Types of Systems

Systems can be categorized by the number of inputs and outputs they have

- Single
- Multiple

Applications:

- System identification
- System modeling
- Controller design

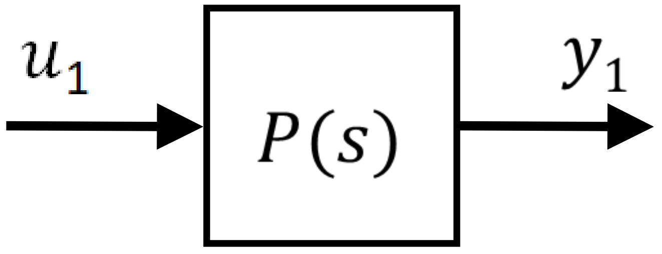
Why Characterize Systems?

Correctly identifying a system based on its inputs/output:

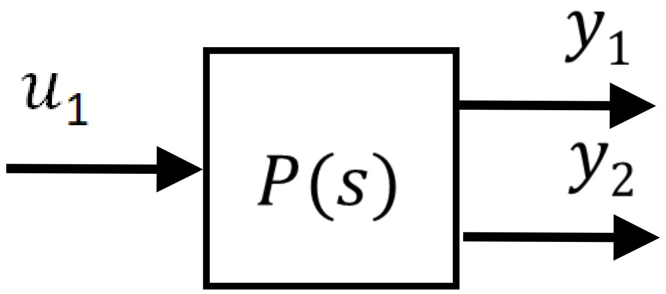
- Enables engineers to design better and more effective controllers
- Necessary for accurate modeling a dynamic systems
- Block diagrams

Single-Input Systems

- Single-input systems have a single control input
- Depending on the number of outputs, sub-categories include:
  - Single-input, single-output (SISO)
  - Single-input, multi-output (SIMO)

