CPSC-406 Report

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Abstract

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1 Introduction

2 Week by Week

2.1 Week 1

2.1.1 DFAs

Exercise 1

1. Which of the following words are accepted/refused by $\mathcal{A}_1$ and $\mathcal{A}_2$? Complete the table.

:w	**accepted by** $\mathcal{A}_1$**?**	**accepted by** $\mathcal{A}_2$**?**
aaa	$\times$	$\checkmark$
aab	$\checkmark$	$\times$
aba	$\times$	$\times$
abb	$\times$	$\times$
baa	$\times$	$\checkmark$
bab	$\times$	$\times$
bba	$\times$	$\times$
bbb	$\times$	$\times$

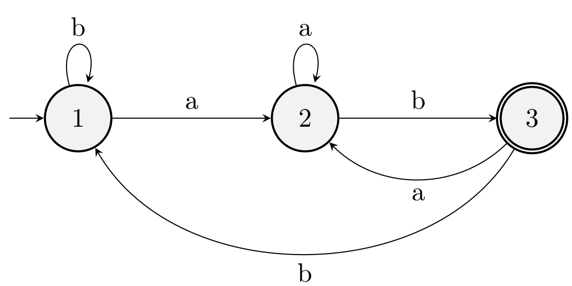
2. More generally, can you completely describe the languages $L(\mathcal{A}_k)$ **accepted by** $\mathcal{A}_k$**, for** $k = 1, 2$**?**

- $L(\mathcal{A}_1)$: The language of all words over $\{a, b\}$ that start with the letter a and end with an odd number of consecutive b's.
- $L(\mathcal{A}_2)$: The language of all words over $\{a, b\}$ that end with the substring aa.

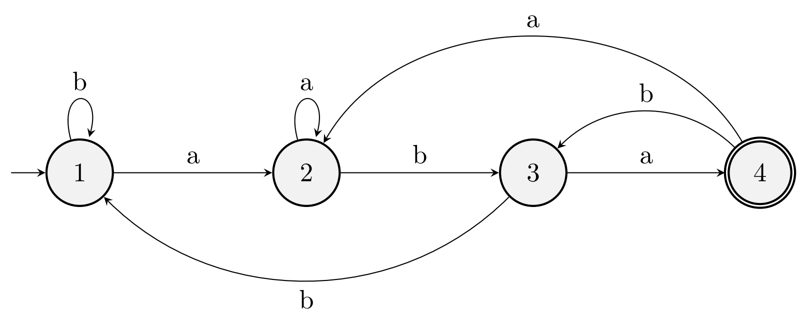
Exercise 2

Design DFAs whose accepted languages are given as follows:

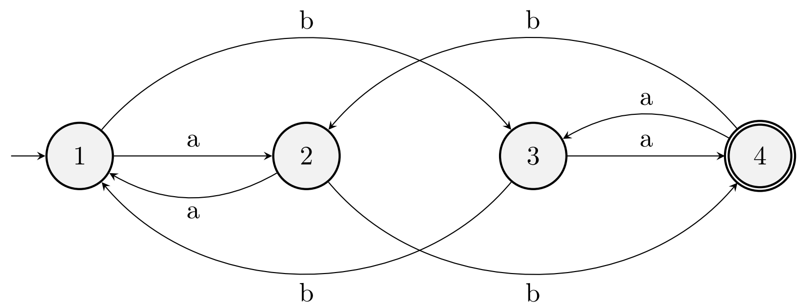
1. All the words that end with ab



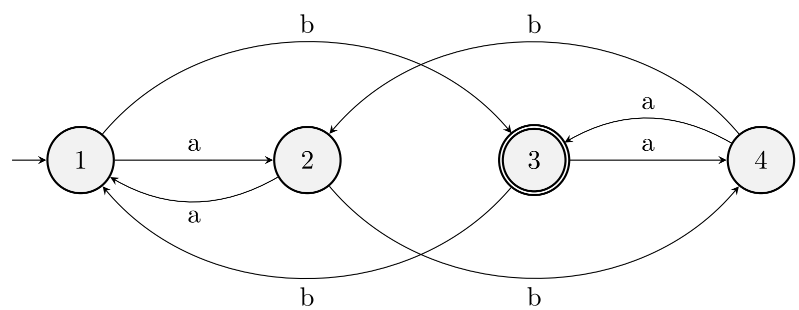
2. All the words that contain aba



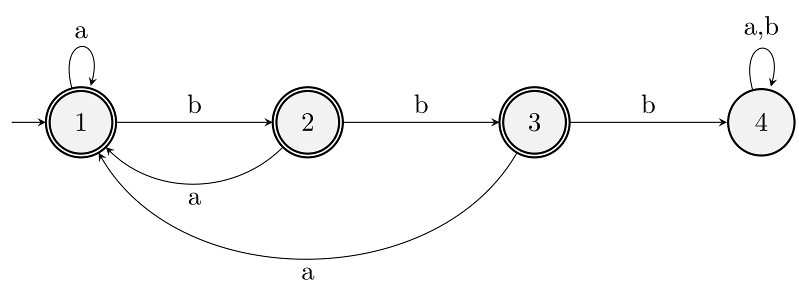
3. All the words that contain an odd number of a's and an odd number of b's



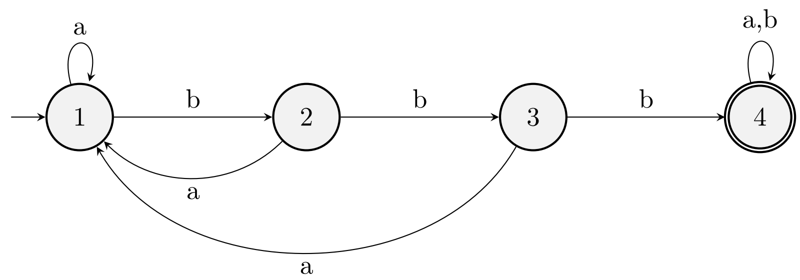
4. All the words that contain an even number of a's and an odd number of b's



5. All the words such that any three consecutive characters contain at least one a



6. All the words that contain bbb



What do you notice when comparing the various automata?

For DFAs that have criteria of "ends with," arrows must leave the accept state because the string could keep going and fail to satisfy DFA characteristics. For DFAs that have criteria of , "contains," the accept state must be a "Trap State" with a self-loop, because once you find the sequence, you can traverse through the DFA while satisfying DFA's accepted language conditions.

2.1.2 Question

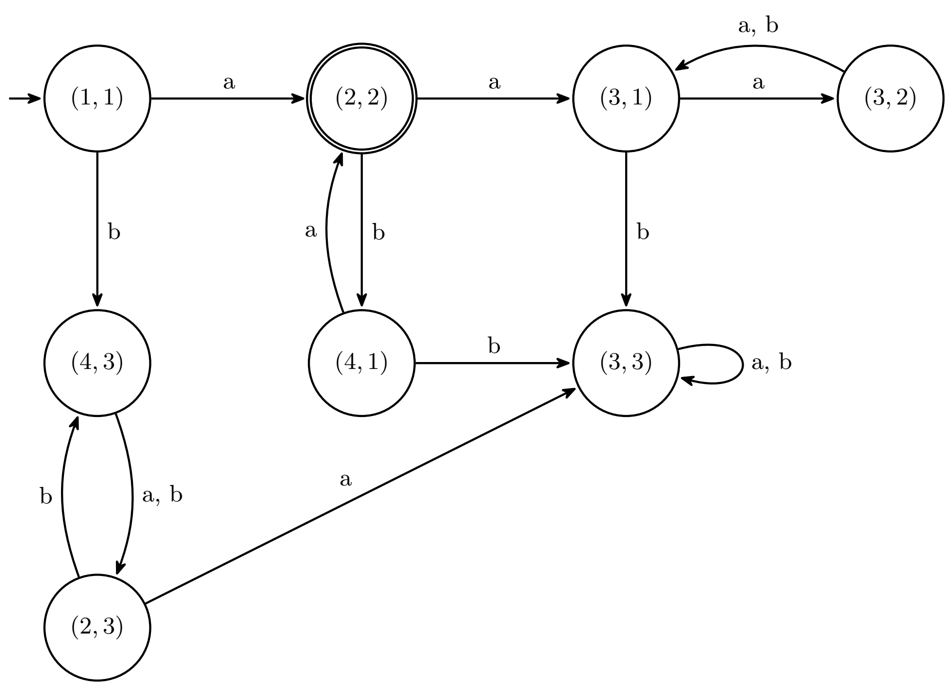
What is the easiest and best way to implement DFAs in programming?

2.2 Week 2

2.2.1 Operations on automata

Exercise 1

1. $L(\mathcal{A}^{(1)})$ can't have aa or bb (two of the same letters in a row). For $L(\mathcal{A}^{(1)})$, any a has to be in an odd numbered state position.

