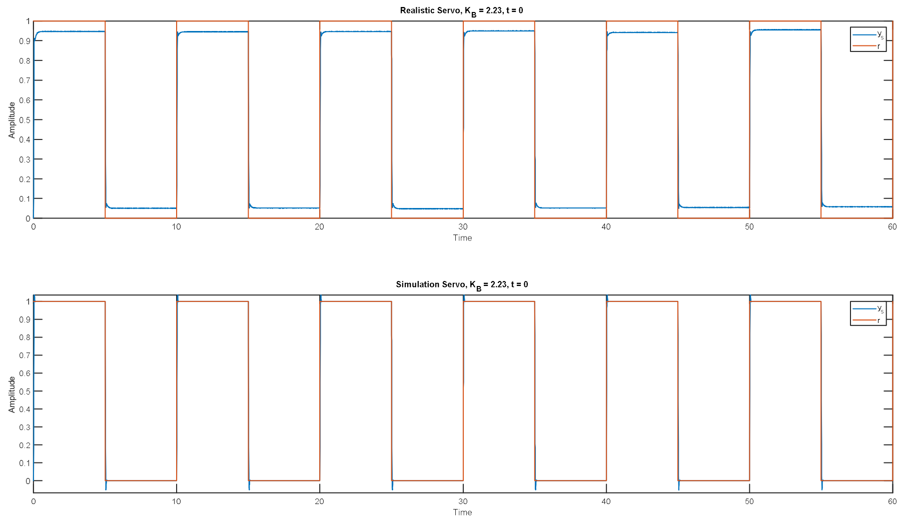

Figure 12: Q2.2

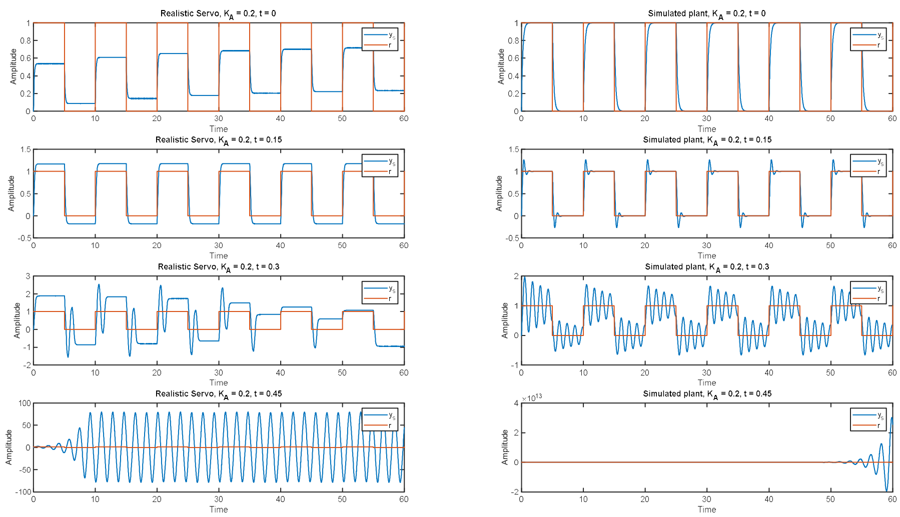

Figure 13: Q2.3

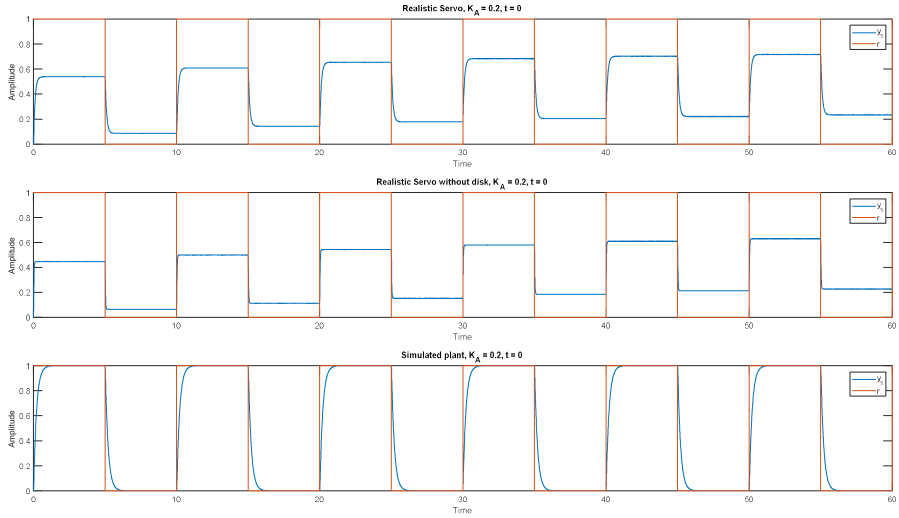