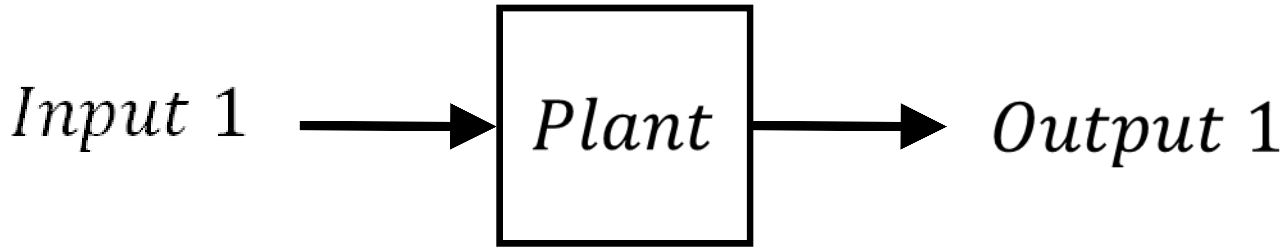


SISO



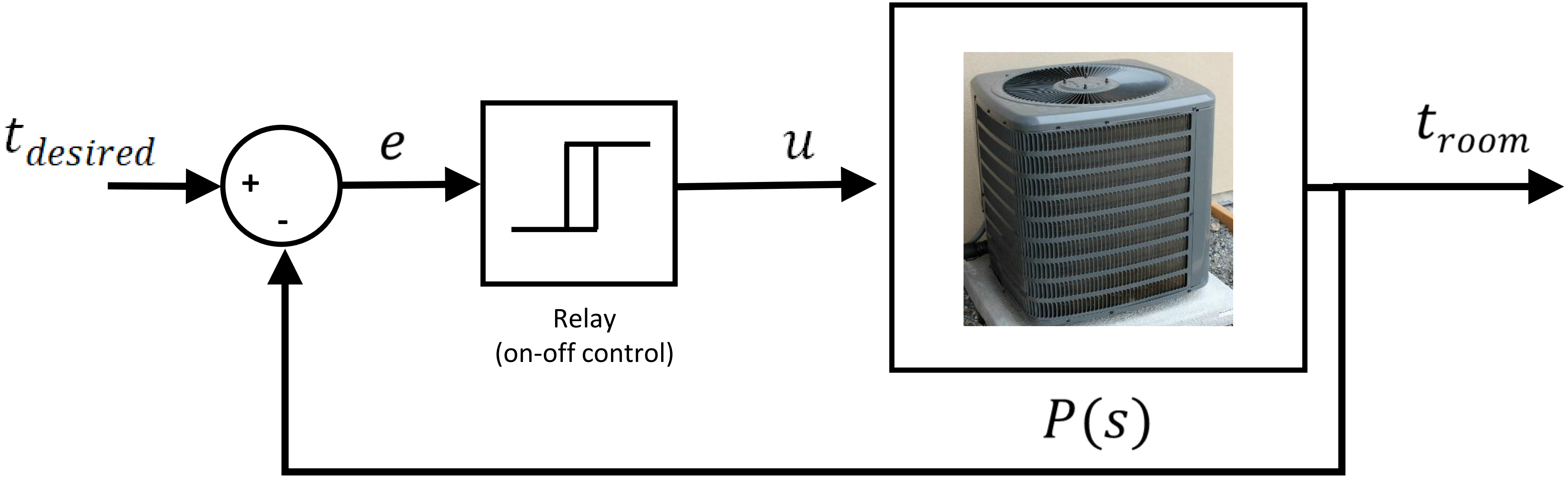
SIMO

Single-Input, Single-Output (SISO) Systems

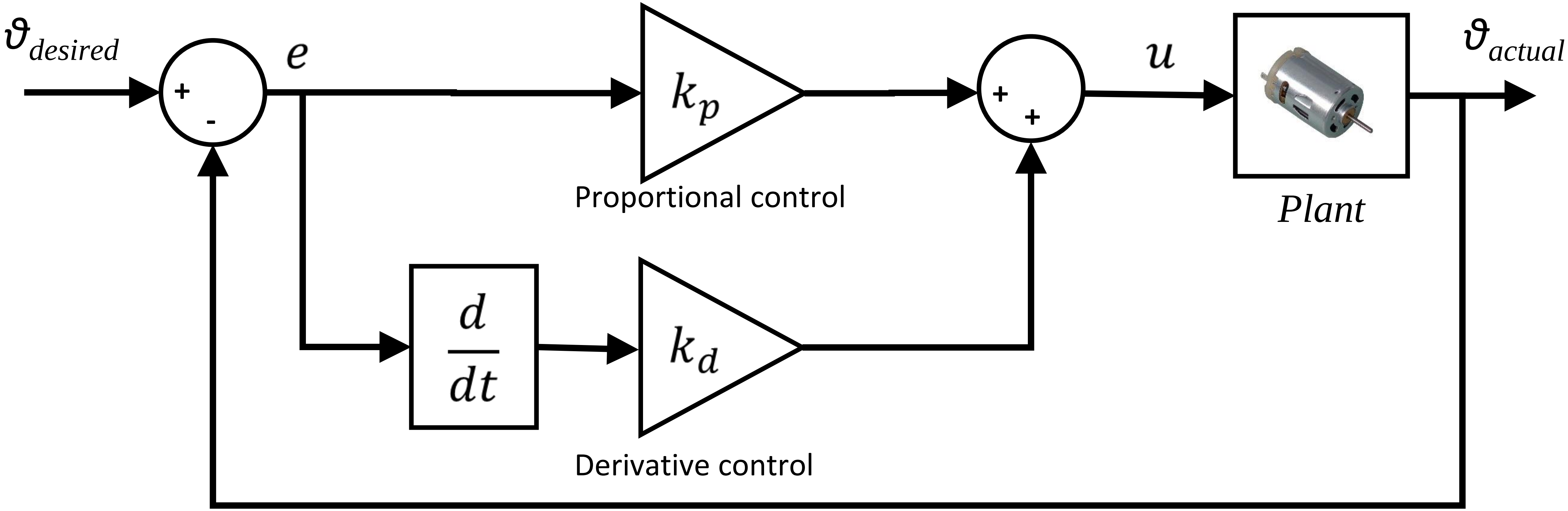


- SISO systems have a **single input** and **single output**
  - Single actuating element
  - Single controlled process
- Relatively **simple**, thus studied extensively in controls literature
  - PID control
- Typically modeled using **transfer functions**
- Examples include:
  - DC motor
  - Heater
  - Solenoid valve

