Dynamics of Interest Rate Swap and Equity Volatilities

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Abstract. While CBOE's VIX index is widely acknowledged as a broad-based investor "fear gauge" for its strong inverse relationship with major equity indexes, one cannot necessarily expect it to translate to the level of future turbulence or investor risk-aversion in fixed-income markets. Indeed, expected volatilities in equity and interest rate markets as measured respectively by CBOE's VIX and their newly-launched swap rate volatility index--SRVX--exhibit significantly distinct behaviors. The two indexes react to different events and risk factors, thereby providing investors with complementary diversification, hedging, and risk-taking tools.

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The rich time-varying dynamics between interest rate and equity volatilities have critical implications for investors involved in the fixed income and equity markets. While the intertwined structure of financial markets leads to some common risk factors across asset classes, volatilities in different markets are distinctive risks to be measured and managed. Over the last few years, Chicago Board Options Exchange's VIX index has become the benchmark for measuring and trading US equity market volatility, and their recently launched swap rate volatility index—SRVX—provides an interest rate counterpart to VIX. This article complements the extensive literature on VIX (see, e.g., the review of Carr and Lee, 2009; or research by Bollerslev, Tauchen and Zhou, 2009, and Corradi, Distaso and Mele, 2013), by shedding light on the properties of SRVX and how a standardized index of forward swap rate volatility can be used in practice.

For retail investors and asset managers alike, stocks and bonds are the traditional mainstay of most portfolios, and given the dominant size of fixed-income markets, the role of interest rate volatility in asset pricing arguably deserves at least as much attention as that of equity volatility. Whereas the financial media have adopted VIX as an investor "fear gauge" for its strong inverse relationship with major equity indexes, one cannot necessarily expect it to translate to the level of future turbulence or investor risk-aversion in fixed-income markets. Indeed, results below confirm that the two volatilities, as measured by VIX and SRVX, exhibit significantly different behaviors and a treatment of one is not complete without the other.

Model-Free Index of Swap Rate Volatility

The design of SRVX possesses some key similarities and differences with respect to that of VIX. Like its older equity cousin, SRVX is based on model-free pricing of a variance swap using quoted option prices and thus reflects the fair value of volatility that is theoretically consistent with prices observed in the swaptions market. Variance swaps, although not traded in practice for forward swaps, offer a pure measure of expected variance as they do not suffer from path-dependence and the resulting P&L noise of swaption-based volatility trading strategies, such as delta-hedged ATM straddles. Mele and Obayashi (2012) show that the fair price of a variance swap maturing at M on a T-year swap can be expressed in terms of observed swaption prices, and is the square of the following index:

$$(1)\quad Index_t(M,T) = 100^2 \times \sqrt{\frac{2}{A_t \times M}\left[\sum_{i:K_i<F_t(M,T)} \mathrm{Receiver}_t(K_i,M,T)\Delta K_i + \sum_{i:K_i \geq F_t(M,T)} \mathrm{Payer}_t(K_i,M,T)\Delta K_i\right]},$$

where $F_t(M,T)$ is the forward swap rate for the maturity-tenor pair (M,T), $\mathrm{Receiver}_t(K_i,.,.)$ and $\mathrm{Payer}_t(K_i,.,.)$ are prices of out-of-the-money receiver and payer swaptions struck at K_i, and A_t is the present value of an annuity of one dollar paid over the forward swap reset dates.

To the reader acquainted with VIX calculations, the structure of Eq. (1) should look familiar but with two key differences. First, the index references not only an option maturity but also an underlying swap tenor to reflect this additional dimension in the rates world, as illustrated by Chart 1;[1] the CBOE-SRVX[SM] index is based on 1 year-10 year swaptions, i.e. $M=1$ and $T=10$, which is one of the most actively traded points on the maturity-tenor surface. Second, the index is constructed as a gauge of basis point volatility as opposed to percentage volatility, which leads to scaling and weighting schemes different from those of VIX. The latter point is not a simple matter of quoting convention, and highlights a fundamental difference between interest rate and equity volatility.

