UNIVERSITY OF GLASGOW

Degrees of MEng, BEng, MSc and BSc in Engineering

CONTROL 4 (ENG4042)

Wednesday 12 December 2012
09:30 – 11:30

Answer ALL questions in Section A and ONE question from Section B and ONE question from Section C.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Data sheet included within paper.

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SECTION A

Q1 (a) State the three properties of operational amplifiers of most use in control engineering. [3]

(b) Draw the op-amp circuit for a non-inverting amplifier and use this diagram to derive the transfer function from first principles. [4]

(c) Sketch the frequency domain gain (amplitude) response of the following filters.

(i) Low pass

(ii) High pass

(iii) Band pass

(iv) Band stop (Notch) [4]

(d) Name the two classes of nonlinearity and give at least two examples of the nonlinearities belonging to each class. [4]

Q2 (a) For a closed loop system with a nominal plant P_0, a phase margin of 80deg at a frequency of 5Hz, and a gain margin of 3dB was derived from a Nyquist diagram of the loop gain.

i. By how much can the plant gain increase before the closed loop becomes unstable? [3]

ii. What additional delay can be added to the closed loop before stability is lost? [2]

(b) Show that closed-loop control implicitly performs an approximate inversion of the plant dynamics, at least for those frequencies where the loop gain is high. [5]

(c) Consider a state space system. What is controller design by pole assignment and how can it be used to design a state feedback controller? [5]

(d) What is controllability in the context of state feedback control and what is meant by stabilisability? [3]

(e) What are the components of a PID controller? Give the control law in the time domain and in the Laplace domain. [2]

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SECTION B

Q3 (a) The transfer function for a specific analogue controller is,

$$C(s) = \frac{(\tau s + 1)}{(\alpha \tau s + 1)}$$

Draw the analogue circuit realisation for this controller using

(i) passive elements only, and [4]

(ii) operational amplifiers. [4]

(b) Extract the transfer function of the analogue control circuit drawn in Figure 1 below with R1 = 1MΩ, R2 = 10kΩ, C = 10μF. [6]

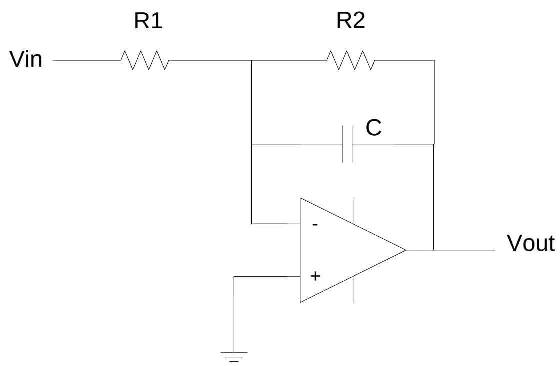

Figure 1

(c) The schematic of a non-inverting amplifier low-pass filter is shown below in Figure 2. Derive the general form of the transfer function for this circuit and then select the resistor and capacitor values to implement the following filter.[6]

$$C(s) = \frac{101}{(0.1s + 1)}$$

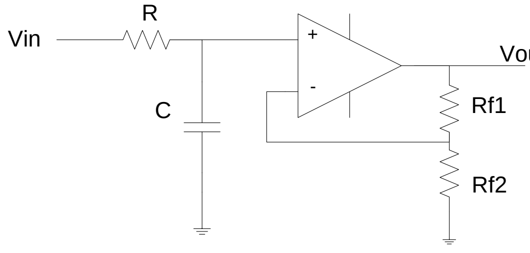

Figure 2

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***Note: You should make use of the table of Laplace and Z-transforms given on the next page (Table 1).**

Q4 (a) Find the equivalent difference equation for the following transfer function,

$$G(s) = \frac{100(5s+1)}{s^2 + 10s + 100}$$

for a sampling interval of T=0.1s using

(i) Forward difference approximation [4]

(ii) Backward difference approximation [4]

(iii) Tustin (bilinear) approximation. [6]

(b) The simplified dynamics of a car's velocity when friction is present can be approximated by the transfer function,

$$G(s) = \frac{1}{1000s + 100}$$

Find the ZOH discrete equivalent for this transfer function with a sampling interval of T = 0.6s. [6]

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Laplace and Z Transforms for Causal Functions

$f(t)$ $t \geq 0$ (causal)	$F(s)$	$F(z)$ ($t=kT$, T = Sample Time, k = Index)
$\delta\ (t)$	1	$1 = z^{-0}$
$\delta\ (t-kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{\alpha \to 0}{Limit} \frac{(-1)^k}{k!} \cdot \frac{\partial^{\ k}}{\partial\, \alpha^{\ k}} \left(\frac{z}{z-e^{-\alpha T}}\right)$
$e^{-a\,t}$	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$t \cdot e^{-a\,t}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z-e^{-aT})^2}$
$e^{-a\,t} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2+b^2}$	$\frac{z \cdot (z-e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-a\,t} \cdot \sin(bt)$	$\frac{b}{(s+a)^2+b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

TABLE 1

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Section C

Q 5 Refer to the closed-loop system shown in Figure 3.

