Waves and Oscillations

Periodic Motion

A motion which repeats itself over and over again after a regular interval of time is known as periodic motion

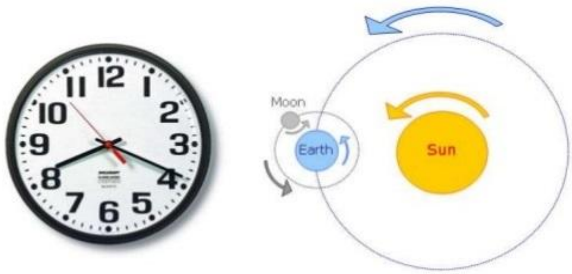


Figure: Periodic motion

Oscillatory Motion

A particle having periodic motion remains half of its time period in one direction and the rest of time period remains in another direction along the same line, then its motion is called oscillatory motion.

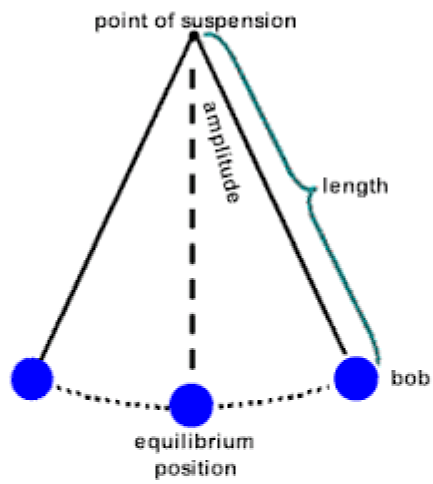


Figure: Simple Harmonic Motion

Simple Harmonic Motion

If the acceleration of an oscillatory particle is proportional to distance from its equilibrium position and always towards the equilibrium position, then that motion is called simple harmonic motion.

All simple harmonic motions are periodic motion but all periodic motions are not simple harmonic motion.

Wave

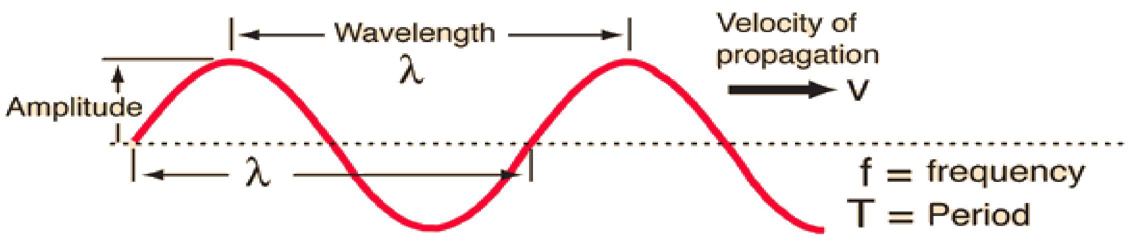
Wave is a form of disturbance which travels through a medium (or vacuum) and is produced due to periodic motion (or vibration) of particles about their mean position. During wave motion, the molecules do not move with the wave but they transfer the energy from one molecule to another.

Properties of wave

(i) Wave has amplitude.
(ii) Wave has frequency.
(iii) Wave has wavelength.
(iv) Wave can be progressive or stationary.
(v) Wave can be transverse or longitudinal.
(vi) Wave produce reflection, refraction, interference and diffraction.
(vii) Wave transfers energy from one place to another place.

Some important terms of waves

i) **Wave length** **ii) Amplitude** **iii) Frequency** **iv) Time- period**



i) **Wave length**: The distance traveled by a wave, when the vibrating particle completes one vibration, is called wavelength of the wave. It is denoted by λ. In figure-1 OP=λ, or BC =λ.

ii) **Amplitude**: The maximum displacement of a vibrating particle from its equilibrium position of rest is called amplitude. In figure-1, PQ = RS =a, the amplitude of the vibrating particle of the transverse wave.

iii) **Frequency**: The number of complete vibrations (or oscillations) of any vibrating (or oscillatory) particle performed in one second is called its frequency of vibrations. Let us consider a vibrating particle completes N number of vibrations at time t. Therefore, the frequency of vibration is $\mathbf{f} = \frac{\mathbf{N}}{\mathbf{t}}$. The unit of frequency is Hertz (Hz).

iv) **Time-period**: The time required by a vibrating or oscillatory particle to complete one vibration is called the time-period of the particle. It is denoted by T. Let us consider a particle complete N number of vibrations in time t. The time-period of the particle is then

$$\mathbf{T} = \frac{\mathbf{t}}{\mathbf{N}}\ \mathbf{second}$$

v) **Phase:** Phase is the position of a point in time (an instant) on a waveform cycle. A complete cycle is defined as the interval required for the waveform to return to its arbitrary initial value.