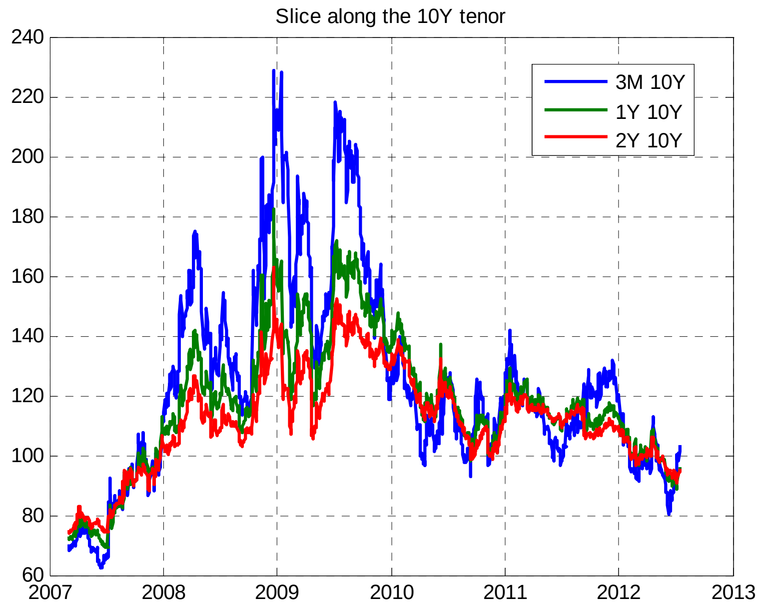

Chart 1: 3m-10y, 1y-10y (SRVX), 2y-10y Index Levels

[1] The historical levels of the SRVX index used in this paper are indicative and subject to changes.

Basis Point Volatility

The concept of basis point volatility naturally arises because absolute changes describe risk more effectively than relative changes in the context of yields and spreads. A rate increase from 10bps to 15bps shares the same percentage change as one from 100bps to 150bps, but, all else equal and accounting for convexity, the latter is a nearly tenfold P&L and risk event. As such, it is more useful for rates traders to know whether a position is likely to experience 5bps moves or 50bps moves over a given horizon, and the basis point volatility formulation of the index addresses this by model-free pricing of a variance swap on *arithmetic changes* in the 1y-10y forward swap rate instead of logarithmic changes as in the case of equity variance swaps.

To illustrate the properties of the basis point volatility index, Mele and Obayashi (2013) show that, under certain conditions, the SRVX may be decomposed into a product of the 1y-10y forward swap rate and a term encapsulating information about uncertainty:

$$Index_t(M,T) = 100^2 \times F_t(M,T) \times Sk_t(M,T),$$

where $Sk_t(M,T)$ is a function only of (M,T), i.e. not the forward swap rate, that condenses forward looking information about interest rate uncertainty contained in the 1y-10y swaption skew.

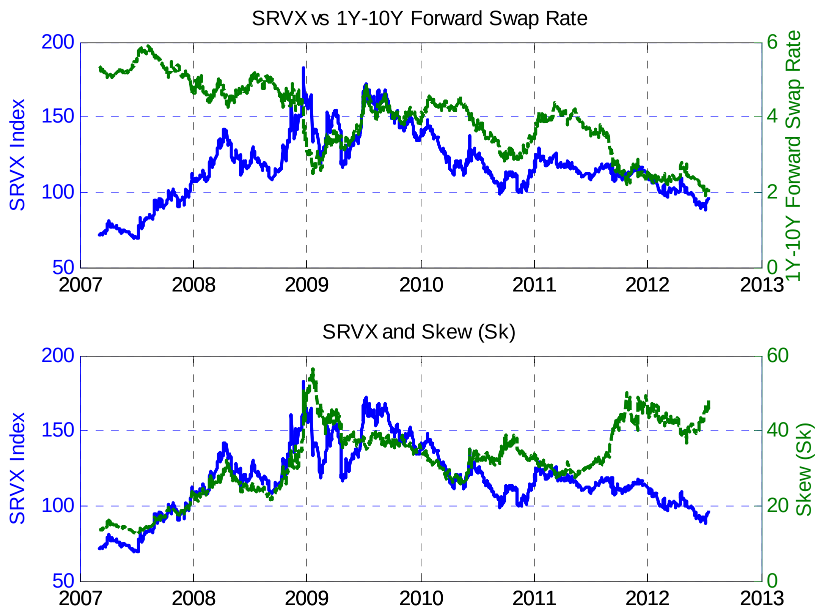

Chart 2: SRVX and its two components

The two panels in Chart 2 illustrate how the two components—forward swap rate and levels of uncertainty—take turns being the dominant driver of the index. Despite a strong rally in rates through the beginning of 2008, the index doubles as the rise in uncertainty outpaces the fall in the forward rate; the period following the first quarter of 2011 demonstrates the opposite phenomenon. As will be made clear in the following section, an understanding of the two components' respective impact on the index is key to interpreting its behavior through various economic episodes.

