تعلم

Ta3lom.net

Probabilities & Statistics

Probability Concept

Probability Concept:

- **event**

  Probability of event is 0.5 = 50%

- **Sample Space ≡ $S \equiv \Omega$ ≡ set of all possible outcomes of an experiment.**
- ***S* consists of all the things that can be happen when one takes a sample.**

Exercise (1):

If a government agency must decide where to locate two new computer research facilities and that (for a certain purpose) it is of interest to indicate how many of them will be located in Texas and how many in California.

$$S = \{(0,0),(1,0),(0,1),(2,0),(0,2),(1,1)\}$$

Number of computers will be located in Texas

Number of computers will be located in California

Exercise (2):

Flip an unbiased coin two times and observe the sequences of heads and tails.

(a) Experiment: flipping (tossing) of the coin two times.

Random experiment ≡ probabilistic experiment ≡ unpredictable experiment ⇒ (≡ outcome is NOT unique)

$$(b)\ S \equiv \Omega = \left\{ \underbrace{(H,H)}_{outcome},(H,T),(T,H),(T,T) \right\}$$

Exercise (3):

Boiling temperature of pure water at sea level is **deterministic**.

Predictable experiment ≡ deterministic experiment ⇒ (≡ outcome is unique)

Exercise (4):

Roll a die one time.

Random experiment

$$\Omega = \{1,2,3,4,5,6\}$$

Exercise (5):

Roll a die and flip a coin.

Random experiment

$$\Omega = \begin{Bmatrix} (1,H),(2,H),(3,H),(4,H),(5,H),(6,H) \\ (1,T),(2,T),(3,T),(4,T),(5,T),(6,T) \end{Bmatrix}$$

Exercise (6):

Experiment consists of measuring in hours the lifetime of a flower.

$$\Omega = \{t : t \geq 0\}$$

one outcome
one and only one

Discrete *S* and Continuous *S*:

If persons checking the Nitrogen Oxide emission of cars are interested in the number of cars they have to inspect before the first one that does NOT meet government regulations, it could be the first, the second,…., the 15th, …., and for all we know they may have to check thousands of cars before they find one that does NOT meet government regulations NOT knowing how far they may have to go.

$$S = \{\text{natural numbers}\} = \{0,1,2,3,....,\infty\}$$

Cars are countable

If they were interested in the N_2O emission of a given car in grams per mile.

$$\begin{aligned} S &= \{\text{all points on a continuous scale}\} \\ &= \{\text{certain interval on the line of real numbers}\} \\ &\equiv \text{contiuum} \end{aligned}$$

- Each outcome in S is called element or member of S or sample point ≡ ω.
- Event: set of sample points, which is a subset of S.

  If A is an event $\Rightarrow$ A has occurred if it contains the outcome that occurred.
- An event is called **elementary** or **simple event** if it contains **exactly one outcome** of the experiment.

Exercise (7):

Experiment consists of tossing 3 coins, and the observed face of each coin is of interest.

$$S=\begin{Bmatrix}(H,H,H),(H,H,T),(H,T,H),(H,T,T)\\(T,H,H),(T,H,T),(T,T,H),(T,T,T)\end{Bmatrix}$$

The subset

$$A=\{(H,H,H),(H,H,T),(H,T,H),(T,H,H)\}$$

contains the outcomes that correspond to the event of obtaining at least 2 heads.

If one of the outcomes in A occurs, then we say that the event A has occurred.

The subset

$$B=\{(H,H,T),(H,T,H),(T,H,H)\}$$

contains the outcomes that correspond to the event of obtaining exactly 2 heads.

OR:

contains the outcomes that correspond to the event of obtaining exactly 1 tail.

Back to Exercise (1) (Texas and California):

$$C=\{(1,0),(0,1)\}$$

Event ≡ Texas and California will get one of the two research facilities.

$$D=\{(0,0),(0,1),(0,2)\}$$

Event ≡ Texas will NOT get either of the two research facilities.

$$E=\{(0,0),(1,1)\}$$

Event ≡ Texas and California will get equally many of the facilities.

- **Empty** set ≡ ϕ ≡ set has NO elements at all.

In many problems, we are interested in events which can be expressed in terms of two or

Suppose that Ω ($\equiv S$) is the sample space and that A_1 and A_2 are events in Ω:

1- The **impossible** event, denoted by ϕ, is defined as the event that contains NO outcomes and therefore can NOT occur.

2- The event $\bar{A}_1$, called the **complement** of A_1, is the event that A_1 does NOT occur and consists of all outcomes in Ω which are NOT in A_1.

3- An event A_1 is said to be a subset of A_2, written as $A_1 \subset A_2$, if the occurrence of A_1 necessarily implies the occurrence of A_2.
In order for this to be true, every outcome in A_1 must belong to A_2.

4- Two events are said to be **equal** if and only if (iff) A_1 is a subset of A_2 and A_2 is a subset of A_1; i.e., $A_1 = A_2$ iff $A_1 \subset A_2$ and $A_2 \subset A_1$.

5- The **intersection** of A_1 and A_2 ($A_1 \cap A_2$) is defined as consisting of the outcomes that belong to BOTH A_1 and A_2; consequently the event $A_1 \cap A_2$ is said to occur iff BOTH A_1 and A_2 occur.

