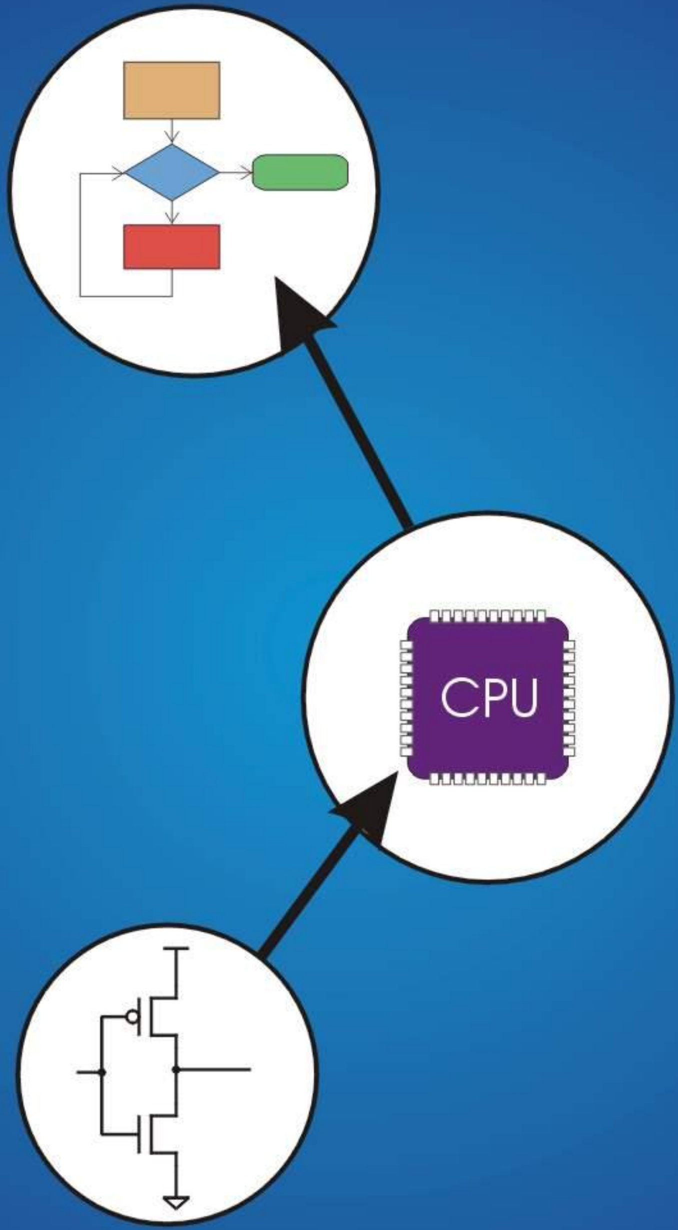

Chapter 2

Bits, Data Types, and Operations

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How do we represent data in a computer?

At the lowest level, a computer is an electronic machine.

- **works by controlling the flow of electrons**

Easy to recognize two conditions:

1. **presence of a voltage – we'll call this state "1"**
2. **absence of a voltage – we'll call this state "0"**

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Computer is a binary digital system.

Digital system:

- finite number of symbols

Binary (base two) system:

- has two states: 0 and 1

Basic unit of information is the *binary digit*, or *bit*.

Values with more than two states require multiple bits.

- **A collection of two bits has four possible states:**
  00, 01, 10, 11
- **A collection of three bits has eight possible states:**
  000, 001, 010, 011, 100, 101, 110, 111
- ***A collection of n bits has 2^n possible states.***

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What kinds of data do we need to represent?

- **Numbers – signed, unsigned, integers, floating point, complex, rational, irrational, …**
- **Text – characters, strings, …**
- **Images – pixels, colors, shapes, …**
- **Sound**
- **Logical – true, false**
- **Instructions**
- **…**

Data type:

- ***representation* and *operations* within the computer**

We'll start with numbers…

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Unsigned Integers

Non-positional notation

- **could represent a number ("5") with a string of ones ("11111")**
- **problems?**

Weighted positional notation

- **like decimal numbers: "329"**
- **"3" is worth 300, because of its position, while "9" is only worth 9**

329

10^2 10^1 10^0

$\boxed{3\text{x}100 + 2\text{x}10 + 9\text{x}1 = 329}$

most significant 101 *least significant*

2^2 2^1 2^0

$\boxed{1\text{x}4 + 0\text{x}2 + 1\text{x}1 = 5}$

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Unsigned Integers (cont.)

