Mean Muon Lifetime and Parity Violation in the Weak Interaction

Brandon Loptman, Cassandra Armstrong, Nicholas Zak-Lee
University of California, Los Angeles

Abstract

The purpose of this experiment is to verify that parity is violated in weak interactions as well as to measure the mean lifetime of muons. This was accomplished by selecting positive cosmic ray muons and allowing them to stop in a copper target. These muons subsequently decay via the weak interaction into positrons. The direction of positron emitted from the decay can be detected by our apparatus. We measured the muon lifetime to be $2.13384 \pm .00611\mu s$, which differs by the currently accepted value by only 0.09241% when considering the upper error bound of our associated with our measurement. Additionally, we were to verify parity violation by showing a noticeable asymmetry between the number of upward and downward positron decays.

I Introduction

Prior to 1956, the laws of nature were thought to remain the same under reflection. That is, the results of an experiment viewed via a mirror were expected to be identical to the results of a mirror-reflected copy of the experimental apparatus. This so-called law of parity conservation was known to be respected by classical gravitation, electromagnetism, and the strong interaction. However, this was to change. In 1956 Chin-Ning Yang and Tsung-Dao Lee were led to wonder whether there had been any experimental test of the assumption of parity conservation. Upon searching the literature, they were surprised to discover that while there was ample evidence for parity invariance in strong, electromagnetic, and gravitational processes there was no such confirmation in the case of the weak interactions. In order to remedy this, Yang and Lee proposed a test. This test was carried out later that year by Chien-Shing Wu.

In Wu's famous experiment radioactive cobalt 60 nuclei were carefully aligned, so that their spins pointed in a particular direction (for sake of illustration, say the z direction). Cobalt 60 undergoes beta decay and Wu recorded the the directions of emitted electrons. Equal numbers of electrons should be emitted parallel and antiparallel to the magnetic field if parity is conserved, but they found that more electrons were emitted in the direction opposite to the magnetic field and therefore opposite to the nuclear spin. This was the first experiment to show parity violation in the weak force. Further work was carried out by Leon Lederman, who performed a very similar experiment that modified an existing cyclotron experiment. Ultimately his findings agreed with Wu's that parity is violated in weak interactions. We take the influences of both Wu and Lederman in our experiment and attempt to replicate their results through the decay of positively charged muons.

II Theory

Mean Muon Lifetime

The muon is an elementary particle and one of the fundamental constituents of matter. Muons are very similar to electrons, apart from the fact that a muon has about 200 times more mass than an electron. Due to their mass muons are unstable and decay by the weak force almost exclusively into an electron or positron and two neutrinos. The decay time probability for muons follows an exponential decay law. The distribution $N(t)$ of muons with a measured lifetime t can be described as

$$N(t) = N_o e^{-t/\tau_{\text{muon}}} \tag{1}$$

where τ_{muon} is the mean muon lifetime and N_0 is a normalization parameter. The currently accepted value of τ_{muon} according to the Particle Data Group (PDG) is 2.19698 μs

Parity Violation in the Weak Interaction

To determine whether parity is violated or not the main quantity we wish to examine is $\langle \vec{\sigma} \cdot \vec{p} \rangle$, where $\vec{\sigma}$ is the spin of some particle and $\vec{p}$ is its momentum. If parity is conserved then one would expect $\langle \vec{\sigma} \cdot \vec{p} \rangle = 0$.

Now, consider a muon with spin $\vec{\sigma}$ oriented downward along the negative z-axis with a constant magnetic field $\vec{B}$ of magnitude B orthogonal to $\vec{\sigma}$ along the positive x-axis. The Hamiltonian for the muon is given by

$$U = \mu_B (\vec{\sigma} \cdot \vec{B}) = \mu_B (\sigma_x B) \tag{2}$$

where μ_B is the Bohr magneton and σ_x is the Pauli matrix given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3}$$

The eigenvalues of σ_x are $\lambda = \pm 1$ with corresponding eigenvectors

$$|x_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4}$$

$$|x_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{5}$$

Using this information we can see that

$$U = \mu_B \sigma_x B = \pm \mu_B B, \tag{6}$$

An important quantity to determine is how $\vec{\sigma} \cdot \vec{p}$ changes as a function of time. This amounts to determining how z spin changes over time. To do this we must rewrite the x-basis in terms of the z-basis. To do so, we need the z Pauli matrix, σ_z, which is given by