6- The **union** of A_1 and A_2 ($A_1 \cup A_2$) is defined as consisting of the outcomes that belong to AT LEAST ONE of the events A_1 and A_2; consequently the event $A_1 \cup A_2$ is said to occur iff either A_1 OR A_2 occurs or iff AT LEAST ONE of them occurs.

7- Any two events A_1 and A_2 that can NOT occur simultaneously $\Rightarrow$ their intersection is the impossible event, A_1 and A_2 are said to be **mutually exclusive events** or **disjoint events** $\Rightarrow$ A_1 and A_2 are disjoint events iff $A_1 \cap A_2 = \phi$.

$\cup$ ≡ or $\cap$ ≡ and

Exercise (8):

Let A and B be two events of a sample space Ω.

(1) $\bar{A}$: A does NOT occur.

(2) $A \cap B$: Both A and B occur.

(3) $A \cup B$: At least one of them occurs.

(4) $A \cap \bar{B}$: A occurs but B does NOT occur.

(5) $\bar{A} \cap \bar{B}$: None of the events will occur.

(6) $(A \cap \bar{B}) \cup (\bar{A} \cap B) = A \Delta B$: A occurs and B does NOT occur or B occurs and A does NOT occur.

Back to Exercise (1) (Texas and California):

$C \cup E = \{(0,0),(1,1),(1,0),(0,1)\}$

$C \cap D = \{(0,1)\}$

$\overline{D} = D' = \{(1,0),(1,1),(2,0)\}$

$\overline{D} \cap D = \phi \qquad \overline{D} \cup D = \Omega$

Venn Diagram:

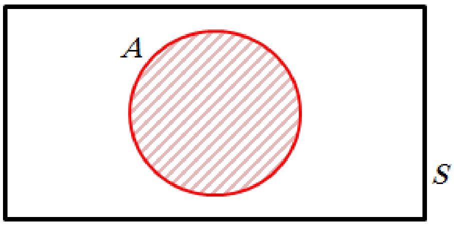
Sample spaces and events, particularly relationships among event, are often depicted by means of Venn diagrams.

- ✓ S is represented by a rectangle
- ✓ Events are represented by regions within the rectangle, usually by circles or parts of circles.

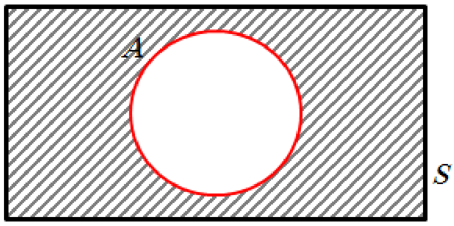
Exercise (9):

If A is the event that a certain student is taking a course in Calculus, and B is the event that the student is taking a course in Physics, what events are represented by the shaded regions of the following four Venn diagrams?

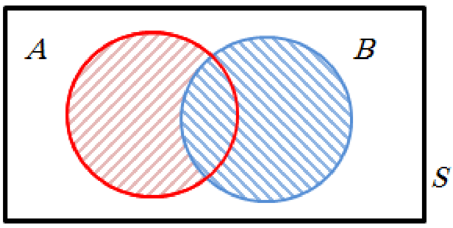
(1)



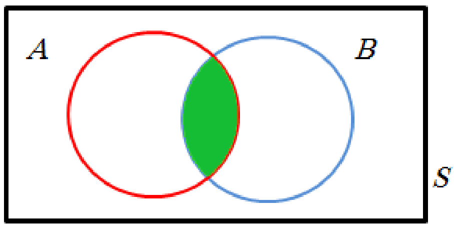
(2)



(3)



(4)



(1) Event: student is taking a course in Calculus.

(2) Event: student is NOT taking a course in Calculus.

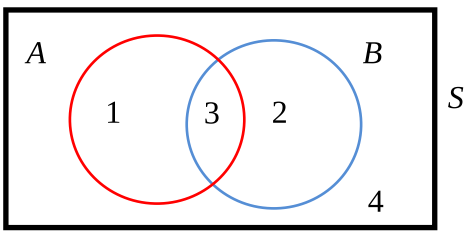
(3) Event: student is taking a course in Calculus or a course in Physics ($A \cup B$).

[AT LEAST ONE OF THEM OCCURS]

(4) Event: student is taking Calculus and Physics together ($A \cap B$).

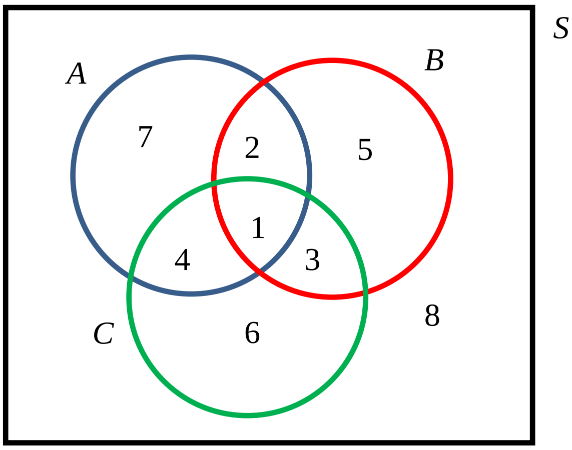
Exercise (10):

Prove that $(A \cup B)' = A' \cap B'$



Exercise (11):

A manufacturer of small motors is concerned with 3 major types of defects. If A is the event that the shaft size is too large, B is the event that the windings are improper, and C is the event that the electrical connections are unsatisfactory, express in words what events are represented by the following regions of the following Venn diagrams.



