
Selling VIX[®] Futures and Options for Portfolio Return Enhancement

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ABSTRACT

The term structure of VIX® futures is generally upward sloping. The persistent VIX contango may result in abnormally strong performance for VIX futures selling or VIX call writing strategies. However, the high volatility of volatility and significant jump risk may expose short uncovered VIX positions to extreme tail risk.

In this paper we consider the performance of VIX futures and call option selling in a portfolio context in 2008 and 2016 as well as over a 10+ year period beginning in 2006. In addition, we consider alternative strategies including VIX futures spread strategies, and the dynamically de-levering VIX futures selling strategies represented by the VPDSM and VPNSM indexes. The VPD Index had annual returns that were much higher than those of the S&P 500 Index in 2009 and 2016 (years without large spikes in VIX), but the VPD returns were lower than those of the S&P 500 in 2008 and 2018, years with significant upward spikes in the VIX Index.

While the goal of the study is not to make a particular recommendation for a VIX-based investment strategy, meaningful portfolio return enhancements are possible over particular time periods with small allocations to VIX selling. A large allocation to uncovered selling of VIX instruments could have substantial volatility jump risk exposure which can result in economically meaningful and potentially catastrophic losses. However, the VPN Index - which sells VIX futures, holds money market instruments and buys VIX calls – had lower volatility and less severe maximum drawdowns than the stock and commodity benchmark indexes studied in the period of analysis.

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1.0 Introduction to the Spot VIX Index and Select Literature

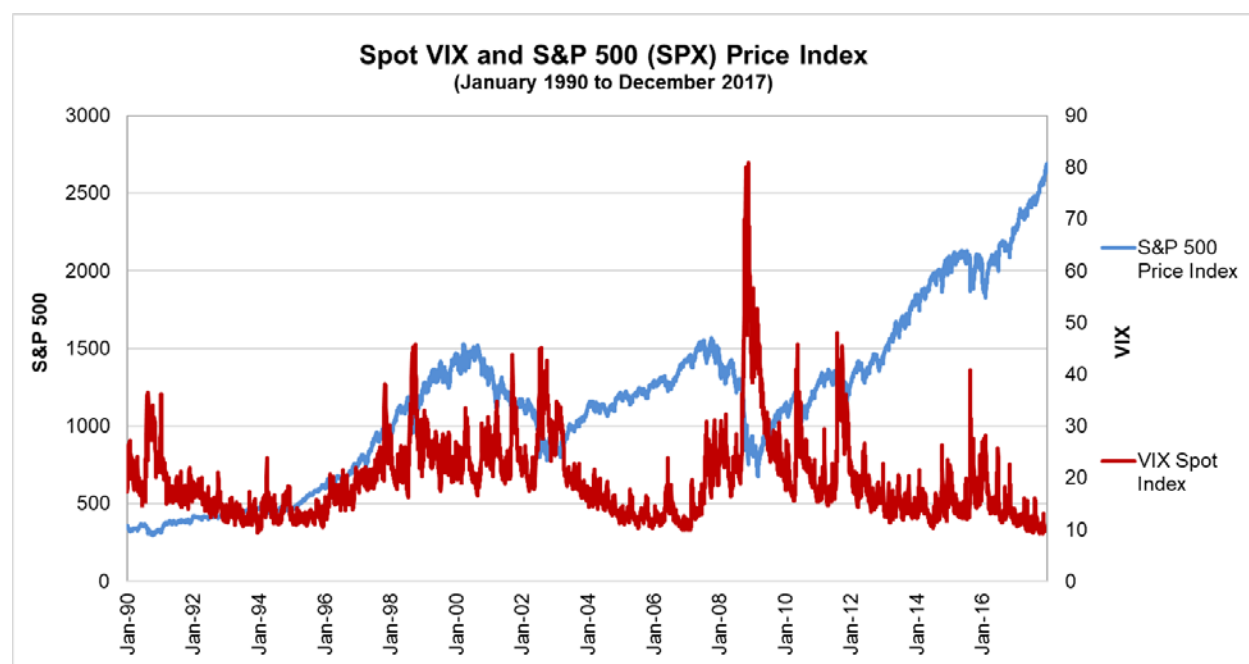
The Cboe Volatility Index® (VIX®) was initially developed in 1993 (see Whaley [1993]) as a forward looking measure of expected market volatility. More specifically, the spot VIX index represents the constant maturity 30-day expected volatility of the entire range of out-of-the-money (OTM) S&P 500® (SPX) put and call options with more than 23 days to expiration and less than 37 days to expiration. In 2014, the methodology was updated to include weekly SPX options, thus reducing the range of tenor of the options used. Additional details and historical data can be found at www.cboe.com/VIX.

As the principal gauge of expected volatility of the equity market, the VIX index has received significant attention in periods of market distress. Additionally, recent years have experienced historically low levels of VIX and low levels of realized volatility, resulting in increased interest in VIX-based trading strategies, particularly short (inverse) VIX futures strategies.

A number of studies have considered the potential diversification benefits of long spot VIX exposure to equity portfolios. Szado [2009] finds that the strong negative correlation between spot VIX and the S&P 500 results in significant diversification benefits from adding a long spot VIX position to an S&P 500 portfolio. Spot VIX can also provide benefits to portfolios containing alternative investments. Dash and Moran [2005] illustrate the potential diversification benefits of adding spot VIX exposure to hedge fund portfolios. Furthermore, Bollen, O'Neill, and Whaley [2017] contend that VIX futures have become the standard tool for volatility risk hedging.

In addition to the diversification benefits of long VIX exposure, an increasing number of researchers have documented the significant portfolio drag of long VIX futures due to frequent contango in the term structure, including Mencia and Sentana [2013], Whaley [2013] and Korovilas [2013], motivating interest in the return enhancing benefits of inverse VIX futures. The historically low levels of VIX in 2017 resulted in an increase in interest in inverse VIX strategies. Furthermore, extant literature suggests that low levels of the VIX index can be predictive of strong negative equity returns (see Giot [2005]). Toikka et al. [2004] of CSFB suggest that the negative correlation between the S&P 500 index and implied volatility is strongest in large down moves. The authors argue that short volatility should be characterized as an asset class. They demonstrate that long volatility is associated with a negative risk premium, therefore over extended time periods, long volatility positions tend to underperform the market. This is consistent with a number of studies on option writing strategies that find excess risk-adjusted returns for short volatility positions. They also note that volatility tends to be mean reverting, and tends to cluster.

Exhibit 1: Spot VIX Index and S&P 500 (SPX) Price Index



1990 to 2017

Mean Daily VIX	19.37
Median Daily VIX	17.50
Highest VIX Close	80.86
Lowest VIX Close	9.14

Average Daily Close by Year

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
S&P 500	1207.2	1310.5	1477.2	1220.0	948.0	1140.0	1267.6	1379.4	1643.8	1931.4	2061.1	2094.7	2448.2
Spot VIX	12.8	12.8	17.5	32.7	31.5	22.5	24.2	17.8	14.2	14.2	16.7	15.8	11.1

Source: Cboe Exchange Inc., Bloomberg

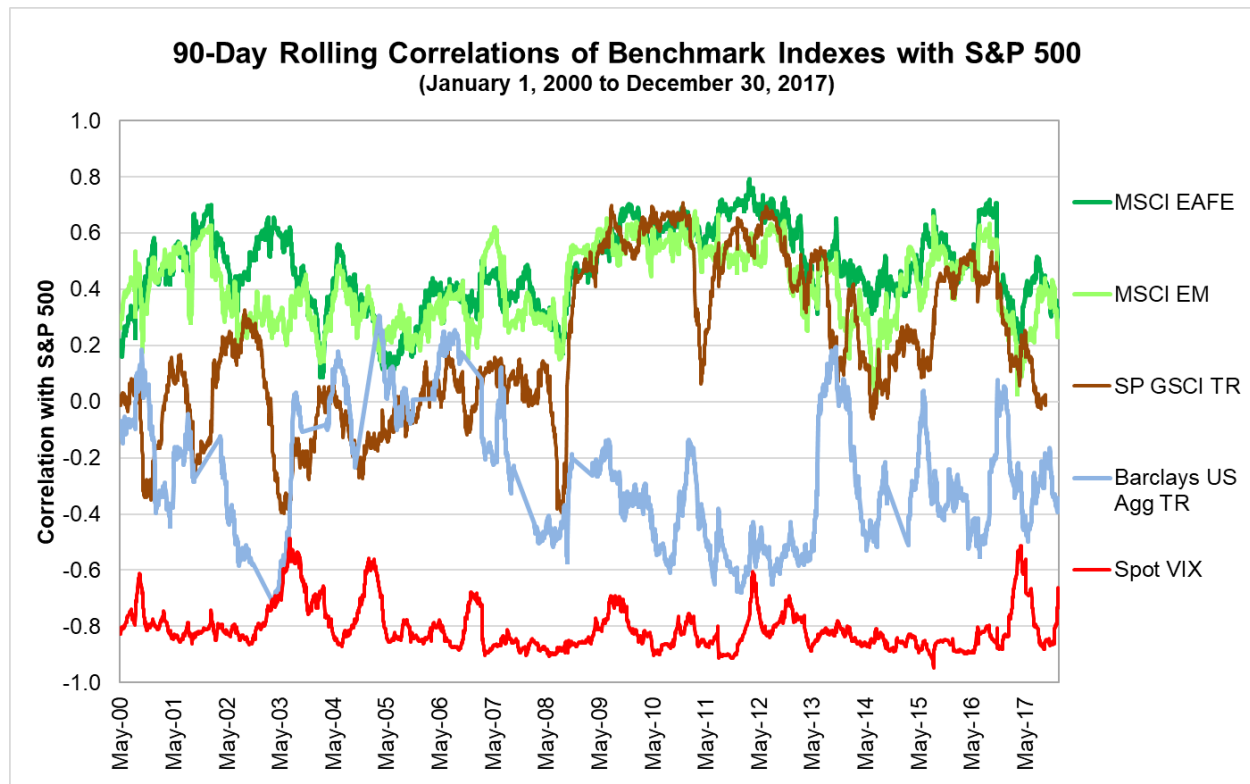
Exhibit 1: This exhibit illustrates the historical levels of the spot VIX and S&P 500 Price Indexes from 1990 to December 2017. Spikes in the level of the VIX are often accompanied by drawdowns in the S&P 500 index.

It is generally well known that the VIX index tends to increase when the equity market experiences large losses. A number of studies have indicated that the spot VIX index tends to be negatively correlated with the SPX (see Daigler and Rossi [2006]). Furthermore, research has suggested that a small exposure to long VIX-based products may provide significant diversification benefits in times of equity market distress (see Szado [2009]).

Exhibit 2 presents 90-day rolling correlations (based on daily returns) between the S&P 500 index and a number of benchmark indexes, including the VIX index. It is clear that the correlations between the indexes and the S&P 500 generally increased during the 2008 financial crisis. The exception was the Barclays Aggregate bond index. Bonds were one of the few asset classes that provided significant diversification benefits in 2008. However, it is

questionable whether bonds could provide similar diversification benefits any time in the near future since yields are currently near historically low levels, leaving little room for them to fall. It is worth noting that not only was the rolling correlation between the VIX index and the S&P 500 consistently negative over the entire period, but the correlation also remained below -0.65 throughout the financial crisis.

Exhibit 2: 90-Day Rolling Correlations of Benchmark Indexes with the S&P 500



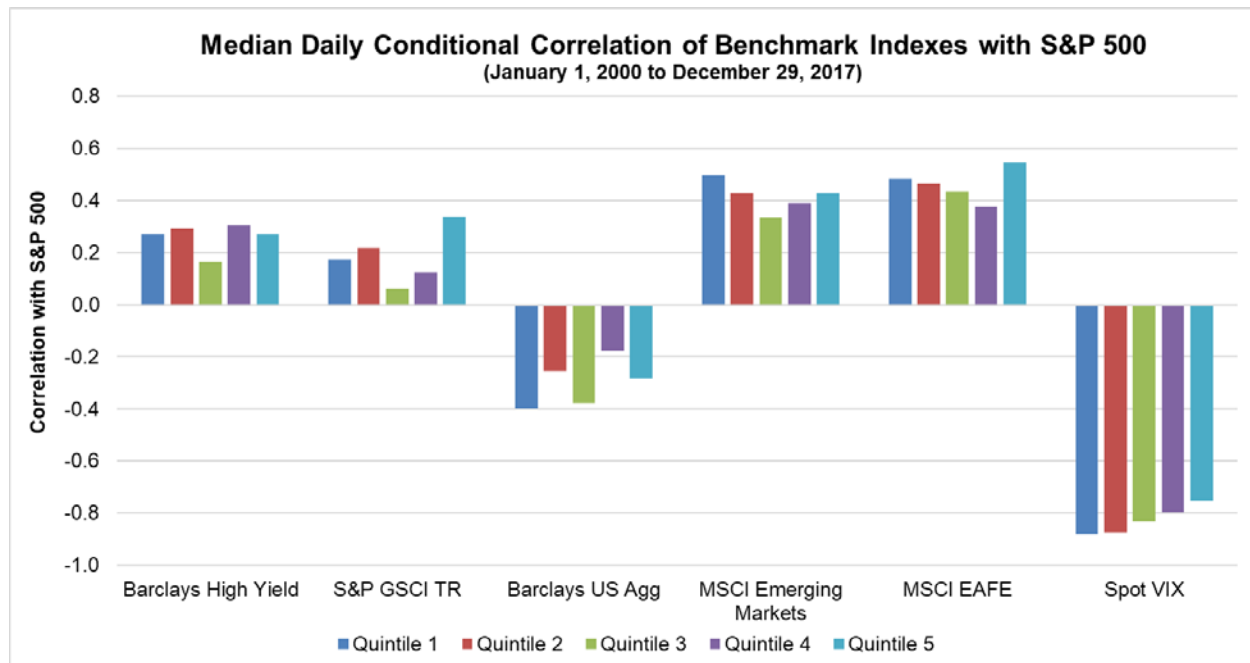
Source: Cboe Exchange Inc., Bloomberg

Exhibit 2: This exhibit provides 90-day rolling correlations of a number of benchmark indexes and spot VIX versus the S&P 500 price index, based on daily data. With the exception of the Barclays U.S. Aggregate bond index and the VIX index, the correlations with the S&P 500 increased significantly in the financial crisis and were quite consistent across the indexes.

While the VIX index is negatively correlated with the S&P 500 index, it is well documented that this correlation is particularly strong for large negative returns in the S&P 500 (see Szado [2009]). Exhibit 3 provides the median within-month correlations of benchmark indexes with the S&P 500 by quintile of S&P 500 monthly returns. Months are ranked based on S&P 500 return and correlations are calculated within each month based on daily returns. The median correlation is calculated for each quintile and presented in the exhibit. The exhibit once again,

illustrates the strong negative correlation between the S&P 500 and the VIX index. It is also clear that the correlation tends to be strongest in the lowest return quintile of S&P 500 monthly returns.

Exhibit 3: Conditional Correlations of Benchmark Indexes with the S&P 500



Source: Cboe Exchange Inc., Bloomberg

Exhibit 3: This exhibit provides median monthly conditional correlations of a number of benchmark indexes with the S&P 500 price index based on daily data. The quintiles are based on monthly S&P 500 returns, ranked from low to high. The correlation between the VIX index and the S&P 500 tends to be strongest in months with the lowest S&P 500 returns.

2.0 Introduction to VIX Futures and Options and VIX-Based Benchmark Indexes

The previous section illustrated the potential diversification benefits of the spot VIX index. However the spot VIX index is not investable. There are a number of alternatives to gain exposure to the VIX index, including VIX futures, VIX options and a variety of VIX-based exchange traded products and funds which are based on VIX futures and options. Key specifications of VIX futures and options are provided in Exhibit 4. Additional unique characteristics of VIX futures and options are described in Szado [2018a]. It is worth noting that VIX options often are priced based on the prices of the corresponding VIX futures contract. As with VIX futures, VIX options typically expire on the Wednesday which is 30 days prior to the

SPX option expiration (which occurs on the third Friday of the month, unless Friday is an exchange holiday). In addition, VIX options are European style options so they can only be exercised at expiration. VIX options expire at the day's market open based on a Special Opening Quotation (SOQ).

Exhibit 4: Key Specifications of VIX Futures and Options

	VIX Futures	VIX Options
Exchange	Cboe Futures Exchange (CFE)	Cboe Options Exchange
Year of Introduction	2004	2006
Ticker	VX and VX01 through VX53	VIX
Multiplier	\$1,000	\$100
Extended Trading Hours CT (Chicago Time)	5:00 p.m. (previous day) to 8:30 a.m. and 3:30 p.m. to 4:00 p.m.	2:00 a.m. to 8:15 a.m.
Regular Trading Hours CT (Chicago Time)	8:30 a.m. to 3:15 p.m.	
Last Trading Date	Trading hours for expiring VX futures contracts end at 8:00 a.m. Chicago time on the Final Settlement Date.	The last trading day (usually a Tuesday) is the business day prior to the Expiration Date of each contract expiration.
Expiration and Settlement	Usually on a Wednesday morning (30 days before a Friday settlement for SPX options)	
Settlement Value	The exercise-settlement value for VIX futures and options (Ticker: VRO) shall be a Special Opening Quotation (SOQ) of VIX calculated from the sequence of opening prices during regular trading hours for SPX of the options used to calculate the index on the settlement date.	
Number of Contract Expirations	Up to six near-term expiration weeks, nine near-term serial months and five months on the February quarterly cycle (February, May, August, November) for the VX futures contract.	Up to six weekly expirations and up to twelve standard (monthly) expirations in VIX options may be listed.
Options Exercise Style	Not applicable.	European - VIX options generally may be exercised only on the Expiration Date.

Source: Cboe Global Markets. www.Cboe.com/VIX

Exhibit 4: This exhibit provides key specifications of VIX futures and options contracts. The specifications are subject to change.

A variety of VIX-based investment products are based on a series of benchmark indexes that represent the returns to VIX futures and options trading strategies, including those published by Cboe Exchange and S&P Dow Jones Indices. These indexes include:

VPD - Cboe VIX Premium Strategy Index - overlays a sequence of short one-month VIX futures on a money market account; the short VIX futures positions are held until expiration and new VIX futures are then sold (see www.cboe.com/VPD).

VPN - Cboe Capped VIX Premium Strategy Index - tracks the performance of a strategy that systematically sells 1-month VIX futures, capped by the purchase of a VIX call option; the short VIX futures position is capped with long VIX calls struck about 25 points higher than the VIX futures price (see www.cboe.com/VPN).

VSTG - Cboe VIX Strangle Index - a premium capture index that overlays short VIX call and put options with a capped long VIX call option position. The position is collateralized by fixing the number of strangles such that 80% of capital is reserved (see www.cboe.com/VSTG).

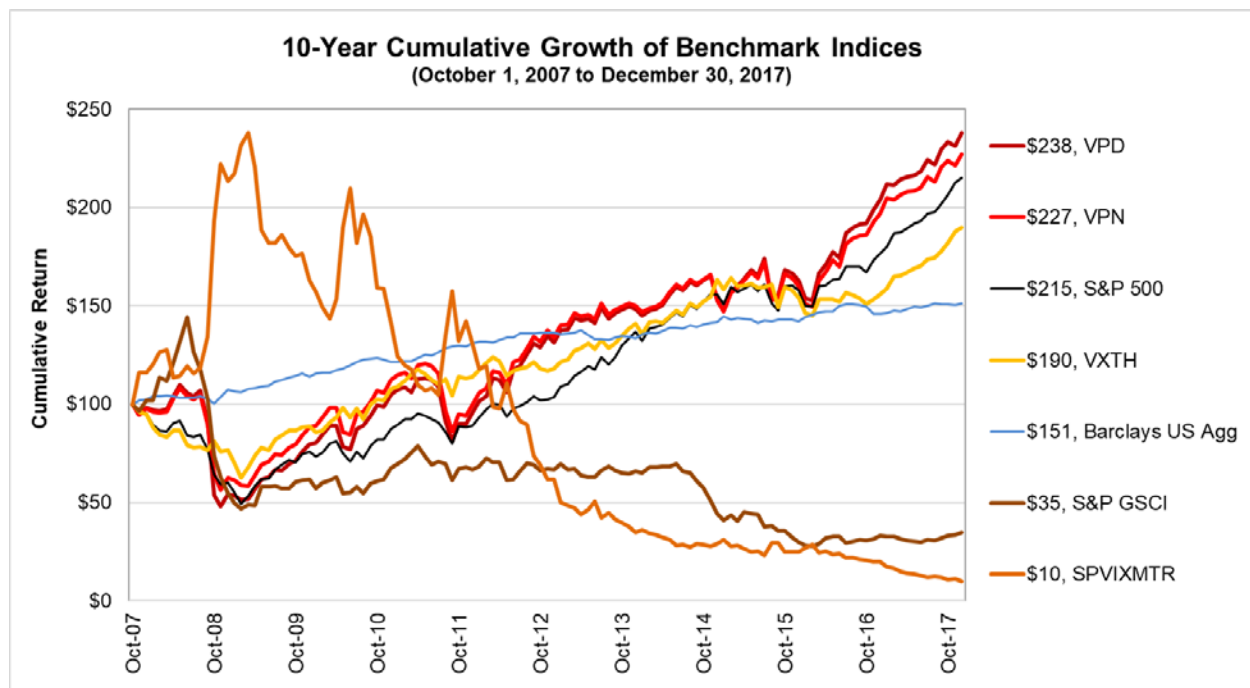
VXTH - Cboe VIX Tail Hedge Index - buys and holds S&P 500 stocks, and also often buys 30-delta call options on the Cboe Volatility Index (VIX) (see www.cboe.com/VXTH).