Volatility in Interest Rate and Equity Markets

The period between 2007 and 2012 provides a rich set of economic events in recent financial history that highlights the varied behaviour of SRVX and VIX. Chart 3 shows the two indexes experiencing large swings in times of global financial distress with periods of marked co-movement and divergence driven by events originating from equity or debt markets. Chart 4 depicts estimates of the correlation between debt and equity volatilities, both realized and implied.

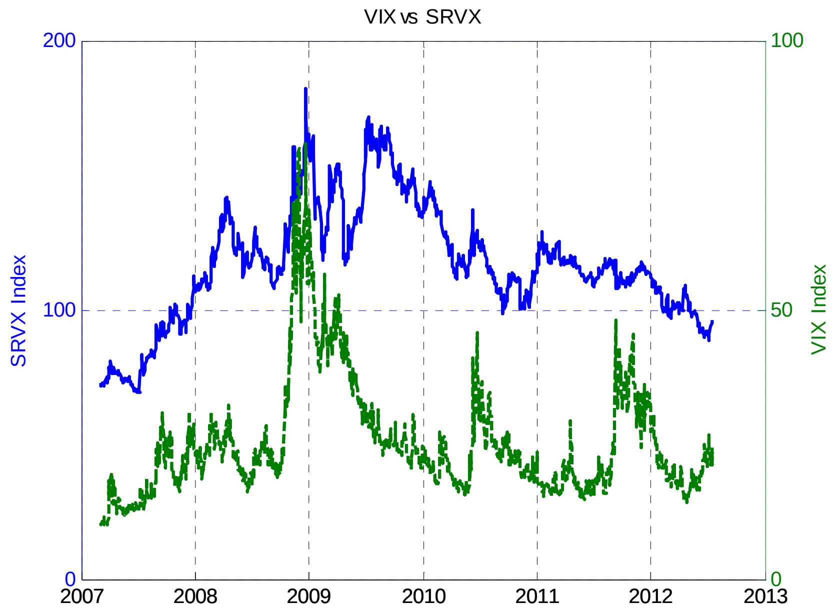

Chart 3: The time series of the VIX and the SRVX

At the onset of the global financial crisis, the SRVX climbs steeper and steadier than the VIX through the first quarter of 2008. This is particularly noteworthy as it occurs in spite of a downward trend in forward swap rates, suggesting a significantly greater increase in uncertainty in fixed income markets than in equity markets during this period. After a brief subsequent decline, both indexes experience a spectacular surge to reach record levels over the fall of 2008 following Lehman's collapse. During this period, the SRVX offered more opportunities for an investor to capture incremental increases in uncertainty over time than the VIX as the latter provided a much narrower window to be on the right side of the trade.

The period from April to June 2009 marks an interesting instance of disconnect between fixed income and equity expected volatilities. Over these months, the VIX experiences a rapid and sustained decline reflecting a stabilization of market expectations fueled by encouraging economic data and a rally in equities as QE1 was expanded. To the contrary, SRVX shoots up once again as rates sell off and uncertainty remains relatively flat. This episode highlights the importance of managing volatilities of different asset classes as distinctive risks, especially for those managing multi-asset portfolios.

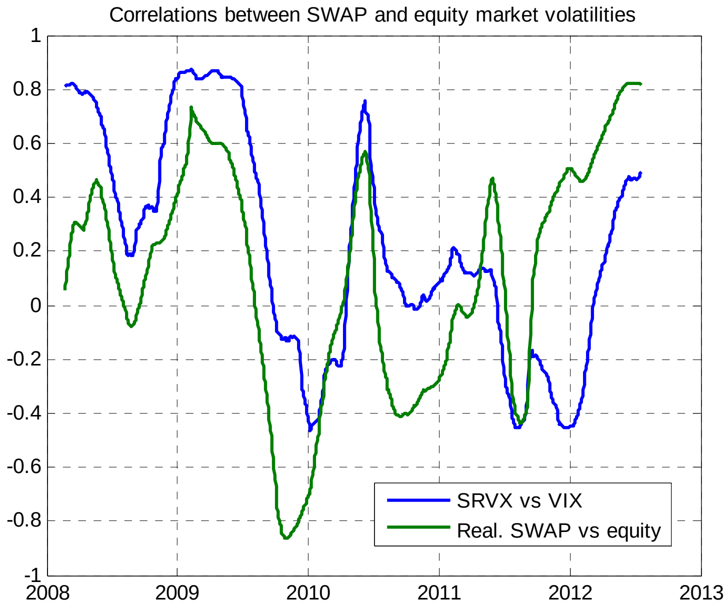

Chart 4: 1Y moving average estimates of the correlations between debt and equity volatilities, realized and expected.