Region (2): contained A and B but NOT C.

$\Rightarrow$ represents the event: shaft size is too large and windings are improper.

Region (1) and (3) together: contained B and C.

$\Rightarrow$ represents the event: windings are improper and electrical connections are unsatisfactory.

Regions (3), (5), (6), and (8) ≡ $\bar{A}$

$\Rightarrow$ represents the event: shaft size is NOT too large.

Exercise (12):

Let A, B, and C be three events:

$\overline{\overline{A}} = A$

$A \cup A = A \cap A = A$

$A \cap \phi = \phi$

$A \cap \Omega = A$

$A \cap \overline{A} = \phi$

$A \cup \phi = A$

$A \cup \Omega = \Omega$

$A \cup \overline{A} = \Omega$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Counting:

At times, it can be difficult to determine the number of elements in a finite sample space by direct enumeration.

$\Rightarrow$ The counting way is used (tree diagram) 

Exercise (13):

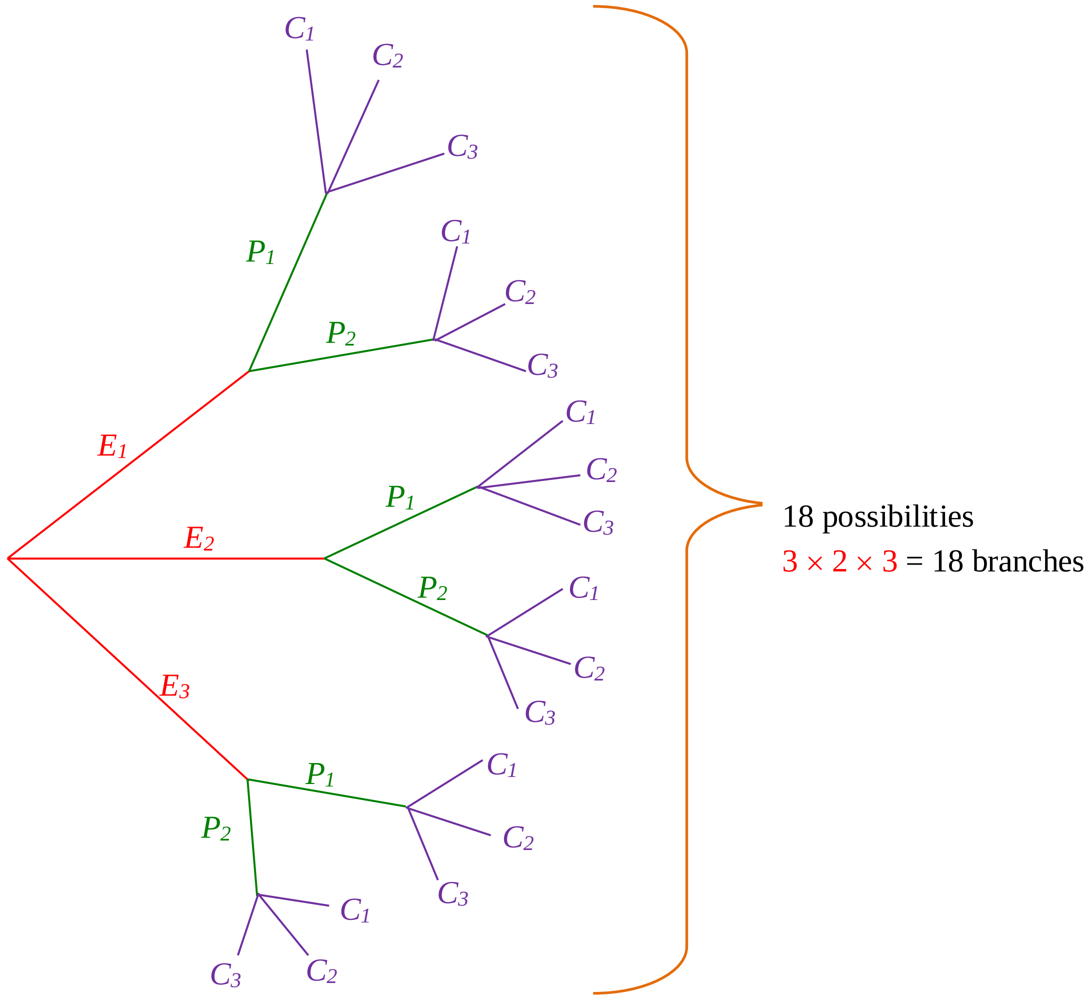
Suppose that a consumer testing service rates lawn mowers as being easy, average, or difficult to operate; as being expensive or inexpensive; and as being costly, average, or cheap to repair. In how many different ways can a lawn mower be rated by this testing service?

