Russell
Andlauer
3/4/15

Section 2.4 : 3, 6a,b,c, 12a,b, 30, 32a

3. What are the terms $a_0, a_1, a_2,$ and a_3 of the sequence $\{a_n\}$, where a_n equals

a) 2^n+1 $\quad 2^0+1=2 \quad 2^1+1=3 \quad 2^2+1=5 \quad 2^3+1=9$

$\boxed{2, 3, 5, 9}$

b) $(n+1)^{n+1}$ $\quad (0+1)^{0+1}=1 \quad (1+1)^{(1+1)}=4 \quad (2+1)^{(2+1)}=27 \quad (3+1)^{(3+1)}=128$

$\boxed{1, 4, 27, 128}$

c) $\frac{n}{2}$ $\quad \frac{0}{2}=0 \quad \frac{1}{2}=\frac{1}{2} \quad \frac{2}{2}=1 \quad \frac{3}{2}=3/2$

$\boxed{0, 1/2, 1, 3/2}$

d) $\frac{n}{2}+\frac{n}{2}$ $\quad 0+0=0 \quad \frac{1}{2}+\frac{1}{2}=1 \quad 1+1=2 \quad \frac{3}{2}+\frac{3}{2}=3$

$\boxed{0, 1, 2, 3}$

6. List the first 10 terms of each of these sequences

a) $a_n = a_{n-3}$ $n\text{-}3$

$a_0 = 10$

$\boxed{10, 7, 4, 1, -2, -5, -8, -11, -14, -17}$

b) $\boxed{a_n = a_{n-1} + a_{n-2} \ldots + a_{n-10}}$

c) $a_n = 3^n - 2^n$

$3^0-2^0 \quad 3^1-2^1 \quad 3^2-2^2 \quad 3^3-2^3 \quad 3^4-2^4 \quad 3^5-2^5$

$1-1 \qquad\qquad 9-4 \quad 27-8 \quad 81-16 \quad 243-32$

$0 \quad 1 \quad 5 \quad 19 \quad 65 \quad 211$

$\boxed{0, 1, 5, 19, 65, 211, 665, 2059, 6305, 19171}$

$3^6-2^6 \quad 3^7-2^7 \quad 3^8-2^8 \quad 3^9-2^9$

$729-64 \quad 2187-128 \quad 6561-256 \quad 19683-512$

$665 \quad 2059 \quad 6305$

Section 2.4

12. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

a) $a_n = 0$ b) $a_n = 1$

$a_n = -3(-1) + 4(-2)$ $a_n = -3(0) + 4(-1)$

$3 + -8$ $= -4$

-5 $a_n = -3(-5) + 4(-6)$

$a_{n-1} = -3(-6) + 4(-7)$ 15 -24

$18 + -28$ $= -9$

-10 $a_n = -3(-10) + 4(-11)$

$a_{n-2} = -3(-11) + 4(-12)$ $30 + -44$

$33 + -48$ $= -14$

$= -15$

When $a_n = 0$ and $a_n = 1$ the solution decreases by 5 each time

30. What are the values of these sums, where $S = \{1, 3, 5, 7\}$

a) $\sum_{j \in S} j$ $j = 1+3+5+7$

$\boxed{j = 16}$

b) $\sum_{j \in S} j^2$ $j = 1^2 + 3^2 + 5^2 + 7^2$

$1 + 9 + 25 + 49$

$\boxed{j = 84}$

c) $\sum_{j \in S} 1/j$ $j = \frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$

$\boxed{j = 4/105}$

d) $\sum_{j \in S} 1$ $j = 1+1+1+1$

$\boxed{j = 4}$

Section 2.4

32. Find the value of each of these sums.

a) $\sum_{j=0}^{8} (1 + (-1)^j)$

$\boxed{= 10}$

$1 + (-1)^0 = 2$
$1 + (-1)^1 = 0$
$1 + (-1)^2 = 2$
$1 + (-1)^3 = 0$
$1 + (-1)^4 = 2$
$1 + (-1)^5 = 0$
$1 + (-1)^6 = 2$
$1 + (-1)^7 = 0$
$1 + (-1)^8 = 2$

Section 4.1: 10 a,c, 12 a,b, 13 a,c, 14 a,c, 28, 30, 38, 40

10. What are the quotient and remainder when

a) $44 / 8$ = Quotient = 5 = 44 div 8
Remainder = 4 = 44 mod 8

c) $-123 / 19$ = Quotient = -6 = -123 div 19
Remainder = 9 = -123 mod 19

12. What time does a 24-hour clock read.

a) 100 hours after 02:00 6 = 102 mod 24

$\boxed{08:00}$ 6+2 = 8

b) 45 hours before it reads 12:00 21 = 45 mod 24

$\boxed{09:00}$ 12 - 21

9:00

Section 4.1

13. Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$

Find the integer c with $0 \leq c \leq 12$ such that

a) $c \equiv 9a \pmod{13}$

$c \equiv 9(4) \pmod{13}$

$c \equiv 36 \bmod 13$

$\boxed{c = 10}$

c) $c \equiv a+b \pmod{13}$

$(4+9) \pmod{13}$

$c = 13 \bmod 13$

$\boxed{c \equiv 0}$

14. Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, $b \equiv 3 \pmod{19}$

Find the integer c with $0 \leq c \leq 18$ such that

a) $c \equiv 13a \pmod{19}$

$13(11) \pmod{19}$

$143 \pmod{19}$

$\boxed{c = 10}$

c) $c \equiv a-b \pmod{19}$

$11-3 \pmod{19}$

$9 \pmod{19}$

$\boxed{c \equiv 9}$

28. Decide whether each of these integers is congruent to 3 modulo 7

a) 37

$37 \not\equiv 3 \pmod 7$

$\boxed{\text{No}}$

b) 66

$66 \equiv 3 \pmod 7$

$\boxed{\text{Yes}}$

c) −17

$-17 \equiv 3 \pmod 7$

$\boxed{\text{Yes}}$

d) −67

$-67 \not\equiv (3 \bmod 7)$

$\boxed{\text{No}}$

Section 4.1

30) Find each of these values

a) $(177 \bmod 31 + 270 \bmod 31) \bmod 31$

$447 \bmod 31$

$\boxed{13}$

b) $(177 \bmod 31 \cdot 270 \bmod 31) \bmod 31$

$47790 \bmod 31$

47771

$\boxed{19}$

38. Show that if n is an integer then $n^2 \equiv 0$ or $1 \pmod 4$

Let n be an integer

$n = 5$

$\cancel{t} = 5^2 \pmod 4$

$n = 4^2 \pmod 4$

$0 = n^2 \pmod 4$

QED

40. Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$

Let n be an odd positive integer

$n = 3$

$3^2 = 1 \pmod 8$

$27 = 1 \pmod 8$

QED