<u>Establish the relation</u>

i) $\mathbf{T} = \frac{1}{\mathbf{f}}$

ii) v = fλ

i) Let us consider a vibrating particle completes N number of vibrations in time, t. Therefore, the frequency of the particle is given by

$$\mathbf{f} = \frac{\mathbf{N}}{\mathbf{t}}\,\mathbf{(Hz)}$$

$$\mathbf{or,}\ \mathbf{N} = \mathbf{ft} \qquad (1)$$

Again, the time period of vibration is given by

$$\mathbf{T} = \frac{\mathbf{t}}{\mathbf{N}}\,\mathbf{(s)} \qquad (2)$$

From equation (1) and (2) we have

$$\mathbf{T} = \frac{\mathbf{t}}{\mathbf{N}} = \frac{\mathbf{t}}{\mathbf{ft}} = \frac{1}{\mathbf{f}}$$

$$\therefore \mathbf{T} = \frac{1}{\mathbf{f}} \qquad (3)$$

This is the required equation.

ii) Let us consider a wave traveling through a medium with a velocity, v. Its frequency, wavelength and time period are f, λ, and T respectively.

Therefore, the velocity of the wave is given by

$$\mathbf{v} = \frac{\mathbf{displacement}}{\mathbf{time}}$$

But according to the definition of wavelength, T is the time required to traverse a distance, λ..

$$\therefore \mathbf{v} = \frac{\lambda}{\mathbf{T}} \qquad (4)$$

From equation (3) and (4)

$$\mathbf{v} = \frac{\lambda}{\frac{1}{\mathbf{f}}} = \mathbf{f}\lambda$$

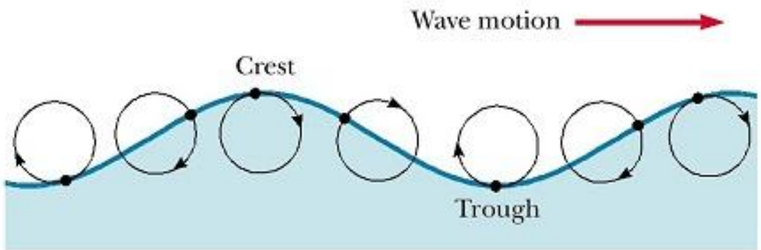
$$\therefore \mathbf{v} = \mathbf{f}\lambda \qquad (5)$$

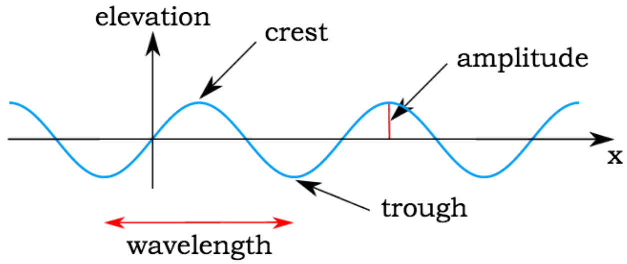
This is the required equation.

<u>According to the direction of motion of the particles in a wave there are two kinds of waves:</u>

i) **Transverse wave** **ii) Longitudinal wave**

i) **Transverse wave**: If the disturbances of the wave particles are at right angles to the direction of propagation then the wave is called a transverse wave.





Example: examples of transverse wave is the wave passing over the surface of water, electromagnetic waves which include light waves are also transverse wave.

ii) Longitudinal wave: A longitudinal wave is one in which the vibration occurs in the same direction as the direction of travel of the wave.

