


# Optimizing Big Integer Multiplication on Bitcoin: Introducing $w$ -windowed Approach

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 [eprint.iacr.org/2024/1236](https://eprint.iacr.org/2024/1236)

 [github.com/distributed-lab/bitcoin-window-mul](https://github.com/distributed-lab/bitcoin-window-mul)

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# Introduction to Bitcoin Script

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**Bitcoin Script** is a scripting language used in Bitcoin to specify conditions on how the UTXO can be spent.

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The standard pay-to-pubkey-hash looks as follows.  
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As a scriptSig, the user provides  $\{ \langle \sigma \rangle \langle pk \rangle \}$ .

# How the Bitcoin Script is executed

Consider the pay-to-pubkey-hash's `scriptSig || scriptPubKey`:

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## Note

One can spend the UTXO iff the output is OP\_1.

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*Significance:* Having effective Bitcoin on-chain Groth16 verification enables the L2 over Bitcoin!

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### Note

In other words, currently, it is *theoretically possible* to build a Groth16 zk-SNARK verification of proof  $\pi$  in a form

**Script:**

$\langle \pi \rangle$   $\langle \text{public statement} \rangle$  OP\_CHECKGROTH16

# Demystifying Math behind BitVM Groth16

- ✓ Arithmetic is allowed only over `u32` integers.

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## Question

Currently, big integer multiplication takes 74.9k OPCODEs to implement. Can we implement a better approach?

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# Big Integer Arithmetic

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# Representing large integers

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## Idea #2

Since we want to minimize the number of limbs, we take the largest  $b$  possible (with  $b = 2^t$  for convenience). Thus, we set  $b := 2^{30}$ .

# Representing large integers

## *Example*

Consider the following 254-bit integer:

$$x = (0xbe48fffd2a6f534dc \\ 5b6a6901840fc0fb65827e6 \\ efd22a8063cded681f5f7b2)$$



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To add this integer to the stack, one uses the following script:

**Script:**

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OP_PUSHBYTES_4 <fbc00f04> OP_PUSHBYTES_4 <9b9f6019>
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**Note:** One needs **9 limbs** to represent a 254-bit integer.

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2. If  $t < 2^{30}$ , set  $z_i \leftarrow t$ ,  $\text{carry} \leftarrow 0$ .
3. If  $t \geq 2^{30}$ , set  $z_i \leftarrow t - 2^{30}$ ,  $\text{carry} \leftarrow 1$ .



# BitInt Addition: Bitcoin Script

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**Algorithm 7:** Adding two integers assuming with no overflow

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**Input** : Two integers on the stack:  $\{\langle x_{\ell-1} \rangle \dots \langle x_0 \rangle \langle y_{\ell-1} \rangle \dots \langle y_0 \rangle\}$ **Output**: Result of addition  $z = x + y$  in a form  $\{\langle z_{\ell-1} \rangle \dots \langle z_0 \rangle\}$ 

```

1 { OP_ZIP } ; /* Convert current stack {⟨xℓ-1⟩ ... ⟨x0⟩ ⟨yℓ-1⟩ ... ⟨y0⟩} to the form
   {⟨xℓ-1⟩ ⟨yℓ-1⟩ ... ⟨x0⟩ ⟨y0⟩} */
2 { ⟨β⟩ } ; /* Push base to the stack */
3 { OP_LIMB_ADD_CARRY OP_TOALTSTACK }
4 for _ ∈ {0, ..., ℓ - 3} do
    /* At this point, stack looks as {⟨xn⟩ ⟨yn⟩ ⟨β⟩ ⟨c⟩} . We need to add carry c
       and call OP_LIMB_ADD_CARRY */
5     { OP_ROT }
6     { OP_ADD }
7     { OP_SWAP }
8     { OP_LIMB_ADD_CARRY OP_TOALTSTACK }
9 end
    /* At this point, again, stack looks as {⟨xn⟩ ⟨yn⟩ ⟨β⟩ ⟨c⟩} . We need to drop the
       base, add carry, and conduct addition, assuming overflowing does not occur */
10 { OP_NIP OP_ADD, OP_ADD }
    /* Return all limbs to the main stack */
11 for _ ∈ {0, ..., ℓ - 2} do
12 | { OP_FROMALTSTACK }
13 end

```

---

# BigInt Multiplication

## *Problem*

Given two 254-bit integers  $x$  and  $y$ , find 508-bit  $z := x \times y$ .