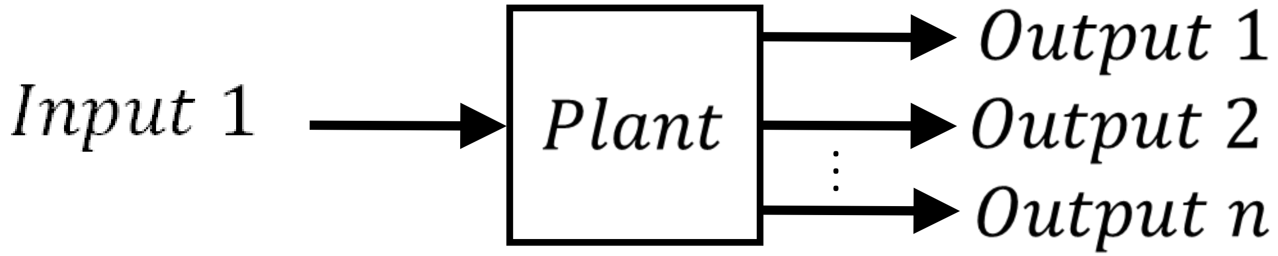
Example: HVAC System



Example: Position Control



Single-Input, Multi-Output (SIMO) Systems



- SIMO systems have a **single input** and **multiple outputs**
  - Single actuating element
  - Multiple controlled processes
- More complex systems
  - LQR Control
- Typically modeled using **state-space** models
- Examples include:
  - Segway
  - Robotic arm

Rotary inverted pendulum

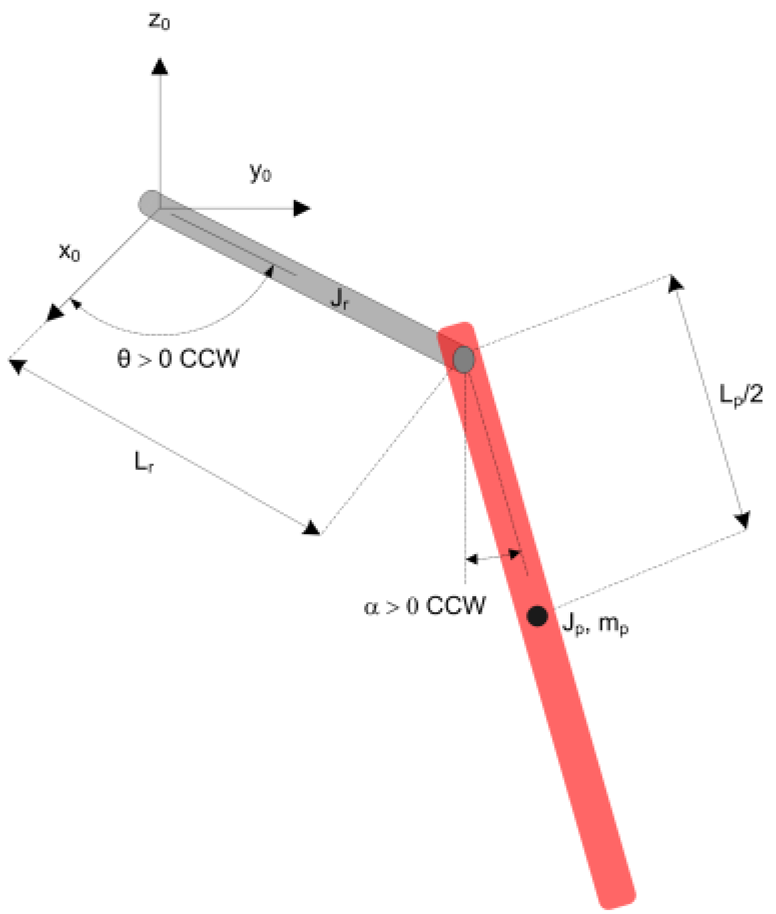
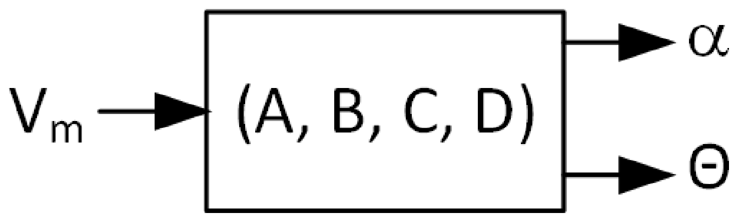
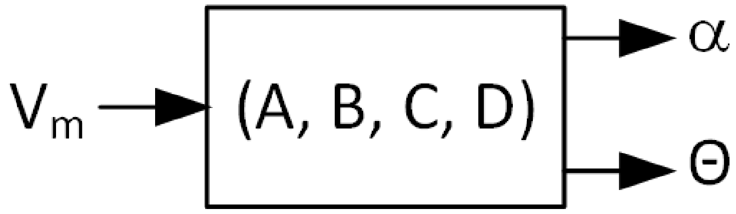


