Professor Wu

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Quantitative Management Modeling

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Module # 3 – Assignment # 2

The Linear Programming Model

1.

a.

1. Number of Collegiate.

2. Number of Minis.

b.

The goal of the problem is to produce the number of each type of backpack to achieve the highest possible total profit. Each Collegiate yields a unit profit of $32 while each Mini yields a unit profit of $24. The objective function is therefore

Maximize Total Profit = $32C + $24M.

c.

The first set of constraints in this problem involve the limited resources (nylon and labor hours). Given the number of backpacks produced, C and M, and the required nylon and labor hours for each, the total resources used can be calculated. These total resources used need to be less than or equal to the amount available. Since the labor available is in units of hours, the labor required for each backpack needs to be in units of hours (3/4 hour and 2/3 hour) rather than minutes (45 minutes and 40 minutes). These constraints are as follows:

Nylon: $3C + 2M \le 5400$ square feet,

Labor Hours: $(3/4)C + (2/3)M \le 1400$ hours.

The final constraint is that they should not produce more of each backpack than the sales forecast. Therefore,

Sales Forecast: $C \le 1000$

$M \leq 1200.$

d.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let $C$ = Number of Collegiates to produce,
$M$ = Number of Minis to produce.

Maximize Total Profit = \$32$C$ + \$24$M$,

subject to

| | |
|---|---|
| Nylon: | $3C + 2M \leq 5400$ square feet, |
| Labor Hours: | $(3/4)C + (2/3)M \leq 1400$ hours, |
| Sales Forecast: | $C \leq 1000$ |
| | $M \leq 1200.$ |

and $C \geq 0$, $M \geq 0$.

2.

a.

The decision variables can be denoted and defined as follows:

$x_{P1L}$ = number of large units produced per day at Plant 1,

$x_{P1M}$ = number of medium units produced per day at Plant 1,

$x_{P1S}$ = number of small units produced per day at Plant 1,

$x_{P2L}$ = number of large units produced per day at Plant 2,

$x_{P2M}$ = number of medium units produced per day at Plant 2,

$x_{P2S}$ = number of small units produced per day at Plant 2,

$x_{P3L}$= number of large units produced per day at Plant 3,

$x_{P3M}$ = number of medium units produced per day at Plant 3,

$x_{P3S}$ = number of small units produced per day at Plant 3.

b.

Letting P (or Z) denote the total net profit per day, the linear programming model for this problem is, Maximize $P = 420\ x_{P1L} + 360\ x_{P1M} + 300\ x_{P1S} + 420\ x_{P2L} + 360\ x_{P2M} + 300\ x_{P2S} + 420\ x_{P3L} + 360\ x_{P3M} + 300\ x_{P3S}$,

subject to

$$x_{P1L} + x_{P1M} + x_{P1S} \leq 750$$

$$x_{P2L} + x_{P2M} + x_{P2S} \leq 900$$

$$x_{P3L} + x_{P3M} + x_{P3S} \leq 450$$

$$20\ x_{P1L} + 15\ x_{P1M} + 12\ x_{P1S} \leq 13000$$

$$20\ x_{P2L} + 15\ x_{P2M} + 12\ x_{P2S} \leq 12000$$

$$20\ x_{P3L} + 15\ x_{P3M} + 12\ x_{P3S} \leq 5000$$

$$x_{P1L} + x_{P2L} + x_{P3L} \leq 900$$

$$x_{P1M} + x_{P2M} + x_{P3M} \leq 1200$$

$$x_{P1S} + x_{P2S} + x_{P3S} \leq 750$$

$$\frac{1}{750}( x_{P1L} + x_{P1M} + x_{P1S} ) - \frac{1}{900}( x_{P2L} + x_{P2M} + x_{P2S} ) = 0$$

$$\frac{1}{750}( x_{P1L} + x_{P1M} + x_{P1S} ) - \frac{1}{450}( x_{P3L} + x_{P3M} + x_{P3S} ) = 0$$

and

$x_{P1L} \geq 0$, $x_{P1M} \geq 0$, $x_{P1S} \geq 0$, $x_{P2L} \geq 0$, $x_{P2M} \geq 0$, $x_{P2S} \geq 0$,

$x_{P3L} \geq 0$, $x_{P3M} \geq 0$, $x_{P3S} \geq 0$.

The above set of equality constraints also can include the following constraint:

$$\frac{1}{900}\left(x_{P2L} + x_{P2M} + x_{P2S}\right) - \frac{1}{450}\left(x_{P3L} + x_{P3M} + x_{P3S}\right) = 0.$$

However, any one of the three equality constraints is redundant, so any one (say, this one) can be deleted.