Universal computation by multiparticle quantum walk with improved error bounds

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Abstract

Quantum walk has proven itself a powerful resource for quantum computing. In the single particle model, Childs [1] showed that on an exponential sized graph, quantum walk is universal for quantum computation. Childs, Gosset, and Webb [2] then showed the universality of multiparticle quantum walk on polynomially sized graphs. In this work, we improve the work of [2] by improving their error bounds.
In particular, [2] used graph scattering with square wavepackets to encode the qubits. By encoding each qubit via a dual-rail encoding, they were able to use single particle and particular two-particle scattering behavior to analyze the many-particle behavior with high precision. However, to ease analysis they used square wavepackets to encode each qubit. In this work, we use the same construction as in [2], but use cutoff Gaussian wavepackets instead. This then leads to a near quadratic improvement in the resulting error bounds.

Single particle quantum walk

For our purposes, we will use continuous time quantum walk. For a given graph G, this means that the time evolution will simply be determined by taking the adjacency matrix of G as the Hamiltonian for the system:

$$U(t) = \exp(-iA(G)t).$$

With this, the underlying Hilbert space is spanned by vertex states $\{|v\rangle, v \in V(G)\}$.
With this time evolution, we will be interested in understanding the eigenstates of $A(G)$. As a particular example, if G is an infinite path (think of a discretization of the real line), we have that the eigenstates are of the so-called "momentum" states:

$$|\tilde{k}\rangle = \sum_{x\in\mathbb{Z}} e^{-ikx}|x\rangle \qquad k \in [-\pi, \pi).$$

These states move at a speed that depends on k, in a similar manner to a one-dimensional free particle.

Multiparticle quantum walk

To generalize quantum walk to multiple particles, we will assume that there is some finite-range interaction between particles, that only depends on the number of particles on two vertices, and the distance between the two vertices. Namely, we will assume that the N-particle Hilbert space for a quantum walk on a graph G is spanned by states of the form

$$\big\{|v_1, v_2, \cdots, v_N\rangle : v_i \in V(G)\big\}.$$

The Hamiltonian governing the time evolution is then given by

$$H_G^N = H_{\text{move}} + H_{\text{int}}$$

where

$$H_{\text{move}} = A(G) \otimes \mathbb{I} \otimes \mathbb{I} \otimes \cdots \otimes \mathbb{I} + \cdots + \mathbb{I} \otimes \cdots \otimes \mathbb{I} \otimes A(G)$$

and

$$H_{\text{int}} = \sum_{d=0}^{R} \sum_{\substack{u,v\in V(G)\\ d(u,v)=d}} U_d(\hat{n}_u, \hat{n}_v)$$

where R is the maximum range of interaction, $\hat{n}_u$ counts the number of particles located on vertex u, and U_d are symmetric polynomials.
Note that this framework easily includes such things as the Bose-Hubbard model and nearest neighbor interactions. Similarly, as the Hamiltonian is symmetric, we can restrict ourselves to bosonic or fermionic particles without problems.

Graph scattering

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Two-particle scattering

Universal Construction

Improvements

References

[1] Andrew M. Childs, *Universal computation by quantum walk*, Physical Review Letters **102** (2009), no. 18, 180501, arXiv:0806.1972.
[2] Andrew M. Childs, David Gosset, and Zak Webb, *Universal computation by multiparticle quantum walk*, Science **339** (2013), no. 6121, 791–794, arXiv:1205.3782.
[3] Zak Webb, *The computational power of many-body systems*, Ph.D. thesis, University of Waterloo, 2016.