Let E_1, E_2, and $E_3 \Rightarrow$ ease of operation

P_1, $P_2 \Rightarrow$ price

C_1, C_2, and $C_3 \Rightarrow$ Cost of repair

$$\Omega = \begin{Bmatrix} (E_1,P_1,C_1),(E_1,P_1,C_2),(E_1,P_1,C_3),(E_1,P_2,C_1),(E_1,P_2,C_2),(E_1,P_2,C_3), \\ (E_2,P_1,C_1),(E_2,P_1,C_2),(E_2,P_1,C_3),(E_2,P_2,C_1),(E_2,P_2,C_2),(E_2,P_2,C_3), \\ (E_3,P_1,C_1),(E_3,P_1,C_2),(E_3,P_1,C_3),(E_3,P_2,C_1),(E_3,P_2,C_2),(E_3,P_2,C_3) \end{Bmatrix}$$



Theorem:

If sets $A_1, A_2\ldots A_k$ contain, respectively, $n_1, n_2\ldots, n_k$ elements, there are $n_1\times n_2\times \ldots\times n_k$ ways of choosing first an element of A_1, then an element of A_2, …, and finally an element of A_k.

Exercise (14):

In how many different ways can a union local with a membership of 25 choose a vice president and a president?

$$\left.\begin{array}{l}\text{vice president} \Rightarrow 25 \text{ ways}\\ \text{president} \Rightarrow 24 \text{ ways}\end{array}\right\} 25\times 24 = 600 \text{ ways in which the whole choice can be made}$$

Exercise (15):

If a test consists of 12 true – false questions, in how many different ways can a student mark the test paper with one answer to each question?

Since each question can be answered in 2 ways

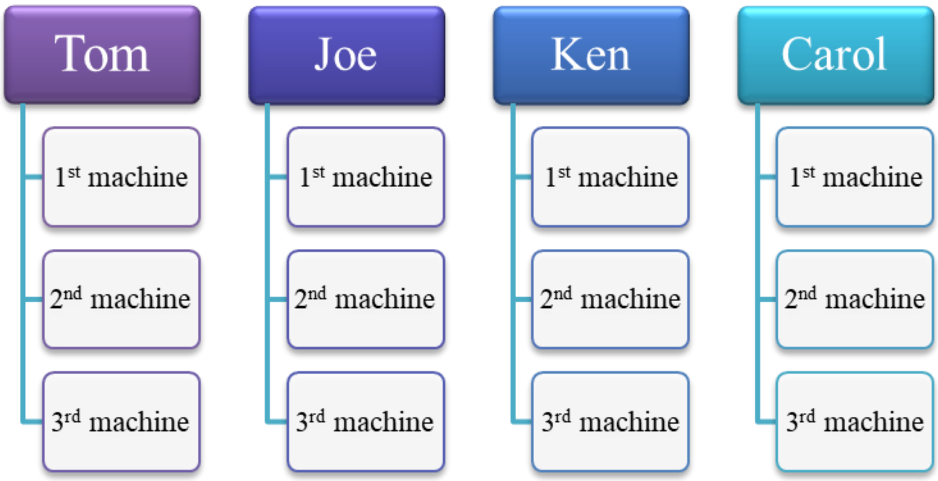
$\Rightarrow 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2=2^{12}=4096$ possibilities

Exercise (16):

A manufacturer is experiencing difficulty getting consistent readings of tensile strength between 3 machines located on the production floor, research lab, and quality control lab respectively. There are also 4 possible technicians – Tom, Joe, Ken, and Carol – who operate at least one of the test machines regularly.

(a) How many operator machine pairs must be included in a designed experiment where <u>every operator tries every machine</u>?

$\Rightarrow 4_{\text{op}}\times3_{\text{machine}}=12$ pairs are required



(b) If each operator – machine pair is required to test 8 specimens, how many test specimens are required for the entire procedure?

NOTE: A specimen is destroyed when its tensile strength is measured.

$12 \text{ pairs}\times8=96$ test

Exercise (17):

Two persons – A and B – want to sit on 3 chairs

$$\left.\begin{array}{l} A: 3 \text{ choices} \\ B: 2 \text{ choices} \end{array}\right\} 3\times 2 = 6 \text{ ways that } A \text{ and } B \text{ can sit on 3 chairs}$$

A		B
A	B	
	A	B
B	A	
	B	A
B		A

Permutations: ORDER is important

Theorem:

The number of ways in which of r objects can be selected from a set of n distinct objects is:

$$_nP_r = n(n-1)(n-2)....(n-r+1)$$
$$= \frac{n!}{(n-r)!} = \binom{n}{r}$$

Exercise (18):

In how many different ways can one make a 1st, 2nd, 3rd, or 4th choice among 12 firms leasing construction equipment?

$n = 12$

$r = 4$

$$_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12\times 11\times 10\times 9\times 8!}{8!} = 12\times 11\times 10\times 9 = 11880 \text{ ways}$$

Exercise (19):

An electronic controlling mechanism requires 5 identical memory chips. In how many ways can this mechanism be assembled by placing the 5 chips in the 5 positions within the controller.