SPVIXMTR - S&P 500 VIX Mid-term Futures Index buys a combination fourth, fifth, sixth and seventh month VIX futures in order to reflect the expectations of the VIX Index level in five months. A portion of the VIX futures are rolled daily to maintain a constant average weighted five-month term (see us.spindexes.com/SPVIXMTR).

SPVIXSTR - S&P 500 VIX Short-term Futures Index buys a combination of first and second month VIX futures in order to reflect the expectations of the VIX Index level in one-month. A portion of the VIX futures are rolled daily to maintain a constant average weighted one-month term (see us.spindexes.com/SPVIXSTR).

Exhibit 5 provides an illustration of the historical performance of a selection of VIX-based indexes along with a selection of benchmark indexes. The high volatility and drawdown of the S&P 500 VIX Mid-term futures index is evident in the exhibit. In contrast, two indexes which sell VIX futures (VPD and VPN) outperformed the S&P 500 on a raw return basis, albeit with increased volatility. The VPD and VPN indexes sell VIX futures and therefore may be exposed to significant tail risk in the case of a major volatility event. To mitigate this tail risk exposure, the VPN index includes a long OTM VIX call option exposure. While the long call results in a drag on performance in periods in which VIX does not increase significantly, it does provide some protection against catastrophic volatility risk. These indexes are discussed in further detail in Szado [2018c].

Exhibit 5: 10-Year Cumulative Growth of Benchmark Indexes



Source: Cboe Exchange Inc., Bloomberg.

Exhibit 5: This exhibit illustrates the cumulative growth of a variety of benchmark total return indexes (pre-tax, with reinvested dividends) over the past 10+ years. It is not surprising that the long VIX futures-based index (SPVIXMTR) experienced significant drawdowns and volatility, due to the generally pervasive contango in the VIX futures term structure over the period. In contrast, the short (inverse) VIX futures indexes (VPN and VPD) experienced strong performance.

3.0 Inverse VIX Futures and Written Call Options in Two Key Years – 2008 and 2016

The analyses in the following sections of the paper focus on trading VIX futures and options. For this purpose, we construct the returns to a series of VIX futures strategies rather than relying on the published benchmarks. The two main VIX futures trading strategy indexes used in the analyses represent the returns to strategies that take short (inverse) positions in:

- 1) One-month VIX futures held until the market close on the day prior to the VIX futures morning expiration, at which time they are rolled into one-month VIX futures, and
- 2) Three-month VIX futures held until the market close on the day prior to the next VIX futures morning expiration, at which time the (now two-month) VIX futures are rolled into three-month VIX futures.

In addition to the two inverse VIX futures trading strategies outlined above, we also similarly create two short VIX option strategies which:

- 1) Write one-month at-the-money (ATM) and,
- 2) Write one-month 25% out-of-the-money (OTM) options.

We will begin by considering the inverse VIX futures strategies. The written VIX options strategies will be discussed later in the paper. For a discussion of corresponding long VIX futures and options strategies, see Szado [2018c].

As indicated in Szado [2018a], the VIX futures term structure is generally in contango. The often significant cost of holding long futures in contango (when spot price is below futures price) suggests that inverse VIX futures and written VIX call option positions may generate abnormal returns due to contango. On the other hand, the convex relationship and strong negative conditional correlation between the VIX index and the S&P 500 index suggests inverse VIX futures and written call options may expose the investor to catastrophic tail risk. Since inverse volatility positions are expected to earn positive excess returns in exchange for exposure to potentially significant risk, an active approach rather than a passive short volatility exposure may be appropriate.

The following section considers the impact of a 5% allocation of inverse VIX futures and a 1% allocation to written VIX options to a traditional stock and bond portfolio and a hypothetical endowment portfolio. The size of the VIX futures allocation was chosen not as typical allocation, but rather as an oversized allocation for the purposes of illustrating hypothetical impacts of a VIX futures allocation. The base portfolios were chosen to represent a traditional 60/40 stock/bond portfolio (60% S&P 500 index and 40% Barclays U.S. Aggregate index) and a hypothetical endowment portfolio which is based on the published annual allocations of a well-known endowment applied to a series of indexes which proxy for the asset classes represented in the allocations (S&P 500 index, Barclays U.S. Aggregate index, Barclays High Yield U.S. Corporate index, HFR Global Hedge Fund index, SG CTA index (formerly Newedge CTA index), S&P GSCI TR index, S&P Listed Private Equity index, Dow Jones Equity REIT index, MSCI EAFE index, MSCI EEM index, and the 3-Month Treasury Bill Secondary Market Rate).

While we consider long-term inverse VIX futures and written options allocations later in the paper, in the following sections we consider two years in which we expect an inverse VIX futures or written options allocation to provide a short-term return enhancement benefits. Two years were chosen to represent periods unfavorable to positions in inverse VIX futures and written call options (2008) in which we expect to see the impact of significant volatility risk exposure during major volatility events and generally favorable to inverse positions in VIX futures and written call options (2016) in which we expect to see return enhancement benefits.

The choice of 2008 as a potentially unfavorable year for inverse VIX futures and written call option exposure is quite obvious due to the high market volatility and large drawdowns of the base portfolios experienced in the last quarter of the year. The choice of a favorable year for inverse VIX exposure is less clear. A number of factors were considered in the choice (see Exhibit 6) including the number of days in which VIX futures were in contango, the average return to

VIX futures, the maximum drawdown of a VIX futures strategy and the slope of the VIX term structure (the futures basis). Overall, 2016 appears to be a relatively favorable year for an inverse VIX futures or written call option exposure. It should be noted that 2016 is not the most favorable year based on any single measure considered but rather exhibits a variety of favorable characteristics. Furthermore, 2016 has the advantage of being more recent than some other alternative years (and thus may possibly be more reflective of the current market microstructure).

Exhibit 6: Select Annual Summary Statistics for Spot VIX, SPX and VIX Futures

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Average Spot VIX Index	17.5	32.7	31.5	22.5	24.2	17.8	14.2	14.2	16.7	15.8	11.1
% in Contango vs Front Month Only	61%	54%	76%	79%	71%	90%	87%	84%	80%	86%	94%
Average VIX Basis (VIX points per day)	-0.003	0.026	-0.006	-0.033	-0.013	-0.042	-0.025	-0.019	-0.014	-0.024	-0.032
Average S&P 500 Realized 30-Day Volatility Risk Premium	1.7%	-2.7%	7.5%	6.0%	3.2%	5.2%	3.5%	3.1%	1.7%	4.5%	4.4%
Spot VIX Average Daily % Change	0.04%	0.07%	-0.07%	-0.02%	0.02%	-0.02%	-0.02%	0.02%	0.00%	-0.02%	-0.01%
Spot VIX Minimum Daily % Change	-6.99%	-17.36%	-10.23%	-12.11%	-12.94%	-4.70%	-3.95%	-4.13%	-5.70%	-5.10%	-3.79%
Spot VIX Maximum Daily % Change	7.16%	16.54%	10.54%	10.47%	16.00%	3.25%	5.21%	4.37%	12.71%	8.51%	4.94%
Spot VIX Annualized Return Volatility	139%	127%	91%	119%	140%	98%	112%	122%	144%	127%	114%
3 Month VIX Futures Average Daily Return	0.23%	0.35%	-0.12%	-0.17%	0.01%	-0.39%	-0.27%	-0.09%	-0.05%	-0.16%	-0.40%
3 Month VIX Futures Minimum Daily Return	-8%	-7%	-7%	-10%	-13%	-11%	-8%	-9%	-9%	-13%	-11%
3 Month VIX Futures Maximum Daily Return	12%	11%	10%	11%	15%	9%	10%	10%	16%	20%	11%
3 Month VIX Futures Maximum Drawdown	-17%	-22%	-47%	-50%	-36%	-69%	-53%	-43%	-37%	-57%	-65%
Average VIX Futures Open Interest	56,019	48,570	31,645	93,591	181,028	320,111	392,584	388,832	332,028	414,813	572,744
Average VIX Futures Volume	4,169	4,301	4,543	17,432	47,744	95,143	158,508	200,521	204,986	238,773	297,694

Source: Cboe Exchange Inc., Bloomberg

Exhibit 6: This exhibit provides a wide variety of annual summary statistics from 2007 through 2017 for the spot VIX index, the S&P 500 index and VIX futures. The year 2008 was chosen as an unfavorable year for inverse VIX futures and written call options allocations due to the large drawdowns in the S&P 500 index and jump in volatility in 2008. The choice of a favorable year to explore the potential return enhancement benefits of inverse VIX futures and written call options allocations is somewhat less obvious. The year 2016 was chosen based on a number of measures including number of days in contango, average return, maximum drawdown and the slope of the VIX term structure (futures basis).

As discussed above, inverse positions in VIX futures may generate abnormal returns due to persistent contango. However, while periods of strong contango may generate high returns to inverse VIX futures strategies if a volatility event does not occur, such strategies may generate catastrophic losses in the case of a volatility event such as a major equity market correction. Furthermore, such losses may occur concurrently with losses in many other asset classes. Thus, one might argue that inverse VIX futures may not be well suited to long term buy-and-hold investment since they are susceptible to overwhelming losses, but rather they may be best suited to strategic/selective uses rather than buy-and-hold investment.

The following section considers the impact of a 5% allocation to inverse one-month or three-month VIX futures or a 1% allocation to written one-month ATM or 25% OTM VIX call options to a traditional stock and bond portfolio and a hypothetical endowment portfolios. Consistent with the analyses above, two years are considered to represent periods generally unfavorable for

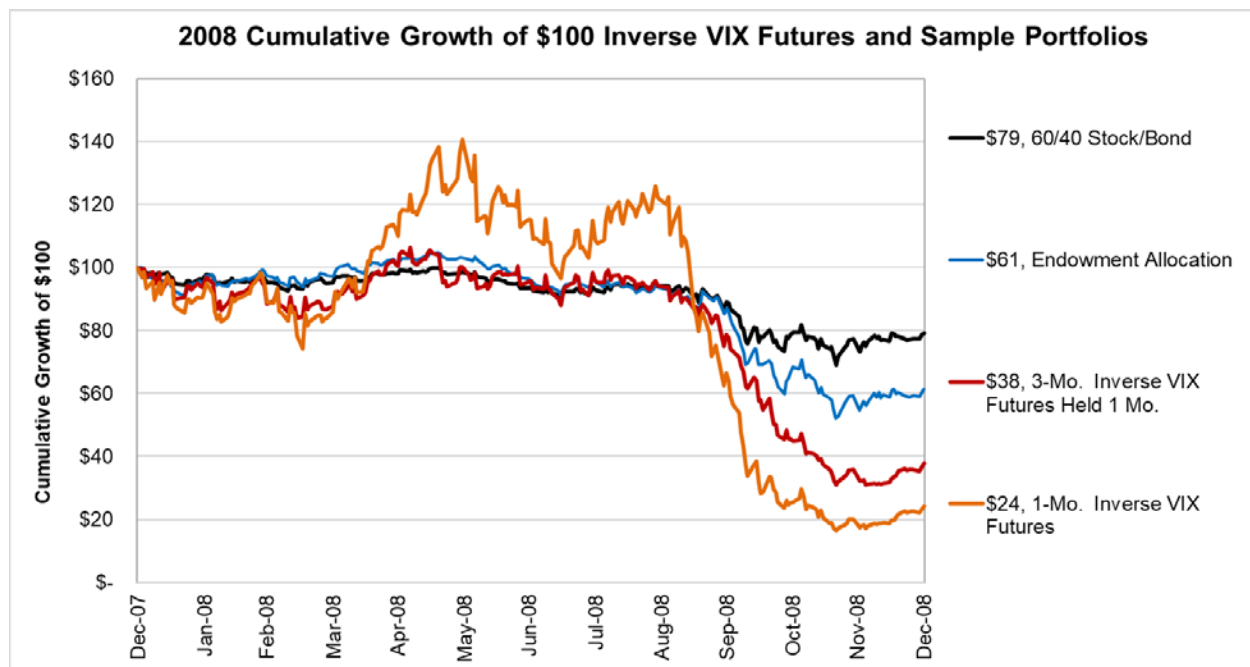
inverse VIX positions (2008) and generally favorable for inverse VIX positions (2016). One would expect that the historic volatility event of 2008 would result in very poor performance for inverse VIX futures and written call option strategies, while the strong and very persistent contango (and the absence of a volatility spike) would result in strong performance in 2016.

3.1 Inverse VIX Futures and VIX Call Writing-Based Strategies in 2008

Exhibit 7 illustrates the 2008 cumulative performance of the two inverse VIX futures portfolios along with the two base portfolios. The high volatility of the inverse VIX futures portfolios is clearly evident in the exhibit. While the inverse one-month VIX futures portfolio experienced significant returns in the first three quarters of the year, it experienced a dramatic drawdown in the last quarter. The magnitude of the losses of the VIX futures strategies in the third quarter significantly eclipses the losses of the base portfolios during one of the worst equity drawdowns in history. This is consistent with the conditional correlations and high volatility of volatility discussed previously in the paper.

Exhibit 8 provides an illustration of the 2008 cumulative growth and summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio lost 21% over the period at a 24% annual standard deviation and a 31% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures performed similarly to each other, increasing the loss of the base portfolio from 21% to 24% and 23%, respectively. In addition, the one- and three-month inverse VIX futures allocations increased standard deviation from 24% to 26% and 25%, respectively and maximum drawdown from 31% to 35% and 33%, respectively. The ex-post skewness was also hurt by adding the inverse VIX futures allocation to the base portfolio. Thus the inverse VIX futures allocation increased risk and reduced returns in 2008, albeit moderately.

Exhibit 7: 100% Allocations to Inverse VIX Futures-Based Strategies in 2008



2008	60/40 Stock/Bond Portfolio	Endowment	1 Mo. Inverse VIX Futures	3 Mo. Inverse VIX Futures Held 1 Mo.
Annual Return	-20.9%	-38.6%	-75.7%	-62.0%
Annualized Standard Deviation	24.1%	31.0%	83.2%	48.2%
Sharpe Ratio	-0.87	-1.25	-0.91	-1.29
Maximum Drawdown	-30.9%	-50.4%	-88.4%	-71.0%
Skewness	0.16	-0.13	-0.52	-0.61
Kurtosis	4.14	3.21	1.30	1.03
Correlation w SPX	1.00	0.92	0.82	0.83
Beta	0.58	0.69	1.66	0.97
Alpha	2.22%	-25.54%	-61.75%	-69.49%
Alpha P-Value	0.50	0.16	0.37	0.08

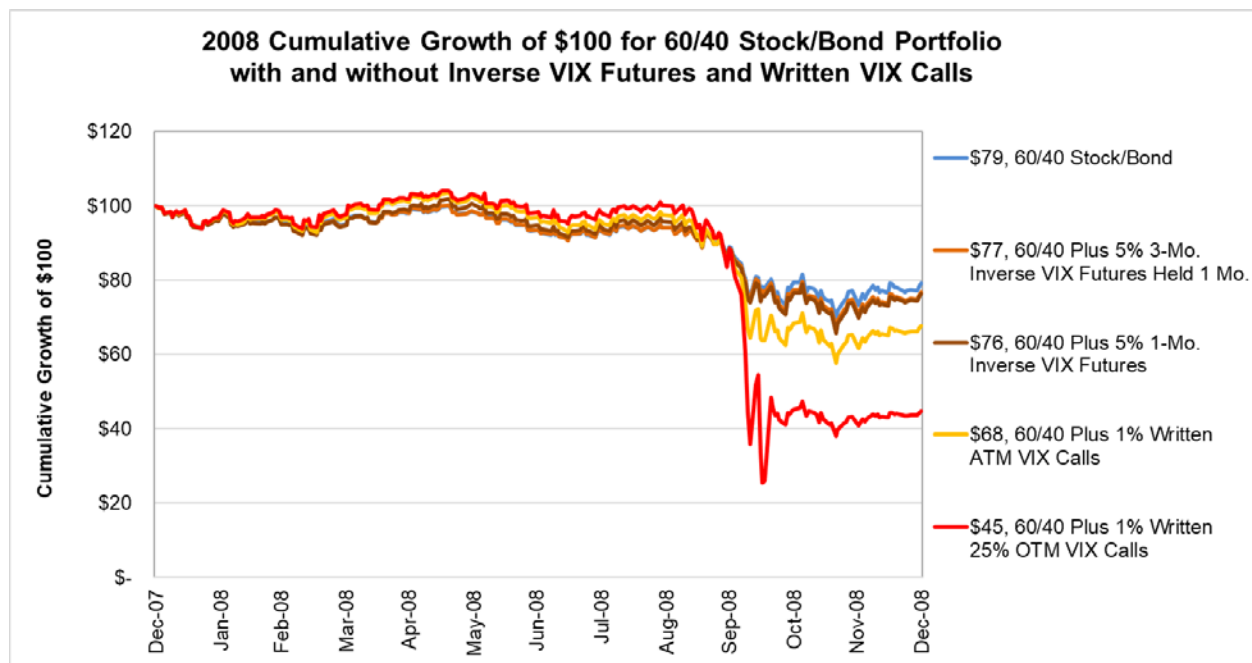
Source: Cboe Exchange Inc., Bloomberg

Exhibit 7: This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios in 2008. The VIX portfolios are both fully collateralized and rebalanced daily. The one-month portfolio takes short positions in one-month VIX futures and rolls out on the close the night before expiration. The Three-month VIX futures portfolio shorts three-month VIX futures and rolls into a new short three-month VIX futures contract after one month at the close before the front month VIX futures contract expires. The traditional portfolios consist of a 60/40 Stock/Bond portfolio and a hypothetical endowment portfolio which invests in a variety of indexes based on the average yearly asset allocation of a representative endowment. It is clear from the exhibit that 2008 was an unfavorable year for inverse VIX futures exposure.

The results for adding a written VIX call option allocation are far more dramatic. The 1% written one-month ATM VIX call allocation reduced returns from -21% to -32% and reduced standard

deviation from 24% to 35%, and maximum drawdown from -31% to -44%, thus doubling losses while increasing risk by half. The 1% allocation to inverse one-month 25% OTM VIX calls was even more extreme, further reducing returns to a 55% loss (from a 21% loss for the base portfolio), and did so at a far higher standard deviation (118% versus 24% for the base portfolio), and a more than doubled maximum drawdown (-76% versus -31% for the base portfolio).

Exhibit 8: 60/40 Portfolio 2008 Performance with Inverse VIX Futures and Written VIX Calls



2008	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1 Mo. Inverse VIX Futures	60/40 Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25% OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annual Return	-20.9%	-23.6%	-23.3%	-55.3%	-32.3%
Annualized Standard Deviation	24.1%	26.0%	24.8%	118.2%	34.7%
Sharpe Ratio	-0.87	-0.91	-0.94	-0.47	-0.93
Maximum Drawdown	-30.9%	-35.4%	-33.3%	-75.6%	-44.1%
Skewness	0.16	0.01	0.05	6.45	0.23
Kurtosis	4.14	3.33	3.74	84.28	9.21
Correlation w SPX	1.00	0.99	1.00	0.52	0.94
Beta	0.58	0.63	0.60	1.51	0.80
Alpha	2.22%	0.42%	-0.84%	49.98%	-4.06%
Alpha P-Value	0.50	0.92	0.81	0.73	0.81

Source: Cboe Exchange Inc., Bloomberg

Exhibit 8: This exhibit illustrates the 2008 performance of a traditional 60/40 portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of crisis, even a small allocation to VIX call writing can significantly reduce returns and increase volatility. A 5% allocation to inverse VIX futures has a smaller impact but also reduces returns while slightly increasing volatility and drawdowns.