The two indexes experience a common spike in the middle of 2010 as a result of a budding European sovereign bond crisis and renewed concerns about macroeconomic developments in the United States. Over the last two years of the sample, the Federal Reserve Bank's success at maintaining both short- and long-term rates extremely low brings the SRVX down even as uncertainty once again increases, which one might argue sets the stage for pronounced hikes in SRVX if the Fed ceases to suppress rates while uncertainty remains elevated.

All Fears are Not Created Equal

The SRVX and the VIX react differently to events originating in debt and equity markets. The VIX has come to be known as the market "fear gauge" for its well-known tendency to increase during times of negative equity market performance. For example, the VIX index increases on average by more than 4 percentage points over days when the S&P drops by more than 2% (see Table 1). The SRVX performs a similar role for fixed income portfolios. For example, Chart 5 reveals that the SRVX mirrors the Dow Jones Investment Grade Corporate Bond Index for most of the sample period.

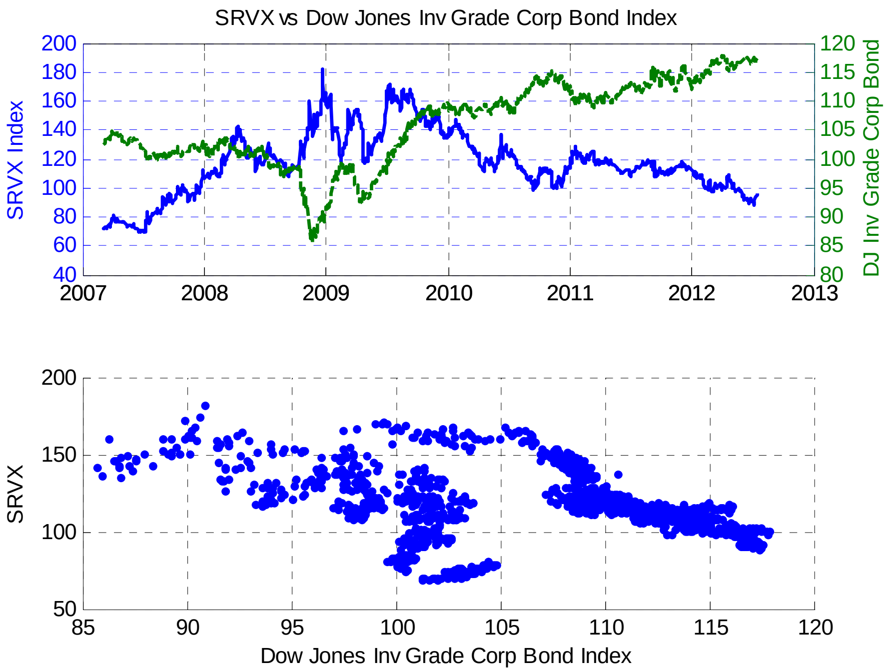

Chart 5: SRVX and debt market developments

Of particular note is the SRVX's reaction to negative tail events experienced by fixed income portfolios. Table 1 documents for example that the SRVX increases, on average, by nearly

2bps when the Dow Jones Investment Grade Corporate Bond Index is down by more than 1 point. The estimates of the SRVX conditional changes (conditional on debt markets events) in Table 1 are both statistically and economically highly significant.

Highlighting the fact that the two indexes react to different risks is the observation that the VIX displays the opposite behavior in days of bond price distress by dropping on average, although this tendency to drop is not always statistically significant. In other words, the SRVX tends to surge while the VIX tends to fall, or, at best, remain flat, over days of negative bond portfolio performance. Interestingly, such a disconnect between the two indexes only occurs during bad times for bond portfolios; on average, the SRVX still increases, and substantially, over days of negative equity market performance, although not as significantly as the VIX, in both statistical and economic terms.