$n = 5$

$r = 5$

$$_5P_5 = \frac{5!}{(5-5)!} = \frac{5\times 4\times 3\times 2\times 1\times 0!}{0!} = 5\times 4\times 3\times 2\times 1 = 120 \text{ ways}$$

Combinations: ORDER is NOT important

Theorem:

The number of ways in which of r objects can be selected from a set of n distinct objects is:

$${}_nC_r = \binom{n}{r} = \frac{n(n-1)(n-2)....(n-r+1)}{r!}$$

$$= \frac{n!}{r!(n-r)!}$$

Exercise (20):

In how many ways cab 3 of 20 laboratory assistants be chosen to assist with an experiment?

$${}_{20}C_3 = \binom{20}{3} = \frac{20!}{3!(20-3)!} = 1140 \text{ ways}$$

Exercise (21):

A calibration study needs to be conducted to see if the readings on 15 test machines are giving similar results. In how many ways can 3 of the 15 be selected for the initial investigation?

$${}_{15}C_3 = \binom{15}{3} = \frac{15!}{3!(15-3)!} = 455 \text{ ways}$$

NOTE: 3 machines selected ≡ 12 machines NOT selected

$$\binom{15}{3} = \binom{15}{12}$$

$$\frac{15!}{3!(15-3)!} = \frac{15!}{12!(15-12)!}$$

Exercise (22):

In how many different ways can the director of a research laboratory choose 2 chemists from among 7 applicants and 3 physicists from among 9 applicants?

Chemists $\Rightarrow {}_7C_2 = 21$ ways

Physicists $\Rightarrow {}_9C_3 = 84$ ways

Probability:

Classical Concept:

If there are n *equally likely* possibilities, of which one must occur and s are regarded as favorable, or such as a "success" then the probability of a "success" is given by S/N.

Exercise (23):

What is the probability of drawing an ace from a well – shuffled deck of 52 playing card?

There are 4 aces

$$\frac{s}{n} = \frac{4}{52} = 0.077$$

Exercise (24):

If 3 of 20 tires in storage are defective and 4 of them are randomly chosen for inspection (that is, each tire has the same chance of being selected), what is the probability that *only one* of the defective tires will be included?

To know Ω

$$\rightarrow n = {}_{20}C_4 = \binom{20}{4} = 4845 \text{ ways for choosing 4 of the 20 tires}$$

$$s = {}_3C_1 \times {}_{17}C_3 = \binom{3}{1}\binom{17}{3} = 2040$$

$$\Rightarrow \text{prob.} = \frac{2040}{4845} = 0.42 = 42\%$$

Axioms of Probability:

Axiom (1): $0 \le P(A) \le 1$ for each event A in S.

Axiom (2): $P(S) = 1 = \frac{n}{n}$

Axiom (3): If A and B are any disjoint events in S

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{s_1}{n} + \frac{s_2}{n}$$

Exercise (25):

For football game:

If $P(\text{team win}) = 0.3$

$P(\text{team loss}) = 0.12$

$P(\text{team equality}) = ?$

$P(\text{team win}) + P(\text{team loss}) + P(\text{team equality}) = 1$

$\rightarrow P(\text{team equality}) = 0.58 = 58\%$

Exercise (26):

If an experiment has three possible and mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probabilities is permissible.

(a) $P(A) = {}^{1}/_{3}$ $P(B) = {}^{1}/_{3}$ $P(C) = {}^{1}/_{3}$

permissible because

$0 \leq P(A) \leq 1$

$0 \leq P(B) \leq 1$

$0 \leq P(C) \leq 1$

and $P(A) + P(B) + P(C) = 1$

(b) $P(A) = 0.64$ $P(B) = 0.38$ $P(C) = -0.02$

NOT permissible because $P(C)$ is negative

(c) $P(A) = 0.35$ $P(B) = 0.52$ $P(C) = 0.26$

NOT permissible

$0 \leq P(A) \leq 1$

$0 \leq P(B) \leq 1$

$0 \leq P(C) \leq 1$

and $P(A) + P(B) + P(C) > 1$

(d) $P(A) = 0.57$ $P(B) = 0.24$ $P(C) = 0.19$

permissible because

$0 \le P(A) \le 1$

$0 \le P(B) \le 1$

$0 \le P(C) \le 1$

and $P(A) + P(B) + P(C) = 1$

Elementary Theory:

If $A_1, A_2, \ldots, A_n$ are mutually exclusive events in a sample space S, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

Exercise (27):

The probability that a consumer testing service will rate a new antipollution device for cars very poor, poor, fair, good, very good, or excellent are 0.07, 0.12, 0.17, 0.32, 0.21, and 0.11. What are the probabilities that it will rate the device?