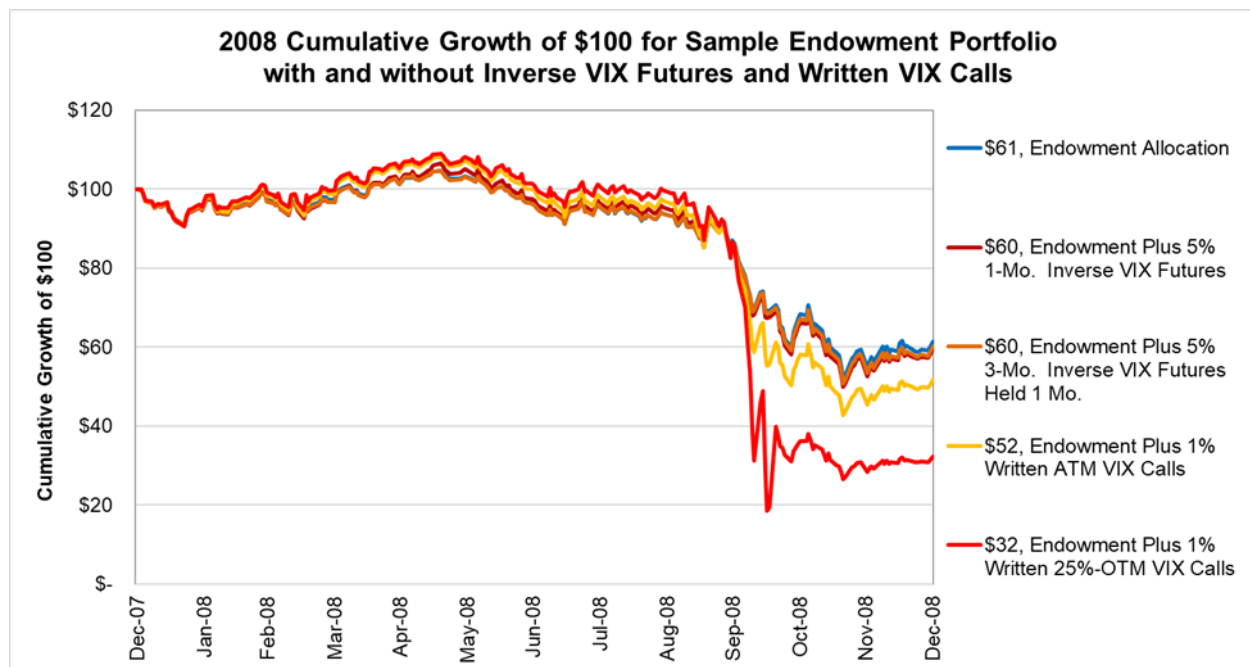
The results above suggest that an inverse VIX futures allocation would have moderately increased losses and risk of a 60/40 portfolio in 2008, while a written call option allocation would have significantly increased losses and risk.

Exhibit 9 presents results which reflect the impact of an addition of inverse VIX futures and written call options to a well-diversified hypothetical endowment portfolio in 2008. The impact of the inverse VIX allocations were similar for both base portfolios. Exhibit 9 provides an illustration of the 2008 cumulative growth and summary statistics of the base hypothetical endowment portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio lost 39% over the period at a 31% annual standard deviation and a -50% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures performed similarly to each other, slightly increasing the loss of the base portfolio from 39% to 40% and 40%, respectively. The one- and three-month VIX futures allocations also slightly increased standard deviation from 31% to 33% and 31.3%, respectively and maximum drawdown from -50% to -53% and -52%, respectively. The ex-post skewness was also worsened by adding the inverse VIX futures allocation to the base endowment portfolio. Thus the inverse VIX futures allocation had little impact on the endowment portfolio, slightly reducing risk and returns in 2008.

As with the 60/40 portfolio, the results for adding a written VIX call option allocation are far more extreme than those from adding inverse VIX futures. The 1% written one-month ATM VIX call allocation significantly reduced returns from -39% to -48% and increased standard deviation from 31% to 41%, and maximum drawdown from -50% to -61%, thus providing significant degradation in risk and return (in contrast to the inverse VIX futures allocations). The 1% allocation to inverse one-month 25% OTM VIX calls further reduced returns to a 68% loss (from a 39% loss for the base portfolio) and more than quadrupled standard deviation (33% versus 140% for the base portfolio), and significantly increased the maximum drawdown (-83% versus -50% for the base endowment portfolio).

Thus, for both base portfolios, an inverse VIX futures allocation had a small negative impact on risk and return while a written call allocation significantly degraded returns and risk, particularly for the OTM calls. At this point, it is worth recalling the results in Exhibit 7. While the results in this section suggest that a 5% allocation to inverse VIX futures had a small negative impact on the base portfolios in 2008, an investment in the inverse VIX strategies outside of a diversified portfolios would have resulted in far larger losses. It is the inclusion in the diversified portfolios and the small size of the allocation that significantly mitigates the negative impact.

Exhibit 9: Endowment Portfolio in 2008 with Inverse VIX Futures and Written VIX Calls



2008	Endowment	Endowment Plus 5% 1 Mo. Inverse VIX Futures	Endowment Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25% OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annual Return	-38.6%	-40.2%	-39.8%	-67.8%	-48.3%
Annualized Standard Deviation	31.0%	32.5%	31.3%	139.6%	41.0%
Sharpe Ratio	-1.25	-1.24	-1.27	-0.49	-1.18
Maximum Drawdown	-50.4%	-53.1%	-51.5%	-83.0%	-60.5%
Skewness	-0.13	-0.21	-0.19	6.93	-0.20
Kurtosis	3.21	2.79	3.07	92.48	5.50
Correlation w SPX	0.92	0.93	0.92	0.50	0.91
Beta	0.69	0.73	0.71	1.70	0.91
Alpha	-25.54%	-26.53%	-27.59%	42.97%	-33.25%
Alpha P-Value	0.16	0.14	0.11	0.81	0.18

Source: Cboe Exchange Inc., Bloomberg

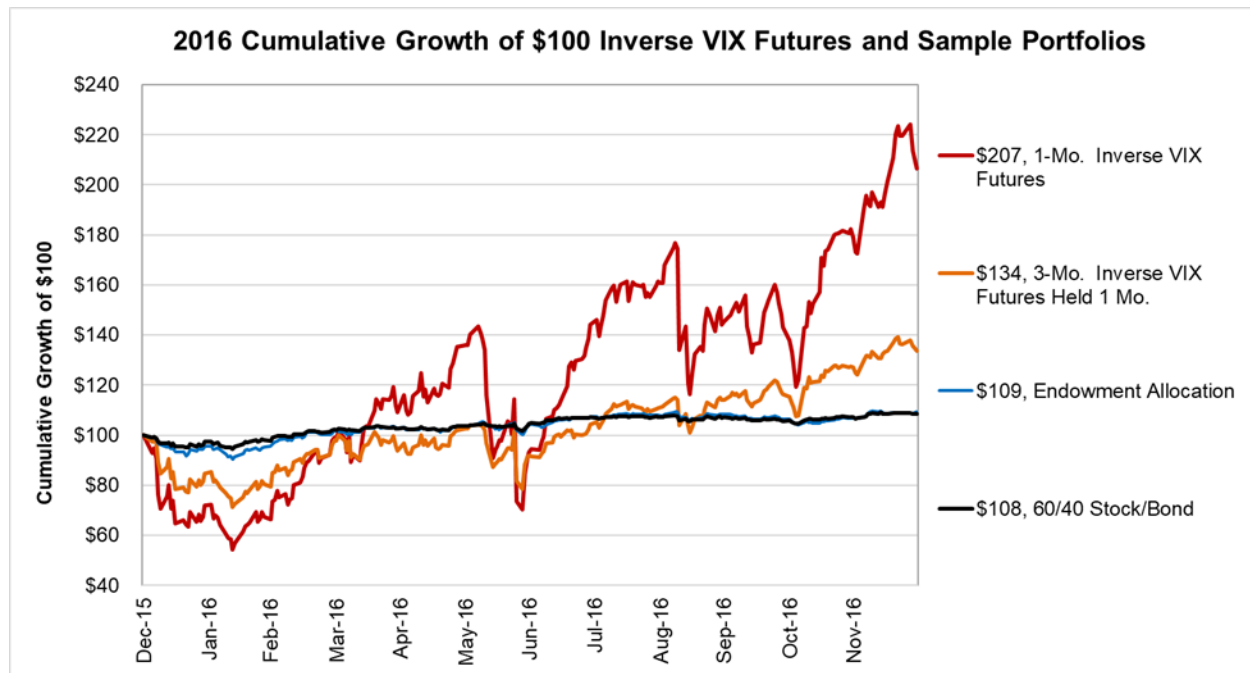
Exhibit 9: This exhibit illustrates the 2008 performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of crisis, even a small allocation to VIX call writing can significantly reduce returns and increase volatility. A 5% allocation to inverse VIX futures has a smaller impact but also reduces returns while slightly increasing volatility and drawdowns.

3.2 Inverse VIX Futures and VIX Call Writing-Based Strategies in 2016

As discussed earlier, 2016 was chosen as a sample year which one would expect would be attractive for an inverse VIX futures or written call option allocation. The following section considers the impact of a 5% allocation to inverse VIX futures or 1% allocation to written VIX call

options on the two base portfolios (the 60/40 portfolio and the hypothetical endowment portfolio).

Exhibit 10: Inverse VIX Futures-Based Strategies in 2016



2016	60/40 Stock/Bond Portfolio	Endowment	1 Mo. Inverse VIX Futures	3 Mo. Inverse VIX Futures Held 1 Mo.
Annual Return	8.4%	9.2%	106.5%	33.7%
Annualized Standard Deviation	7.6%	10.0%	88.2%	47.4%
Sharpe Ratio	1.11	0.92	1.21	0.71
Maximum Drawdown	-5.4%	-9.6%	-51.0%	-28.6%
Skewness	-0.29	-0.48	-1.63	-1.12
Kurtosis	2.23	2.56	9.46	9.17
Correlation w SPX	0.99	0.82	0.83	0.84
Beta	0.57	0.62	5.58	3.05
Alpha	2.06%	2.52%	67.37%	5.02%
Alpha P-Value	0.24	0.77	0.35	0.89

Source: Cboe Exchange Inc., Bloomberg

Exhibit 10: This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios in 2016. The VIX portfolios are both fully collateralized and rebalanced daily. The one-month portfolio takes short positions in one-month VIX futures and rolls out on the close the night before expiration. The Three-month VIX futures portfolio shorts three-month VIX futures and rolls into a new short three-month VIX futures contract after one month at the close before the front month VIX futures contract expires. The traditional portfolios consist of a 60/40 Stock/Bond portfolio and a hypothetical endowment portfolio which invests in a variety of indexes based on the average yearly asset allocation of a representative endowment. It is clear from the exhibit that 2016 was a generally favorable year for inverse VIX futures exposure.

Exhibit 10 provides a graphical representation and summary statistics of the 2016 cumulative performance of the one-month and three-month inverse VIX futures strategies along with the two base portfolios. It is evident that 2016 was an overall favorable year for an inverse VIX futures exposure from a total return perspective, albeit a very high volatility. While the futures strategies performed very poorly at the start of the year, both futures strategies ended the year strongly while experiencing many very large drawdowns over the course of the year. In fact, the one-month inverse VIX futures strategy experienced a standard deviation of 88% over the year, with a maximum drawdown of -51% (versus 8% and -5% for the 60/40 portfolio).

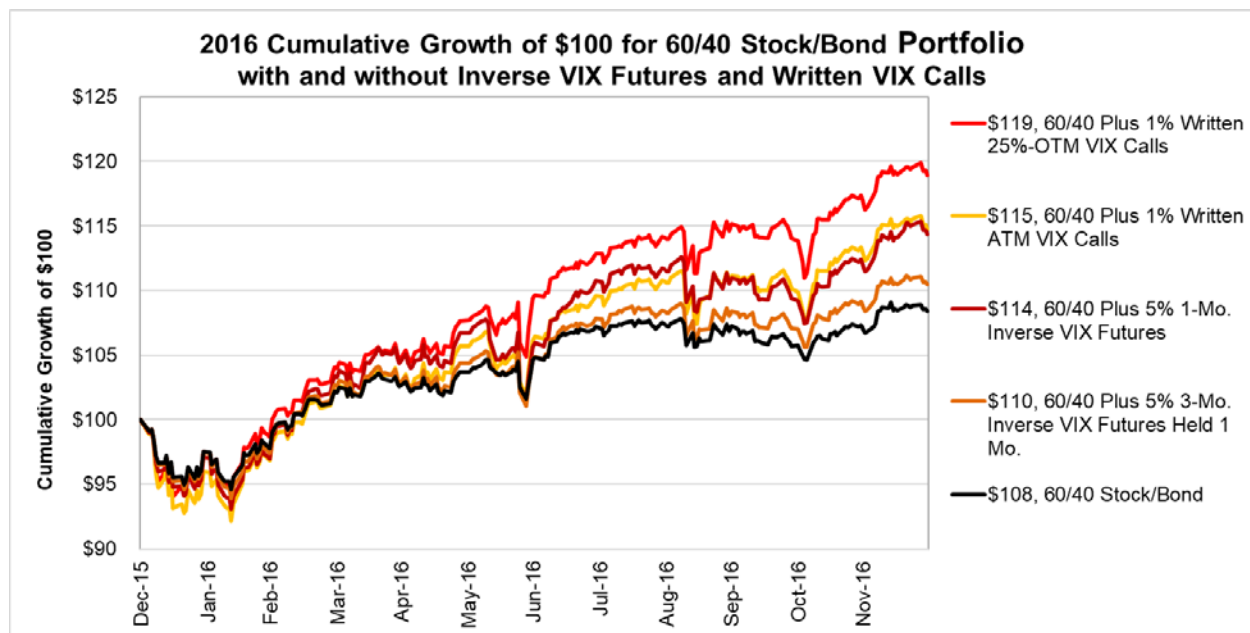
The following section provides an analysis of the impact of an addition of an inverse VIX futures or written call option allocation to the two base portfolios in 2016. We begin by considering a 5% allocation to inverse VIX futures and 1% allocation to written VIX call options in 2016. As mentioned earlier, 2016 was chosen as an example of a favorable year for an inverse VIX allocation as we expect that an inverse VIX futures or written call exposure would benefit from the relatively strong and consistent contango in VIX futures and relative lack of major volatility spikes (see Exhibit 7).

Exhibit 11 provides an illustration of the 2016 cumulative growth and summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio gained 8% over the period at an 8% annual standard deviation and a -5% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures outperformed the base portfolio, increasing the returns of the base portfolio from 8% to 14% and 10%, respectively. However, the one- and three-month VIX futures allocations increased standard deviation from 8% to 11% and 9%, respectively and maximum drawdown from -5% to -7% and -6%, respectively. The ex-post skewness was also reduced by adding the inverse VIX futures allocation to the base portfolio. Thus, while the inverse VIX futures allocations improved returns in 2016, they did so at the expense of a small increase in risk. The return to risk tradeoff (as measured by the Sharpe ratio) of the base portfolio was improved by the addition of inverse VIX futures (from 1.11 to 1.30 and 1.14, respectively). Similar improvements are seen in alpha (although the alphas are not statistically significant at 10%).

The results for adding a written VIX call option allocation are similar, yet somewhat more extreme, particularly for the OTM calls. The 1% written one-month ATM VIX call allocation increased returns from 8% to 15% and standard deviation from 8% to 11%, and maximum drawdown from -5% to -8%, thus providing improvements in returns at the cost of increased risk. The 1% written OTM VIX call allocation increased returns from 8% to 19% and standard deviation from 8% to 11%, and maximum drawdown from -5% to -6%. The return to risk tradeoff (as measured by the Sharpe ratio) of the base portfolio was greatly improved by the addition of written VIX call options (from 1.11 to 1.30 and 1.72, respectively).

Thus, in 2016 the addition of inverse VIX futures or written call options to the 60/40 base portfolio resulted in meaningful increases in absolute returns and in risk-adjusted returns based on the Sharpe ratio and alpha (although most are statistically insignificant at 10%, with the exception of the 60/40+OTM call portfolio).

Exhibit 11: 60/40 Portfolio in 2016 with Inverse VIX Futures and Written VIX Calls

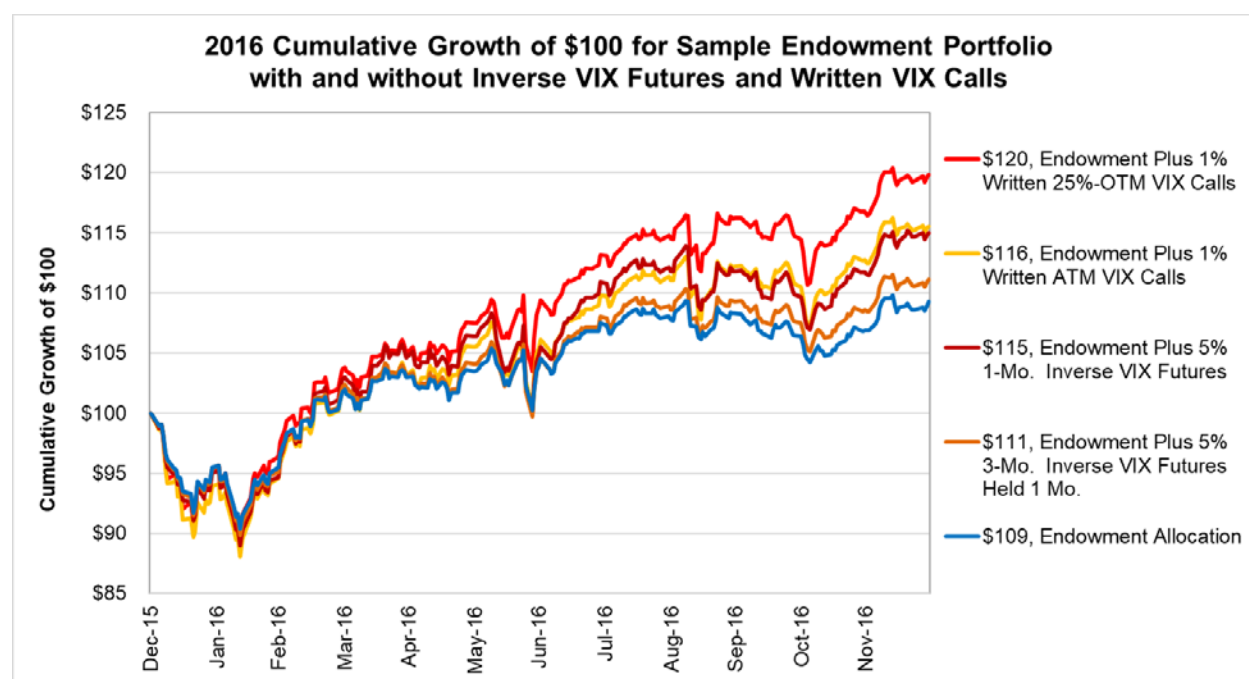


2016	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1 Mo. Inverse VIX Futures	60/40 Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25% OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annual Return	8.4%	14.3%	10.4%	18.9%	14.7%
Annualized Standard Deviation	7.6%	11.0%	9.1%	11.0%	11.3%
Sharpe Ratio	1.11	1.30	1.14	1.72	1.30
Maximum Drawdown	-5.4%	-6.9%	-6.1%	-6.0%	-7.8%
Skewness	-0.29	-0.96	-0.51	-0.59	-0.70
Kurtosis	2.23	5.22	3.52	3.89	3.71
Correlation w SPX	0.99	0.96	0.98	0.95	0.95
Beta	0.57	0.81	0.68	0.79	0.82
Alpha	2.06%	6.02%	2.94%	12.03%	6.36%
Alpha P-Value	0.24	0.17	0.26	0.02	0.21

Source: Cboe Exchange Inc., Bloomberg

Exhibit 11: This exhibit illustrates the 2016 performance of a traditional 60/40 portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of low volatility and generally positive market returns, a small allocation to VIX call writing can significantly increase returns with a moderate increase in volatility. A 5% allocation to inverse VIX futures has a smaller impact but also provides a meaningful increase in return with a small increase in volatility and drawdowns.

Exhibit 12: Endowment Portfolio in 2016 with Inverse VIX Futures and Written VIX Calls



2016	Endowment	Endowment Plus 5% 1 Mo. Inverse VIX Futures	Endowment Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25% OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annual Return	9.2%	15.0%	18.9%	19.8%	15.5%
Annualized Standard Deviation	10.0%	13.0%	10.0%	12.7%	13.1%
Sharpe Ratio	0.92	1.15	1.89	1.56	1.19
Maximum Drawdown	-9.6%	-11.0%	-4.4%	-10.0%	-12.0%
Skewness	-0.48	-1.00	0.82	-0.71	-0.74
Kurtosis	2.56	5.32	6.15	3.68	3.33
Correlation w SPX	0.82	0.87	0.31	0.87	0.88
Beta	0.62	0.86	0.24	0.85	0.87
Alpha	2.52%	6.29%	21.61%	12.43%	6.74%
Alpha P-Value	0.77	0.50	0.12	0.17	0.46

Source: Cboe Exchange Inc., Bloomberg

Exhibit 12: This exhibit illustrates the 2016 performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of relative calm and positive performance, small allocations to inverse VIX futures or VIX call writing increase returns significantly with a moderate increase in volatility. A 5% allocation to inverse three-month VIX futures doubles returns with no increase in volatility and a reduction in drawdown.

We now consider the addition of inverse VIX futures and written call options to a well-diversified portfolio that contains other asset classes beyond equity and fixed income. Exhibit 12 presents an analysis of an addition of inverse VIX futures and written call options to a well-

diversified hypothetical endowment portfolio in 2016. As in 2008, in 2016 the impact of the inverse VIX allocations were similar for both base portfolios. Exhibit 12 provides an illustration of the 2016 cumulative growth and summary statistics of the base hypothetical endowment portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio gained 9% over the period at a 10% annual standard deviation and a -10% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures both significantly outperformed the base portfolio, increasing the gains of the base portfolio from 9% to 15% and 19%, respectively. While the one-month inverse VIX futures allocation increased standard deviation from 10% to 13% and maximum drawdown from -10% to -13%, the three-month inverse VIX futures allocation increased returns without a change in standard deviation and actually reduced maximum drawdown from -10% to -4%. Similarly, the ex-post skewness was reduced for the one-month inverse VIX futures allocation but improved by adding the three-month inverse VIX futures allocation to the base endowment portfolio. From a risk-adjusted return basis, the addition of inverse VIX futures provided a large improvement in the performance (as measured by the Sharpe ratio) of the base portfolio (from 0.92 to 1.15 and 1.89, respectively).