2. 

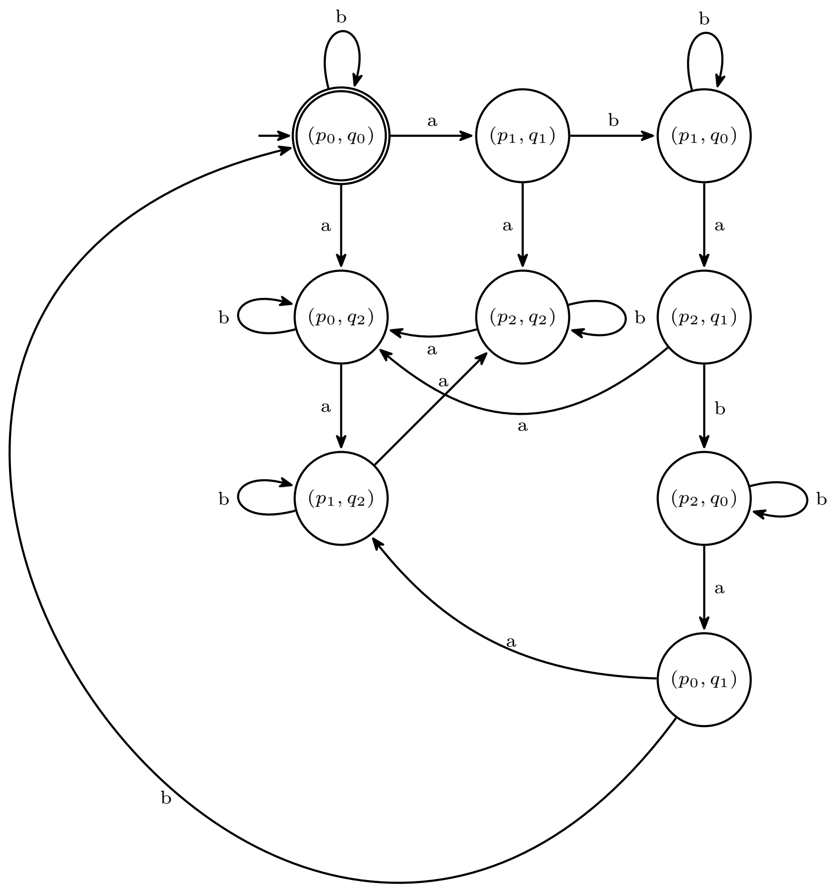
3. To find the intersection of two languages ($L_1 \cap L_2$), you create a new machine that simulates how $\mathcal{A}^{(1)}$ and $\mathcal{A}^{(2)}$ work at the same time. When you take in an input a or b, both numbers in the pair update

at the same time based on their original rules. For intersection ($\cap$), you only have a double circle if both machines are in an accepting state.

4. For union ($\cup$), you have a double circle if at least one of the machines is in an accepting state.

Exercise 2

1. For $L(\mathcal{B}^{(1)})$, the total number of a is a multiple of 3, including zero. For $L(\mathcal{B}^{(2)})$, every a is directly followed by at least one b, and the string must not end with a.



2.

3. Since the only accepting state is (p_0, q_0), a string is accepted if and only if it leads both $L(\mathcal{B}^{(1)})$ and $L(\mathcal{B}^{(2)})$ to an accepting state.

4. A state will be accepting if the first part is p_0 or if the second part is not q_0.

Exercise 2.2.7 Basis:
Let the length of the string be $n = 0$. The empty string, ϵ, is of length 0.

$$\hat{\delta}(q, \epsilon) = q$$

$\therefore$ The basis is true.

Inductive Hypothesis:
Assume that for any string w of length n, $\hat{\delta}(q, w) = q$.

Inductive Step:
Consider a string x of length $n + 1$. The string can be written as $x = wa$, where w is a string of length n and a is a single input symbol ($a \in \Sigma$).

$$\begin{aligned}
\hat{\delta}(q, wa) &= \delta(\hat{\delta}(q, w), a) && \text{– by definition of } \hat{\delta} \\
&= \delta(q, a) && \text{– by Inductive Hypothesis: } \hat{\delta}(q, w) = q \\
&= q && \text{– by given property: } \delta(q, a) = q \text{ for all } a
\end{aligned}$$

2.2.2 Question

What other mathematical properties aside from intersection and union could be applied to automata?

3 Synthesis

4 Evidence of Participation

5 Conclusion

References

[BLA] Author, Title, Publisher, Year.