(a) By means of a sketch of a typical Nyquist diagram, indicate the vector margin for the loop gain, and the alternative vector margin for the inverse loop gain, and explain how these can be used to characterise the relative stability of the closed-loop. [5]

(b) Explain why 'peaking' in the sensitivity function S_0 and the complementary sensitivity function T_0 should be avoided. The explanation should be based on performing the following analysis.

(i) Derive the closed-loop equations of the system, and discuss how peaking would affect the system's response to *r*, *d* and *n*, which are defined in Figure Q4. [4]

(ii) Show that the vector margin for the inverse loop gain is equal to the inverse of the peak value of $|T_0|$. By a symmetry argument, briefly describe the effect of $|S_0|$ on the vector margin for the loop gain. [6]

(iii) Show that T_0 is equal to the sensitivity of S_0 to changes in the plant P_0. By a symmetry argument, briefly describe the effect of S_0 on the sensitivity of T_0. [5]

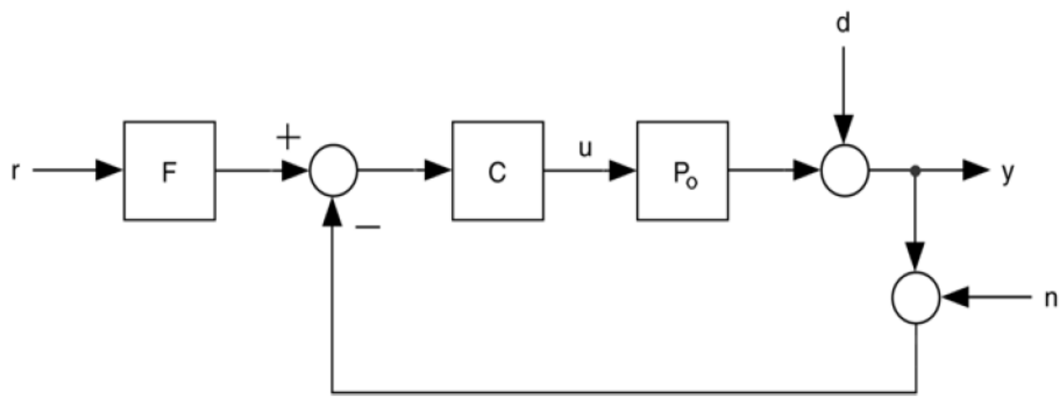

Figure 3

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Q6 (a) The control system in Figure 3 has a plant $P_0(s)=\frac{B_0(s)}{A_o(s)}$ and a controller $C(s)=\frac{G(s)}{H(s)}$. Using the definitions of the sensitivity function and the complementary sensitivity functions, derive the expression for the characteristic polynomial of the closed loop and discuss its significance. Based on this, explain what is meant by feedback controller design using pole assignment. [7]

Consider the plant $P_0(s)=\frac{1}{s+3}$

(b) Sketch the frequency response of this plant in form of a Bode plot. What is the bandwidth of the open loop system? [2]

(c) Describe how a proportional controller can be used to increase the bandwidth of the closed loop system by considering how it affects the loop gain. Mention two limitations associated with using only a proportional component in the controller and how the other terms of the PID controller could be used to overcome these limitations. [5]

(d) Given the plant $P_0(s)=\frac{1}{s+3}$, a strictly proper 1^{st} order controller C(s) should be designed using the pole placement design method. The closed loop rise time t_r should be 1sec and the overshoot M_p should be 0.1.

(i) Using the following formulae to relate time-domain specifications to parameters of the closed loop polynomial, specify the desired characteristic polynomial A_{cl}.

$$\omega_n \cong \frac{1.8}{t_r},\ \xi=-\frac{\ln M_p}{\sqrt{\pi^2+\left(\ln M_p\right)^2}};\ M_p=\exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right),\ 0\le\xi<1.$$ [2]

(ii) Calculate the parameters of the controller by solving the Diophantine equation. [4]

End of question paper