An *n*-bit unsigned integer represents 2^n values: from 0 to 2^n-1.

2^2	2^1	2^0	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

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Unsigned Binary Arithmetic

Base-2 addition – just like base-10!

- add from right to left, propagating carry

carry

```
  10010      10010      1111
+  1001    +  1011    +    1
  -----      -----    ------
  11011      11101     10000

             10111
           +   111
             -----
```

Subtraction, multiplication, division,…

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Signed Integers

With n bits, we have 2^n distinct values.

- **assign about half to positive integers (1 through 2^{n-1}-1) and about half to negative (-(2^{n-1}-1) through -1)**
- **that leaves two values: one for 0, and one extra**

Positive integers

- **just like unsigned – zero in *most significant* (MS) bit**
  00101 = 5

Negative integers

- **sign-magnitude – set MS bit to show negative, other bits are the same as unsigned**
  10101 = -5
- **one's complement – flip every bit to represent negative**
  11010 = -5
- **in either case, MS bit indicates sign: 0=positive, 1=negative**

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Two's Complement

Problems with sign-magnitude and 1's complement

- **two representations of zero (+0 and –0)**
- **arithmetic circuits are complex**
  - **How to add two sign-magnitude numbers?**
    - **e.g., try 2 + (-3)**
  - **How to add two one's complement numbers?**
    - **e.g., try 4 + (-3)**

Two's complement **representation developed to make circuits easy for arithmetic.**

- **for each positive number (X), assign value to its negative (-X), such that X + (-X) = 0 with "normal" addition, ignoring carry out**

```
  00101  (5)         01001  (9)
+ 11011  (-5)      + _____  (-9)
  00000  (0)         00000  (0)
```

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Two's Complement Representation

If number is positive or zero,

- normal binary representation, zeroes in upper bit(s)

If number is negative,

- start with positive number
- flip every bit (i.e., take the one's complement)
- then add one

```
   00101  (5)                 01001  (9)
   11010  (1's comp)                 (1's comp)
+      1                   +      1
  ------                     ------
   11011  (-5)                       (-9)
```

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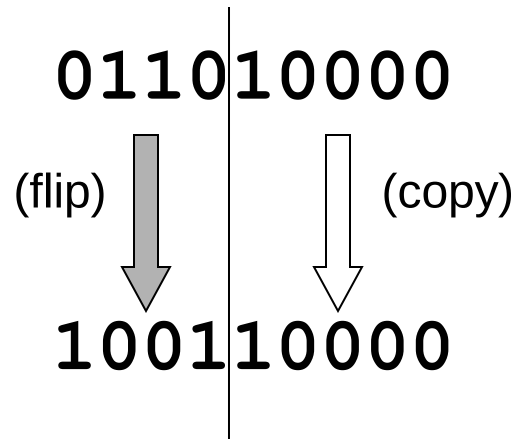
Two's Complement Shortcut

To take the two's complement of a number:

- **copy bits from right to left until (and including) the first "1"**
- **flip remaining bits to the left**

```
  011010000
  100101111   (1's comp)
+         1
  100110000
```



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Two's Complement Signed Integers

MS bit is sign bit – it has weight -2^{n-1}.

Range of an n-bit number: -2^{n-1} through $2^{n-1} - 1$.

- **The most negative number (-2^{n-1}) has no positive counterpart.**

-2^3	2^2	2^1	2^0	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

-2^3	2^2	2^1	2^0	
1	0	0	0	-8
1	0	0	1	-7
1	0	1	0	-6
1	0	1	1	-5
1	1	0	0	-4
1	1	0	1	-3
1	1	1	0	-2
1	1	1	1	-1

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Converting Binary (2's C) to Decimal

1. **If leading bit is one, take two's complement to get a positive number.**
2. **Add powers of 2 that have "1" in the corresponding bit positions.**
3. **If original number was negative, add a minus sign.**

$$X = 01101000_{two}$$
$$= 2^6+2^5+2^3 = 64+32+8$$
$$= 104_{ten}$$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Assuming 8-bit 2's complement numbers.