Table 1:
Behavior of SRVX and VIX over down days in debt & equity markets

Dow Jones Invest Grade Corp Bond Index down by:	No of obs.	SRVX average Δ change (standard error)	VIX average Δ change (standard error) $\times 100$
< 0	621	0.37bps (0.09)	-0.54bps (0.09)
< -0.5pt	138	1.38bps (0.27)	-1.06bps (0.25)
< -1.0pt	21	1.63bps (0.80)	-1.72bps (1.08)
S&P 500 down by:			
< 0	618	0.16bps (0.10)	1.39bps (0.09)
< -2%	114	0.80bps (0.28)	4.24bps (0.31)
< -5%	14	2.69bps (1.02)	9.11bps (1.26)

Volatility of Volatility

How volatile is the SRVX, and why should investors care? Realized volatility of SRVX presumably affects the ongoing evolution of the distribution of expected future values of SRVX, which will in turn be a driving factor of options on SRVX that may become traded in the future.

We calculate one-month realized volatilities of SRVX and, for purposes of comparison, also that of VIX as

$$VolVol_t = \sqrt{\frac{252}{21}\sum_{i=1}^{21}\ln^2\frac{I_{t+1-i}}{I_{t-i}}},$$

where I_t is the closing value of any of the two indexes as of day t.

To allow for a better comparison of swap and equity market *VolVols*, we consider the realized volatility of the 3m-10 year version of the SRVX against that of the 3 month VIX (VXV). Chart 6 shows that the Swap $VolVol_t$ varies significantly over time, and that it peaks at the same time as the SRVX during periods of market distress, such as the fall of 2008 (see also Table 2). Otherwise, there does not seem to be a strong relationship between the level and volatility of SRVX similar to what is known for the VIX and its realized volatility, which in part explains the demand for both futures and options. It also appears that with the exception of periods of acute market turmoil, the two *VolVols* are driven by different events, which alludes to the likelihood of options on SRVX allowing investors to trade risks not spanned by those on VIX.

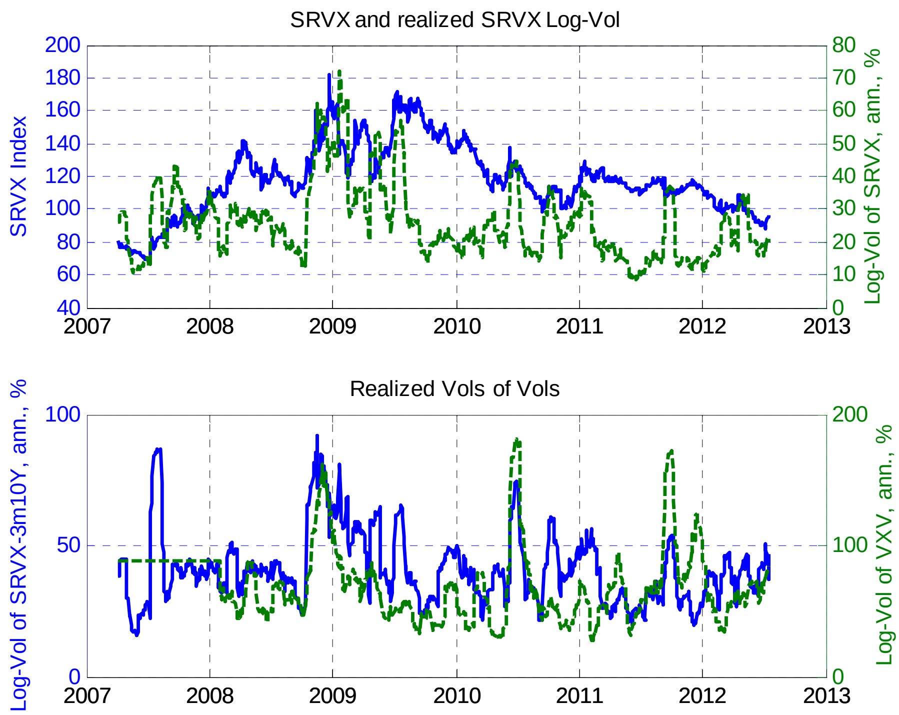

Chart 6: SRVX and volatilities of volatilities in debt and equity markets

Table 2:
Behavior of realized volatilities of SRVX and VIX over days of increased expected future volatility in debt & equity markets

