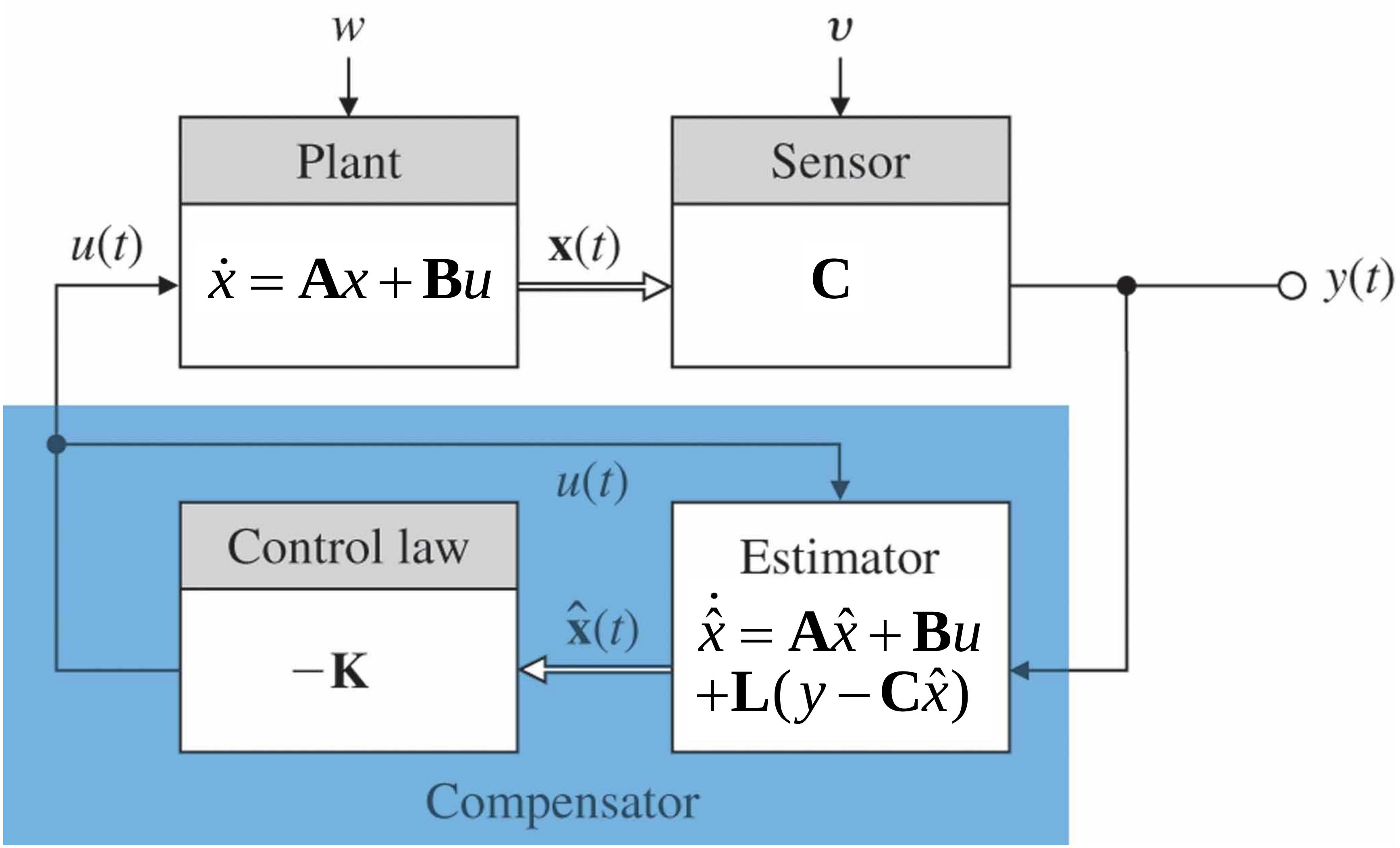
State-estimator feedback control



Combining State Feedback with an Observer

A reasonable conjecture arising from the last two sections is that it would be a good idea, in the presence of unmeasurable states, to proceed by estimating these states via an observer and then to complete the feedback control strategy by feeding back these estimates in lieu of the true states. Such a strategy is indeed very appealing, because it separates the task of observer design from that of controller design. A-priori, however, it is not clear how the observer poles and the state feedback interact. The following theorem shows that the resultant closed-loop poles are the combination of the observer and the state-feedback poles.