Figure 1: Rotary inverted pendulum model



(b) QUBE-Servo 2 with Pendulum Module

Rotary inverted pendulum



$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

- Four states:

$$x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$$

$$y = \begin{bmatrix} \theta & \alpha \end{bmatrix}^T.$$

$$A = \frac{1}{J_T} \begin{bmatrix} 0 & 0 & J_T & 0 \\ 0 & 0 & 0 & J_T \\ 0 & \frac{1}{4} m_p^2 L_p^2 L_r g & -\left(J_p + \frac{1}{4} m_p L_p^2\right) D_r & \frac{1}{2} m_p L_p L_r D_p \\ 0 & -\frac{1}{2} m_p L_p g \left(J_r + m_p L_r^2\right) & \frac{1}{2} m_p L_p L_r D_r & -\left(J_r + m_p L_r^2\right) D_p \end{bmatrix}$$

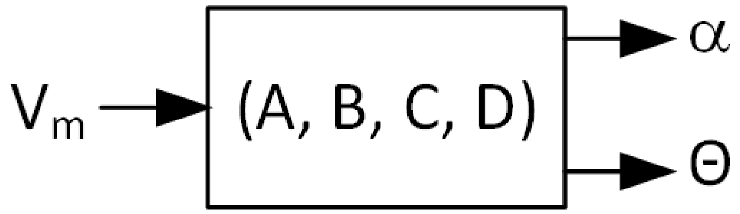
$$B = \frac{1}{J_T} \begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4} m_p L_p^2 \\ -\frac{1}{2} m_p L_p L_r \end{bmatrix}$$

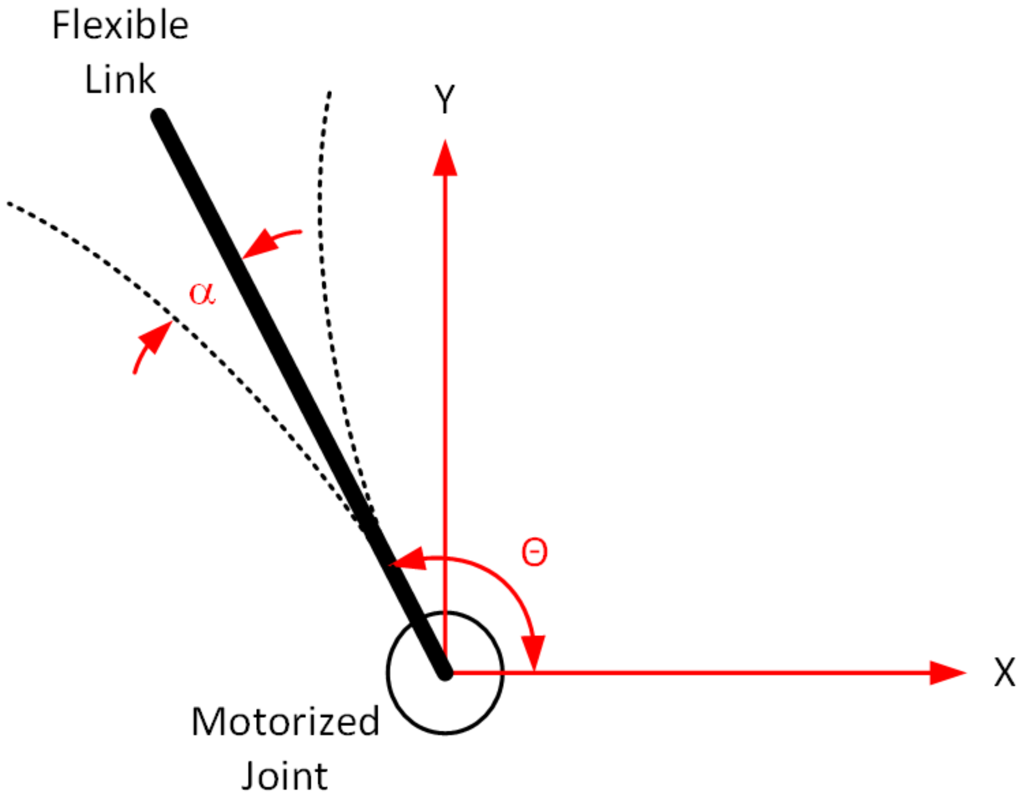
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Example: Canadarm