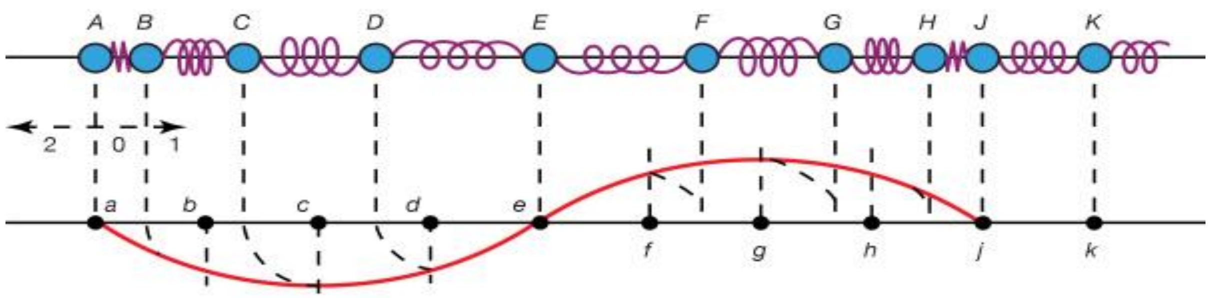


Fig. Propagation of sound (Motion of spring)

Example: The most common example of a longitudinal wave is a sound wave. This is produced by alternate compressions and refractions of the air. Another example of longitudinal wave is spring wave. This is found when one end of the spring is fixed to a rigid support and the either end is slowly moved back and forth.

Difference between transverse waves and longitudinal waves

Transverse Waves	Longitudinal Waves
1. Transverse waves consist of crests and troughs.	1. Longitudinal waves consist of compressions and rarefactions.
2. There are no pressure variations.	2. There is a pressure variation throughout the medium.
3. They can be propagated through solids and surfaces of liquids but not in gases.	3. They can be propagated through solids, liquids as well as through gases.
4. In transverse waves, the particles of the medium vibrate at right angles to the direction of wave propagation.	4. In longitudinal waves, the particles of the medium vibrate parallel to the direction of the wave propagation.
5. There is no change in the density of medium.	5. There is a change in the density throughout the medium.
6. Light wave is an example of transverse wave.	6. Sound wave is an example of longitudinal wave.

What do you mean by Crests and Troughs ?

In transverse wave, the points where the particles of the medium have maximum displacement in the positive direction are called crests.

On the other hand, the points where the particles of the medium have maximum displacement in the negative direction are called troughts.

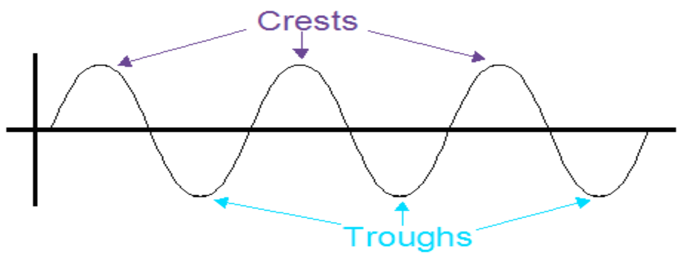


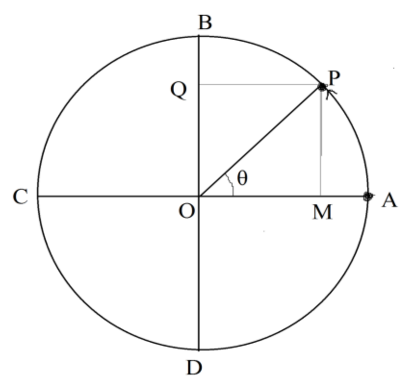
Figure: Crests and troughs of a wave

Equation of Simple Harmonic Motion

Let a particle move round a point O in a circular path ABCD of radius 'a' as shown in figure.

Let it come to point P from point A at time t. From P a normal PM is drawn on the diameter CA.