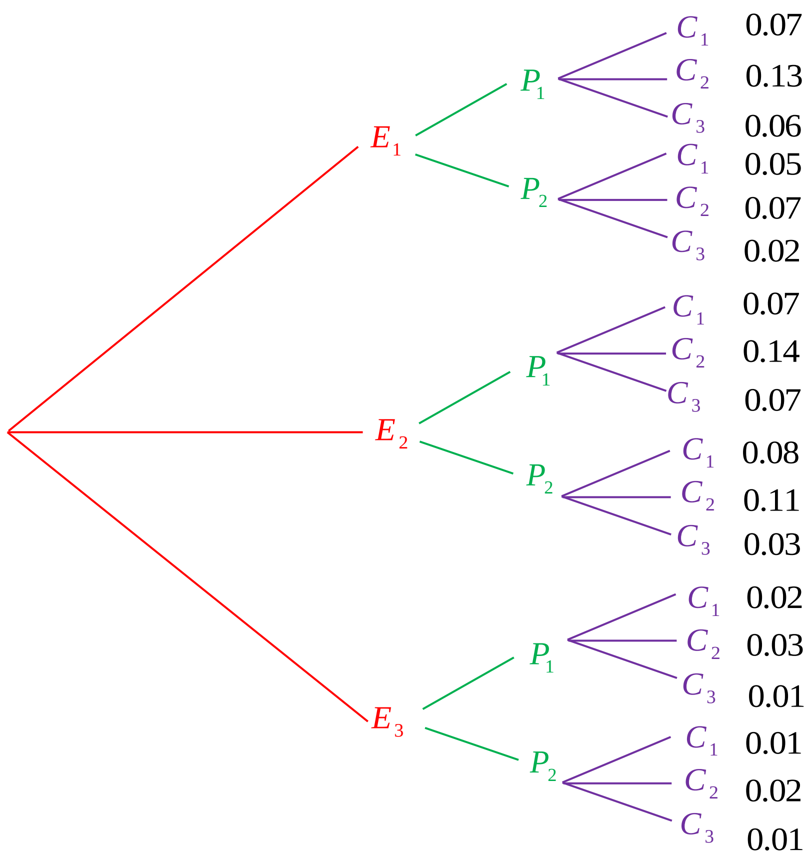
(a) Very poor, poor, fair, or good

$0.07 + 0.12 + 0.17 + 0.32 = 0.68$

(b) Good, very good, or excellent

$0.32 + 0.21 + 0.11 = 0.64$

Back to Exercise (13) (Rating of Lawn Mowers):



Find:

(1) $P(E_1) = 0.07 + 0.13 + 0.06 + 0.05 + 0.07 + 0.02 = \boxed{0.40}$

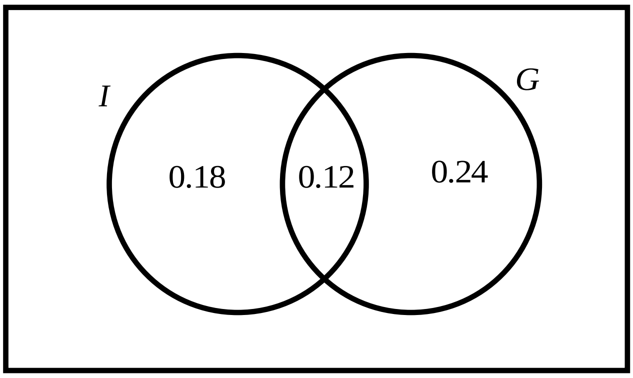
(2) $P(P_1) = 0.07 + 0.13 + 0.06 + 0.07 + 0.14 + 0.07 + 0.02 + 0.03 + 0.01 = \boxed{0.60}$

(3) $P(C_1) = 0.07 + 0.05 + 0.07 + 0.08 + 0.02 + 0.01 = \boxed{0.30}$

(4) $P(E_1 \cap P_1) = 0.07 + 0.13 + 0.06 = \boxed{0.26}$

(5) $P(E_1 \cap C_1) = 0.07 + 0.05 = \boxed{0.12}$

Exercise (28):



The above Venn diagram concerns the job applications of recent engineering school graduates. The letters I and G stand for getting a job in industry or getting a job with the government.

$P(I) = 0.18 + 0.12 = 0.30$

$P(G) = 0.12 + 0.24 = 0.36$

$P(I \cup G) = 0.18 + 0.12 + 0.24 = 0.54$

I and G are NOT mutually exclusive events

$$\Rightarrow P(I \cup G) \neq P(I) + P(G)$$

NOTE:

$P(I \cup G) = P(I) + P(G) = 0.66$

$\downarrow$

which exceeds the correct value by $\mathbf{0.12} \equiv P(I \cap G)$

$\downarrow$

error: results by adding $P(I \cap G)$ twice

$\rightarrow$ to get the correct answer

$$\boxed{P(I \cup G) = P(I) + P(G) - P(I \cap G)}$$

I and G are NOT disjoint events (have common region)

Elementary Theory:

If A and B are any events in S, then

$$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{\substack{\text{if } A \text{ and } B \text{ are} \\ \text{disjoint} \to P(A\cap B)=0}}$$

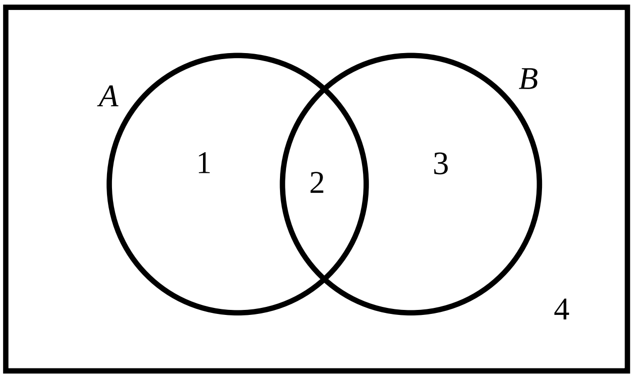
Exercise (29):

Prove using Venn diagram that

$(A \cap B) \cup (A \cap B') = A$

or

$P(A \cap B) + P(A \cap B') = P(A)$



$A \Rightarrow 1+2$

$B \Rightarrow 2+3$

$B' \Rightarrow 1+4$

$A \cap B \Rightarrow 2$

$A \cap B' \Rightarrow 1$

$(A \cap B) \cup (A \cap B') \Rightarrow 1+2 \equiv A$

Back to Exercise (13) (Rating of Lawn Mowers):