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{7}$$

The eigenvalues of σ_z are $\lambda = \pm 1$ with corresponding eigenvectors

$$|z_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

$$|z_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{9}$$

Using equations 8 and 9 it can be readily shown that

$$|x_+\rangle = \frac{1}{\sqrt{2}} |z_+\rangle + \frac{1}{\sqrt{2}} |z_-\rangle \tag{10}$$

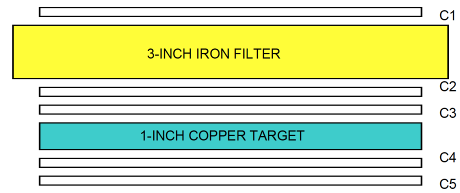

Figure 1: Schematic sketch of our experimental apparatus.

$$|x_-\rangle = \frac{1}{\sqrt{2}} |z_+\rangle - \frac{1}{\sqrt{2}} |z_-\rangle. \tag{11}$$

For ease of calculation, suppose that the initial state of the system is $|z_+\rangle$. Then we have

$$\begin{aligned}
\Psi(t = 0) &= |z_+\rangle \\
&= \frac{1}{2} \left(|z_+\rangle + |z_-\rangle \right) + \frac{1}{2} \left(|z_+\rangle - |z_-\rangle \right) \\
\implies \Psi(t) &= \frac{1}{2} \left(e^{-i\omega t} + e^{i\omega t} \right) |z_+\rangle + \frac{1}{2} \left(e^{-i\omega t} - e^{i\omega t} \right) |z_-\rangle \\
&= \cos(\omega t) |z_+\rangle - i \sin(\omega t) |z_-\rangle.
\end{aligned} \tag{12}$$

where $\omega = \frac{\mu_B B}{\hbar}$. The average spin along the z-axis as a function of time is is given by

$$\begin{aligned}
\langle \Psi | \sigma_z | \Psi \rangle &= \cos^2(wt) \langle z_+ | \sigma_z | z_+ \rangle + \sin^2(wt) \langle z_- | \sigma_z | z_- \rangle \\
&= \cos^2(wt) - \sin^2(wt) \\
&= \cos(2wt).
\end{aligned} \tag{13}$$

Therefore,

$$\langle \vec{\sigma} \cdot \vec{p} \rangle \sim \cos(2wt). \tag{14}$$

Integrating the above function over a full period we find $\langle \vec{\sigma} \cdot \vec{p} \rangle = 0$. If parity is not conserved then $\langle \vec{\sigma} \cdot \vec{p} \rangle$ would attain some nonzero value.

III Procedure

To find the muon lifetime and parity violation, we used a set-up consisting of five counters $C1$, $C2$, $C3$, $C4$, and $C5$, one 3 inch iron filter, and a 1 inch copper target. A counter was on top of the system followed by the iron filter, two more counters, the copper target, and finally two last counters (as shown in figure 1). The counters are made of polystyrene, a material

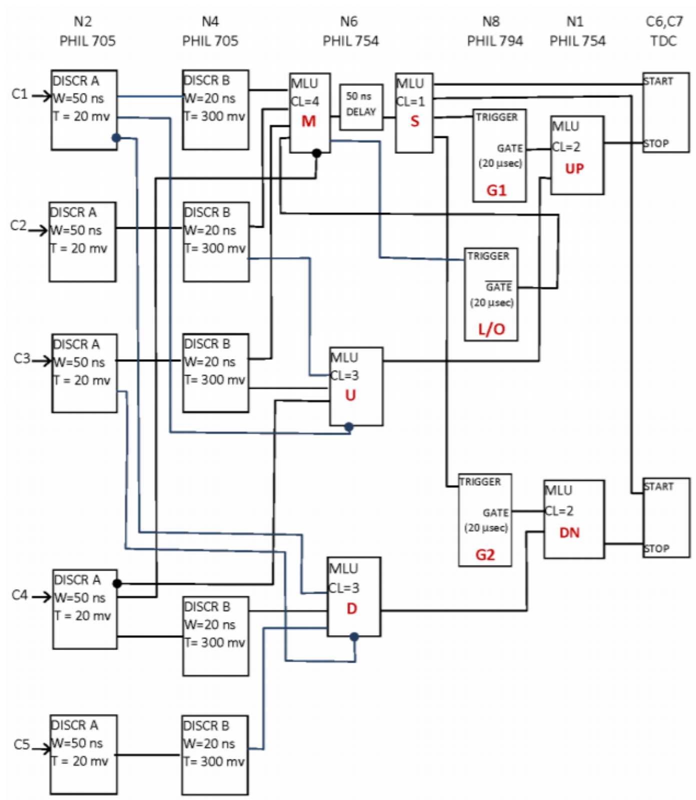

Figure 2: The logic setup of our experiment. The five counters were each connected to a discriminator A with a specified output width and threshold. The discriminator produces a signal of the specified width (W) when it receives a signal that crosses the set threshold (T). We set each discriminator to best filter out noise while still picking up signals from muon hits.