- Arm is made of several flexible links and motorized joints
  - Each link/joint represents a SIMO
- Modeled using state-space
- Coupled dynamics
  - Joint movement/flexible link dynamically affect each other

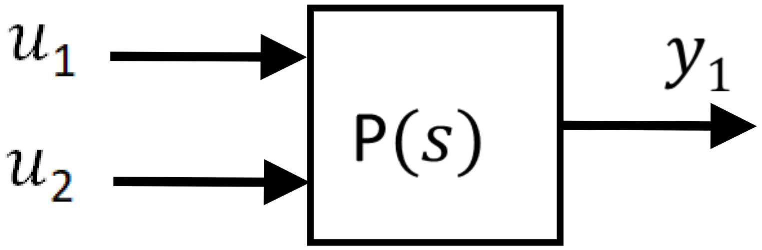
Example: Canadarm (cont.)



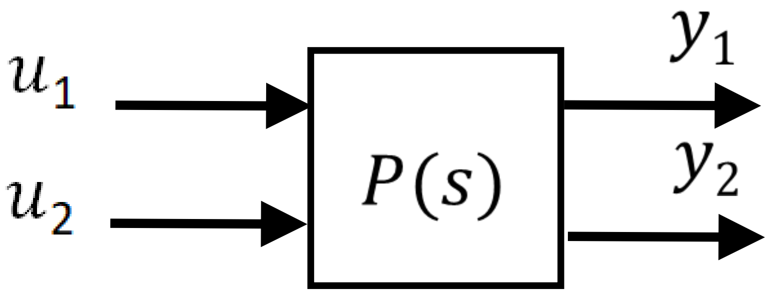


Multi-Input Systems

- Multi-input systems have a multiple control inputs
- Depending on the number of outputs, sub-categories include:
  - Multi-input, single-output (MISO)
  - Multi-input, multi-output (MIMO)