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More Examples

$$X = 00100111_{two}$$
$$= 2^5+2^2+2^1+2^0 = 32+4+2+1$$
$$= 39_{ten}$$

$$X = 11100110_{two}$$
$$-X = 00011010$$
$$= 2^4+2^3+2^1 = 16+8+2$$
$$= 26_{ten}$$
$$X = -26_{ten}$$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Assuming 8-bit 2's complement numbers.

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Converting Decimal to Binary (2's C)

First Method: *Division*

1. **Find magnitude of decimal number. (Always positive.)**
2. **Divide by two – remainder is least significant bit.**
3. **Keep dividing by two until answer is zero, writing remainders from right to left.**
4. **Append a zero as the MS bit; if original number was negative, take two's complement.**

$X = 104_{ten}$

104/2	=	52 r0	*bit 0*
52/2	=	26 r0	*bit 1*
26/2	=	13 r0	*bit 2*
13/2	=	6 r1	*bit 3*
6/2	=	3 r0	*bit 4*
3/2	=	1 r1	*bit 5*
1/2	=	0 r1	*bit 6*

$X = 01101000_{two}$

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Converting Decimal to Binary (2's C)

Second Method: *Subtract Powers of Two*

1. **Find magnitude of decimal number.**
2. **Subtract largest power of two less than or equal to number.**
3. **Put a one in the corresponding bit position.**
4. **Keep subtracting until result is zero.**
5. **Append a zero as MS bit; if original was negative, take two's complement.**

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$X = 104_{ten}$

104 - 64	=	40	*bit 6*
40 - 32	=	8	*bit 5*
8 - 8	=	0	*bit 3*

$X = 01101000_{two}$

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Operations: Arithmetic and Logical

Recall:
a data type includes *representation* and *operations*.

We now have a good representation for signed integers, so let's look at some arithmetic operations:

- **Addition**
- **Subtraction**
- **Sign Extension**

We'll also look at overflow conditions for addition.

Multiplication, division, etc., can be built from these basic operations.

Logical operations are also useful:

- **AND**
- **OR**
- **NOT**

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Addition

As we've discussed, 2's comp. addition is just binary addition.

- **assume all integers have the same number of bits**
- **ignore carry out**
- **for now, assume that sum fits in n-bit 2's comp. representation**

`01101000`	(104)	`11110110`	(-10)
+ `11110000`	(-16)	+ ________	(-9)
`01011000`	(98)		(-19)

Assuming 8-bit 2's complement numbers.

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Subtraction

Negate subtrahend (2nd no.) and add.

- assume all integers have the same number of bits
- ignore carry out
- for now, assume that difference fits in n-bit 2's comp. representation

	01101000	(104)		11110110	(-10)
−	00010000	(16)	−		(-9)
	01101000	(104)		11110110	(-10)
+	11110000	(-16)	+		(9)
	01011000	(88)			(-1)

Assuming 8-bit 2's complement numbers.

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Sign Extension

To add two numbers, we must represent them with the same number of bits.

If we just pad with zeroes on the left:

4-bit	**8-bit**
`0100` (4)	`00000100` (still 4)
`1100` (-4)	`00001100` (12, not -4)

Instead, replicate the MS bit -- the sign bit:

4-bit	**8-bit**
`0100` (4)	`00000100` (still 4)
`1100` (-4)	`11111100` (still -4)

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Overflow

If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.

`01000`	(8)	`11000`	(-8)
`+ 01001`	(9)	`+ 10111`	(-9)
`10001`	(-15)	`01111`	(+15)

We have overflow if:

- **signs of both operands are the same, and**
- **sign of sum is different.**

Another test -- easy for hardware:

- **carry into MS bit does not equal carry out**

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Logical Operations

Operations on logical TRUE or FALSE

- **two states -- takes one bit to represent: TRUE=1, FALSE=0**

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

A	NOT A
0	1
1	0

View *n*-bit number as a collection of *n* logical values

- **operation applied to each bit independently**

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Examples of Logical Operations

AND

- useful for clearing bits
  - AND with zero = 0
  - AND with one = no change

```
     11000101
AND  00001111
     --------
     00000101
```

OR

- useful for setting bits
  - OR with zero = no change
  - OR with one = 1

```
     11000101
OR   00001111
     --------
     11001111
```

NOT

- unary operation -- one argument
- flips every bit

```
NOT  11000101
     --------
     00111010
```

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Hexadecimal Notation

It is often convenient to write binary (base-2) numbers as hexadecimal (base-16) numbers instead.