SRVX up by:	No of obs.	SRVX Volatility average Δ change (standard error) $\times 100$	VIX Volatility average Δ change (standard error) $\times 100$
> 0	698	0.15bps (0.09)	0.54bps (0.38)
> 3bps	112	2.74bps (0.33)	2.83bps (0.88)
> 5bps	56	3.73bps (0.55)	1.73bps (1.33)
VIX up by:			
> 0	679	0.06bps (0.09)	1.30bps (0.44)
> 5bps ($\times 100$)	61	0.59bps (0.51)	14.55bps (1.87)
> 7bps ($\times 100$)	30	0.71bps (0.57)	20.15bps (2.60)

Volatility Risk Premiums

Similar to equity variance swaps and VIX derivatives, contracts that settle to the realized variance of swap rates have the potential to diversify and hedge portfolios. This potential is reflected in the risk premium investors are willing to pay to "hold" volatility. We define this risk-premium as the difference between the expected interest rate volatility to prevail under the (swap) market probability (i.e., the SRVX) and that under the objective probability. While the latter expectation is not observed, it can be proxied by the realized volatility, defined as:

$$Vol_t \equiv \sqrt{\frac{252}{21 \cdot n} \sum_{i=1}^{21 \cdot n} \Delta F_{t+1-i}^2},$$

where ΔF_t denotes the change of the forward swap rate at time *t*, and *n* is the number of months in the whole trading period.[2] Accordingly, we define the swap variance risk premium as:

$$\pi_{t+M} \equiv SRVX_t^2(M,T) - Vol_{t+M}^2 .$$

[2] The square of Vol_t is also referred to as "quadratic variation" (or in continuous time, "integrated variance") by mathematicians. To calculate realized volatility, we estimate missing Forward Swap rates through linear interpolations.

Thus, π_{t+M} links to the P&L at time $t+M$ of a short position in a variance swap contract originated at time *t*. We calculate swap volatility risk premiums for the 3m-10 year version of the SRVX for purposes of comparing its dynamics with those of the three-month VIX (VXV).

Chart 7 depicts realized volatility and the price of insurance against it for debt and equity markets, and Chart 8 depicts the time series behavior of two standardized volatility risk premiums, defined as the differences of the volatilities in Chart 7, and both standardized to have a standard deviation of one. While the risk premiums are positive most of the time, thereby providing compensation to sellers of volatility, they become sharply negative in times of distress, such as the third quarter of 2008 when the credit crisis broke out. The period of time extending approximately from mid-2009 to mid-2012 shown in Chart 8 illustrates that debt and equity volatility premiums can follow very different paths for prolonged periods of time, and even assume opposite signs. This indicates that selling interest rate volatility is an effective and diversifying alternative for investors searching for yield in the volatility space.

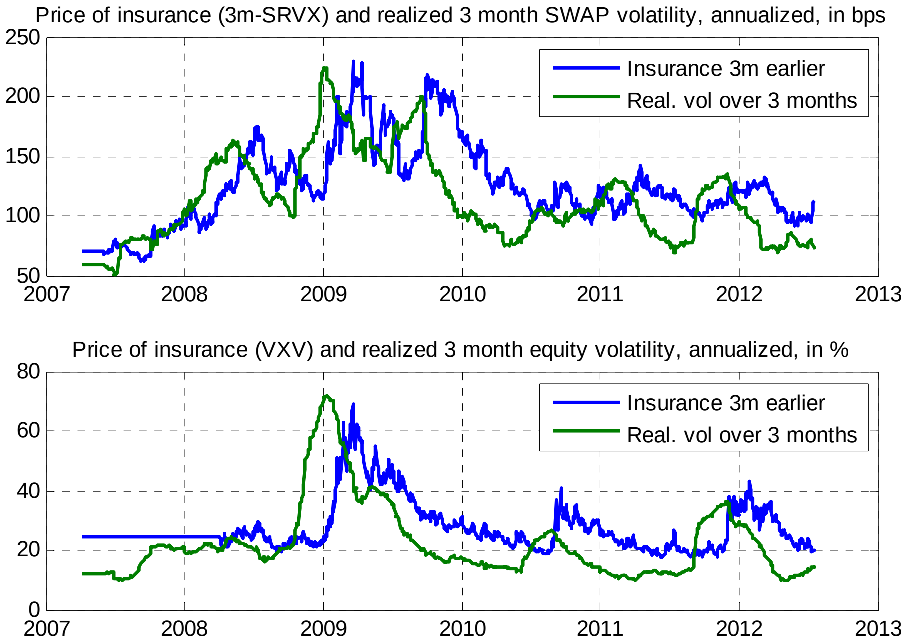

Chart 7: Price of insurance and realized volatility in debt (top panel) and equity (bottom panel) markets.