As with the 60/40 portfolio, the results for adding a written VIX call option allocation are similar to those from adding inverse VIX futures. The 1% written one-month ATM VIX call allocation improved returns from 9% to 16% while increasing standard deviation from 10% to 13%, and maximum drawdown from -10% to -12%, thus providing improvements in return with an increase in risk very similar to the one-month inverse VIX futures allocation. The 1% allocation to inverse one-month 25% OTM VIX calls further increased returns to 20% (from 9% for the base portfolio), but did so at a higher standard deviation (13% versus 10% for the base portfolio), and at an essentially identical maximum drawdown (-10.0% versus -9.6% for the base endowment portfolio).

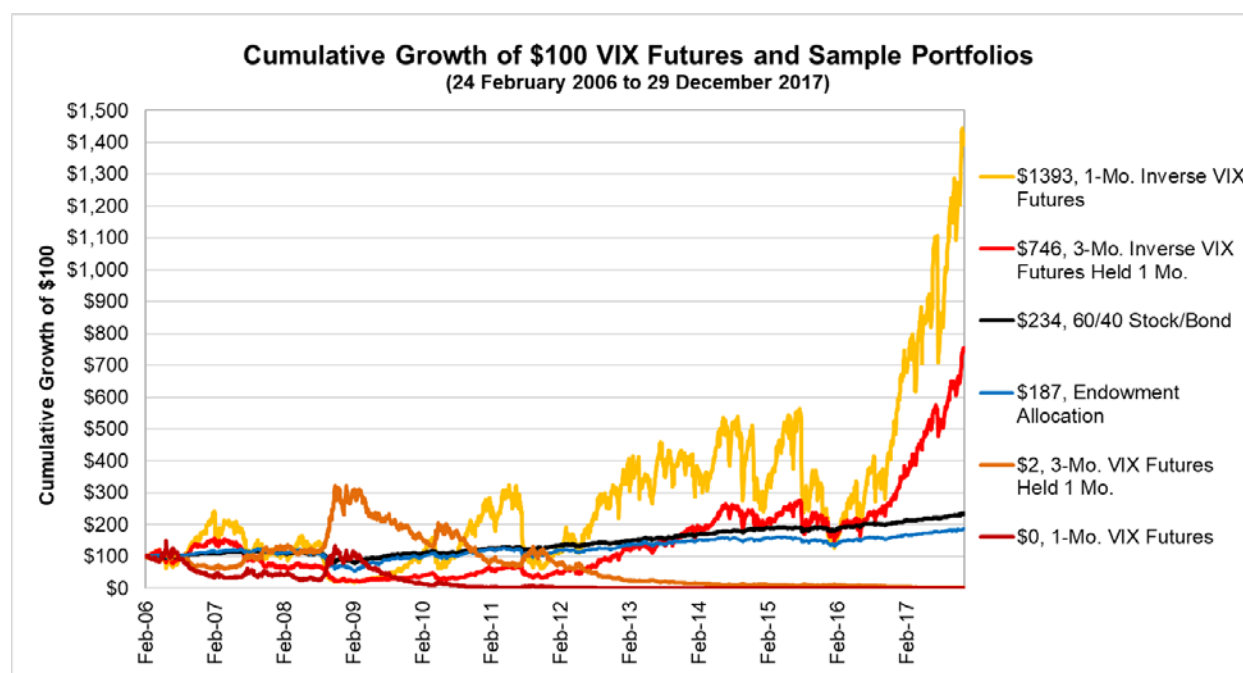
Thus, for both base portfolios, an inverse VIX futures or call allocation resulted in an economically significant increase in returns and a smaller increase in risk, resulting in much higher Sharpe ratios. Similar results are found when considering alpha (although all measured alphas are statistically insignificant at 10%). Unlike the previous analyses, in which the VIX call allocations had more extreme impacts than the VIX futures allocations, the 1% written VIX call allocations in 2016 were very similar to the 5% inverse VIX futures allocations in 2016.

4.0 Performance of Long Term Buy-and-hold Inverse VIX-Based Strategies from 2006 to 2017

As mentioned above, the relatively persistent contango in VIX futures and the high volatility of volatility would likely make a long term buy-and-hold standalone long or inverse VIX futures position a questionable proposition for most investors. For this reason, the previous analysis

focused on VIX futures or call option investments in a portfolio context over single year time periods. For the sake of completeness, this section expands the analysis by considering the performance of these strategies over the 10+ years since the introduction of VIX options.

Exhibit 13: Inverse VIX Futures-Based Strategies Buy-and-Hold performance for over 10 years from 2006 to 2017



24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	Endowment	1 Mo. VIX Futures	3 Mo. VIX Futures Held 1 Mo.	3 Mo. Inverse VIX Futures Held 1 Mo.	3 Mo. Inverse VIX Futures Held 1 Mo.
Annual Return	1.8%	2.0%	-55.8%	-28.8%	25.0%	18.5%
Annualized Standard Deviation	11.4%	16.5%	77.9%	43.6%	77.9%	43.6%
Sharpe Ratio	0.08	0.07	-0.73	-0.68	0.31	0.40
Maximum Drawdown	-35.3%	-58.1%	-100.0%	-99.4%	-92.3%	-86.5%
Skewness	-0.10	-0.24	1.04	0.64	-1.05	-0.64
Kurtosis	12.35	10.15	5.47	4.02	5.47	4.02
Correlation w SPX	0.99	0.89	-0.74	-0.74	0.74	0.74
Beta	0.58	0.75	-2.94	-1.66	2.94	1.66
Alpha	2.70%	-1.79%	-31.58%	-10.92%	34.67%	14.01%
Alpha P-Value	0.00	0.57	0.15	0.37	0.12	0.25

Source: Cboe Exchange Inc., Bloomberg

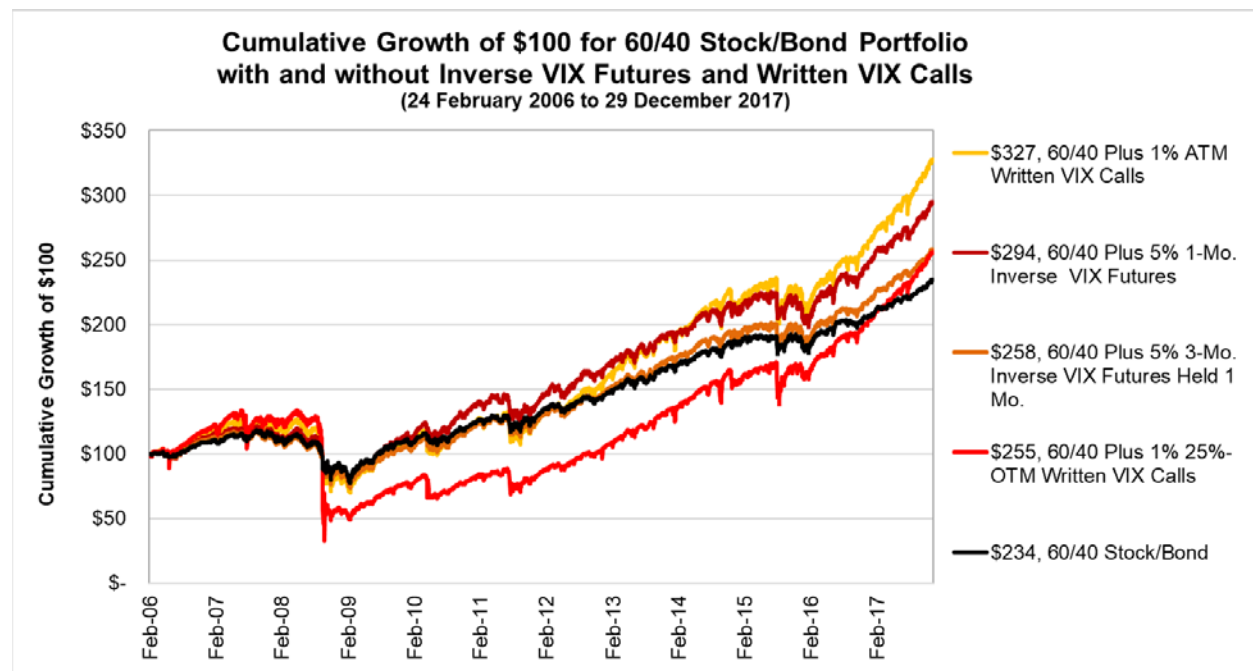
Exhibit 13: This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios and two long VIX futures-based portfolios over the entire, almost 12 year, period of study. The volatility and drawdowns of the four VIX futures-based portfolios suggest that VIX futures may be best suited for shorter term strategic portfolio risk management or return enhancement rather than long term buy-and-hold strategies.

Exhibit 13 provides a graphical representation and summary statistics of the cumulative performance of the one-month and three-month long and inverse VIX futures strategies along with the two base portfolios since the inception of VIX options. It is evident that the overall period was favorable for an inverse VIX futures exposure from a total return perspective and unfavorable for a long VIX futures exposure. However, the volatility of the long and inverse VIX exposures were exceedingly high (ranging from 32% to 78%) as were the maximum drawdowns (ranging from -90% to -100%). The maximum drawdowns for the one-month and three-month long VIX futures strategies were catastrophically high at -100% and -99%, respectively. The following sections will consider the full period performance of long and inverse VIX futures and call options in a portfolio context.

4.1 Inverse VIX Futures and Written Options-Based Strategies Portfolio Allocations from 2006 to 2017

This section considers the performance of inverse VIX futures and VIX call writing strategies within 60/40 and hypothetical endowment portfolios over the 10+ years since the introduction of VIX options. Exhibit 14 illustrates the 10+ year cumulative growth and provides summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio generated a 7.5% annualized return over the period at an 11% annualized standard deviation with a -35% maximum drawdown. Both of the 5% inverse VIX futures allocations moderately increased the annualized return (from 7.5% to 9.5% and 8.3%, respectively) and standard deviation (from 11% to 14% and 12%, respectively) of the base portfolio. The maximum drawdown also increased from -35% to -39% and -38%, respectively. The ex-post skewness was also degraded by adding the inverse VIX futures allocations to the base portfolio. Thus over the full period, the inverse VIX futures allocations improved absolute returns and risk. Based on the Sharpe ratio and alpha, the inverse VIX futures allocation improved risk-adjusted returns. The Sharpe ratio increased slightly from 0.65 to 0.69 and 0.67, respectively and alpha increased from 2.8% to 4.4% and 3.4%, respectively.

Exhibit 14: 60/40 Portfolio Performance with Inverse VIX Futures and Written VIX Calls from 2006 to 2017



24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1 Mo. Inverse VIX Futures	60/40 Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25% OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annual Return	7.5%	9.5%	8.3%	8.3%	10.5%
Annualized Standard Deviation	11.4%	13.7%	12.4%	38.6%	17.4%
Sharpe Ratio	0.65	0.69	0.67	0.21	0.60
Maximum Drawdown	-34.8%	-38.5%	-37.7%	-75.6%	-44.8%
Skewness	-0.10	-0.41	-0.27	15.45	-0.26
Kurtosis	12.38	7.43	9.43	629.17	19.36
Correlation w SPX	0.99	0.98	0.99	0.58	0.93
Beta	0.58	0.69	0.63	1.14	0.83
Alpha	2.82%	4.40%	3.41%	4.01%	4.51%
Alpha P-Value	0.000	0.000	0.000	0.763	0.105

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 14: This exhibit illustrates the performance of a 60/40 traditional portfolio with and without a small allocation to inverse VIX futures or written VIX call options. The returns of the 60/40 portfolio are improved with the addition of VIX call writing or inverse VIX futures, although the VIX call writing strategies significantly increase both volatility and drawdowns.

The results for adding a written ATM VIX call option allocation are generally similar to the results for the inverse VIX futures allocations. The 1% one-month written ATM VIX call allocation increased returns from 7.5% to 10.5% and standard deviation from 11% to 17%, and maximum drawdown from -35% to -45%. In contrast, while the 1% allocation to written one-month 25% OTM VIX calls had a similar return increase to the other inverse VIX allocations, it increased standard deviation far more than the written VIX call or inverse VIX futures allocations. Returns increased reduced from 7.5% to 8.3% while standard deviation tripled from 11% to 39%, and

maximum drawdown more than doubled from -35% to -76%. From a risk-adjusted risk-adjusted return perspective, the results are mixed. The written ATM and OTM VIX call allocations reduced the Sharpe ratio of the base portfolio 0.65 to 0.60 and 0.21, respectively, while alpha increased from 2.8% to 4.5% and 4.1%, respectively, (although the alpha for the written OTM call portfolios is statistically insignificant at 10%).

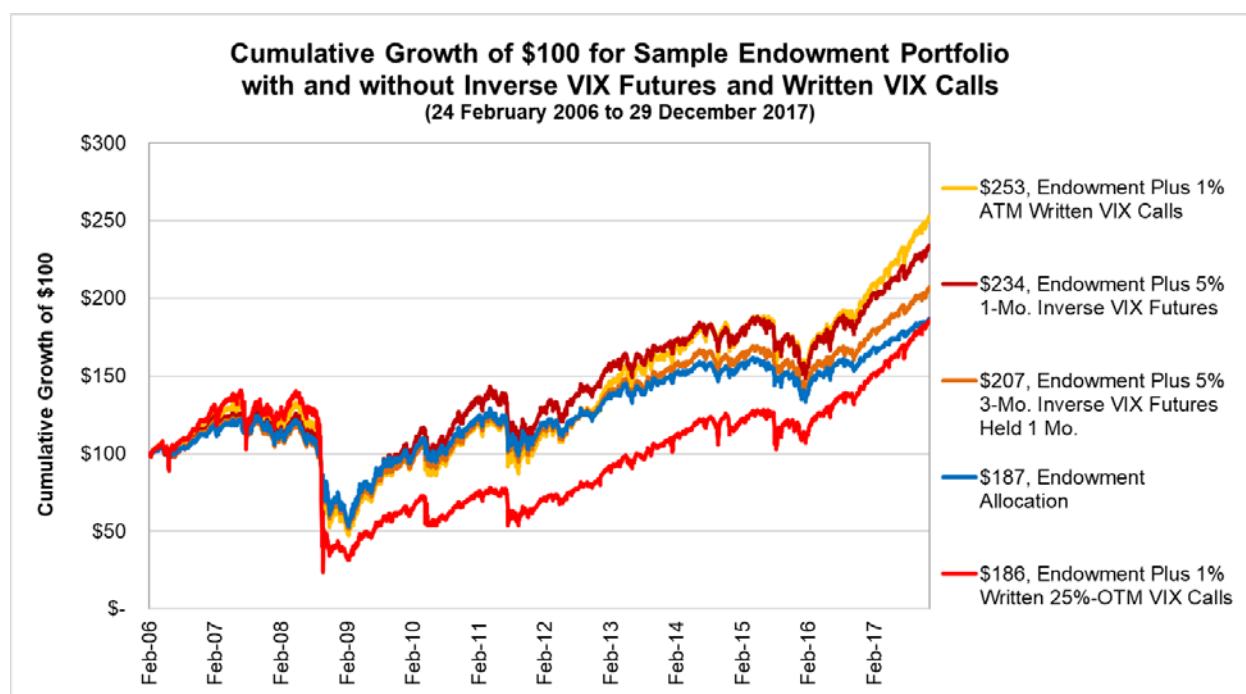
The results above suggest that an inverse VIX futures or written call allocation would have improved performance from a raw return perspective at the expense of increased risk. However, the results are mixed from a risk-adjusted return perspective. This suggests that permanent buy-and-hold allocations to inverse VIX futures or written calls may provide small improvement in returns but at the expense of increased risk over the long term.

This section assesses the impact of an added allocation of inverse VIX futures or call options to a well-diversified hypothetical endowment portfolio over the full 10+ year period. Exhibit 15 provides an illustration of the full period cumulative growth and summary statistics of the base hypothetical endowment portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) or a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio experienced a 5% annualized return over the period at a 17% annualized standard deviation and a -57% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures outperformed the base portfolio, experiencing moderately higher returns (8% and 6% versus 5% for the base portfolio) although standard deviations were also somewhat higher (18.3% and 17.1% versus 16.5% for the base portfolio). The maximum drawdown also slightly increased from -57% to -59% and -58%, respectively. The ex-post skewness was also worsened by adding the inverse VIX futures allocations to the base endowment portfolio. From a risk-adjusted return perspective, the inverse VIX futures allocations improved performance of the endowment portfolio with the Sharpe ratio increasing from 0.33 to 0.41 and 0.37 and alpha increasing from -1.4% to +0.2% and -0.7%, respectively.

The results for adding a written VIX call option allocation are quite different from the inverse VIX futures allocations. The 1% written one-month ATM or OTM VIX call allocation reduced returns from 5% to 3% and 2%, respectively, and increased standard deviation from 17% to 16% and 14%, respectively, and maximum drawdown from -57% to -43% and -50%, respectively. From a risk-adjusted return perspective, the inverse VIX call allocations reduced performance of the endowment portfolio based on the Sharpe ratio (dropping from 0.33 to 0.16 for both written call strategies). However the results for alpha are mixed, with alpha decreasing from -1.4% to -2.8% and increasing from -1.4% to +0.2%, respectively (although the alphas are not statistically significant).

Thus, while the inverse VIX futures allocations improved absolute and risk-adjusted returns for both base portfolios, the call allocations results are mixed for both absolute returns and risk-adjusted returns.

Exhibit 15: Endowment with Inverse VIX Futures and Written VIX Calls from 2006 to 2017



24 Feb 2006 to 29 Dec 2017	Endowment	Endowment Plus 5% 1 Mo. Inverse VIX Futures	Endowment Plus 5% 3 Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25% OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annual Return	5.4%	7.5%	6.3%	2.2%	2.6%
Annualized Standard Deviation	16.5%	18.3%	17.1%	13.7%	16.2%
Sharpe Ratio	0.33	0.41	0.37	0.16	0.16
Maximum Drawdown	-57.4%	-59.0%	-58.3%	-50.2%	-42.9%
Skewness	-0.24	-0.44	-0.34	0.04	-0.40
Kurtosis	10.18	7.66	8.90	11.36	28.10
Correlation w SPX	0.89	0.91	0.91	0.71	0.44
Beta	0.75	0.85	0.79	0.50	0.37
Alpha	-1.44%	0.24%	-0.69%	-2.84%	0.17%
Alpha P-Value	0.648	0.939	0.820	0.482	0.978

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 15: This exhibit illustrates the performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. The returns of the endowment portfolio are improved with the addition of inverse VIX futures with a small increase in volatility. In contrast, VIX call writing reduces returns with a small impact on volatility.

5.0 VIX Spread Strategies

Spread trading is very popular in futures trading. VIX futures are no exception. The amount of VIX futures open interest held in the form of futures spreads is a significant portion of total open interest and has grown significantly since 2012 (see Szado [2018a]). While there are a wide variety of VIX futures spread strategies, the following analysis focuses on a simple term

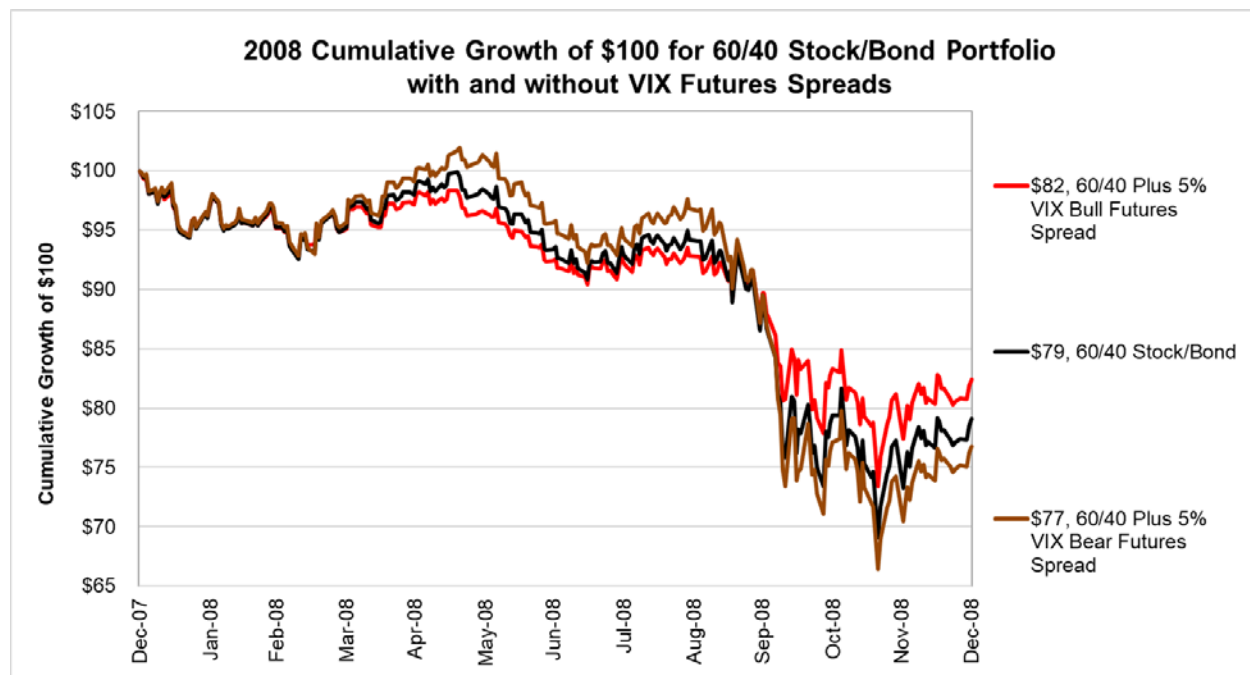
structure spread strategy which takes a position in the one-month VIX futures contract versus an opposite position in the equal number of three-month VIX futures, with both contracts held for one-month before rolling. The following sections consider the addition of a bull or bear VIX Futures Spread to the two base portfolios in 2008, in 2016 and in the overall 10+ year period. The bull VIX Futures Spread consists of a long one-month VIX futures position and a short three-month VIX futures position, while a bear VIX Futures Spread is short one-month VIX futures and long three-month VIX futures. It is worth noting that the terms “bull” and “bear” do not refer to bull or bear equity exposure, but rather to the exposure to volatility through VIX futures. A bull spread in VIX futures might be considered in some way comparable to a bear spread in S&P 500 futures since one would expect VIX futures to rise as S&P 500 futures fall. The spread strategies can be implemented to take advantage of a wide variety of characteristics of VIX futures, including the relative betas, volatility or cost of carry of futures contracts of different tenor or expectations of changes in the term structure. Spread strategies may also be implemented as a less “expensive” means of accessing VIX exposure since the volatility of volatility exposure is expected to be less in a term spread than in a direct inverse futures exposure, or the cost of carry drag caused by steep contango may be less in a term spread than in a direct long futures position.