Separation Theorem

Theorem: (*Separation theorem*). Consider the state space model and assume that it is completely controllable and completely observable. Consider also an associated observer and state-variable feedback, where the state estimates are used in lieu of the true states:

$$u(t) = \bar{r}(t) - \mathbf{K}\hat{x}(t)$$

$$\mathbf{K} \triangleq \begin{bmatrix} k_0 & k_1 & \dots & k_{n-1} \end{bmatrix}$$

Then

(i) the closed-loop poles are the combination of the poles from the observer and the poles that would have resulted from using the same feedback on the true states - specifically, the closed-loop polynomial $A_{cl}(s)$ is given by

$$A_{cl}(s) = \det\left(sI - \left(\mathbf{A} - \mathbf{BK}\right)\right)\det\left(s\mathbf{I} - \left(\mathbf{A} - \mathbf{LC}\right)\right)$$

(ii) The state-estimation error $\tilde{x}(t)$ cannot be controlled from the external signal $\bar{r}(t)$.

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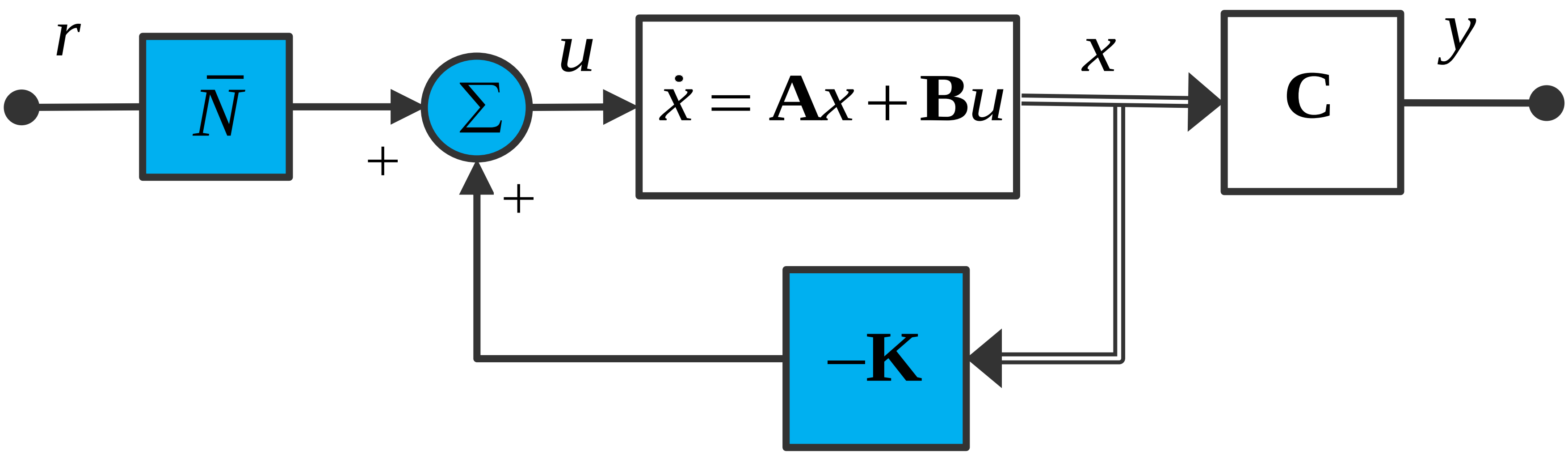
The above theorem makes a very compelling case for the use of state-estimate feedback. However, it should be noted that the location of closed-loop poles is only one among many factors that come into control-system design. Indeed, state-estimate feedback is not a panacea, and is subject to the same issues of sensitivity to disturbances, model errors, etc. as all feedback solutions. In particular, all of the schemes (state-estimate feedback, transfer functions) turn out to be essentially identical.