Figure 14: Q2.4

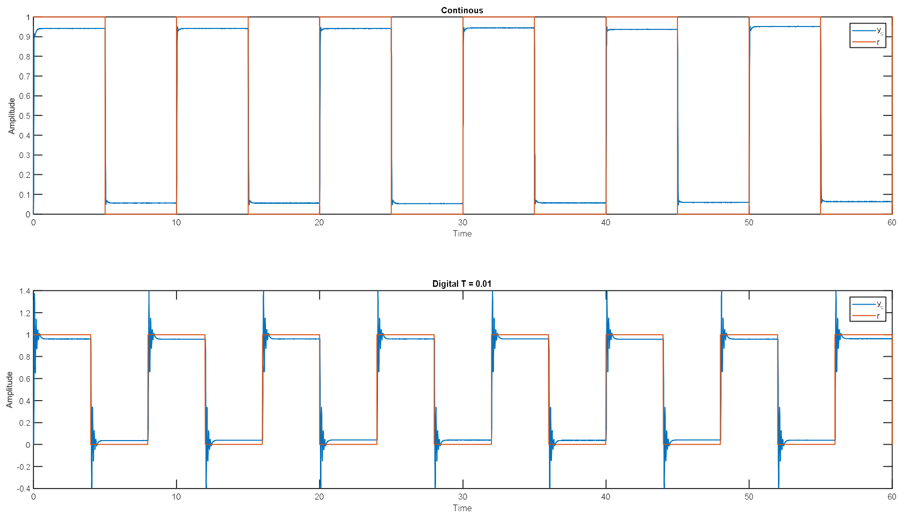

Figure 15: Q2.5

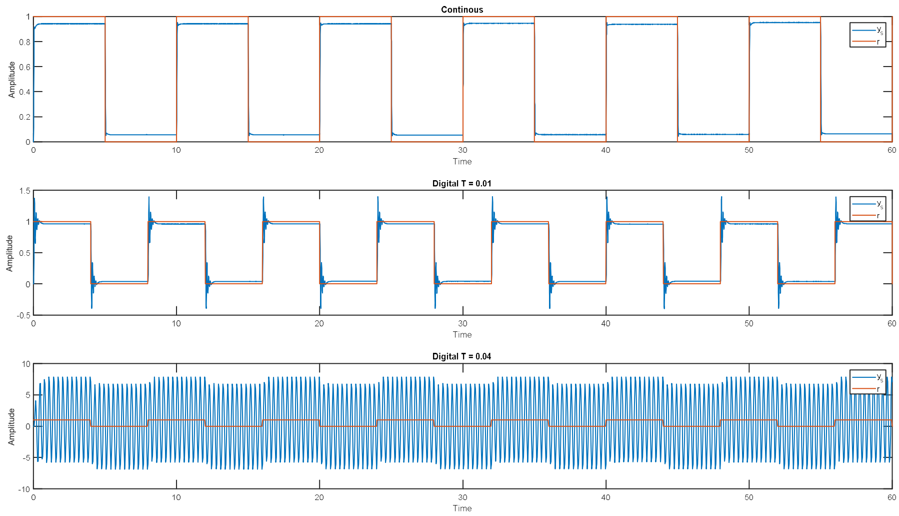

Figure 16: Q2.6