5.1 VIX Spread Strategies in 2008

Exhibit 16 provides results pertaining to the 2008 performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads (as defined above). Previously discussed summary statistics for the 60/40 portfolio with a 5% long or inverse VIX futures allocation are also provided for reference.

The base 60/40 portfolio lost 21% over the period at a 24% annual standard deviation and a 31% maximum drawdown. The portfolios with a 5% allocation to bull VIX futures spreads performed similarly to long one-month VIX futures, although the bull VIX futures portfolio underperforms the one-month futures portfolio. The spread portfolio experienced a -18% return at a 20% standard deviation versus a -13% return and 18% standard deviation for the portfolio with the one-month long VIX futures allocation. Risk-adjusted return for the bull spread allocation portfolio is also lower with an alpha of 1.7% versus 5.5%, although both are not statistically significant. In contrast, the performance of the portfolios with allocations to bear VIX futures spreads and to inverse one-month VIX futures performed almost identically by most of the calculated measures.

Exhibit 16: 60/40 Portfolio with and without VIX Futures Spreads in 2008



2008	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	-20.9%	-17.6%	-23.2%	-13.4%	-23.6%
Annualized Standard Deviation	24.1%	20.4%	26.0%	18.4%	26.0%
Sharpe Ratio	-0.87	-0.86	-0.89	-0.73	-0.91
Maximum Drawdown	-30.9%	-26.6%	-34.8%	-22.1%	-35.4%
Skewness	0.16	0.22	0.17	0.36	0.01
Kurtosis	4.14	3.69	4.91	4.08	3.33
Correlation w SPX	1.00	0.98	0.98	0.97	0.99
Beta	0.58	0.49	0.62	0.44	0.63
Alpha	2.22%	1.73%	0.78%	5.52%	0.42%
Alpha P-Value	0.501	0.745	0.914	0.357	0.923

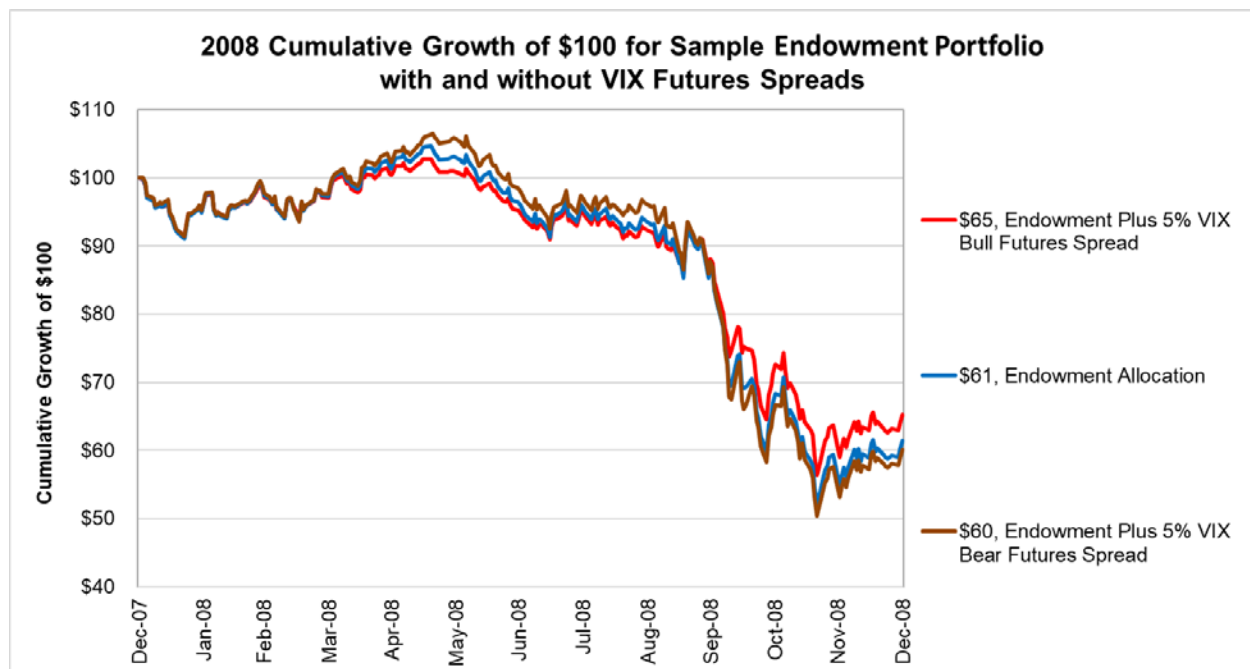
Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 16: This exhibit illustrates the 2008 performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bull spread had slight improvements in return, standard deviation, and drawdown when compared to the 60/40 portfolio.

Similar results are found in Exhibit 17 for the endowment-based portfolios. Once again, the portfolio with the bull spread allocation underperformed the long one-month VIX futures allocation portfolio with a lower return (-35% versus -31%) and higher standard deviation (27% versus 25%), while the bear spread allocation portfolio performed virtually identically to the long one-month VIX futures allocation portfolio.

Thus, in 2008 both the bull VIX futures spread allocations and the long one-month VIX futures allocation improved performance of the base 60/40 and endowment portfolios in 2008, however the bull spread allocation was less effective than the long one-month VIX futures allocation. In contrast, the bear VIX futures spread allocations and the inverse one-month VIX futures allocation both slightly reduced performance of the base 60/40 and endowment portfolios in 2008, and did so virtually identically to one another.

Exhibit 17: Endowment Portfolio with and without VIX Futures Spreads in 2008



2008	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	-38.6%	-34.7%	-39.9%	-31.1%	-40.2%
Annualized Standard Deviation	31.0%	27.0%	32.5%	25.2%	32.5%
Sharpe Ratio	-1.25	-1.29	-1.23	-1.23	-1.24
Maximum Drawdown	-50.4%	-45.2%	-52.7%	-41.5%	-53.1%
Skewness	-0.13	-0.08	-0.17	-0.02	-0.21
Kurtosis	3.21	3.38	3.13	3.61	2.79
Correlation w SPX	0.92	0.89	0.92	0.87	0.93
Beta	0.69	0.59	0.73	0.54	0.73
Alpha	-25.54%	-24.21%	-26.16%	-19.88%	-26.53%
Alpha P-Value	0.158	0.170	0.166	0.269	0.138

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 17: This exhibit illustrates the 2008 performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The endowment portfolio plus a 5% allocation to a VIX futures bull spread had slight improvements in return and standard deviation when compared to the endowment portfolio.

5.2 VIX Spread Strategies in 2016

Exhibit 18 provides a graphical representation and summary statistics of the 2016 performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads. Previously discussed 2016 summary statistics for the 60/40 portfolio with a 5% long or inverse VIX futures allocation are also provided for reference. The base 60/40 portfolio gained 8% over the period at an 8% annual standard deviation and a -5% maximum drawdown. The portfolio with a 5% allocation to bear VIX futures spreads outperformed the base portfolio but slightly underperformed the one-month inverse VIX futures allocation portfolio from an absolute return perspective, increasing the returns of the base portfolio from 8.4% to 13.6% (versus 14.3% for the inverse futures allocation portfolio). However, the bear VIX futures spread allocation portfolio increased standard deviation by less than the one-month inverse VIX futures allocation (from 8% to 9% versus 11%), while maximum drawdown was increase by approximately the same amount for both VIX allocations. maximum drawdown from -5% to -7% and -6%, respectively. From a risk-adjusted return perspective, both the bear VIX futures spread and inverse VIX futures allocations improved the 60/40 portfolio's Sharpe ratio and alpha, although the bear spread allocation provided a greater improvement. Sharpe ratio was increased from 1.11 to 1.49 and 1.30, respectively and alpha was increase from 2.1% to 7.6% and 6.0%, respectively (although only the bear spread allocation alpha is not statistically significant at 10%).

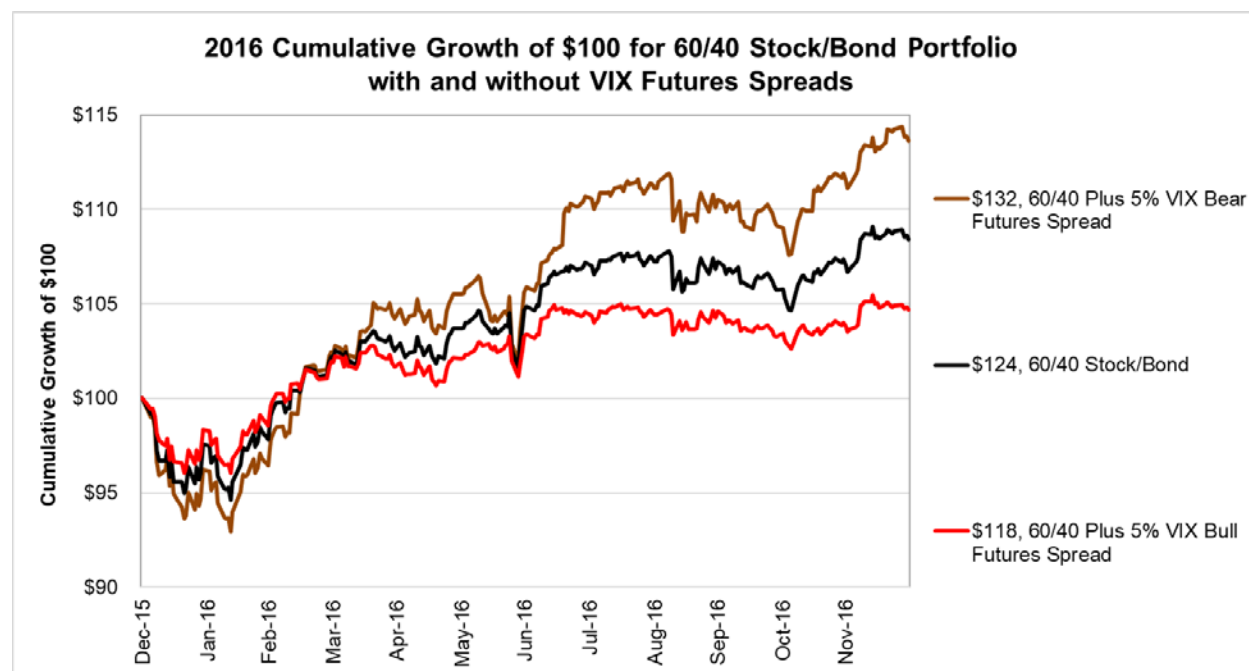
As expected, the bull VIX futures allocation and the long one-month VIX futures allocation both reduced the performance of the 60/40 portfolio in 2016. The bull spread allocation reduced returns by less than the long one-month futures allocation (from 8% to 5% versus 2%, respectively), although it also reduced standard deviation by less (from 8% to 6% versus 4%, respectively). From a risk-adjusted return perspective, both allocations reduced performance, yet the bull spread allocation portfolio reduced performance by less than the long one-month futures allocation. Sharpe ratio was reduced from 1.11 to 0.79 and 0.52, respectively and alpha was decrease from 2.1% to -0.7% and -1.4%, respectively (although none of the alphas are statistically significant at 10%).

Similar results are found in Exhibit 19 for the endowment based portfolios. Once again, the portfolio with the bull spread allocation underperformed the long one-month VIX futures allocation portfolio with a lower return (6% versus 3%) at a higher standard deviation (9% versus 7%), while both had higher returns and lower standard deviations than the base endowment portfolio. Both portfolios also experienced lower Sharpe ratios and alphas than the base portfolio although, once again, the bull spread allocation outperformed the long one-month VIX futures allocation portfolio.

The bear spread allocation and inverse one-month VIX futures allocation also had similar impacts on the endowment portfolio as they did on the 60/40 portfolio. Both allocations increased absolute and risk-adjusted return, while slightly increasing standard deviation and maximum drawdown. Once again, the inverse one-month VIX futures allocation increased

absolute return by more than the bear spread allocation, although the bear spread allocation provided a greater increase in risk-adjusted return (Sharpe ratio and alpha).

Exhibit 18: 60/40 Portfolio with and without VIX Futures Spreads in 2016



2016	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	8.4%	4.7%	13.6%	2.3%	14.3%
Annualized Standard Deviation	7.6%	5.9%	9.1%	4.4%	11.0%
Sharpe Ratio	1.11	0.79	1.49	0.52	1.30
Maximum Drawdown	-5.4%	-4.0%	-7.1%	-2.9%	-6.9%
Skewness	-0.29	0.05	-0.39	0.57	-0.96
Kurtosis	2.23	1.58	2.29	1.77	5.22
Correlation w SPX	0.99	0.96	0.94	0.81	0.96
Beta	0.57	0.43	0.65	0.27	0.81
Alpha	2.06%	-0.73%	7.57%	-1.36%	6.02%
Alpha P-Value	0.240	0.762	0.093	0.714	0.168

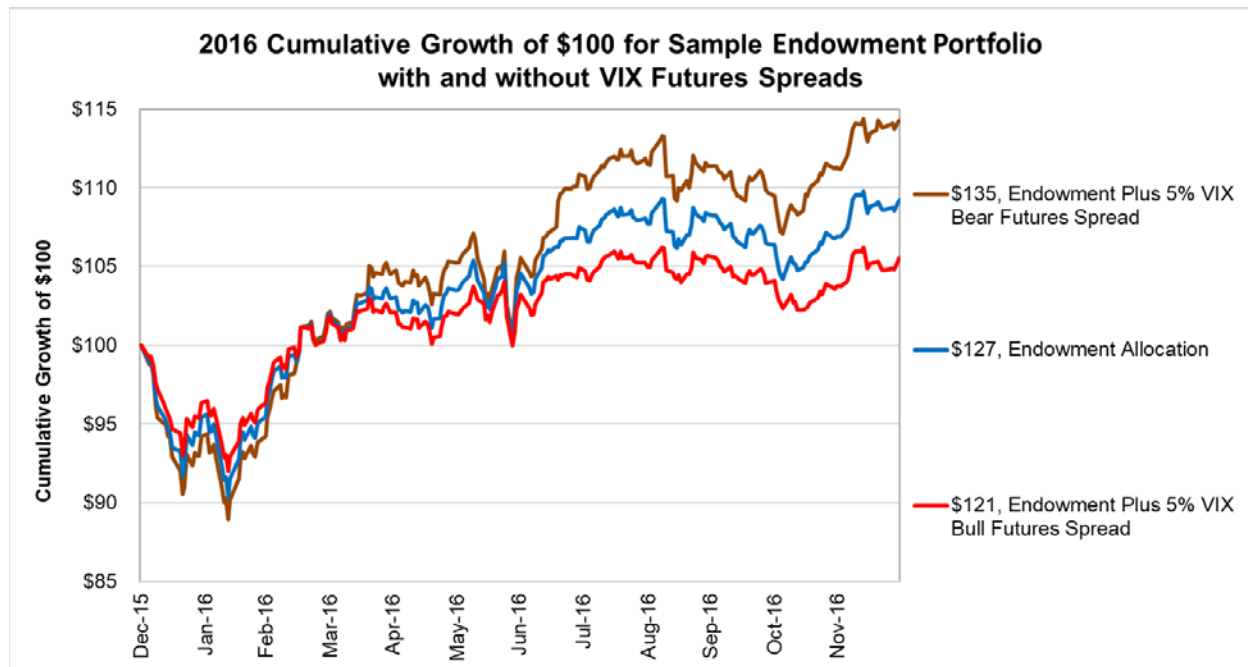
Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 18: This exhibit illustrates the 2016 performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.

Thus, in 2016 both the VIX futures spread allocations generated higher risk-adjusted returns than their corresponding one-month VIX futures allocations (for both base portfolios). In contrast, in 2008 the bear VIX futures spread allocations provided essentially identical impacts

on absolute and risk-adjusted returns, while the bull VIX futures spread allocations provided somewhat lower improvement in absolute and risk-adjusted returns when compared to one-month VIX futures allocations.

Exhibit 19: Endowment Portfolio with and without VIX Futures Spreads in 2016



2016	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	9.2%	5.5%	14.3%	3.2%	15.0%
Annualized Standard Deviation	10.0%	8.5%	11.2%	7.3%	13.0%
Sharpe Ratio	0.92	0.65	1.28	0.44	1.15
Maximum Drawdown	-9.6%	-8.0%	-11.1%	-6.4%	-11.0%
Skewness	-0.48	-0.36	-0.53	-0.27	-1.00
Kurtosis	2.56	2.17	2.58	1.79	5.32
Correlation w SPX	0.82	0.75	0.83	0.58	0.87
Beta	0.62	0.48	0.71	0.32	0.86
Alpha	2.52%	-0.22%	7.85%	-0.77%	6.29%
Alpha P-Value	0.765	0.979	0.390	0.928	0.504

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 19: This exhibit illustrates the 2016 performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of the endowment portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.

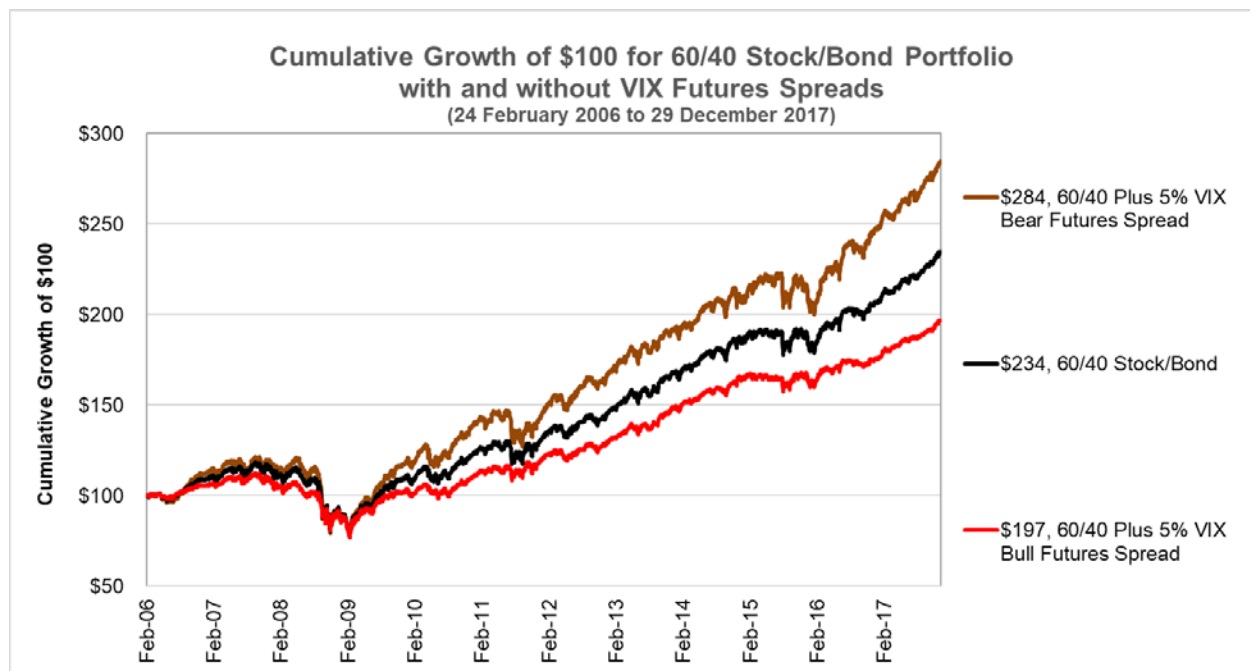
5.3 VIX Spread Strategies from 2006 to 2017

This section considers the performance of the bull and bear VIX futures spread strategies over a 10+ year period since the introduction of VIX options (24 February 2006 to 31 December 2017). Exhibit 20 provides a graphical representation and summary statistics of the cumulative performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads while Exhibit 21 provides the corresponding results for the cumulative performance of the base endowment portfolio with and without an allocation to VIX futures spreads. The exhibits also include summary statistics for the base portfolios with a 5% long or inverse VIX futures allocation.