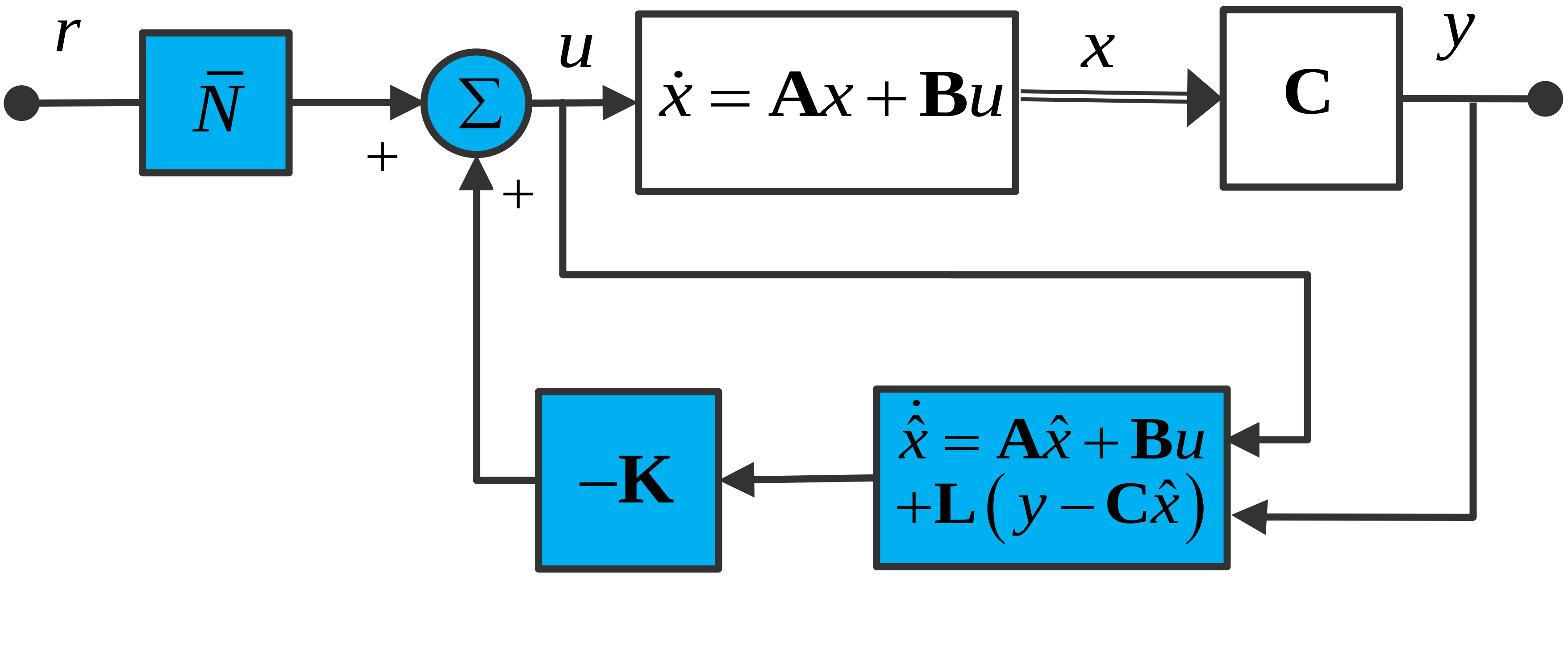
Summary

- the transfer function of a state-space system represents its fully observable and fully controllable part.
- For a fully controllable and fully observable system, state-estimator feedback control is essentially equivalent to a two-degree of freedom feedback control structure.

State feedback control



State-estimator feedback control



State-estimator feedback control

- **Plant**

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t); \quad x(0) = x_0$$
$$y(t) = \mathbf{C}x(t)$$

- **Controller (with reference input)**

$$\dot{\hat{x}}(t) = \left(\mathbf{A} - \mathbf{LC} - \mathbf{BK}\right)\hat{x}(t) + \mathbf{B}\bar{N}r(t) + \mathbf{L}y(t); \quad \hat{x}(0) = \mathbf{0}$$
$$u(t) = -\mathbf{K}\hat{x}(t) + \bar{N}r(t)$$

Continous Control system