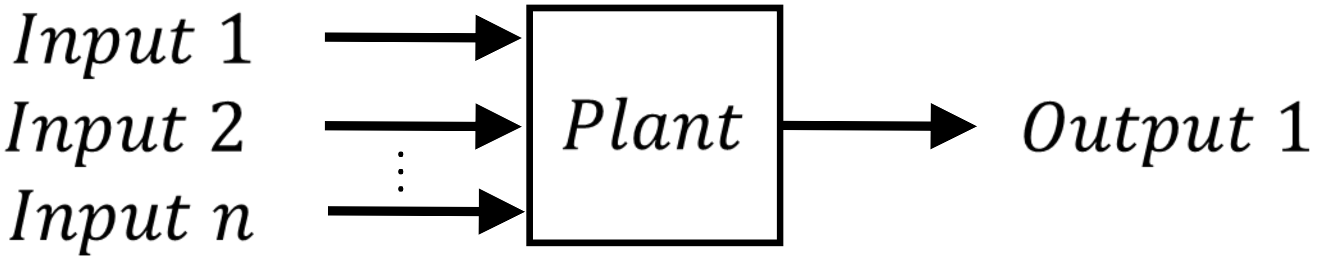


MISO



MIMO

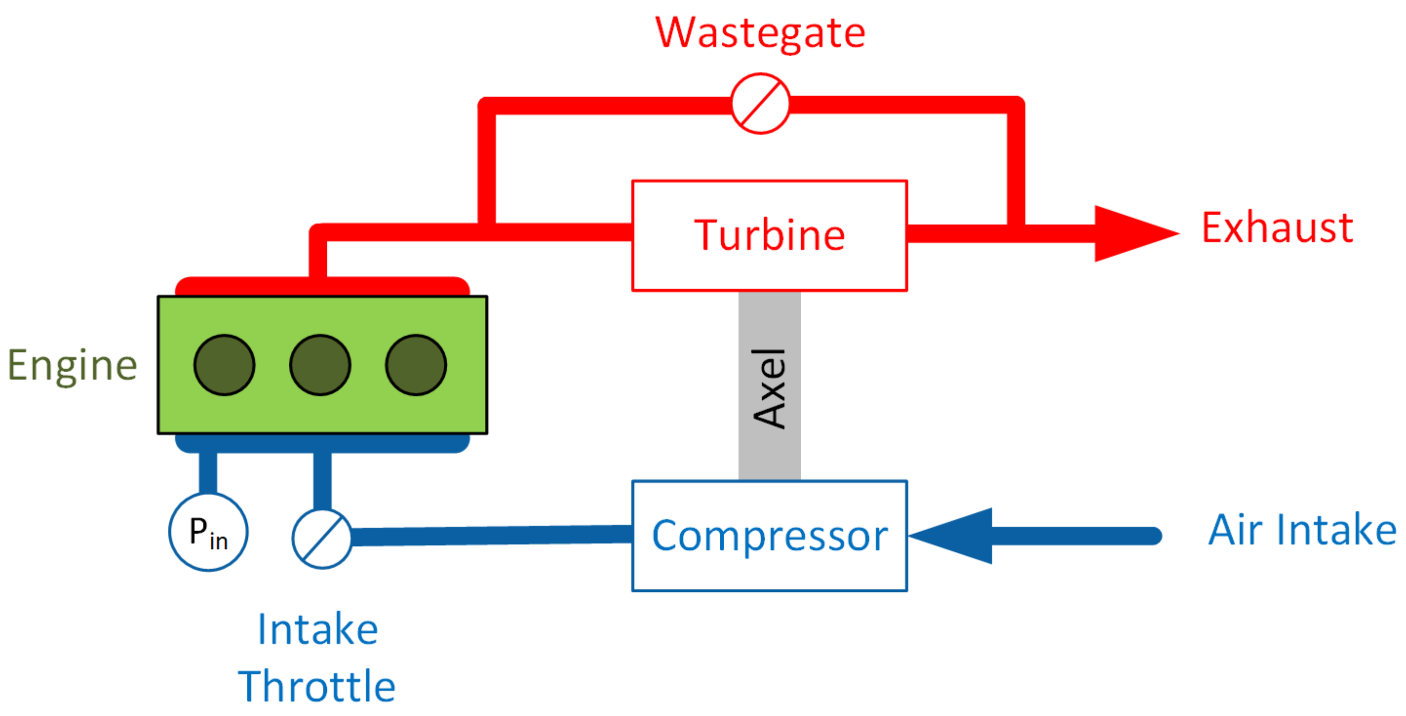
Multi-Input, Single-Output (MISO) Systems



- MISO systems have **multiple inputs** and a **single output**
  - Multiple actuating elements
  - Single controlled process
- Relatively **complex**
- Advanced controllers
  - LQR control
  - Valve Position Control (mid-ranging)
- Typically modeled using **state-space** models
- Examples include:
  - Automotive turbocharger