Consistent with the previously presented results, the bull VIX spread allocation and the long one-month VIX futures allocation both reduce the performance of the 60/40 base portfolio (see Exhibit 20). This is true both from an absolute return perspective and a risk-adjusted return perspective. However, the bear VIX futures spread allocation portfolio outperforms the long VIX futures allocation portfolio. The performance of the bear VIX futures spread allocation portfolio lies close to mid-way between the performance of the 60/40 portfolio and the performance of the long VIX futures allocation portfolio by most measures. For example, the returns of the base, bull spread allocation and long futures allocation portfolios are 8%, 6% and 4%, respectively, the Sharpe ratios are 0.65, 0.62 and 0.54, respectively and alphas are 2.8%, 2.0% and 1.1%, respectively. The risk of the portfolios follow a similar pattern with standard deviations of 11%, 10% and 8%, respectively and maximum drawdowns of -35%, -31% and -27%. This is consistent with the 2008 results shown above in which the long one-month futures allocation portfolio provided greater protection than the bull spread allocation portfolio.

The bear spread and inverse VIX futures allocations both improved absolute returns and risk-adjusted returns over the 10+ years since the inception of VIX option trading, while also moderately increasing risk. While the bear spread allocation increased return by less than the inverse VIX futures allocation (from 7.5% to 9.2% and 9.5%, respectively), the bear spread allocation increased risk-adjusted return by more (Sharpe ratio increased from 0.65 to 0.73 and 0.69 while the alpha increased from 2.8% to 4.7% and 4.4%, respectively). The risk of the of the bear VIX futures spread allocation portfolio was close to mid-way between the risk of the 60/40 portfolio and the long VIX futures allocation portfolio as measured by the standard deviation and the maximum drawdown.

Exhibit 20: 60/40 Portfolio with and without VIX Futures Spreads

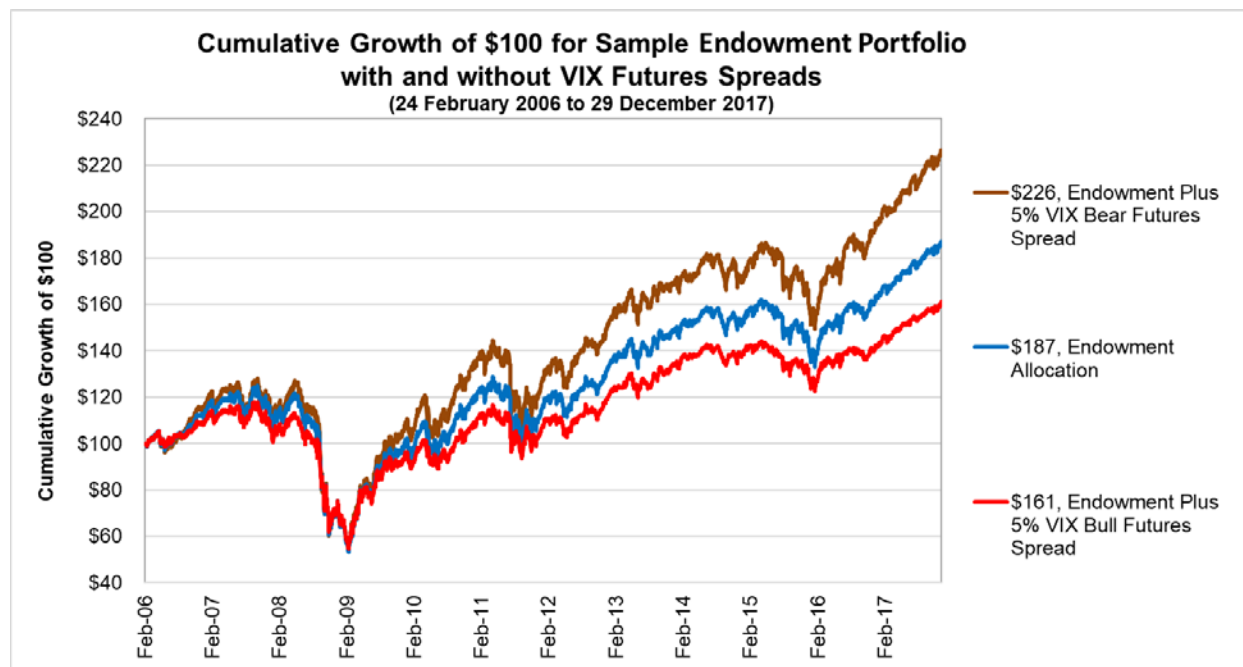


24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	7.5%	5.9%	9.2%	4.4%	9.5%
Annualized Standard Deviation	11.4%	9.5%	12.6%	8.1%	13.7%
Sharpe Ratio	0.65	0.62	0.73	0.54	0.69
Maximum Drawdown	-34.8%	-31.4%	-35.9%	-26.5%	-38.5%
Skewness	-0.10	0.08	-0.18	0.35	-0.41
Kurtosis	12.38	12.32	12.37	16.25	7.43
Correlation w SPX	0.99	0.97	0.97	0.92	0.98
Beta	0.58	0.47	0.63	0.38	0.69
Alpha	2.82%	1.96%	4.66%	1.12%	4.40%
Alpha P-Value	0.000	0.037	0.000	0.408	0.000

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 20: This exhibit illustrates the performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads over the full period of the study. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.

Exhibit 21: Endowment Portfolio with and without VIX Futures Spreads



24 Feb 2006 to 29 Dec 2017	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	5.4%	4.1%	7.1%	2.8%	7.5%
Annualized Standard Deviation	16.5%	14.4%	17.2%	13.1%	18.3%
Sharpe Ratio	0.33	0.28	0.41	0.21	0.41
Maximum Drawdown	-57.4%	-53.8%	-57.5%	-50.4%	-59.0%
Skewness	-0.24	-0.10	-0.35	0.02	-0.44
Kurtosis	10.18	10.78	9.45	12.41	7.66
Correlation w SPX	0.89	0.86	0.90	0.80	0.91
Beta	0.75	0.63	0.79	0.54	0.85
Alpha	-1.44%	-2.02%	0.47%	-2.78%	0.24%
Alpha P-Value	0.648	0.516	0.885	0.397	0.939

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 21: This exhibit illustrates the performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads over the full period of the study. The strategy of the endowment portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.

Thus, over the full period of study, the bull and bear spread allocation portfolios both provided greater risk-adjusted returns than their corresponding one-month futures allocation portfolios. Furthermore, over the long term, long VIX futures and bull spreads historically reduced risk-adjusted performance while inverse VIX futures and bear spreads improved risk-adjusted performance, providing an economically significant increase in ex post alpha.

Similar results are found in Exhibit 21 for the endowment based portfolios. Once again, the portfolio with the bull spread allocation outperformed the long one-month VIX futures

allocation portfolio with a higher return (6% versus 4%), but at a higher standard deviation (10% versus 8%), while both had lower returns and standard deviations than the base endowment portfolio. Both portfolios also provided lower Sharpe ratios and alphas than the base portfolio and, once again, the bull spread allocation outperformed the long one-month VIX futures allocation portfolio.

The bear spread allocation and inverse one-month VIX futures allocation also had similar impacts on the endowment portfolio as they did on the 60/40 portfolio. Both allocations increased absolute and risk-adjusted return, while slightly increasing standard deviation and maximum drawdown. The inverse one-month VIX futures allocation increased absolute return by more than the bear spread allocation, although the bear spread allocation provided a greater increase in risk-adjusted return based on alpha (both provided the same Sharpe ratio).

6.0 Inverse VIX Futures with Dynamic De-Levering - the VPD and VPN Indexes

This section considers the performance of two benchmark indexes that provide exposure to inverse VIX futures with a dynamically varying degree of leverage, with or without additional risk mitigation from the addition of long VIX calls. The Cboe VIX Premium Strategy Index (VPD) sells a varying number of one-month VIX futures contracts versus a money market account investment representing the full value of a VIX futures contract. The number of futures contracts sold is determined on the roll date such that 75 percent of the capital will be preserved in the face of a 25-point increase in the VIX futures price (see www.cboe.com/VPD). Specifically, the number of one-month VIX futures contracts sold on the roll date is determined by the following formula:

$$N_{\text{NEW}} = M_t * (1 + R_t - 0.75) / (1000 * 25)$$

where M_t is the amount in the money market account at the roll and R_t is the effective three-month Treasury bill rate to the next roll.

Since the notional value of the short VIX futures position is determined by the level of the futures price and the number of contracts sold, the level of VIX futures will have a significant impact on the degree of leverage. The futures position is typically de-levered since it is generally over-collateralized. For example, if the VIX futures are at 25.0 at the roll, approximately 0.25 futures contracts will be sold versus a money market position representing the full value of one futures contract (de-levered) and if the VIX futures are at 100.0 at the roll, the short futures position will be approximately fully collateralized (unlevered). At VIX futures levels significantly above 100.0, the short VIX futures will be partially collateralized (levered). Thus, the short VIX futures position is expected to be de-levered unless VIX futures are at extremely high levels.

The Cboe Capped VIX Premium Strategy Index (VPN) is a risk-mitigated version of the VPD index in which the short VIX futures positions are protected by purchasing OTM one-month VIX calls

(see www.cboe.com/VPN). The number of futures contracts sold is determined by the same methodology as used for the VPD index. The risk mitigation is accomplished by purchasing OTM VIX calls at a strike price 25 points above the VIX futures price (or nearest available strike price below the calculated level).

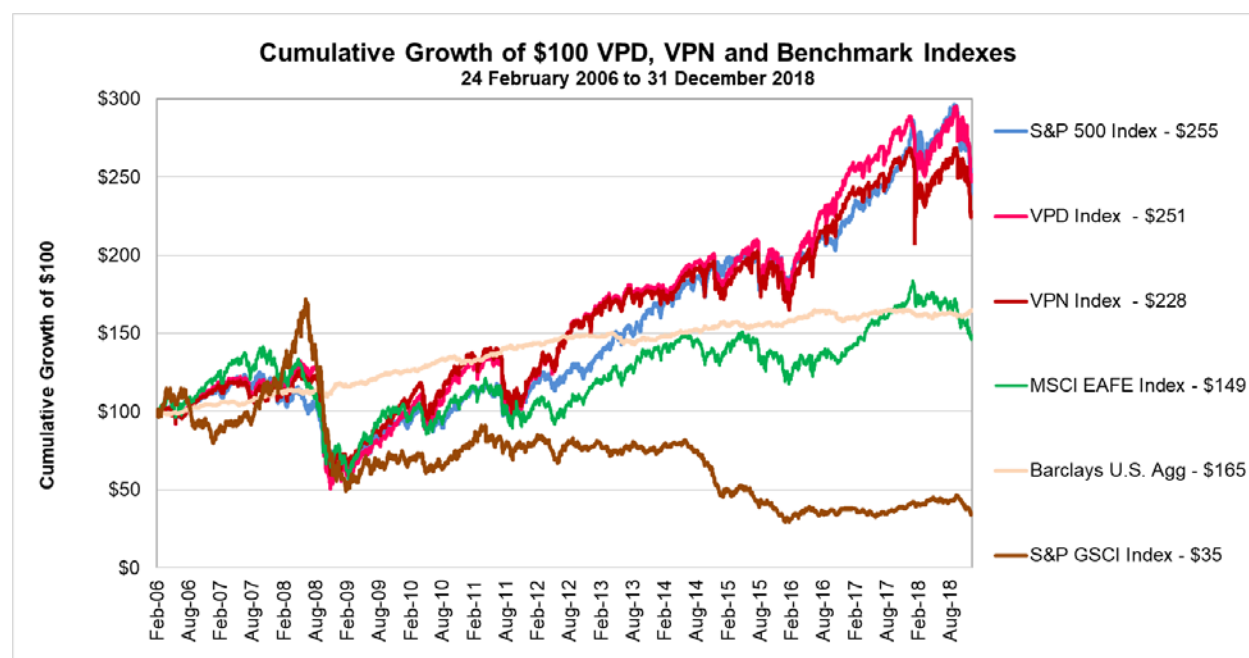
In contrast, two indexes which sell VIX futures (VPD and VPN) outperformed the S&P 500 on a raw return basis, albeit with increased volatility. The VPD index sells hypothetical fully collateralized VIX futures and is therefore exposed to significant tail risk in the case of a major volatility event. To mitigate this tail risk exposure, the VPN index includes a long OTM VIX call option exposure. While the long call results in a drag on performance in periods in which VIX does not increase significantly, it does provide some protection against catastrophic volatility events.

6.1 Performance of the VPD and VPN Indexes from 2006 to 2018

This section considers the 12+ year performance of the VPD and VPN Indexes over the period of study from the inception of VIX options 24 February 2006 to the end of 2018 relative to a variety of benchmark indexes. In addition, the performance is compared to a 60/40 portfolio with a 5% allocation to inverse one-month or three-month (held for one month) VIX futures for the period from 24 February 2006 to the end of 2017.

Exhibit 22 provides an illustration of the 12+ year cumulative growth and summary statistics of the VPD and VPN indexes and four traditional benchmark total return indexes (S&P 500, Barclays U.S. Aggregate bond index, MSCI EAFE equity index, and S&P GSCI commodity index). The VPD and VPN Indexes generated a 7.4% and 6.5% annualized return, at a 22% and 19% annualized standard deviation, and a -62% and -53% maximum drawdown, respectively. While the VPD index generated a higher return than the VPN index, the VPN index experienced lower volatility and a smaller maximum drawdown and slightly higher risk-adjusted returns in terms of the Sharpe ratio (0.34 versus 0.33), although VPD had a higher alpha (1.51% versus 1.31%). This suggests that the protective long call positions may be effective at reducing risk in an efficient manner. Only the S&P 500 index generated higher returns than the VPD and VPN Indexes, albeit only slightly higher. With the exception of the Barclays U.S. Aggregate bond index, the traditional indexes exhibited generally similar standard deviations to those of the VPD and VPN indexes. From a risk-adjusted return perspective, the VPD and VPN indexes had similar Sharpe ratios to the S&P 500, but far lower than the Barclays U.S. Aggregate bond index, although the alphas of the VPD and VPN indexes were only surpassed by that of the Barclays U.S. Aggregate bond index (although only the Barclays Aggregate Index alpha is statistically significant).

Exhibit 22: VPD and VPN Indexes and Benchmark Indexes from 2006 to 2018



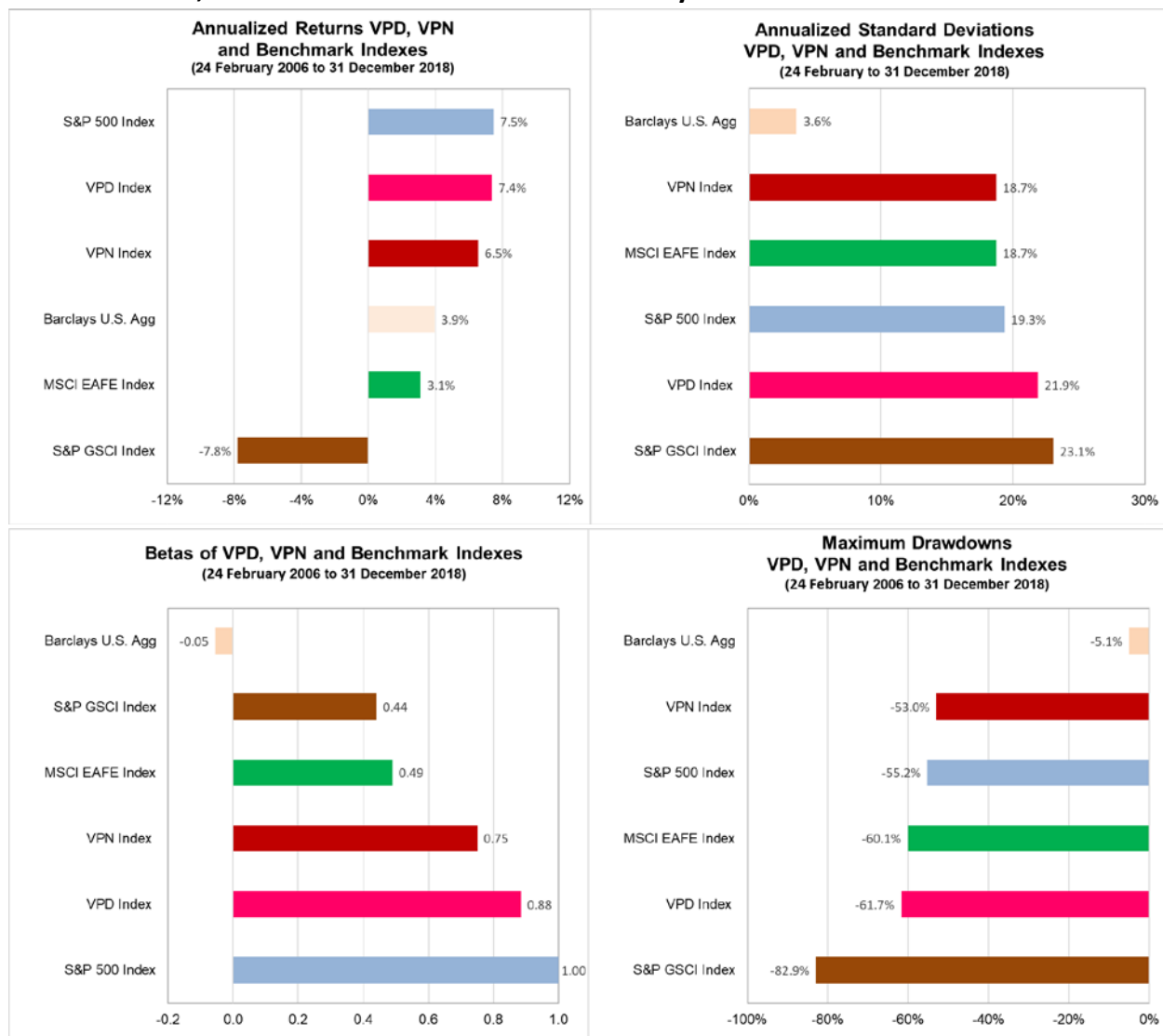
24 Feb 2006 to 31 Dec 2018	VPD Index	VPN Index	S&P 500 Index	Barclays U.S. Agg	S&P GSCI Index	MSCI EAFE Index
Annualized Return	7.4%	6.5%	7.5%	3.9%	-7.8%	3.1%
Annualized Standard Deviation	21.9%	18.7%	19.3%	3.6%	23.1%	18.7%
Sharpe Ratio	0.33	0.34	0.38	1.06	-0.34	0.16
Maximum Drawdown	-61.7%	-53.0%	-55.2%	-5.1%	-82.9%	-60.1%
Skewness	-1.26	-1.29	-0.12	-0.09	-0.18	-0.27
Kurtosis	36.22	26.83	11.38	1.96	3.11	7.72
Correlation w SPX	0.78	0.78	1.00	-0.29	0.37	0.51
Beta	0.88	0.75	1.00	-0.05	0.44	0.49
Alpha	1.51%	1.31%		4.47%	-9.61%	0.39%
Alpha P-Value	0.692	0.692		0.000	0.109	0.932

Source: Cboe Exchange Inc., Bloomberg

Exhibit 22: This exhibit illustrates the performance of the VPD and VPN Indexes as well as a variety of traditional benchmark indexes from the inception of the VPD and VPN Indexes to the end of 2018. The VPD and VPN Indexes exhibited similar volatility to that of the traditional indexes (with the exception of the Barclays U.S. Aggregate Bond Index) and higher returns than all indexes except for the S&P 500 index. Only the Barclays U.S. Aggregate Bond Index exhibited a significantly higher Sharpe ratio and alpha than the VPN Index.

Exhibit 23 provides select summary statistics for the VPD and VPN indexes and traditional benchmark indexes from the inception of the VPD and VPN indexes through 2018. The VPD and VPN indexes generated slightly lower returns than the S&P 500 index at similar levels of risk and similar Sharpe ratios while also generating positive alphas relative to the S&P 500 index.

