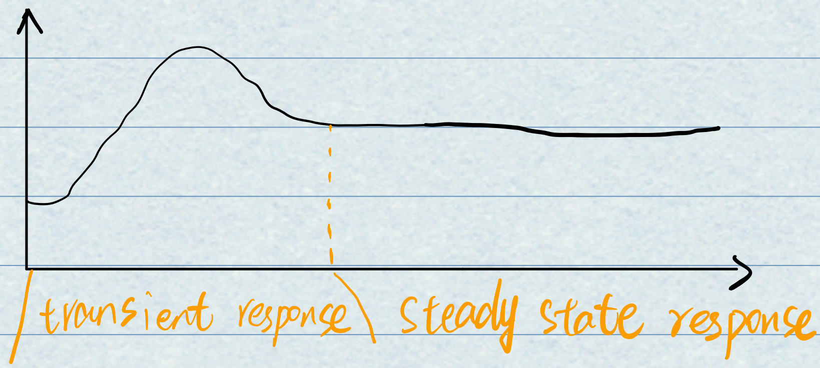
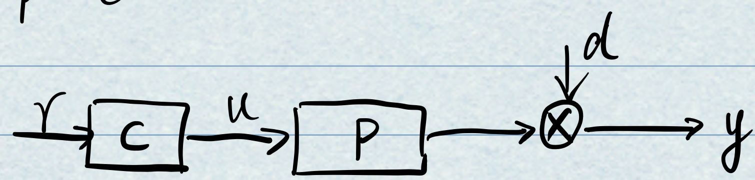

transient response
steady state response

Open loop Control



r: reference

C: controller

u: control signal

P: plant

d: disturbance

y: output

Control aim: minimise control error e

$$e = r - y \,, \quad \boxed{y = u \cdot p + d = cpr + d}$$

$$\therefore \; e = r - (cpr + d)$$

best controller: $c = 1/p$. then $e = -d$

How do plant changes the system

assume $d=0$, $C = 1/P$

nominal openloop transfer function : $T_{OL} = \frac{y}{r} = \frac{CPr}{r} = 1$

if $\tilde{P} = P + \delta P$,

then $\tilde{T}_{OL} = T_{OL} + \delta T_{OL} = \frac{C\tilde{P}r}{r} = 1 + \frac{\partial P}{P}$

$$\therefore \quad \frac{\delta T_{OL}}{T_{OL}} = \frac{\partial P}{P}$$

define $\frac{\partial T_{OL}}{T_{OL}} = S \frac{\partial P}{P}$, S: senstivity

For a open loop case. $\boxed{S=1}$, which means:

a 10% change in P will result in a 10% change in the overall system

Example : Pendulum

$m = 5kg, \quad l = 0.4 m, \quad c = 2 Nm/rad, \quad g = 9.81 m/s^2$

$I = \frac{1}{3}(mL^2)$

nature frequency : $w_n^2 = \frac{mgl}{2I} \Rightarrow w_n = 6.06$ rad/s

damping: $2\zeta w_n = \frac{c}{I} \Rightarrow \zeta = 0.61$

$$P(s) = \frac{1/I}{s^2 + 2\zeta w_n s + w_n^2} = \frac{3.75}{s^2 + 7.5s + 36.8} = \frac{Y(s)}{U(s)}$$

What is the steady state plant gain?

final value theory (终值定理) of laplace transform

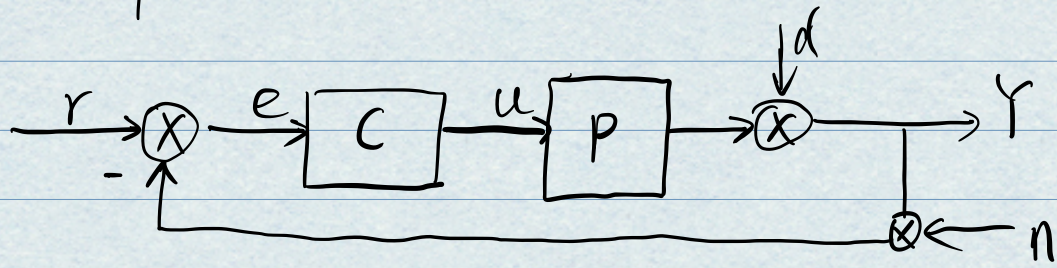
$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

input : step signal, $u(t) = 1, \ t > 0$; $u(s) = 1/s$

$$Y(s) = P(s) \cdot U(s) = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot P(s) = 0.1 = \frac{1}{9.81}$$

