Hydrodynamics

1D Hydrodynamics (Equation 2)

The 1D conservation form is:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$

Where:

$$\mathbf{U} = \{\rho, \rho v, E\}, \quad \mathbf{F} = \left\{\rho v, \rho v^2 + P, (E+P)v\right\}$$

Nondimensionalization

Define the characteristic scales:

$$L \quad \text{(Length scale)}, \quad U \quad \text{(Velocity scale)}, \quad T = \frac{L}{U} \quad \text{(Time scale)}.$$

The nondimensional variables are:

$$x' = \frac{x}{L}, \quad t' = \frac{t}{T}, \quad v' = \frac{v}{U}, \quad \rho' = \frac{\rho}{\rho_0}, \quad P' = \frac{P}{P_0}, \quad E' = \frac{E}{E_0}$$

Substitute these into the 1D conservation equations:

1. Mass conservation:

$$\frac{\partial \rho'}{\partial t'} + \frac{\partial}{\partial x'}\left(\rho' v'\right) = 0$$

2. Momentum conservation:

$$\frac{\partial (\rho' v')}{\partial t'} + \frac{\partial}{\partial x'}\left(\rho' v'^2 + P'\right) = 0$$

3. Energy conservation:

$$\frac{\partial E'}{\partial t'} + \frac{\partial}{\partial x'}\left(v'(E' + P')\right) = 0$$

2D Hydrodynamics (Equation 14)

The 2D conservation form is:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0$$

Where:

$$\mathbf{U} = \{\rho, \rho v_x, \rho v_y, E\}\,, \quad \mathbf{F} = \begin{pmatrix} \rho v_x \\ \rho v_x^2 + P \\ \rho v_x v_y \\ (E+P)v_x \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho v_y \\ \rho v_x v_y \\ \rho v_y^2 + P \\ (E+P)v_y \end{pmatrix}$$

Nondimensionalization

Define the characteristic scales:

$$L_x, L_y \quad \text{(Length scales in } x \text{ and } y\text{)}, \quad U \quad \text{(Velocity scale)}, \quad T = \frac{L_x}{U} \quad \text{(Time scale)}.$$

The nondimensional variables are:

$$x_i' = \frac{x_i}{L_i}, \quad t' = \frac{t}{T}, \quad v_x' = \frac{v_x}{U}, \quad v_y' = \frac{v_y}{U}, \quad \rho' = \frac{\rho}{\rho_0}, \quad P' = \frac{P}{P_0}, \quad E' = \frac{E}{E_0}$$

Substitute these into the 2D conservation equations:
1. Mass conservation:

$$\frac{\partial \rho'}{\partial t'} + \frac{1}{L_x}\frac{\partial}{\partial x'}(\rho' v_x') + \frac{1}{L_y}\frac{\partial}{\partial y'}(\rho' v_y') = 0$$

2. Momentum conservation in the x-direction:

$$\frac{\partial(\rho' v_x')}{\partial t'} + \frac{1}{L_x}\frac{\partial}{\partial x'}(\rho' v_x'^2 + P') + \frac{1}{L_y}\frac{\partial}{\partial y'}(\rho' v_x' v_y') = 0$$

3. Momentum conservation in the y-direction:

$$\frac{\partial(\rho' v_y')}{\partial t'} + \frac{1}{L_x}\frac{\partial}{\partial x'}(\rho' v_x' v_y') + \frac{1}{L_y}\frac{\partial}{\partial y'}(\rho' v_y'^2 + P') = 0$$

4. Energy conservation:

$$\frac{\partial E'}{\partial t'} + \frac{1}{L_x}\frac{\partial}{\partial x'}\left(v_x'(E' + P')v_x'\right) + \frac{1}{L_y}\frac{\partial}{\partial y'}\left(v_y'(E' + P')v_y'\right) = 0$$