- **fewer digits -- four bits per hex digit**
- **less error prone -- easy to corrupt long string of 1's and 0's**

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

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Converting from Binary to Hexadecimal

Every four bits is a hex digit.

- **start grouping from right-hand side**

`011 1010 1000 1111 0100 1101 0111`

`3 A 8 F 4 D 7`

This is not a new machine representation, just a convenient way to write the number.

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Addition in Hexadecimal Notation

- **2's complement representation**
- **assume all integers have the same number of bits**
- **ignore carry out**

```
   x025B  (603)          x7D96  (32150)
+  x26DE  (9950)      +  x7412  (29714)
   -----                 -----
   x2939  (10553)        xF1A8  (-19)
```

$\texttt{xF1A8} = \texttt{1111 0001 1010 1000}_{(2)}$

Overflow

Assuming 16-bit 2's complement numbers.

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Fractions: Fixed-Point

How can we represent fractions?

- **Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."**
- **2's comp addition and subtraction still work.**
  - **if binary points are aligned**

$2^{-1} = 0.5$

$2^{-2} = 0.25$

$2^{-3} = 0.125$

```
  00101000.101 (40.625)
+ 11111110.110 (-1.25)
  00100111.011 (39.375)
```

No new operations -- same as integer arithmetic.

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Very Large and Very Small: Floating-Point

Large values: 6.023×10^{23} -- requires 79 bits

Small values: 6.626×10^{-34} -- requires >110 bits

Use equivalent of "scientific notation": $F \times 2^E$

Need to represent F (*fraction*), E (*exponent*), and sign.

IEEE 754 Floating-Point Standard (32-bits):

1b	*8b*	*23b*
S	Exponent	Fraction

$$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent}-127}, \; 1 \le \text{exponent} \le 254$$

$$N = (-1)^S \times 0.\text{fraction} \times 2^{-126}, \; \text{exponent} = 0$$

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Floating Point Example

Single-precision IEEE floating point number:

$$\underbrace{1}_{\textit{sign}}\,\underbrace{01111110}_{\textit{exponent}}\,\underbrace{10000000000000000000000}_{\textit{fraction}}$$

- **Sign is 1 – number is negative.**
- **Exponent field is 01111110 = 126 (decimal).**
- **Fraction is 0.100000000000… = 0.5 (decimal).**

Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} = -0.75$.

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Text: ASCII Characters

ASCII: Maps 128 characters to 7-bit code.

- both printable and non-printable (ESC, DEL, …) characters

00	nul	10	dle	20	sp	30	0	40	@	50	P	60	`	70	p
01	soh	11	dc1	21	!	31	1	41	A	51	Q	61	a	71	q
02	stx	12	dc2	22	"	32	2	42	B	52	R	62	b	72	r
03	etx	13	dc3	23	#	33	3	43	C	53	S	63	c	73	s
04	eot	14	dc4	24	$	34	4	44	D	54	T	64	d	74	t
05	enq	15	nak	25	%	35	5	45	E	55	U	65	e	75	u
06	ack	16	syn	26	&	36	6	46	F	56	V	66	f	76	v
07	bel	17	etb	27	'	37	7	47	G	57	W	67	g	77	w
08	bs	18	can	28	(	38	8	48	H	58	X	68	h	78	x
09	ht	19	em	29	)	39	9	49	I	59	Y	69	i	79	y
0a	nl	1a	sub	2a	*	3a	:	4a	J	5a	Z	6a	j	7a	z
0b	vt	1b	esc	2b	+	3b	;	4b	K	5b	[	6b	k	7b	{
0c	np	1c	fs	2c	,	3c	<	4c	L	5c	\	6c	l	7c	|
0d	cr	1d	gs	2d	-	3d	=	4d	M	5d	]	6d	m	7d	}
0e	so	1e	rs	2e	.	3e	>	4e	N	5e	^	6e	n	7e	~
0f	si	1f	us	2f	/	3f	?	4f	O	5f	_	6f	o	7f	del