$\therefore$ Ideal controller : $C = \frac{1}{P_{ss}} = 9.81$

Closed loop control



n: measurement noise
e: error signal

Control aim: minimize control error e

$$e = r - (y+n)$$

assume $d=0$, $n=0$

* reference response: $r \to y$: $y = cpr - cpy = \frac{cp}{1+cp} r$

$\therefore G = \frac{y}{r} = \frac{cp}{1+cp}$

* disturbance response: $d \to y$

$y = \frac{1}{1+cp} d$, $G = \frac{y}{d} = \frac{1}{1+cp}$

* noise response : $n \to y$

$y = \frac{-cp}{1+cp} n$, $G = \frac{y}{n} = \frac{-cp}{1+cp}$

* overall response

$$\boxed{y = \frac{cp}{1+cp} r + \frac{1}{1+cp} d - \frac{cp}{1+cp} n}$$

Effect of changes to the plant P

$$\tilde{P} = P + \partial P, \quad T_{cl} = \frac{CP}{1+CP}, \quad \frac{\partial T_{cl}}{T_{cl}} = S\,\frac{\partial P}{P}$$

$$\therefore S = \frac{\partial T_{cl}/T_{cl}}{\partial P/P} = \frac{\partial T_{cl}}{\partial P}\cdot\frac{P}{T_{cl}}$$

$$\frac{\partial T_{cl}}{\partial P} = \frac{\partial \frac{CP}{1+CP}}{\partial P} = \frac{\partial CP}{\partial P}\cdot\frac{1}{1+CP} + \frac{\partial \frac{1}{1+CP}}{\partial P}\cdot CP$$

$$= \frac{C}{1+CP} - \frac{CP\cdot C}{(1+CP)^2} = \frac{C}{(1+CP)^2}$$

$$S = \frac{C}{(1+CP)^2}\cdot\frac{P}{\frac{CP}{1+CP}} = \frac{1}{1+CP}$$

$$\therefore \boxed{S = \frac{1}{1+CP}}$$

$$y = \underbrace{\frac{CP}{1+CP}}_{T} r + \underbrace{\frac{1}{1+CP}}_{S} d - \underbrace{\frac{CP}{1+CP}}_{T} n$$

T: complimentary senstivity , $S+T=1$

Example : Pendulum

$m = 0.5\ kg$

$l = 0.4\ m$

$C = 2\ Nm/rad$

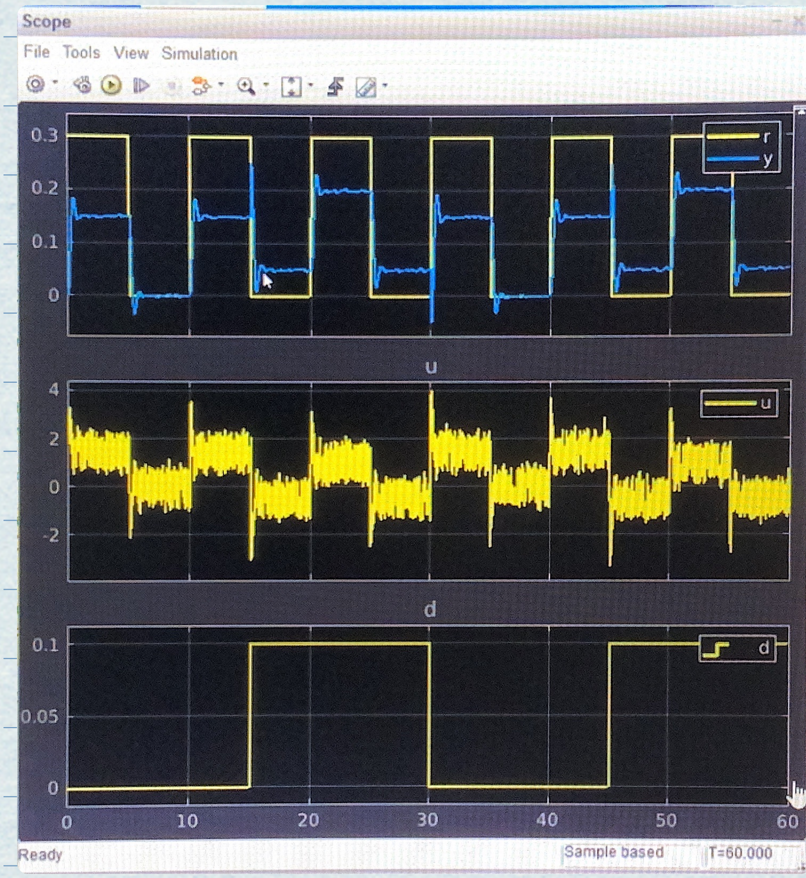
$$P(s) = \frac{3.75}{s^2 + 7.5s + 36.8}$$

