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Assignment on:
propositional Logic
Submitted To:
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Propositional or Sentence

An expression consisting of some symbols, letters and words is called a sentence if it is true or false.

For Example :

1 Jaipur is capital of Rajasthan

2. $2+3=5$

3. Mumbai is in America.

4. $9<6$

5. "Wish you happy life" is not a proposition because truth or false is not certain.

Truth Value

If any proposition is true then its truth value is denoted by T and if the proposition is false then its truth value is denoted by F

Example :

(1) 1 is less than 3 T
(2) 14 is odd number F

types of proposition

(1) Simple proposition : the proposition having one subject and one predicate is called a simple proposition.

Example :

(1) this flower is pink.
(2) Every even number is divisible by 2.

(2) Compound proposition :
Two or more simple proposition when combined by various connectivities into a single composition sentence is called compound proposition.

Example :

(1) the earth is around and revolves around the sun.
(2) A triangle is equilateral iff its three sides are equal.

Logical Connectives

The particular words and symbols used to join two or more proposition into a single composite form or compound proposition are called logical connectives.

Logical Connectives words	symbol	uses
And / conjuction / join	$\wedge$	$p \wedge q$
or / disjunction / meet	$\vee$	$p \vee q$
Negation	~~$-$~~ ~	$\sim p$
Equivalent	$\leftrightarrow$	$p \leftrightarrow q$

(1) Conjuction :

Any two proposition can be combined by the word "AND" to form a compound proposition said to be the conjuction.

Truth Table.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example :

(1) Delhi is in India & 2+2 = 4 (T)

(2) Delhi is in India and 2+2 = 5 (F)

(3) Delhi is in Russia and 2+2 = 4 (F)

(2) Disjunction:

Any two proposition can be combined by word "or" to form a compound proposition is said to be the dijunction.

Example:

Truth Table.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

(1) Delhi is in india or 2+2 = 4 (T)
(2) Delhi is in india or 2+2=5 (T)
(3) Delhi is in Russia or 2+2=4 (T)
(4) Delhi is in Russia or 2+2=6 (F)

3. Negation:

The negation proposition of any given proposition p is the proposition whose truth value is opposite to p.

Example:

p	~p
T	F
F	T

(1) p= "This flower is pink."
(2) ~p= "this flower is not pink."

Equivalence

If p and q are statements, The compound statement p if and only if q, denoted by p ↔ q, is called equivalence. The connective if and only if is denoted by the symbol ↔.

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies and Contradiction.

A proposition is said to be an tautologies if it contain only T in last column of truth table and if it contain only F in last column of truth table then it is called contradiction.

Example 1: Show that the following proposition is tautologies $\{(p \vee \sim Q) \wedge (\sim p \vee \sim Q)\} \vee Q$.

sol

p	Q	$\sim p$	$\sim Q$	$p \vee \sim Q$	$\sim p \vee \sim Q$	$(p \vee \sim Q) \wedge (\sim p \vee \sim Q)$	$\{(p \vee \sim Q) \wedge (\sim p \vee \sim Q)\} \vee Q$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	T

Logically Equivalent :

we can say that p and q are logically Equivalent, or simply equivalent, if $p \leftrightarrow$ is a tautology. when an equivalence is shown to be a tautology. we denote that p is equivalent to q by $p \equiv q$.

Example: The binary operation $\vee$ has the commutative property; that is, $p \vee q \equiv q \vee p$. the truth table for $(p \vee q) \leftrightarrow (q \vee p)$ shown that the statement is a tautology.

p	q	$p \vee q$	$q \vee p$	$(p \vee q) \leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

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Conditional Statement.

Many statements are of the form "If p then q" such statement are said to be the Conditional statement and denoted by $p \Rightarrow q$ or $p \rightarrow q$.

Truth Table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional statement.

A statement "p if and only if q" such statement are said to be bi-conditional statement and denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$.

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse, inverse and Contrapositive of Conditional statement.

(1) Converse of Conditional Statement:
let $p \to q$ be a conditional statement then $q \to p$ is called its converse.

Example:
if x is a labourer then he is poor
its converse
if x is poor then he is labourer.

(2) Inverse of Conditional statement:
let $p \to q$ be a conditional statement then $\sim p \to \sim q$ is called its inverse.

Example: if Harsa read book then he will get knowledge.
its Inverse
if Harsa not read book then he will not get knowledge.

(3) Contrapositive of Conditional statement:
let $p \to q$ be a conditional statement then $\sim q \to \sim p$ is called contrapositive.

Example: if $f(x)$ is differentiable then it is continuous.
Its Contrapositive:
If $f(x)$ is not continuous then it is not differentiable.

predicates

Two or more statement have some feature common the we take common feature as predicate:

Example:

$B(x) = x$ is a bachelor

$x = $ John $\rightarrow$ John is a bachelor.

→ Hence x is a place holder until it is replaced by name of the object it will not be a statement.

→ predicate can have n-no of place holder.

$T(x, y) = x$ is taller than y.

$S(x, y, z) = x$ sits in between y & z.