

Table of Contents

List of Figures	viii
1 Introduction	1
1.1 Quantum walk	1
1.2 Many-body systems	1
1.3 Computational complexity	1
1.4 Hamiltonian Complexity	1
1.5 Layout of thesis	1
2 Single-particle scattering on graphs	2
2.1 Introduction and motivation	2
2.1.1 Infinite path	2
2.1.2 Scattering off of a graph	2
2.2 Using scattering for simple computation	2
2.2.1 Momentum switches	2
2.2.2 Applying an encoded unitary	2
2.2.3 Degree-3 graphs are sufficient	2
2.3 Finite truncation	2
2.4 Universal computation?	2
2.4.1 Encoded two-qubit gates	2
2.4.2 Single-qubit blocks	2
2.4.3 Combining blocks	2
3 Multi-particle scattering on graphs	3
3.1 Multi-particle quantum walk	3
3.2 Two-particle scattering on an infinite path	3
3.3 Applying an encoded $C\theta$ -gate	3
3.3.1 Finite truncation	3
3.3.2 Construction of $C\theta$ -gate	3
3.4 Impossibility of some momentum switches	3
3.5 Universal Computation	3
3.5.1 Two-qubit blocks	3
3.5.2 Combining blocks	3
3.6 Improvements and Modifications	3

4	Ground energy of quantum walk	4
4.1	Encoding computations as states	4
4.1.1	History states	4
4.2	Determining ground energy of a sparse adjacency matrix is QMA-complete .	4
4.2.1	Kitaev Hamiltonian	4
4.2.2	Transformation to Adjacency Matrix	4
5	Ground energy of multi-particle quantum walk	5
5.1	Introduction	5
5.1.1	Containment in QMA	5
5.1.2	Reduction to frustration-free case	5
5.2	Constructing the underlying graph for QMA-hardness	5
5.2.1	Gate graphs	5
5.2.1.1	The graph g_0	5
5.2.1.2	Gate graphs	5
5.2.1.3	Frustration-free states for a given interaction range	6
5.2.2	Gadgets	6
5.2.2.1	The move-together gadget	6
5.2.2.2	Two-qubit gate gadget	6
5.2.2.3	Boundary gadget	6
5.2.3	Gate graph for a given circuit	6
5.2.3.1	Occupancy constraints graph	6
5.3	Proof of QMA-hardness for MPQW ground energy	6
5.3.1	Overview	6
5.3.2	Configurations	6
5.3.2.1	Legal configurations	6
5.3.3	The occupancy constraints lemma	6
5.3.4	Completeness and Soundness	6
5.4	Open questions	6
6	Conclusions	7
6.1	Open Problems	7
	References	8