Now from figure, $\angle POA = \theta$, PM= OQ = y ,
Radius OP = a



Now, $\sin\theta = \frac{PM}{OP} = \frac{y}{a}$

or, y = a sin θ (1)

We know that, angular velocity, $\omega = \frac{\theta}{t}$

$\therefore\ \theta = \omega t$

Putting the value of θ in eqn.(1) We have,
$\therefore y = a \sin\omega t$

$\boxed{y = a\sin\omega t}$ →Displacement equation of Simple harmonic motion

Velocity, $v = \frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t) = a\,\omega\cos\omega t$

Acceleration, $A = \frac{dv}{dt} = \frac{d}{dt}(a\,\omega\cos\omega t) = -a\,\omega^2\sin\omega t$

Progressive wave

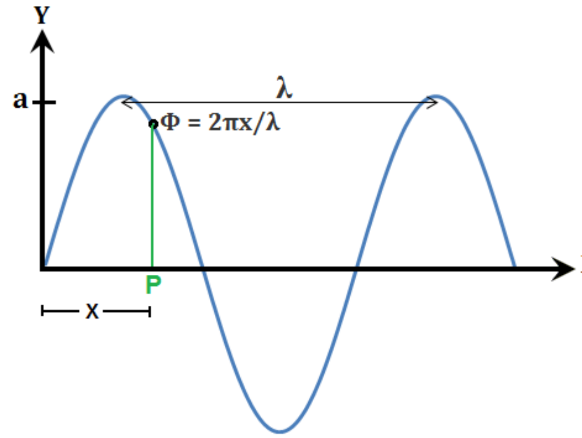
When a wave propagating from one layer to another layer of a wide medium progresses continuously in the forward direction, then it is called progressive wave.

Stationary wave

Stationary waves are produced by superposition of two progressive waves of equal amplitude and frequency, travelling with the same speed in opposite direction.

Equation of Progressive Wave

The wave which transmits energy from one point to another point of a medium is called progressive wave. In such wave, disturbance travels forward and handover to next particle after a certain time. During the propagation of the wave, all the particles of the medium vibrates with the same amplitude and wavelength, but vibration begins a little later than the other particle immediately before it. So, phase lag gets increased along the direction of propagation.



Let us consider a wave travelling from left to right along the x-axis.
Consider a particle at the origin O vibrate simple harmonically and its
displacement (y) at time t is given by:

$$y = asin\theta$$
$$y = asin\omega t$$

Where a is the amplitude, ω is the angular velocity of the wave.
If phi (φ) be the phase difference of the particle at P then,

When distance = λ, then the phase difference = 2π

distance = 1, then the phase difference = $\frac{2\pi}{\lambda}$

distance = x, then the phase difference = $\frac{2\pi}{\lambda}x$

So, phase difference, $\varphi = \frac{2\pi}{\lambda} \times$ Path difference

$$y = \text{asin}(\omega t - \varphi)$$
$$y = \text{asin}(\omega t - \frac{2\pi}{\lambda}x)$$
$$y = \text{asin}(2\pi f t - \frac{2\pi}{\lambda}x)$$
$$y = \text{asin}(2\pi \frac{v}{\lambda}t - \frac{2\pi}{\lambda}x)$$
$$y = \text{asin}\frac{2\pi}{\lambda}(vt - x)$$

Hence

$$\omega = 2\pi f = \frac{2\pi}{T}$$
$$v = f\lambda$$

If the wave move at the negative direction of X-axis, then the equation become

$$y = \text{asin}\frac{2\pi}{\lambda}(vt + x)$$

Problem:
The equation of a progressive wave is $y = 5\sin\frac{2\pi}{3}(6t - x)$; using this equation, calculate the (i) amplitude, (ii) wavelength, (iii) velocity (iv) frequency (v) Time period

Solution:
We know, the equation if progressive wave is $y = a\sin\frac{2\pi}{\lambda}(vt - x)$ and hence the given equation is $y = 5\sin\frac{2\pi}{3}(6t - x)$
Comparing these two equations, we get
(i) Amplitude a = 5 m
(ii) Wavelength λ = 3 m
(iii) Velocity v = 6 m/s
(iv) Frequency f = ?

We know $v = f\lambda$

So, $f = v/\lambda$ = 6/3 = 2 Hz

(v) Time period T = ?
T = 1/f = 1/2 = 0.5 sec

Problem 1: The equation of a progressive wave is y = 3 sin (100πt – 1.57 x); using this equation calculate the amplitude, wavelength, frequency and velocity of the wave.

Problem 2: A wave has a frequency of 10000 Hz and a wave length of 2 cm. Calculate the velocity of the wave.