that when a muon passes through, electrons are ionized and emit a photon when recoupled. The material that these counters are made of is called a scintillator. With a photomultiplier tube (PMT), photons emitted due to muons passing through the counters are detected. The iron filter and copper target select out muons of a specific momentum. The muons lose energy to ionization as they pass through the iron filter. When the muons reach the copper target, they ionize further and those that lose all their energy to ionization are stopped in the target. These muons that stop in the copper target decay into a positron and a neutrino. Using a logic system (figure 2), we could find which muons stopped in the copper target and how many of those muons decayed into upward moving positrons and how many decayed into downward moving positions.

The signal produced by a muon stopping in the copper target is $M = \overline{C4} \otimes C3 \otimes C2 \otimes C1$, which means that a signal is produced when there is a coincidence between scintillation detected in $C1$, $C2$, and $C3$ and no scintillation detected in $C4$. This signal is created by inputting B-discriminator output signals from $C1$, $C2$, and $C3$ into a coincidence module, while inputting a veto signal from the C4 A-discriminator. The muons trapped in the copper target then decayed to produce a positron and a neutrino. The positron produced from these decay events went either up in an up event or down in a down event. The up events were detected using the logic signal $U = \overline{C1} \otimes C2 \otimes C3 \otimes \overline{C4}$ while the down events were detected with the logic signal $D = \overline{C1} \otimes \overline{C3} \otimes C4 \otimes C5$. To record and time these up or down events, we started two triggers (G1 for UP and G2 for DN) after an M event occurred and used a lockout signal (L/O) to ensure no new M events triggered before an up or down event occurred or until 20μs had gone by. If a U or D event occurs within at least 20μs of the M event, the event is recorded as either an up (UP) or down (DN) event with the TDC. With these events, we recorded the number of up and down events and the time of each up or down event after the initial muon capture. To determine the mean muon lifetime, we did not use a magnetic field. We later turned on a magnetic field to study parity violation. The magnetic field was constructed such that the period of precession of the muons is close to their lifetime of 2.2μs.

All of the data from our experiment was analyzed in the data analysis framework ROOT.

IV HV Plateaus

In order to have our detectors fully efficient on triggering on real charged particles and trigger on as little spurious electrical noise as possible, two quantities must be adjusted: the PMT high voltage (HV) and the discriminator threshold voltage. The discriminator threshold voltage was adjusted according to the voltages given in figure 2, as previously described. Ideally, for a fixed discriminator setting there should be a range of HV values for which the triggering efficiency is roughly constant, i.e. a plateau.

For our experiment, we plateaued our detectors in two steps. First, for each PMT we measured the trigger rate over 10 second intervals for a range of HV values between 1200 and 1500 volts. This data was then plotted as a function of HV and examined qualitatively in order to determine the voltages for which each PMT was approximately constant. For most of our detectors, the plateau was difficult to make out, if not evident at all. This resulted in crude, but reasonable, estimates of the proper HV values to be used in the second plateauing step.

Next, we operated three PMTs in coincidence with each other, holding two at approximately their proper HV values and varying the other. Doing so, the HV of any one of them can be optimized. To illustrate, let us denote the three PMTs being used as $C1$, $C2$, and $C3$. To optimize the HV of $C2$, we would find the ratio

$$R = \frac{C1 \otimes C2 \otimes C3}{C1 \otimes C3}. \tag{15}$$

As the HV of $C2$ is raised (and it becomes fully efficient at detecting muons) the ratio of the coincidences will become constant. The HV at which R becomes constant is the desired value of HV for that PMT. This procedure was followed for each of our detectors, again measuring the trigger rates over 10 second intervals for a range of HV values, resulting in the plots in figure 3. Again, these plots were examined qualitatively to determine the appropriate HV

PMT No.	Optimal HV (Volts)
1	1400
2	1350
3	1400
4	1375
5	1450

Table 1: Optimal HV value for each PMT based on figure 3.