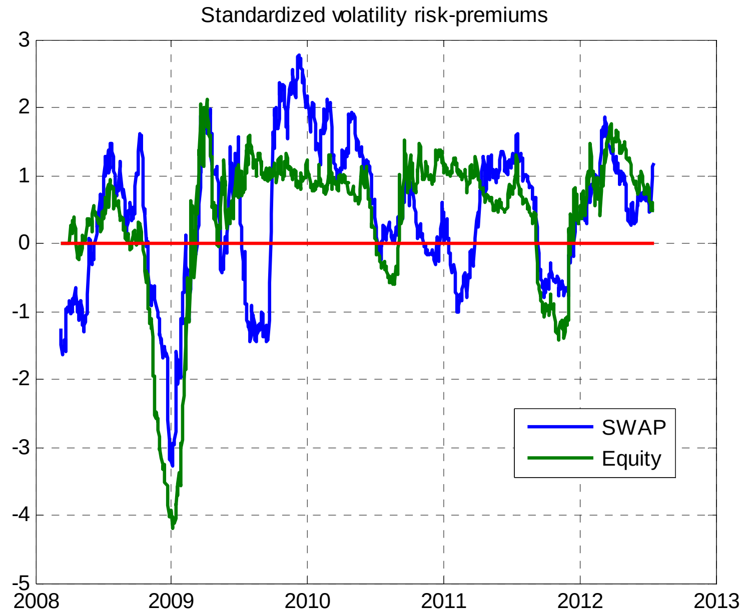

Chart 8: Swap and equity volatility risk-premiums, standardized.

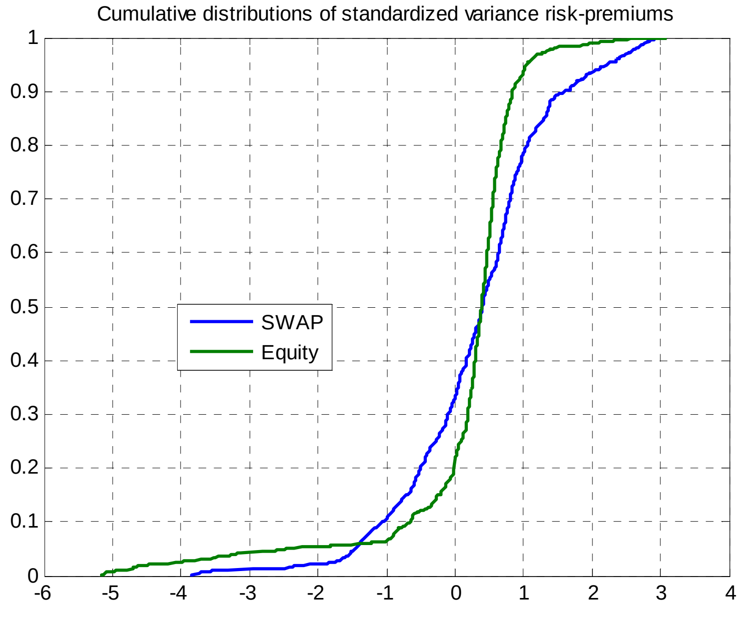

Chart 9: Cumulative distributions of standardized variance risk premiums in debt and equity markets.

Chart 9 depicts the cumulative distributions of the variance risk premiums π_{t+M} for short 3-month variance positions in the 3 month 10 year forward swap rate and the S&P 500 rate of return counterpart from 2008 to 2012. The premiums are scaled by their standard deviation. During this period, the variance risk-premium for swap rates has a smaller left tail and greater positive mass. This smaller negative skew may render it an attractive alternative to selling equity volatility for more risk averse investors.

SRVX and the Current Environment

At present, there is growing concern about the fallout from an inevitable end to the sustained rally in bond portfolios and high prices being propped up, for the time being, by the Federal Reserve's vow to keep interest rates low for as long as it takes for significant improvement in the macroeconomy to materialize. As fixed income investors fall complacent through repeated assurances of continued quantitative easing, many are likely to be unprepared for the day when the music stops and precipitates a potentially swift sell-off. Being a forward-looking measure that displayed a prescient climb leading up to the pinnacle of the last crisis, fixed income investors may be well-advised to keep a steady eye on SRVX.

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