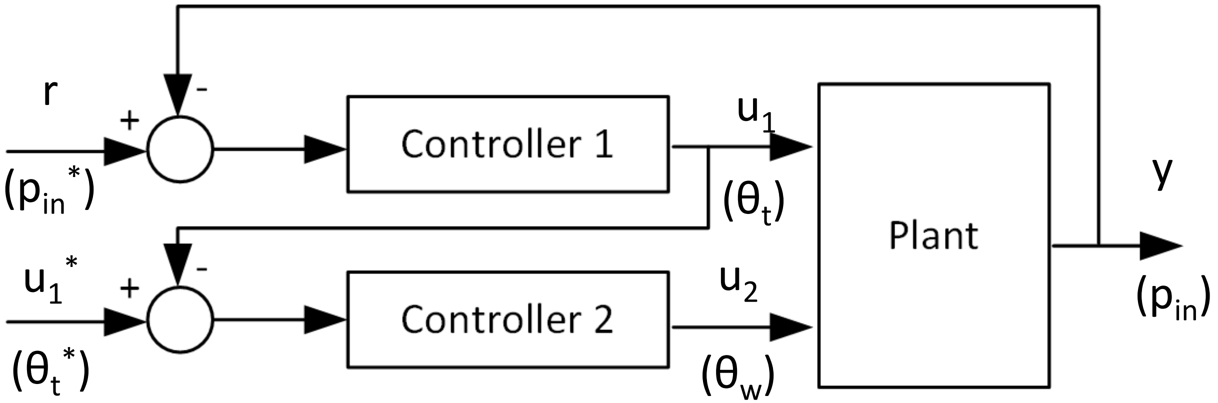
Example: Turbocharger

- Exhaust gasses turns turbine and connected compressor
- Extra air forced into combustion chamber
  - Increase efficiency & power output
  - Improved fuel consumption
- Need to maintain certain intake pressure (P_{in})



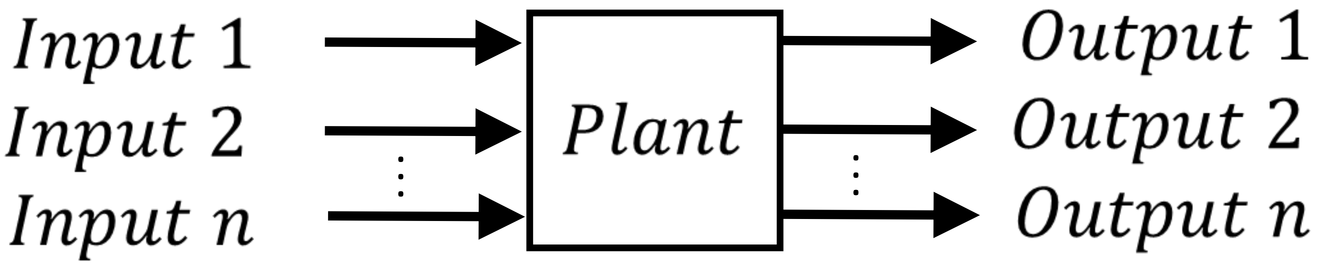
Example: Turbocharger (cont.)

- Valve Position Control (Mid-ranging)
- Trim (fast) actuator u_1 (intake throttle) tracks output y to required pressure (r)
- Coarse (slow) actuator u_2 (wastegate valve) maintains u_1 at setpoint u_1^* such that u_1 does not saturate