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## Algorithm 2: Double-and-add method for integer multiplication

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**Input** :  $x, y$  — two integers being multiplied

**Output** : Result of the multiplication  $x \times y$

```
1 Decompose  $y$  to the binary form:  $(y_0, y_1, \dots, y_{N-1})_2$ 
2  $r \leftarrow 0$ 
3  $t \leftarrow x$ 
4 for  $i \in \{0, \dots, N-1\}$  do
5   if  $y_i = 1$  then
6      $r \leftarrow r + t$ 
7   end
8    $t \leftarrow 2 \times t$ 
9 end
Return : Integer  $r$ 
```

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**Problem:** If the cost of addition in  $A$  and of doubling is  $D$ , the total algorithm's cost is  $NA + ND$ .

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## Definition

The  $w$ -**width** form of a scalar  $k \in \mathbb{Z}_{\geq 0}$  is the representation:

$$k = \sum_{i=0}^{L-1} k_i \times 2^{wi}, \quad 0 \leq k_i < 2^w,$$

where the decomposition length is  $L = \lceil N/w \rceil$ .

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Using multiplication in such form, we will reduce the complexity down to:

$$[2^{w-1}A + 2^{w-1}D] + \left[ \frac{N}{w}A + ND \right]$$

# $w$ -width BigInt Multiplication

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## Algorithm 3: Double-and-add method for integer multiplication

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**Input** :  $x, y$  — two integers being multiplied

**Output**: Result of the multiplication  $x \times y$

- 1 Decompose  $y$  to the  $w$ -width form:  $(y_0, y_1, \dots, y_{L-1})_w$
  - 2 Precompute  $\{0x, 1x, 2x, \dots, (2^w - 1)x\}$ . Denote  $\mathcal{T}[j] = jx$ .
  - 3  $q \leftarrow 0$
  - 4 **for**  $i \in \{L-1, \dots, 0\}$  **do**
  - 5     **for**  $\_ \in \{1, \dots, w\}$  **do**
  - 6          $q \leftarrow 2q$
  - 7     **end**
  - 8      $q \leftarrow q + \mathcal{T}[y_i]$
  - 9 **end**
- Return** : Integer  $q$
-

## Further Optimization

However, which value of  $w$  to choose? In our research, we optimized the cost function:

$$C(w) = 2^{w-1}(C_A + C_D) + \frac{NC_A}{w} + NC_D$$



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### Theorem

*Optimal value is given by  $\hat{w}^2 2^{\hat{w}} = \frac{2N}{\log 2} \cdot \frac{C_A}{C_A + C_D}$*

For BN254, we have  $N = 254$ ,  $\hat{w} = 4$ , so we need  $72A + 262D$  operations instead of  $254A + 254D$ .

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**Naive approach:** Extend  $x, y$  to  $2N$  bit and conduct the algorithm as usual.

**Better approach:** Extend number of bits of  $x$  and  $y$  gradually throughout the loop.

# Results

Approach	Overflowing Multiplication	Widening Multiplication
Cmpeq	N/A	201,879
BitVM bigint	106,026	200,334
BitVM Karatsuba	N/A	74,907
Our <i>w</i> -width method	55,710	71,757

- Besides better results in terms of OPCODEs #, this method allows to optimize MSM (batch multiplication).
- As a result, our implementation is included in the BitVM codebase and the total Groth16 script size is reduced 2×.

# Thank you for your attention



[distributedlab.com](https://distributedlab.com)



[github.com/distributed-lab/bitcoin-window-mul](https://github.com/distributed-lab/bitcoin-window-mul)