Find the probability that a lawn mower will be rated easy to operate and/or having a high average cost of repairs, i.e., $P(E_1 \cup C_1)$

$$P(\underbrace{E_1 \cup C_1}_{\substack{\text{NOT mutually} \\ \text{exclusive events}}}) = \overset{\substack{0.07+0.13+0.06 \\ +0.05+0.07+0.02}}{0.4} + \underset{\substack{0.07+0.05+0.07 \\ +0.08+0.02+0.01}}{0.3} - \overset{0.07+0.05}{0.12} = 0.58$$

Exercise (30):

If the probabilities are 0.87, 0.36, and 0.29 that, while under warranty, a new car will require repairs on the engine, drive train, or both. What is the probability that a car will require one or the other or both kinds of repairs under the warranty?

$\Rightarrow \cup \Rightarrow 0.87+0.36-0.29=0.94$

Theorem:

If A is any event in *S*, $P(A')=1-P(A)$

Proof:

$$P(A')=1-P(A)$$
$$P(A\cup A')=P(A)+P(A')-\underbrace{P(A\cap A')}_{=0}$$
$$P(S)=P(A)+P(A')$$
$$1\quad=P(A)+P(A')$$
$$\boxed{P(\phi)=1-P(S)=0}$$

Back to Exercise (13) (Rating of Lawn Mowers):

Find:

(a) The probability that a lawn mower will NOT be rated easy to operate

$$\underbrace{P(E_1')}_{P(E_2)+P(E_3)}=1-P(E_1)=1-0.4=0.6$$

(b) The probability that a lawn mower will be rated as either NOT being easy to operate or NOT having a high average repair cost

$$P(E_1'\cup C_1')=P\left((E_1\cap C_1)'\right)=1-P(E_1\cap C_1)=1-0.12=0.88$$

PROVE THAT

$$P(\overline{A}\cup\overline{B})=P(\overline{A\cap B})$$

Conditional Probability:

Theorem:

If *A* and *B* are any events in *S*, and $P(B) \neq 0$, conditional probability of *A* given *B* is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Exercise (31):

If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have $\overbrace{\text{high fidelity}}^{A}$ and high selectivity is 0.18, what is the probability that a system with high fidelity will also have $\overbrace{\text{high selectivity}}^{B}$?

$$P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{0.18}{0.81}=0.22$$

Exercise (32):

If the probability that a research project will be $\overbrace{\text{well planned}}^{A}$ is 0.8 and the probability that it will be well planned and well executed is 0.72, what is the probability that a research project that is well planned will also be $\overbrace{\text{well executed}}^{B}$?

$$P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{0.72}{0.80}=0.90$$

Back to Exercise (13) (Rating of Lawn Mowers):

Find $P(E_1\,|\,C_1)$:

$$P(E_1\,|\,C_1)=\frac{P(E_1\cap C_1)}{P(C_1)}=\frac{0.07+0.05}{0.3}=0.4$$

Exercise (33):

The supervisor of a group of 20 construction workers wants to get the opinion of 2 of them (to be selected random) about certain new safety regulations. If 12 of them favor the new regulations and the other 8 are against it. What is the probability that both of the workers chosen by the supervisor will be against the new safety regulations?

$$\frac{{}_8C_2}{{}_{20}C_2}=\frac{\binom{8}{2}}{\binom{20}{2}}=0.15=\frac{8}{20}\times\frac{7}{19}$$

Theorem:

If *A* and *B* are independent events, then

$P(A \cap B) = P(A).P(B)$

Exercise (34):

What is the probability of getting two heads in two flips of a balanced coin?

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

independent events

Exercise (35):

Two cards are drawn @ random from an ordinary deck of 52 playing cards. What is the probability of getting two aces if:

(a) The first card is replaced before the second card is drawn.

$\frac{4}{52} \times \frac{4}{52} \cong 0.006$

independent events

(b) The first card is NOT replaced before the second card is drawn.

$\frac{4}{52} \times \frac{3}{51} = 0.0045$

NOT independent events

Independence is violated when the sampling is without replacement

Exercise (36):

$P(C) = 0.56, P(D) = 0.4,$ and $P(C \cap D) = 0.24$. Are the events C and D independent?

If C and D are independent

$P(C \cap D) = P(C).P(D)$

$0.24 \overset{?}{=} 0.4 \times 0.56$

$0.24 \neq 0.4 \times 0.56$

$\therefore C$ and D are NOT independent (dependent) events

Exercise (37):

Let A be the event that raw material is available when needed and B be the event that the machining time is less than hour. If $P(A)=0.8$ and $P(B)=0.7$, assign probability to the event $P(A\cap B)$.