θ_t: intake throttle value angle
θ_w: wastegate value angle

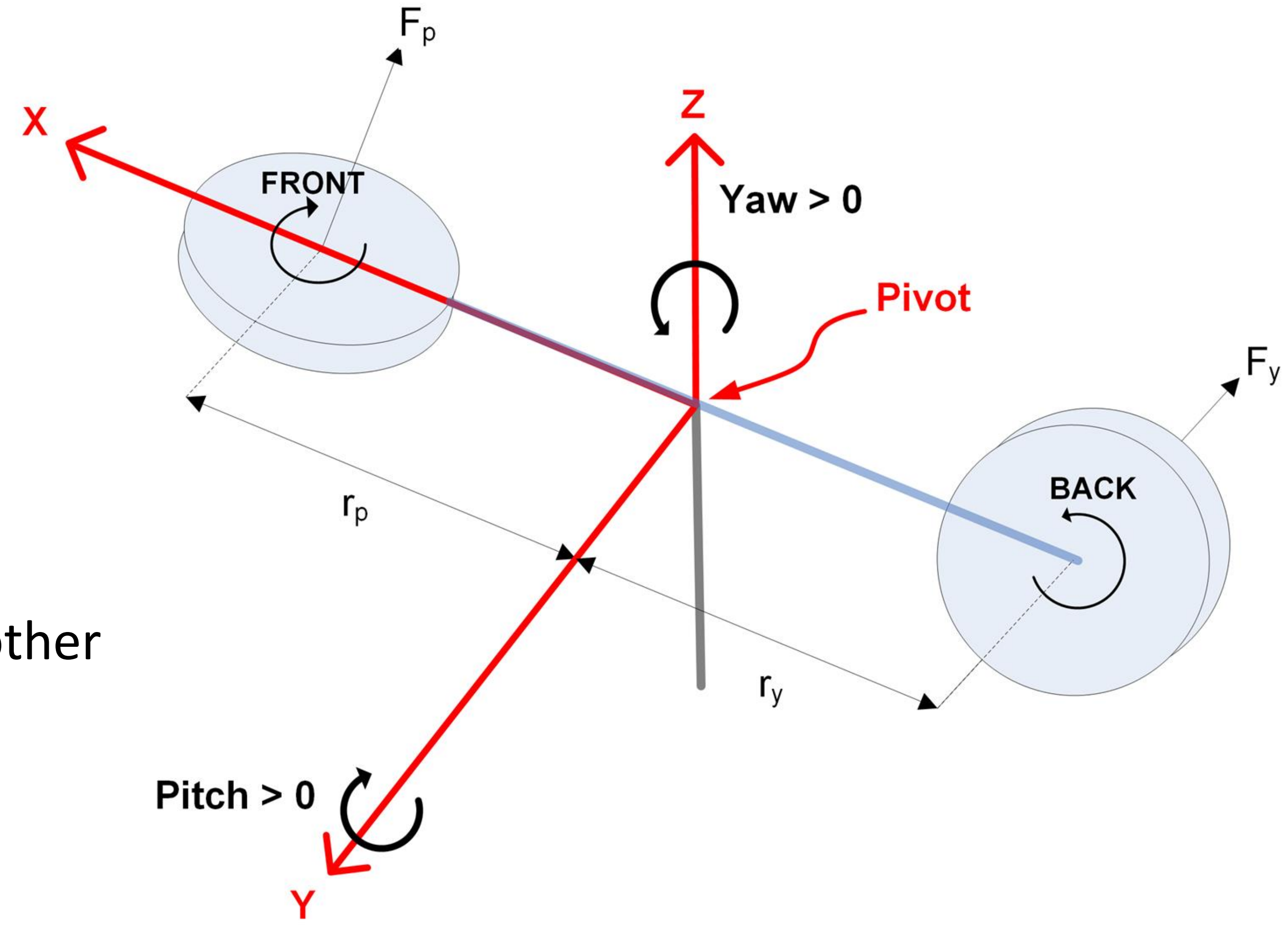
Multi-Input, Multi-Output (MIMO) Systems



- MIMO systems have **multiple inputs** and **multiple outputs**
  - Multiple actuating elements
  - Multiple controlled processes
- **Complex** systems
- Advanced controllers
  - LQR control
- Modeled using **state-space** models
- Examples include:
  - Helicopter
  - Petrochemical Plants

Example: Helicopter

- Inputs
  - Front rotor
  - Read rotor
- Outputs
  - Pitch
  - Yaw
- Coupled dynamics
  - Pitch/yaw affect each other



SISO vs. MIMO

In certain cases complex systems can be modeled as multiple SISO systems

- Why? SISO systems are easier to model!
- When? Nonexistent or negligible coupled dynamics.
- Easier to tune multipole SISO systems, instead of a single MIMO system

State-space form and MIMO systems

- The state space form can be readily extended to MIMO systems

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

NOTE: The dynamic properties of the underlying system (defined by $\boldsymbol{A}$ are not affected by the choice of inputs and outputs.

- If $u(t)$ has more than one element (**multiple inputs**), then $\boldsymbol{B}$ will have as many columns as inputs, e.g.

$B = \begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$ corresponds to a system with 2 inputs and 3 states

State-space form and MIMO systems

- If $u(t)$ has more than one element (**multiple inputs**), then $\boldsymbol{B}$ will have as many columns as inputs, e.g.
  $\boldsymbol{B} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$ corresponds to a system with 2 inputs and 3 states
- The state feedback vector $\boldsymbol{K}$ will have as many rows as there are inputs, e.g.
  $$\boldsymbol{K} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$$
- The design methodology, especially the closed loop dynamics remain unchanged
  $$A_{cl} = A - BK$$

State-space form and MIMO systems

- If $y(t)$ has more than one element (**multiple outputs**), then $\boldsymbol{C}$ will have as many rows as outputs, e.g. $\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ corresponds to a system with 2 outputs and 3 states.
- The observer vector $\boldsymbol{L}$ will have as many columns as there are outputs, e.g.
$$\boldsymbol{L} = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{bmatrix}$$
- The design methodology, especially the observer error dynamics remain unchanged
$$A_o = A - LC$$