$$P_{ss} = \lim_{s \to 0} P(s) = 0.1$$

* while $C = 1/P_{ss} = 10$

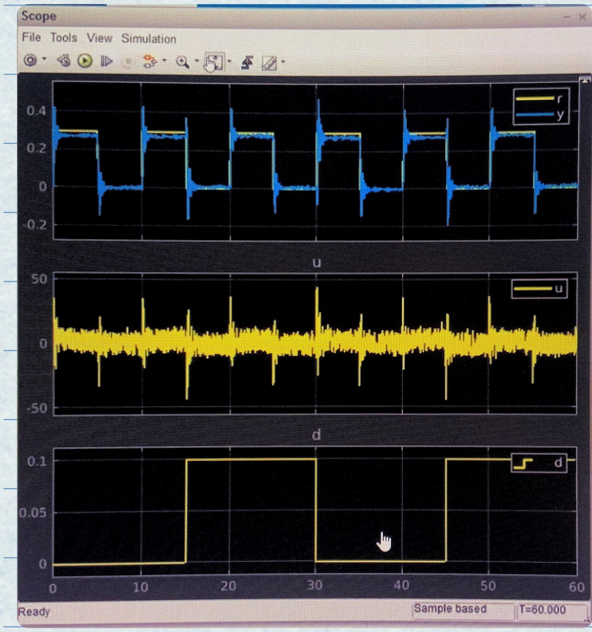
Loop gain $L = C \cdot P_{ss} = 1$, $y = \frac{CP}{1+CP} r + \frac{1}{1+CP} d - \frac{CP}{1+CP} n$

$$\therefore \quad y = \frac{1}{2} r + \frac{1}{2} d - \frac{1}{2} n$$



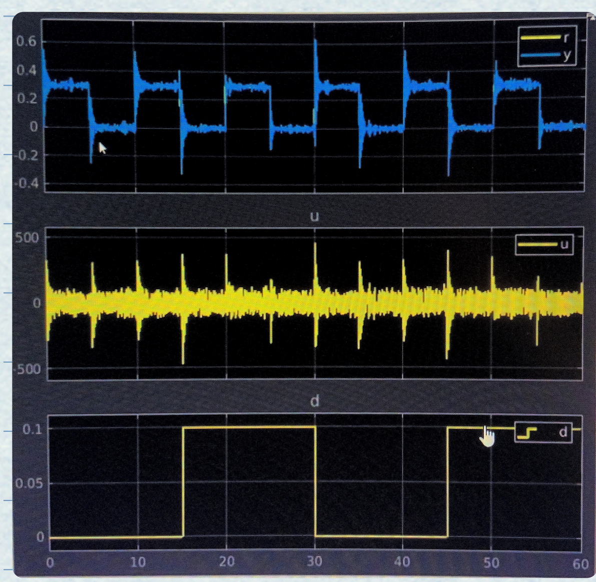
u : noise

* while $c = 100$, $L = cp = 10$, $y = \frac{10}{11} r + \frac{1}{11} d - \frac{10}{11} n$



hardly see the effect of d

* while $c = 1000$, $L = cp = 100$. $y = \frac{100}{101} r + \frac{1}{101} d - \frac{1}{101} n$



closed loop control and plant inversion

$S = \frac{C}{1+CP}$, for large controller gain, $C \gg 1$, $S = \frac{1}{P}$

* which means we don't have to choose $C = 1/P$ matches the plant only pick a large C

* high gain feedback control is equivalent to plant inversion