DEQUANTIFYING COMPLEXITY:

CLASSICALLY ACCESSIBLE PROBLEMS FOR QUANTUM CLASSES

Midsouth Theory Day

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QUANTUM IS SPOOKY!

Entanglement, many-worlds, measurement-problem

Infinite dimensional spaces

DON'T LISTEN TO THE HYPE

Essentially just linear algebra

Albeit on exponential sized vector-spaces

BASIC DEFINITIONS

Quantum state $|\psi\rangle \in \mathrm{H} = \mathbb{C}^{2^n}$

- 2-norm of $|\psi\rangle$ is 1
- $\langle\psi|$ is corresponding linear form

Unitary U

- 2-norm preserving linear operator on H
- Describes evolution of quantum states

Hamiltonian H

- Hermitian linear operator on H
- Eigenvalues correspond to energy

- $U(t) = \exp(-iHt)$

BQP: A promise problem $(\Pi_{\text{yes}}, \Pi_{\text{no}})$ is in **BQP** if $\exists$ a uniform family of poly-sized quantum circuits $\{C_n\}_{n\in\mathbb{N}}$:

- if $x \in \Pi_{\text{yes}}$ then $\Pr\left[C_{|x|} \text{ accepts } |x\rangle\right] > \frac{2}{3}$
- if $x \in \Pi_{\text{no}}$ then $\Pr\left[C_{|x|} \text{ accepts } |x\rangle\right] < \frac{1}{3}$

QMA: A promise problem $(\Pi_{\text{yes}}, \Pi_{\text{no}})$ is in **QMA** if $\exists$ a uniform family of poly-sized quantum circuits $\{C_n\}_{n\in\mathbb{N}}$:

- if $x \in \Pi_{\text{yes}}$ then $\exists\, |\phi_x\rangle, \Pr\left[C_{|x|} \text{ accepts } |x, \phi_x\rangle\right] > \frac{2}{3}$
- if $x \in \Pi_{\text{no}}$ then $\forall\, |\psi\rangle, \Pr\left[C_{|x|} \text{ accepts } |x, \psi\rangle\right] < \frac{1}{3}$

Think about definition of **P** and **NP** using circuits.

BQP $\approx$ P

QMA $\approx$ NP

(Include possibility of error, make circuits quantum, and use a quantum proof)

USUAL COMPLETE PROBLEMS

BQP: Usually deals with applying a unitary to a fixed quantum state.

QMA: Usually related to smallest eigenvalue of a given Hamiltonian.

Known relations to common classes

- **BPP** $\subset$ **BQP**
- **QMA** $\subset$ **PP**

There exist oracles relative to which:

- **BQP** $\not\subset$ **MA**
- **NP** $\not\subset$ **BQP**

QUANTUM RESOURCES ARE HARD TO COMPARE TO CLASSICAL!

Try to create more "classical" problems

QUICK ASIDE ON QUANTUM WALK

Quantum generalization of Random Walk

Require evolution to be reversible

Use adjacency matrix as Hamiltonian!

$$U(t) = \exp(-iA(G)t)$$

SUCCINCTLY REPRESENTED GRAPHS!

Each vertex of G corresponds to a single dimension

A d-degree graph G is **row-computable** if:

- $\exists f : V(G) \times [d] \to V(G)$
- $f(u, i)$ gives the ith neighbor of u

We will be interested in d-degree, row-computable, 2^n vertex graphs.

s, t-TRANSPORT PROBLEM

Given two vertices s and t, does a QW take $|s\rangle$ to $|t\rangle$?

Input:

- A d-degree, row-computable, N-vertex graph G
- Two vertices s and t
- A total evolution time, T
- Minimal probability, p
- Promise on acceptance difference, ϵ^{-1}

BQP-COMPLETE TO DETERMINE WHETHER:

- $$|\langle s| \exp(-iA(G)T)|t\rangle|^2 > p + \epsilon$$
- $|\langle s| \exp(-iA(G)T)|t\rangle|^2 < p$

ADJACENCY MATRIX SMALLEST EIGENVALUE PROBLEM

Given a graph, what is its smallest eigenvalue?

Input:

- A d-degree, row-computable, N-vertex graph G
- Minimal energy, a
- Promise on acceptance difference, ϵ^{-1}

QMA-COMPLETE TO DETERMINE WHETHER:

- $\lambda_1(A(G)) < a$
- $\lambda_1(A(G)) > a + \epsilon$

LOTS OF OPEN QUESTIONS

Other "classical" problems

NEXP-complete row-computable succinct graph problems

Adjacency matrix smallest eigenvalue on explicit graphs

THANK YOU