Since A and B are independent events

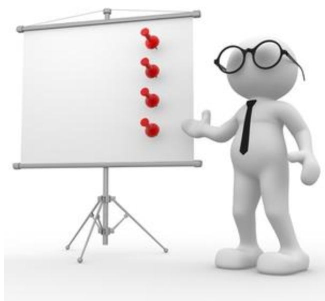
$\Rightarrow P(A\cap B)=P(A)\times P(B)=0.8\times 0.7=0.56$

Exercise (38):

What is the probability of NOT rolling any 5's in four rolls of a balanced die?

$$\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}=\left(\frac{5}{6}\right)^4=0.48$$

Some Rules:



(1) $P(@\ \textbf{least one event occurs})=P(A\cap\overline{B})+P(B\cap\overline{A})+P(A\cap B)$

(2) $P(@\ \textbf{most one event occurs})=P(A\cap\overline{B})+P(B\cap\overline{A})+P(\overline{A}\cap\overline{B})$

$$=P(\overline{A\cap B})$$
$$=1-P(A\cap B)$$

(3) $P(A-B)=P(A)-P(A\cap B)$

(4) $P(A\cap\overline{B})=P(A-B)$

$$=P(A)-P(A\cap B)$$

Exercise (39):

If $P(A)=0.2; P(B)=0.3; P(A\cap B)=0.1$

Find:

(1) $P(A\cup\overline{B})$

$$P(A\cup\overline{B})=P(A)+P(\overline{B})-P(A\cap\overline{B})$$
$$=P(A)+P(\overline{B})-P(A-B)$$

$$\begin{aligned} &= P(A) + P(\overline{B}) - P(A) + P(A \cap B) \\ &= 0.2 + 0.7 - 0.2 + 0.1 \\ &= 0.8 \end{aligned}$$

(2) $P(\overline{A} \cap B)$

$$\begin{aligned} P(\overline{A} \cap B) &= P(B \cap \overline{A}) \\ &= P(B - A) \\ &= P(B) - P(A \cap B) \\ &= 0.3 - 0.1 \\ &= 0.2 \end{aligned}$$

(3) $P(\overline{A} \mid B)$

$$P(\overline{A} \mid B) = \frac{P(B \cap \overline{A})}{P(B)} = \frac{0.2}{0.3} = 0.67$$

(4) ***P*(only one of the two events occurs)**

$$\begin{aligned} &= P(A \cap \overline{B}) + P(B \cap \overline{A}) \\ &= P(A - B) + P(B - A) \\ &= P(A) - P(A \cap B) + P(B) - P(B \cap A) \\ &= 0.2 - 0.1 + 0.3 - 0.1 \\ &= 0.3 \end{aligned}$$

Exercise (40):

If $A \subset B$ and $P(A) = 0.3$

$P(B) = 0.4$

Find $P(\overline{A} - \overline{B})$.

$$\begin{aligned} P(\overline{A} - \overline{B}) = P(\overline{A} \cap B) &= P(B \cap \overline{A}) \\ &= P(B - A) \\ &= P(B) - P(\underbrace{A \cap B}_{=A;A \subset B}) \\ &= P(B) - P(A) \\ &= 0.4 - 0.3 = 0.1 \end{aligned}$$

Exercise (41):

If A and B are independent events, $P(A)=0.6, P(A\bigcup B)=0.8$

Find $P(A-B)$.

$$P(A-B)=P(A)-P(A\bigcap B)$$
$$=P(A)-P(A)P(B)$$
$$P(A\bigcup B)=P(A)+P(B)-P(A\bigcap B)$$
$$0.8=P(A)+P(B)-P(A)P(B)$$
$$0.8=0.6+\underbrace{P(B)-0.6P(B)}_{0.4P(B)}$$
$$0.2=0.4P(B)\Rightarrow P(B)=0.5$$
$$\rightarrow P(A-B)=0.6-0.6\times 0.5=0.3$$

Exercise (42):

If A and B are independent events

$$P(A\bigcap B)=0.18$$
$$P(A)=2P(B)$$

Find $P(A)$.

$$P(A\bigcap B)=P(A).P(B)$$
$$0.18=P(A)\times\frac{1}{2}P(A)\Rightarrow\left(P(A)\right)^2=0.36\rightarrow P(A)=0.6$$

Exercise (43):

In certain teaching center, if 80% of the students are of the scientific branch, 75% of the students have cars, 70% of the students are of the scientific branch and have cars. If one student is chosen randomly, what is the probability that the student that is of the scientific branch also does NOT have a car?

$$A\rightarrow\text{scientific branch}, P(A)=0.8$$
$$B\rightarrow\text{has car}, P(B)=0.75$$
$$P(A\bigcap B)=0.7$$
$$P(\overline{B}\,|\,A)=\frac{P(\overline{B}\bigcap A)}{P(A)}=\frac{P(A-B)}{P(A)}=\frac{P(A)-P(A\bigcap B)}{P(A)}=\frac{0.8-0.7}{0.8}=0.125$$

Exercise (44):

If 85% of the citizens in Amman prefer news, 90% of the citizens in Amman prefer ads, 95% prefer news or ads, a citizen is chosen randomly, what is the probability that the citizen that does NOT prefer news also prefers ads?

Exercise (45):

50 students in a class, 30 of them study chemistry, 10 of them study physics and do NOT study chemistry, one student is chosen randomly, what is the probability that the student studies @ least one of the courses?

Exercise (46):

The probability that Tom scores a goal is 0.7, and the probability that Matt scores a goal is 0.8.

(1) What is the probability that @ least one of them scores a goal?

(2) What is the probability that just one of them scores a goal?

(3) What is the probability that @ most one of them scores a goal?