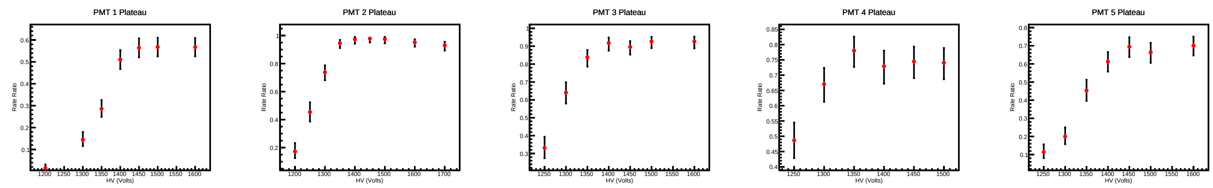

Figure 3: HV Plateau curves for each PMT using the coincidence method of plateauing.

for each PMT. These values are summarized in table 1 were used for the entire duration of the experiment.

V Data Analysis and Results

Determination of the Mean Muon Lifetime

The first objective of our experiment was to measure the mean muon lifetime. This was accomplished by first constructing histograms for both TDCs using the data taken without the magnet installed or turned on. Let the histogram for TDC6 (up events) be denoted U and the histogram for TDC7 (down events) be denoted D. Both U and D were constructed on the interval of 1 to 20 μs (in TDC time) with 38 bins. The interval was chosen to begin at $1\mu s$ in order to mitigate the effects of PMT noise that tends to accumulate for early TDC times.

In order to have a higher total number of events, and thus yield better statistics for our lifetime measurement, the U and D data sets were added together. The composite histogram, $U + D$, was then fit using ROOT to the decaying exponential function

$$N(t) = Ae^{-t/\tau} + B, \tag{16}$$

where τ is our experimental value for the mean muon lifetime, A is the normalization parameter, B is the number of background events per time bin, and t is time. The U, D, and $U+D$ histograms are shown in figure 4. Following this procedure, we found the mean muon lifetime to be $2.13384 \pm .00611$. Comparing our result to the value of the lifetime reported by the PDG we see that their value is just outside the upper error bound of our measurement. Quantitatively, this amounts to a deviation of 0.09241% from the currently accepted value of the mean muon lifetime, which is $\tau_{\text{muon}} = 2.19698\mu s$.

Determination of Parity Violation in the Weak Interaction

Our second objective was to verify parity violation in the weak interaction. This is done by measuring an asymmetry in the number of up and down events recorded by the TDCs. Using the data taken with the magnet installed and turned on we again construct histograms U, D, and $U + D$ in the same manner as before. Additionally, we will also construct a histogram $U - D$, which is the asymmetry between the number of up and down events. These histograms are shown in 5. Finally, we will plot the muon polarization $(U - D)/(U + D)$ by dividing $U - D$ by $U + D$. The histogram division and associated errors are calculated by the TGraphAsymmErrors class in ROOT.

Because of the presence of the magnetic field we expect the muons to precess, leading us to fit the following general sinusoidal function to the asymmetry plot:

$$A\cos(\omega t + \phi) + B, \tag{17}$$

where A, B, ω, and ϕ are chosen to fit the data. Fitting this function to $(U - D)/(U + D)$ we get figure 6. Qualitatively, we can observe that our data mimics the behavior of a sinusoid very well as evident by the best fit curve passing through each data point on the asymmetry plot. Additionally, quantitatively we can compare the value of ω produced by the fit to what we would expect from our experimental apparatus. From the fit we get that $\omega = 2.17285 \pm .02074$. The magnet in our experiment is set up such that the period of precession is close to the muon lifetime of 2.2μs, which corresponds to an angular frequency of 2.85599 Hz. Although, the value of ω generated by the curve fitting differs from its intended value it is still reasonably similar to it.

From our asymmetry plot we can readily observe the violation of parity in the weak interaction. This is apparent as the curve fit to the data does not agree with that of the one predicted by the theory. Compared to the theory, our sinusoid is shifted up on the y-axis which indicates a greater number of up events than down events recorded by our TDCs. In the same vein, it is obvious that for our sinusoid, $\langle \vec{\sigma} \cdot \vec{p} \rangle \neq 0$. This is because the sinusoid is always non-negative so any integral over the function must be greater than zero. Thus, we again see a disagree-

