### UltraGroth: Interactive Groth16

August 29, 2025

#### Dmytro Zakharov Distributed Lab

distributedlab.com/

github.com/rarimo/ultragroth



# Why we should care?

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Using quite unsophisticated math,  $128 \times 10000 = 1.28 \text{ mln}$ .

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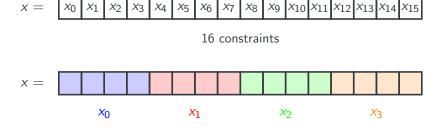
**Result:** We pay 65.5k constraints once and then every 128-bit range checks costs only 8 constraints instead of 128!

### Illustration

Let us illustrate this visually for a 16-bit range check over x!

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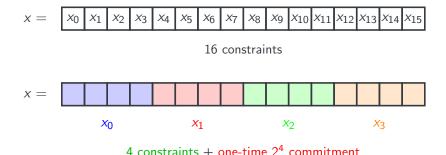
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4 constraints + one-time 2<sup>4</sup> commitment

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**Example:** 10000 such range checks would cost  $16 \times 10000 = 160k$  constraints for a regular R1CS while  $2^4 + 4 \times 10000 \approx 40k$  constraints over ZK system with lookups.

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- Non-native field arithmetic: e.g., optimized ECDSA verification for Rarimo passport verification.
- And surely, zero-knowledge Machine Learning Bionetta.

	Constraints #	68.4K							
		00.41	66.7K	106.8K	126.8K	108.4K	187.7K	1.03M	2.50M
Bionetta P.	Proof Size (KB)	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20
	PK (MB)	48.40	50.60	80.60	106.30	81.90	156.20	0.95GB	1.90GB
(Citradion)	/K (KB)	3.78	3.79	3.78	3.78	3.78	3.78	4.05	4.20
P.	Prove (s)	0.57	0.73	0.74	1.08	0.89	1.79	6.27	15.22
V	/erify (s)	0.006	0.005	0.005	0.006	0.006	0.005	0.006	0.006
C	Constraints #	29.0K	5.9K	522.4K	779.4K	543.0K	1.56M	12.01M	31.78M
Bionetta P.	Proof Size (KB)	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81
(Groth16)	PK (MB)	21.30	10.20	396.20	560.20	409.30	1.2GB	≈9.0GB	≈23.8GB
(Grounto)	/K (KB)	3.65	3.65	3.65	3.65	3.65	3.65	<b>≈</b> 4.0	<b>≈</b> 4.0
P	Prove (s)	0.12	0.27	2.19	2.20	2.22	4.72	≈180	<b>≈</b> 480
V	/erify (s)	0.006	0.006	0.006	0.006	0.006	0.005	≈0.005	≈0.006

### Up to x12.7 boost in # of constraints!

**Surprising result**: if the circuit consists of L range-checks, each costing b constraints, using lookup protocol, you can reduce  $\mathcal{O}(n)$  constraints (n = Lb) down to  $\mathcal{O}(n/\log n)$ .

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### Theorem (Some stuff from ZKDL Camp)

The inclusion check  $\{z_i\}_{i\in[n]}\subseteq\{t_i\}_{i\in[v]}$  is satisfied if and only if there exists the set of multiplicities  $\{\mu_i\}_{i\in[v]}$  where  $\mu_i=\#\{j\in[n]:z_j=t_i\}$  such that for  $\gamma\leftarrow$ \$  $\mathbb{F}$ :

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*High-level idea:* We can: (1) compute  $\{\mu_i\}_{i\in[v]}$  off-circuit, (2) write circuit in n+2v constraints, given  $\gamma$  signal is passed randomly.

### Circom-like Implementation

```
signal input t[M];
                               // The lookup table
1
        signal random input gamma; // Random challenge value
        signal input z[N]; // The array of values to check
3
4
5
        var sum_z, sum_t = 0;
        for (var i = 0: i < N: i++) {
6
            inv_z[i] \le 1 / (z[i] + gamma);
            sum_z += inv_z[i]; // Compute the left-hand side
8
        }
9
10
11
        for (var j = 0; j < M; j++) {
12
            mu[j] <-- 0; // Compute the multiplicities off-circuit
            for (var k = 0: k < N: k++) {
13
                mu[j] += (t[j] == z[k]);
14
15
            inv_t[i] <== mu[j] / (t[j] + gamma);</pre>
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            sum_t += int_v[i]; // Compute the right-hand side
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        sum_z === sum_t; // Check both sides are equal
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# UltraGroth Explained

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#### One important consequence

The protocol is **safe**. It is sound and zero-knowledge! And it is now proven in **three** different independent papers.

#### UltraGroth Performance

Now, let us recap the **Groth16** performance over the circuit of size n and statement size  $\ell$ .

• Prover work: MSM of size  $\mathcal{O}(n)$  over  $\mathbb{G}_1$  and  $\mathbb{G}_2$ .

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- Verifier work: 4 pairings  $+ \mathcal{O}(\ell) \mathbb{G}_1 \exp s + 1$  hashing.

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**Recap:** Proof in Groth16 consists of three points  $g_1^{a(\tau)}$ ,  $g_1^{c(\tau)}$ ,  $g_2^{b(\tau)}$ :

$$a(X) = \alpha + \sum_{i \in [n]} z_i \ell_i(X) + r\delta, \quad b(X) = \beta + \sum_{i \in [n]} z_i r_i(X) + s\delta,$$

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The verification equation is:

$$e(\pi_A, \pi_B) = e(g_1^{\alpha}, g_2^{\beta}) \cdot e(g_1^{i(\tau)}, g_2^{\gamma}) \cdot e(\pi_C, g_2^{\delta}).$$

for  $\pi_A = g_1^{a(\tau)}$ ,  $\pi_C = g_1^{c(\tau)}$ ,  $\pi_B = g_2^{b(\tau)}$ , i(X) is a polynomial derived from the public statement.

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**Note**: This construction can be easily generalized for d > 1 rounds.

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- Proved completeness, soundness, and zero-knowledge for general d-round UltraGroth. Formalized everything properly.

- Implemented a single-round UltraGroth (essentially, a Mirage protocol). Credits to Artem Sdobnov, Vitalii Volovyk, Yevhenii Sekhin, and Illia Dovgopoly.
  - o Forked rapidsnark.
  - Forked snarkjs for witness export/verify functions and smart-contract autogeneration.
  - o Thanks to Ivan Lele, we even have a Swift SDK for that!
- Proved completeness, soundness, and zero-knowledge for general d-round UltraGroth. Formalized everything properly.
- Applied UltraGroth to Bionetta and obtained incredible results.

# Any Questions?



