

NAPAKA V SIMPSONOVEM PRAVILU

Trditev 1. Napaka Simpsonovega pravila na intervalu $[a, b]$ je enaka

$$(1) \quad -\frac{1}{90}h^5 f^{(4)}(\xi), \quad \text{za nek } \xi \in [a, b].$$

Dokaz. Taylorjeva polinoma stopnje 4 za funkcijo f v točki a z $\Delta = h$ oz. $\Delta = 2h$ sta:

$$(2) \quad \begin{aligned} f(a+h) &\approx f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f^{(3)}(a) + \frac{h^4}{4!}f^{(4)}(a), \\ f(a+2h) &\approx f(a) + 2hf'(a) + 2h^2f''(a) + \frac{4h^3}{3}f^{(3)}(a) + \frac{h^4}{3}f^{(4)}(a). \end{aligned}$$

Seštejemo in dobimo, da je $\frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]$ približno

$$(3) \quad 2hf(a) + 2h^2f'(a) + \frac{4}{3}h^3f''(a) + \frac{2}{3}h^4f^{(3)}(a) + \frac{5}{18}h^5f^{(4)}(a).$$

Integriramo Taylorjev polinom

$$p_4(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f^{(3)}(a) + \frac{(x-a)^4}{4!}f^{(4)}(a)$$

stopnje 4 za $f(x)$ in dobimo

$$(4) \quad \int_a^{a+2h} f(x) dx \approx 2hf(a) + 2h^2f'(a) + \frac{4}{3}h^3f''(a) + \frac{2}{3}h^4f^{(3)}(a) + \frac{4}{15}h^5f^{(4)}(a).$$

Primerjamo (3) in (4) in pri osnovnem Simpsonovem pravilu dobimo napako (1). □