PREMIK V BROYDENOVI METODI

Trditev 1 (Približek za Jacobijevo matriko v Broydenovi metodi)**.**

$$B_{r+1} = B_r + \frac{\underline{f}(\underline{x}^{(r+1)})(\Delta\underline{x}^{(r)})^T}{\|\Delta\underline{x}^{(r)}\|_2^2}.$$

Dokaz. Iščemo B_{r+1}, da veljajo naslednje trditve:

(1) $B_r\Delta\underline{x}^{(r)} = -\underline{f}(\underline{x}^{(r)})$.
(2) $B_{r+1}\Delta\underline{x}^{(r)} = \underline{f}(\underline{x}^{(r+1)}) - \underline{f}(\underline{x}^{(r)})$.
(3) $B_{r+1} - B_r$ ima najmanjšo možno spektralno normo.

Iz (1) in (2) sledi

$$(B_{r+1} - B_r)\Delta\underline{x}^{(r)} = \underline{f}(\underline{x}^{(r+1)}). \tag{0.1}$$

Matrika $B_{r+1} - B_r$ je zato enaka

$$\frac{\underline{f}(\underline{x}^{(r+1)})(\Delta\underline{x}^{(r)})^T}{\|\Delta\underline{x}^{(r)}\|_2^2}.$$

Res, ta matrika zadošča (0.1) in ima normo $\frac{\|\underline{f}(\underline{x}^{(r+1)})\|_2}{\|\Delta\underline{x}^{(r)}\|_2}$. Vsaka matrika, ki zadošča (0.1) pa ima normo vsaj $\frac{\|\underline{f}(\underline{x}^{(r+1)})\|_2}{\|\Delta\underline{x}^{(r)}\|_2}$. □