Modern Control – State Space Control

Tutorial Questions

State space and transfer function description

Q1 Consider the linear state space system

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \quad x(0)=x_0$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t)$$

Show how a linear transfer function *G(s)* can be derived.

Q2 Given the transfer function

$$G(s) = \frac{5}{s^2 + 3s + 12}$$

derive the state space description in controller / observer canonical form.

Q3 In the state space description given in question Q2, what is the requirement in terms of the properties of the matrices / vectors A, B, C and D for which the system is stable. What are the corresponding requirements for the transfer function G(s).

Controllability and Observability

Q4 What is controllability in the context of state feedback control and what is meant by stabilisability?

Q5 What is observability in the context of state estimation and what is meant by detectability?

Q6 Describe a test for controllability / observability of a state space model. Consider the following systems and derive whether they are controllable / observable.

System 1:

$$\dot{x} = \begin{bmatrix} -4 & 4 \\ 8 & 0 \end{bmatrix} x + \begin{bmatrix} 12 \\ -12 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

System 2:
$$\dot{x} = \begin{bmatrix} -13 & -2 \\ -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$
$$y = \begin{bmatrix} -1 & 1 \end{bmatrix} x$$

State feedback control

Q7 Describe the concept of state feedback control using a block diagram and derive the differential equation of the closed loop system.

Q8 How can a reference input be introduced to the state feedback control structure in question Q7?

Q9 What is controller design by pole assignment and how can it be used to design a state feedback controller?

Q10 Consider the state space system
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -12 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
Show if this system is controllable.

Q11 For the system in Q10, design a state feedback controller K using the pole placement method in such a way that the closed system has an overshoot of M_p=1% and a rise time of t_r=0.2 seconds. Consider a reference input r and show how the corresponding gain $\bar{N}$ can be derived so that the output reaches the reference in steady state. You can use the following equations to derive the natural frequency and the damping of the desired closed loop system:
$$\omega_n \cong \frac{1.8}{t_r} \quad \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + \left(\ln M_p\right)^2}};\ 0 \le \xi < 1$$

State estimator feedback control

Q12 What is meant by state-estimator feedback control. Use a block diagram to illustrate your explanations and mark the elements which form the compensator. What are the advantages of using a state estimator?

Q13 Consider the state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Show if this system is observable.
Design a state estimator L using the pole placement method in such a way that the observer poles are located at -10 and -20.

Q14 What is meant by the Separation Theorem in the context of state-estimator feedback control?