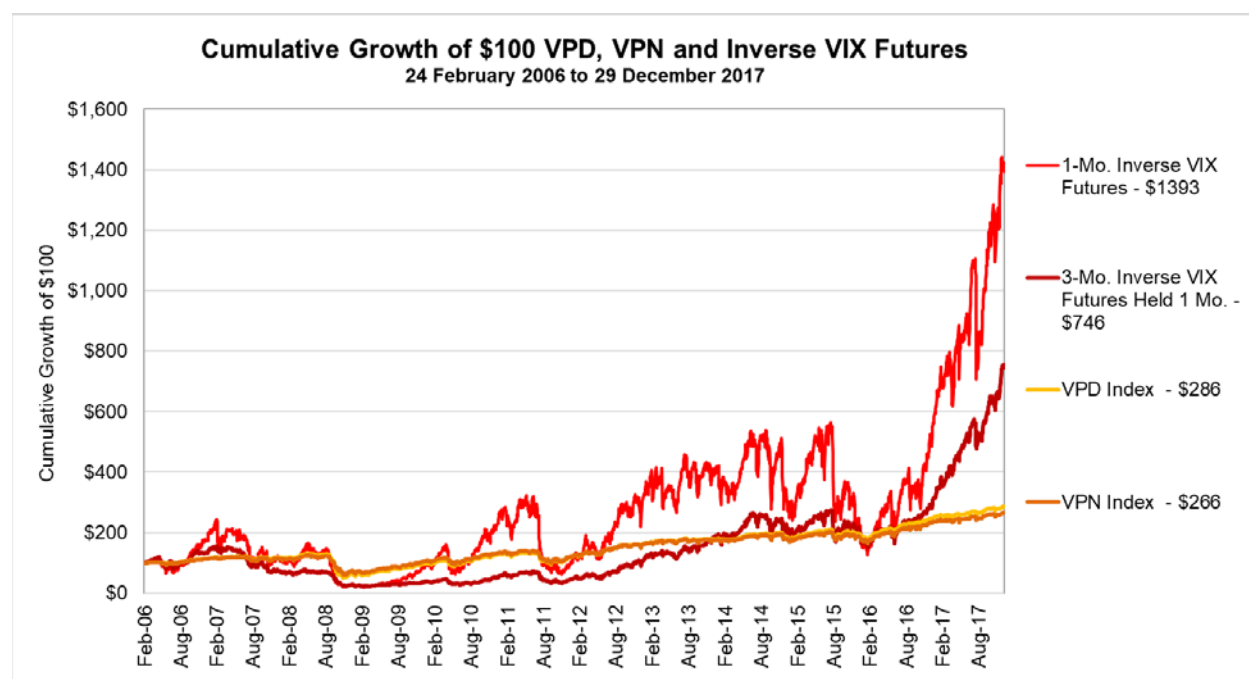
Exhibit 23: VPD, VPN and Benchmark Indexes Summary Statistics from 2006 to 2018



Source: Bloomberg, CFE, Cboe Exchange Inc.

Exhibit 23: This exhibit illustrates the performance of the VPD and VPN indexes as well as a variety of traditional benchmark indexes from the inception of VIX options on 24 February 2006 to the end of 2018. The VPD and VPN indexes exhibited similar volatility to the traditional indexes (with the exception of the Barclays U.S. Aggregate Bond index) and higher returns than all indexes except for the S&P 500 index. The VPN Index (that is collateralized and buys VIX calls) had less severe maximum drawdowns than the stock and commodity indexes.

Exhibit 24: VPD and VPN Indexes and Inverse VIX Futures from 2006 to 2017



24 Feb 2006 to 29 Dec 2017	VPD Index	VPN Index	1 Mo. Inverse VIX Futures	3 Mo. Inverse VIX Futures Held 1 Mo.
Annual Return	9.2%	8.5%	24.6%	18.2%
Annualized Standard Deviation	21.4%	17.8%	78.0%	43.6%
Sharpe Ratio	0.42	0.47	0.31	0.42
Maximum Drawdown	-61.7%	-53.0%	-92.3%	-86.5%
Skewness	-0.84	-0.72	-1.04	-0.64
Kurtosis	29.93	6.99	5.44	4.00
Correlation w SPX	0.80	0.80	0.74	0.74
Beta	0.87	0.73	2.94	1.66
Alpha	2.28%	2.39%	24.02%	9.69%
Alpha P-Value	0.542	0.441	0.118	0.254

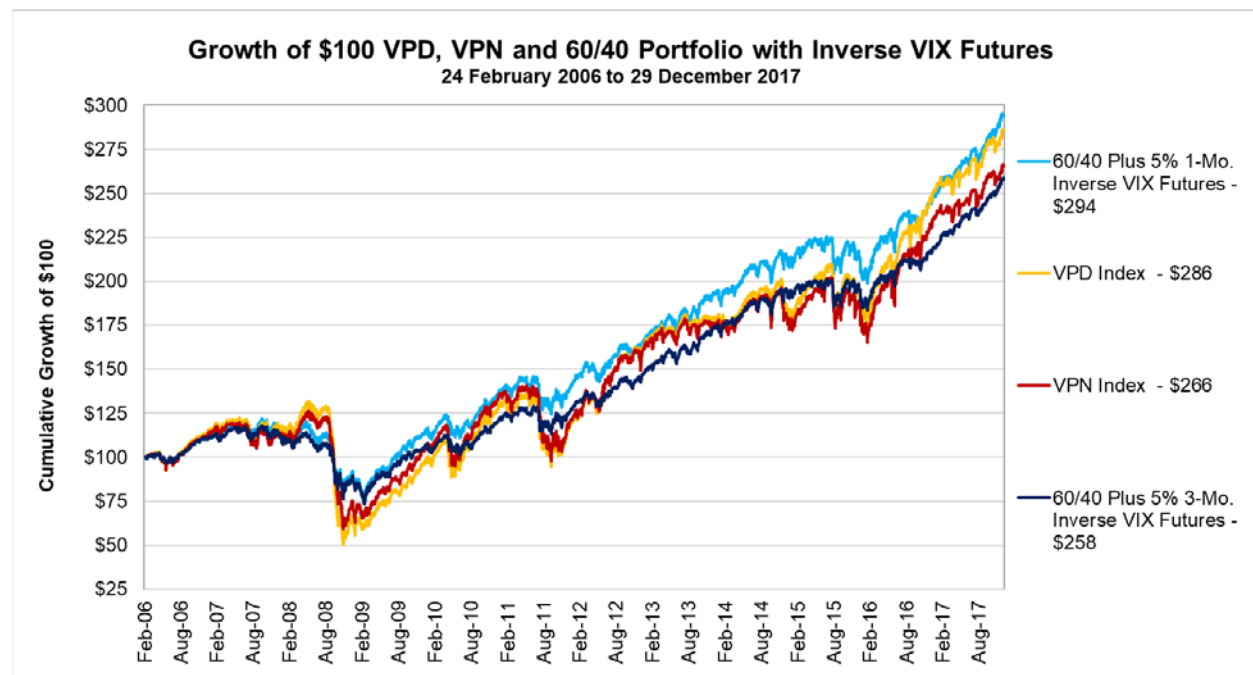
Source: Cboe Exchange Inc., Bloomberg

Exhibit 24: This exhibit illustrates the performance of the VPD and VPN Indexes as well as the one-month and three-month inverse VIX futures strategies. The risk-mitigating impact of the overcollateralization of the VPD and VPN indexes and the long call purchasing of the VPN indexes is clearly evident in the exhibit, as the standard deviations and maximum drawdowns for the VPD and VPN indexes were much lower than for the other two inverse VIX futures strategies.

Exhibit 24 provides details on the performance of the VPD and VPN Indexes relative to inverse one-month and three-month VIX futures strategies from 24 February 2006 to 29 December 2017. The de-levering and resulting reduced risk of the VPD and VPN indexes is clearly evident in the exhibit. The returns of the VPD and VPN Indexes are about half of that of the three-

month inverse VIX futures strategy and less than half of the return of the one-month inverse VIX futures strategy. Similarly, the standard deviations of the VPD and VPN Indexes are less than half of that of the three-month inverse VIX futures strategy and less than half of the return of the one-month inverse VIX futures strategy. Interestingly, the standard deviation of the VPD and VPN Indexes are about one quarter of the standard deviation of the one-month inverse VIX futures strategy (which matches the tenor of the VPD and VPN indexes).

Exhibit 25: VPD and VPN Indexes and 60/40 with Inverse VIX Futures from 2006 to 2017



24 Feb 2006 to 29 Dec 2017	VPD Index	VPN Index	60/40 Plus 5% 1 Mo. Inverse VIX Futures	60/40 Plus 5% 3 Mo. Inverse VIX Futures	60/40 Plus 1% 25% OTM Written VIX Calls	60/40 Plus 1% ATM Written VIX Calls
Annual Return	9.2%	8.5%	9.4%	8.2%	8.1%	10.4%
Annualized Standard Deviation	21.4%	17.8%	13.7%	12.4%	38.6%	17.4%
Sharpe Ratio	0.42	0.47	0.68	0.65	0.21	0.59
Maximum Drawdown	-61.7%	-53.0%	-38.5%	-37.7%	-75.6%	-44.8%
Skewness	-0.84	-0.72	-0.40	-0.27	15.43	-0.26
Kurtosis	29.93	6.99	7.42	9.42	628.03	19.33
Correlation w SPX	0.80	0.80	0.98	0.99	0.58	0.93
Beta	0.87	0.73	0.69	0.63	1.14	0.83
Alpha	2.28%	2.39%	3.04%	2.36%	2.77%	3.12%
Alpha P-Value	0.542	0.441	0.000	0.000	0.763	0.105

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 25: This exhibit illustrates the performance of the VPD and VPN Indexes as well as four 60/40 stock/bond portfolios with allocations to inverse VIX futures and calls. The VPD and VPN Indexes exhibited similar returns to the 60/40 portfolios with inverse VIX futures allocations at much higher volatility and maximum drawdowns.

Exhibit 25 details the performance of the VPD and VPN Indexes relative to a 60/40 portfolio with a 5% allocation to inverse VIX futures and 1% allocation to written VIX calls. The returns and standard deviations of the VPD and VPN Indexes are much nearer those of the 60/40 portfolios with 5% allocations to inverse VIX futures than to those of the inverse VIX futures strategies presented in Exhibit 24.

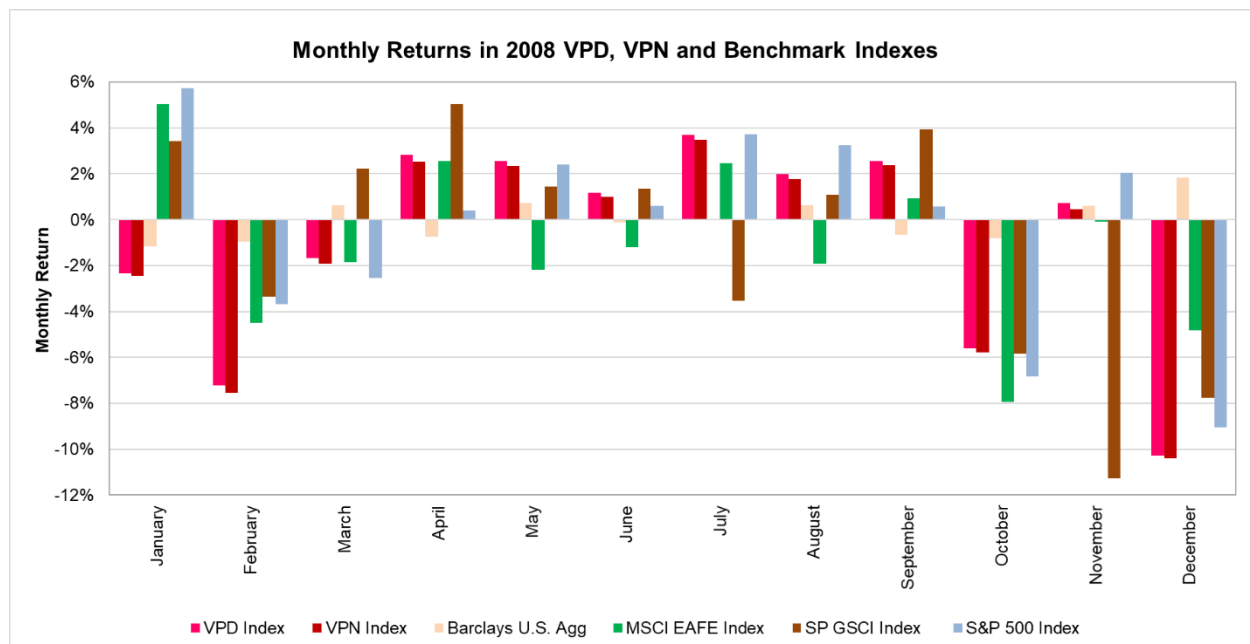
The VPD and VPN indexes generated virtually identical returns to the 60/40 portfolios with one- and three-month inverse allocations at 9.2% and 8.5%, versus 9.4% and 8.2%, respectively. However, they did so at much higher standard deviations of 21% and 18% versus 14% and 12%, respectively. A similar pattern is found in the maximum drawdowns. While the VPD and VPN indexes generated much lower Sharpe ratios than the 60/40-based portfolios, the alphas were generally quite similar. It is worth noting that the 60/40-based portfolios benefit from the diversification of the 40% allocation to bonds. Adding such an allocation to a VPD and VPN index-based portfolio may improve risk-adjusted performance accordingly.

6.2 Performance of the VPD and VPN Indexes in 2018

The VIX index was near record low levels near the end of 2017. In the first half of 2018, the VIX index experienced higher levels and a very sharp spike in the month of February. In the following sections, we consider the performance of the VPD and VPN Indexes in 2018 followed by performance in February 2018.

Exhibit 26 provides monthly returns in 2018 for the VPD and VPN Indexes as well as select benchmark indexes. The VPD and VPN Indexes exhibited significant drawdowns in February and December as the spot VIX index spiked. Despite the large spikes in the spot VIX index in 2018, the volatility of the VPD and VPN Indexes was generally similar to that of the S&P 500 index.

Exhibit 26: VPD and VPN Indexes and Benchmark Indexes Monthly Returns in 2018



2018 Monthly Returns	VPD Index	VPN Index	Barclays U.S. Agg	MSCI EAFE Index	SP GSCI Index	S&P 500 Index
January	-2.33%	-2.47%	-1.15%	5.03%	3.42%	5.72%
February	-7.21%	-7.54%	-0.95%	-4.48%	-3.34%	-3.69%
March	-1.67%	-1.92%	0.64%	-1.84%	2.22%	-2.54%
April	2.83%	2.52%	-0.74%	2.55%	5.04%	0.38%
May	2.57%	2.35%	0.71%	-2.17%	1.45%	2.41%
June	1.19%	0.98%	-0.12%	-1.18%	1.36%	0.62%
July	3.68%	3.48%	0.02%	2.48%	-3.53%	3.72%
August	1.99%	1.77%	0.64%	-1.90%	1.08%	3.26%
September	2.54%	2.37%	-0.64%	0.92%	3.93%	0.57%
October	-5.59%	-5.77%	-0.79%	-7.95%	-5.84%	-6.84%
November	0.74%	0.46%	0.60%	-0.10%	-11.28%	2.04%
December	-10.29%	-10.41%	1.84%	-4.82%	-7.75%	-9.03%

Source: Cboe Exchange Inc., Bloomberg

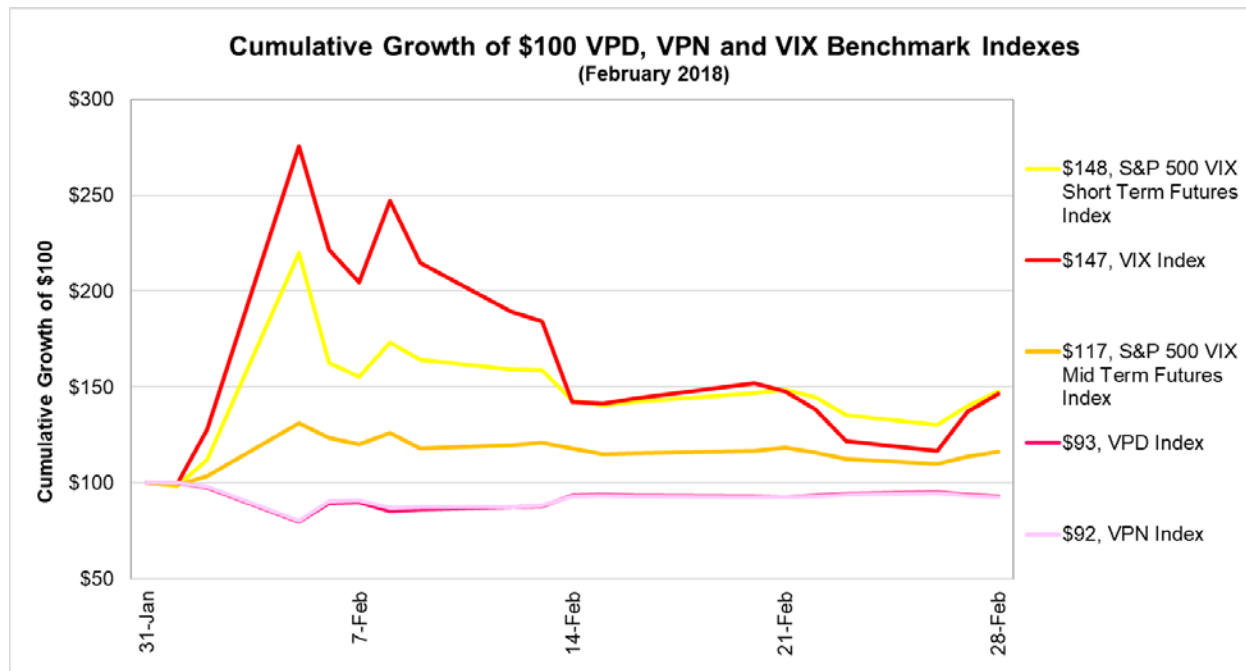
Exhibit 26: This exhibit illustrates the monthly performance of the VPD and VPN Indexes as well as a variety of traditional benchmark indexes in 2018. The VPD and VPN Indexes increased in every month from April through September, but exhibited significant drawdowns in February and December. In 2018, the volatility of the VPD and VPN Indexes was generally similar to that of the S&P 500 index.

6.3 The Volatility Spike of February 2018

Exhibit 27 illustrates the performance of the VPD and VPN Indexes as well the spot VIX index and two VIX futures indexes over the month of February 2018. The February 5, 2018 spike in the VIX index is clearly visible. On that day, the VIX index had its largest ever daily return of over 100%. While the spot VIX index level more than doubled in a single day, the increases in the VIX futures indexes were far less extreme while the VPD and VPN Indexes experienced a meaningful

decreased in value. However, the VPD and VPN Indexes experienced a decrease in value of far smaller magnitude than the increase in front-month VIX futures, largely due to their de-levered and risk-mitigating positions.

Exhibit 27: VPD and VPN Indexes and VIX Futures Index Performance in February 2018

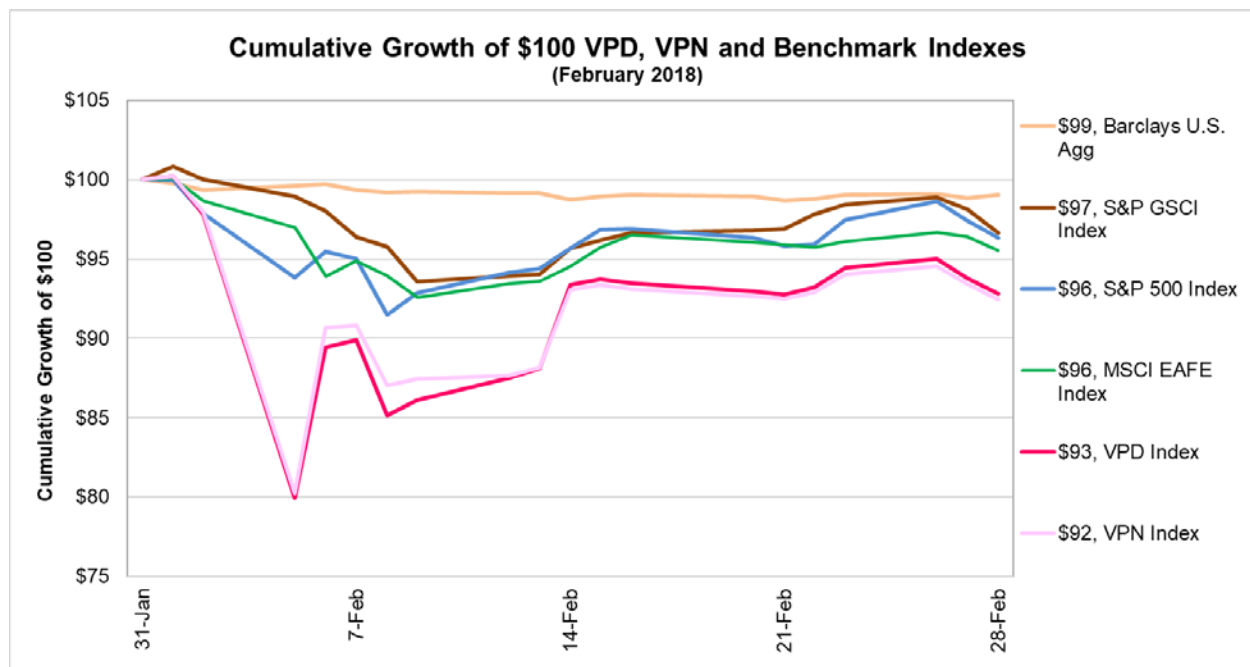


February 2018	VPD Index	VPN Index	S&P 500 VIX Mid Term Futures Index	S&P 500 VIX Short Term Futures Index
Period Return	-7.2%	-7.5%	16.5%	47.6%
Annualized Standard Deviation	88.2%	88.1%	110.1%	379.1%
Sharpe Ratio	-0.08	-0.09	0.15	0.13
Maximum Drawdown	-20.2%	-20.0%	-16.0%	-40.8%
Skewness	-1.44	-1.17	2.94	3.42
Kurtosis	7.06	7.37	10.90	13.85
Correlation w SPX	0.81	0.76	-0.80	-0.75
Beta	2.61	2.47	-3.24	-10.49
Period Alpha	4.55%	3.71%	7.93%	38.40%
Alpha P-Value	0.763	0.821	0.676	0.594

Source: Cboe Exchange Inc., Bloomberg

Exhibit 27: This exhibit illustrates the performance of the VPD and VPN Indexes as well the spot VIX index and two VIX futures indexes over the month of February 2018. The February 5, 2018 spike in the VIX index is clearly visible in which the VIX index had its largest ever daily change of over 100%. While the spot VIX index level more than doubled in a single day, the increases in the VIX futures indexes were far less extreme. Due to their de-levered and risk-mitigating positions, the VPD and VPN Indexes experienced a decrease in value of far smaller magnitude than the increase in front-month VIX futures.