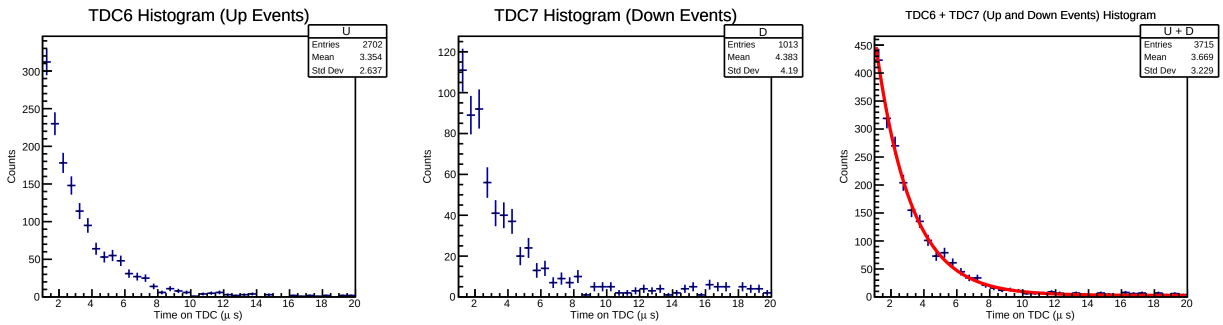

Figure 4: Various histograms for the data runs without the magnet installed and turned off. The red curve in $U + D$ is the best fit of the function given by equation 16. The errors were automatically calculated by ROOT.

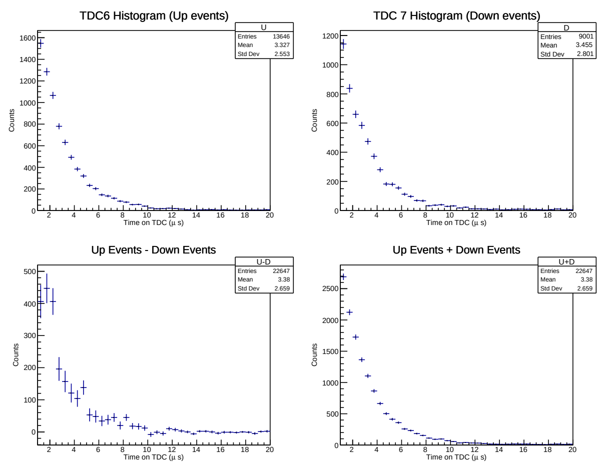

Figure 5: Various histograms for data runs with the magnet installed and turned on. The errors were automatically calculated by ROOT.

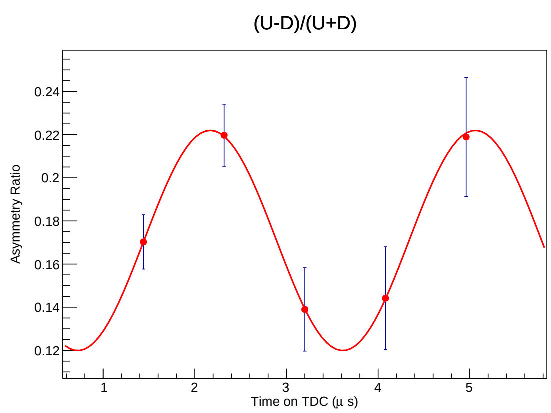

Figure 6: Plot of the asymmetry between U and D and their associated errors. The plot is over approximately two periods. The curve in red is fit from the data points.

ment with the theory, for if parity were conserved $\langle \vec{\sigma} \cdot \vec{p} \rangle$ would be zero.

Error Analysis

The main source of error present during this experiment is the presence of spurious PMT noise. PMT noise can be present in both the muon lifetime measurement and the asymmetry measurement. Noise from the detectors causes the TDCs to go off when genuine stopping signal (from an actual muon decay) has not been triggered, adding erroneous data to U and D, skewing our results. The effect of the PMT noise can be minimized by simply ignoring measurements for very early TDC times as this is where its effect is most prevalent.

Another source of error was caused by the power supply for our electromagnet. While the magnet was supposed to receive a current of 3.84 A, the current output by the power supply would wander slightly. These fluctuations in current were typically only $\pm.01$ A, but nonetheless could introduce error as it would cause the muons to precess with slightly different periods over the course of a data run. However, it is difficult to determine how much one would expect the precession period to differ from what was expected, since the exact geometry of the magnet was unknown.

Overall, we are confident in our results and do not believe they were greatly impacted by these possible errors. This is largely because the results of our experiment were in line with what has been demonstrated by other reputable physicists in the past.