Exhibit 28: VPD, VPN and Benchmark Performance in February 2018



February 2018	VPD Index	VPN Index	S&P 500 VIX Mid Term Futures Index	S&P 500 VIX Short Term Futures Index	S&P 500 Index	Barclays U.S. Agg	S&P GSCI Index	MSCI EAFE Index
Period Return	-7.2%	-7.5%	16.5%	47.6%	-3.7%	-0.9%	-3.3%	-4.5%
Annualized Standard Deviation	88.2%	88.1%	110.1%	379.1%	27.3%	3.5%	16.4%	18.4%
Sharpe Ratio	-0.08	-0.09	0.15	0.13	-0.14	-0.31	-0.21	-0.25
Maximum Drawdown	-20.2%	-20.0%	-16.0%	-40.8%	-8.5%	-1.3%	-7.2%	-7.5%
Skewness	-1.44	-1.17	2.94	3.42	-1.02	-0.24	-0.33	-0.88
Kurtosis	7.06	7.37	10.90	13.85	0.56	-1.15	-0.43	0.73
Correlation w SPX	0.81	0.76	-0.80	-0.75	1.00	0.08	0.35	0.32
Beta	2.61	2.47	-3.24	-10.49	1.00	0.01	0.21	0.22
Period Alpha	4.55%	3.71%	7.93%	38.40%		-0.91%	-2.56%	-3.69%
Alpha P-Value	0.763	0.821	0.676	0.594		0.370	0.565	0.466

Source: Cboe Exchange Inc., Bloomberg

Exhibit 28: This exhibit illustrates the performance of the VPD and VPN Indexes as well four traditional benchmark indexes over the month of February 2018. It is clear that the February 5, 2018 spike in the VIX index did not benefit the VPD and VPN Indexes in this case as the VPD and VPN Indexes exactly tracks the S&P 500 index through the first half of the month. This is illustrative of one of the limits of market expectation-based dynamic allocation strategies in which unanticipated events such as the February 5, 2018 spike in the VIX index may not be captured since they are not expected.

Exhibit 28 illustrates the performance of the VPD and VPN Indexes as well four traditional benchmark indexes over the month of February 2018. It is clear that the 5 February 2018 spike in the VIX index resulted in a meaningful drawdown (-20% maximum drawdown from the start of February to February 5th) in the VPD and VPN indexes, although the drawdown was of far

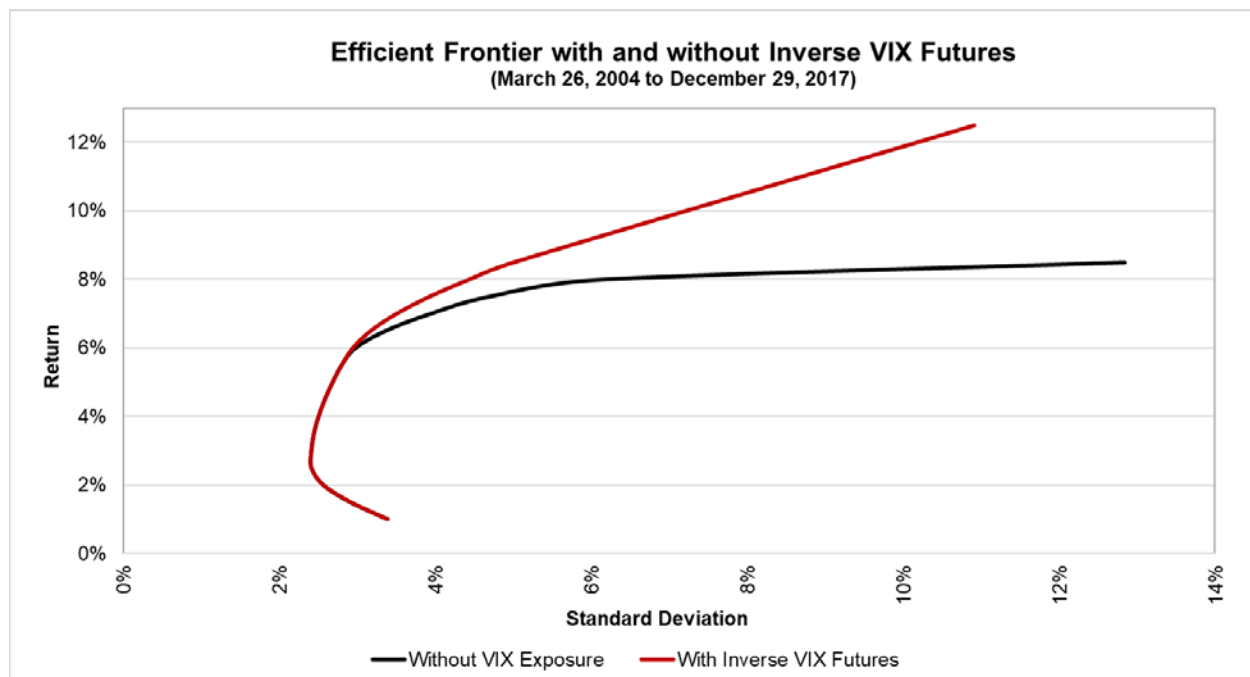
smaller magnitude than the increase in the VIX index. While a 20% drawdown over the course of 5 calendar days is certainly economically meaningful, at 1/5 the size of the one-day 5th of February 2018 increase in the spot VIX index the impact of the reduced leverage of the VPD and VPN Indexes is clearly evident. The size of the one-day loss of the VPD and VPN indexes (-18%) may exclude them from consideration as standalone investments for most investors and suggest that they may be more appropriate as return enhancers or diversifiers in well diversified portfolios.

7.0 Return Enhancement Benefits – Expansion of Efficient Frontier

Much of the discussion in the previous sections focused on the return enhancement benefits of inverse VIX futures and written VIX calls. This section approaches the analysis from a slightly different perspective by considering impacts on efficient frontiers over the entire 10+ year period since the inception of VIX options. Exhibits 28 through 31 provide efficient frontiers with and without the inclusion of inverse VIX futures, written VIX call options and bull and bear VIX futures spreads. The efficient frontiers are constructed by considering long-only exposure to the following ten total return indexes representing a wide range of asset classes: SP500 (Equity), Barclays U.S. Aggregate (Bonds), Barclays High Yield U.S. Corporate (High Yield Bonds), HFR Global Hedge Fund Index (Hedge Funds), SG CTA Index, (formerly Newedge CTA Index) (Managed Futures), S&P GSCI (Commodity), S&P Listed Private Equity Index (Private Equity), Dow Jones Equity REIT Index (Real Estate), MSCI EAFE Index USD (EAFE), and MSCI EEM Index USD (Emerging Markets). Each of the following four exhibits present two efficient frontiers; one calculated by considering inclusion of the ten indexes listed above and the second calculated by considering inclusion of the ten indexes plus one or two of the indexes representing the VIX-based trading strategies discussed in the previous sections of the paper.

Exhibit 29 illustrates efficient frontiers with and without the consideration of inverse one-month and long three-month VIX futures, Exhibit 30 provides efficient frontiers with and without written one-month VIX ATM and 25% OTM call options and Exhibit 31 provides efficient frontiers with and without bear VIX one-month/three-month futures spreads. In all three cases, the addition of inverse VIX futures or written call option exposure expands the efficient frontier at the upper return/standard deviation ranges, illustrating possible return enhancement abilities of inverse VIX exposures. At return levels below 6%, the standard deviations are reduced at any given return level, particularly at lower return levels.

Exhibit 29: Efficient Frontier with and without Inverse VIX Futures

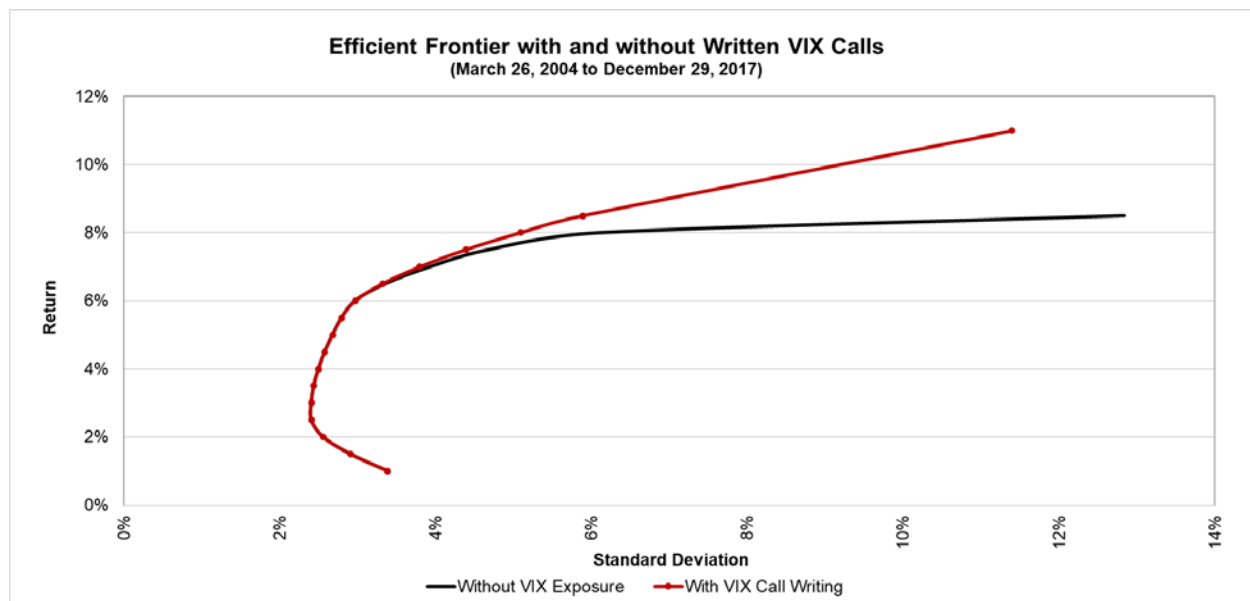


Source: Bloomberg, CFE, Cboe Exchange Inc.

Exhibit 29: This exhibit illustrates the diversification benefits of including long VIX futures in a sample universe of diversified investments in the construction of an efficient frontier. The efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. All indexes which comprise the sample theoretical endowment portfolio are included in the optimization and short positions are not allowed. The addition of inverse VIX futures expands the upper end of the efficient frontier. For portfolio returns below 6%, the optimal allocation to inverse VIX futures was zero. For higher return levels, small allocations to inverse VIX futures expand the efficient frontier. At the 12.5% portfolio return level, the inverse VIX futures allocation was 11.7%. At the 10% standard deviation level, the addition of inverse VIX futures increases returns by approximately 4%.

Exhibit 32 illustrates efficient frontiers with and without the consideration of inverse one-month and inverse three-month VIX futures, Exhibit 49 provides efficient frontiers with and without written one-month VIX ATM and 25% OTM call options and Exhibit 50 provides efficient frontiers with and without bear VIX one-month/three-month futures spreads. In all three cases, the addition of VIX futures or call option exposure expands the efficient frontier at the upper return/standard deviation ranges, illustrating possible return enhancement abilities of inverse VIX exposures.

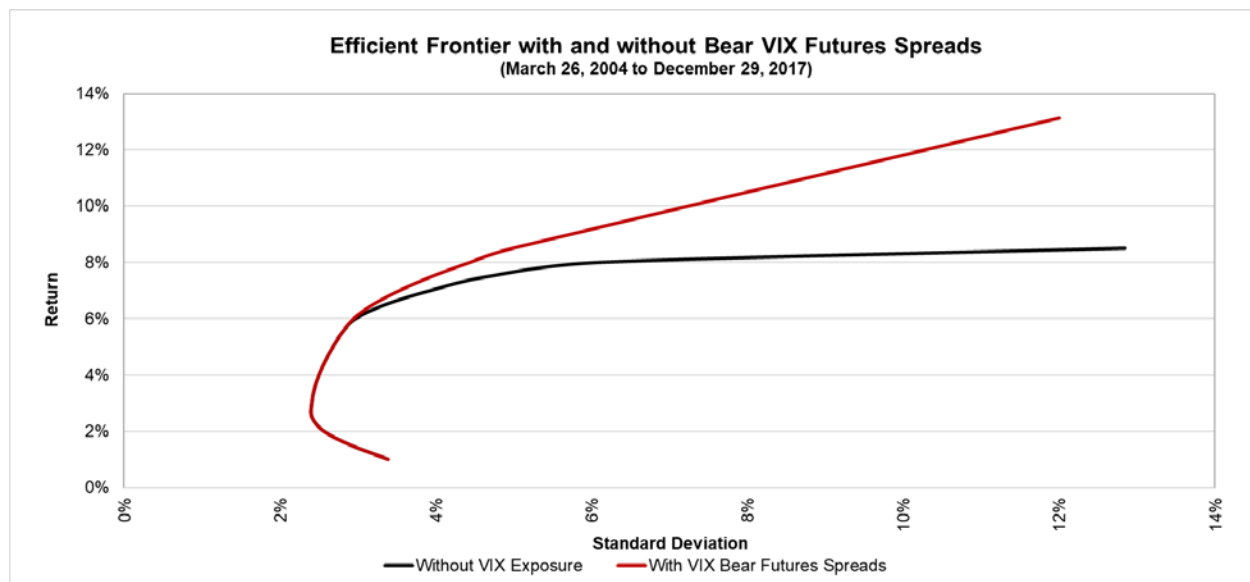
Exhibit 30: Efficient Frontier with and without Written VIX Calls



Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 30: This exhibit illustrates the diversification benefits of including written VIX calls in a sample universe of diversified investments in the construction of an efficient frontier. The efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. All indexes which comprise the sample theoretical endowment portfolio are included in the optimization and short positions are not allowed. The addition of written VIX calls expands the upper end of the efficient frontier. For portfolio returns below 6.5%, the optimal allocation to written VIX calls was zero. For higher return levels, small allocations to written VIX calls expand the efficient frontier. At the 11% portfolio return level, the written VIX call allocation was 1.1%. At the 10% standard deviation level, the addition of written VIX calls increases returns by over 2%.

Exhibit 31: Efficient Frontier with and without Bear VIX Futures Spreads

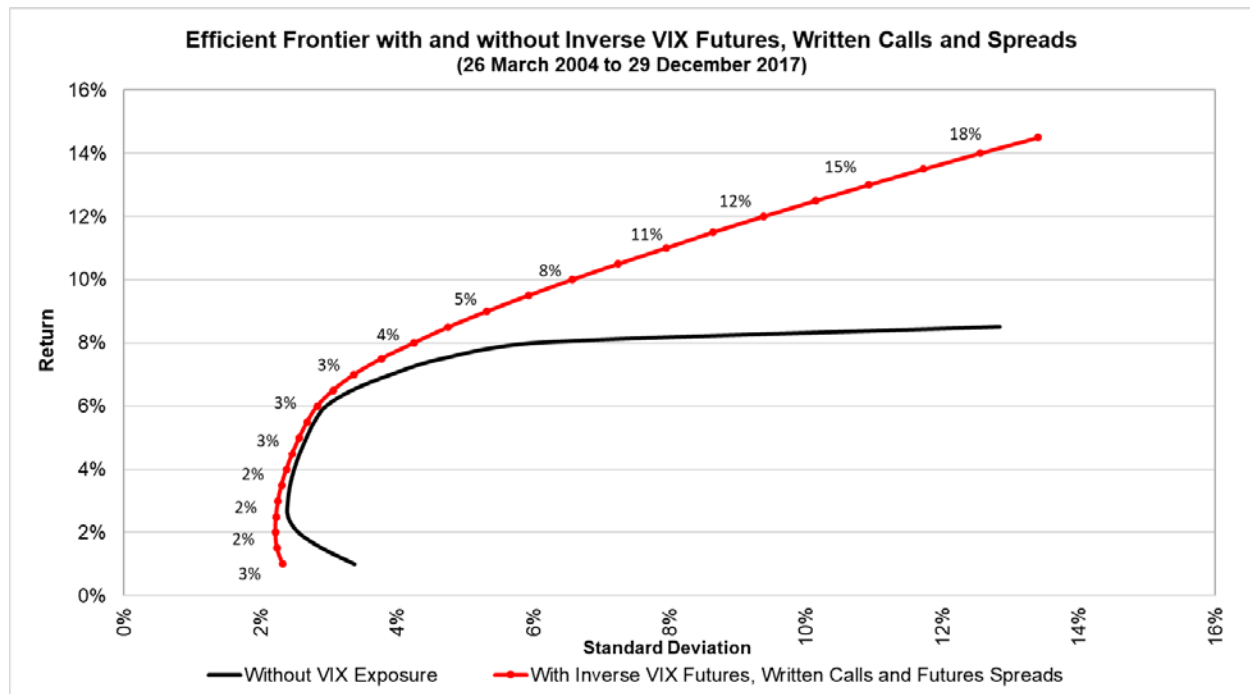


Source: Bloomberg, CFE, Cboe Exchange Inc.

Exhibit 31: This exhibit illustrates the diversification benefits of including bear VIX futures spreads in a sample universe of diversified investments in the construction of an efficient frontier. The efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. All indexes which comprise the sample theoretical endowment portfolio are included in the optimization and short positions are not allowed. The addition of bear VIX futures spreads expands the upper end of the efficient frontier. For portfolio returns below 6%, the optimal allocation to bear VIX futures spreads was zero. For higher return levels, small allocations to bear VIX futures spreads expand the efficient frontier. At the 13% portfolio return level, the bear VIX futures spread allocation was 4.9%. At the 10% standard deviation level, the addition of bear VIX futures spreads increases returns by approximately 4%.

Exhibit 32 illustrates efficient frontiers with and without the consideration of inverse one-month VIX futures, inverse three-month VIX futures, written one-month VIX ATM calls, written one-month VIX 25% OTM calls, bull VIX one-month/three-month futures spreads and bear VIX one-month/three-month futures spreads. In addition, labels are provided on the efficient frontier that includes inverse VIX exposure to indicate the total percentage allocation to inverse VIX plus VIX spread exposure at each point. It is clear that the addition of inverse VIX futures, written VIX call option and VIX futures spread exposure expands the efficient frontier across the full range of returns and standard deviations, illustrating possible return enhancement abilities of inverse VIX exposures. It is worth noting that the addition of bull and bear VIX futures spreads to the efficient frontier calculations effectively allows the inclusion of long VIX futures exposures through the spreads thus potentially expanding the efficient frontier beyond the frontiers that only allow inverse VIX exposure.

Exhibit 32: Efficient Frontier with and without Inverse VIX Futures, Written VIX Calls, and Bull and Bear VIX Futures Spreads



Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 32: This exhibit illustrates the diversification benefits and return enhancement benefits of including inverse VIX futures, written VIX calls, and bull and bear VIX futures spreads in a sample universe of diversified investments in the construction of an efficient frontier. The efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. All indexes which comprise the sample theoretical endowment portfolio are included in the optimization and short positions are not allowed. The addition of inverse VIX futures and written VIX calls and bull and bear VIX futures spreads expands the entire efficient frontier, particularly at the pper end and lower end of the curve. The labels adjacent to the red efficient frontier indicate the cumulative allocation to the four VIX trading strategies at each point. For example, at the 12.5% portfolio return level, the inverse VIX futures allocation was 12.4%. At the 12% standard deviation level, the addition of inverse VIX futures, written VIX calls, and bull and bear VIX futures spreads increases returns by over 5%.

It is worth noting that an ex post efficient frontier is, by its very nature, more of an academic exercise than a realistic representation of the performance that could have been expected of any particular portfolio manager. Efficient frontiers are created with perfect hindsight, utilizing optimization technics, and thus represent the best possible return/standard deviation combinations that were possible over the time period based on the investments included in the optimization procedure.

8.0 Conclusions

The goal of this study is not to make a particular recommendation for a VIX-based investment strategy but rather to detail some important characteristics of VIX futures and options investments and investigate the potential effectiveness of inverse VIX futures or written VIX calls as return enhancers over short periods of rising equity markets. While the results indicate that such strategies may be effective over particular time periods, the potentially substantial volatility jump risk exposure which can result in meaningful and potentially catastrophic losses.

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