UT-BATTELLE

Analytical and Numerical Modeling of the Detection Statistics from a Fissile Sample

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CHALMERS

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Overview

Number Distributions

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  - Induced fission.
  - Change of multiplicities.
  - Multiplicities and coincidences can give isotopic composition and mass of the sample.
  - A full number distribution offers deeper insight than a few factorial moments.

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- These events will lead to a certain number distribution for a sample that will vary with mass and composition.

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Factorial moments and probability distributions

The multiplicities (factorial moments) of a function f are obtained as derivatives of its PGF $g(z)$ evaluated at $z = 1$:

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- Nested functions, lower order derivatives recurring.
- Longer expressions, which on the other hand can be expressed recursively

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$$p_1(0) = (1-p)z + pq_f[h(z)]\Big|_{z=0} = p\sum_{n=0}^{N} p_f(n)[p_1(0)]^n. \quad (7)$$

8-th degree polynomial to be solved for $p_1(0)$, note the p-dependence for $p_1(0)$.

Including absorption

The event of absorption can be included into the fission distribution:

$$\widetilde{p}_f(n) = \frac{p' - p}{p'}\delta_{n,0} + \frac{p}{p'}p_f(n). \tag{8}$$

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The first master equation will then read as:

$$h(z) = (1 - p')z + p'\widetilde{q}_f[h(z)]. \tag{9}$$

where $\widetilde{q}_f(z)$ is the generating function of the $\widetilde{p}_f(n)$ of Eq. (8).

Including detection

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- The derivatives needed for finding factorial moments as well as the statistics change in a simple way:

$$\frac{d^n h_d(z)}{dz^n} = \frac{d^n h(z)}{dz^n} \cdot (\epsilon)^n \quad , \quad \frac{d^n H_d(z)}{dz^n} = \frac{d^n H(z)}{dz^n} \cdot (\epsilon)^n \tag{12}$$

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- For the factorial moments the full change is

$$\widetilde{\nu}_{d,n} = (\epsilon)^n \cdot \widetilde{\nu}_n. \tag{13}$$

Gamma Distribution

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$$g(z) = (1 - p) + p r_f(z) q_f[g(z)] \quad (14)$$

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With PGFs for gammas produced in spontaneous and induced fission

$$g(z) = \sum_n f_1(n) z^n \quad , \quad G(z) = \sum_n F(n) z^n. \tag{16}$$

Absorption and detection

Photon absorption will be accounted for by the probability l_γ that describes the leakage probability for one single photon:

$$l(z) = l_\gamma z + (1 - l_\gamma). \tag{17}$$

Note that for photons the absorption of neutrons also play a fundamental role, while the opposite is not true.

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The next step in the simulation of the statistics obtained from measurements is to incorporate the process of detection:

$$\varepsilon_\gamma(z) = \epsilon_\gamma z + (1 - \epsilon_\gamma). \tag{18}$$

Using this equation we can obtain the detection statistics as:

$$g_d(z) = g\big[l\{\varepsilon(z)\}\big] \quad , \quad G_d(z) = G\big[l\{\varepsilon(z)\}\big]. \tag{19}$$

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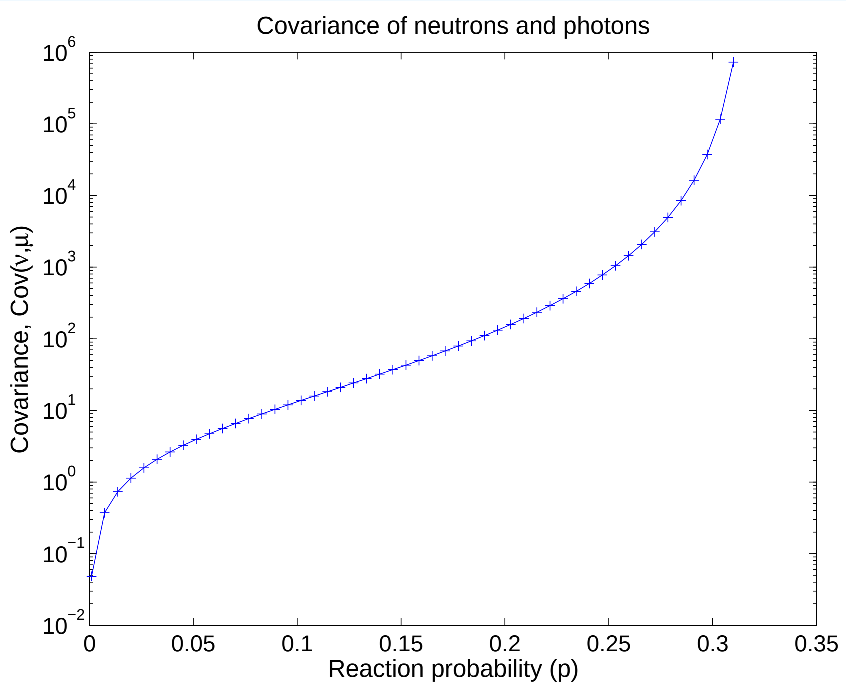
- Multiplication of neutrons on the other hand is the reason for additional photon multiplication as well:

$$\begin{aligned}\mathbf{Cov}\left\{\widetilde{\boldsymbol{\nu}}, \widetilde{\boldsymbol{\mu}}\right\} = \tfrac{p}{1-p}\left(\nu_{s,1}\nu_{r,1} + \nu_{s,2} - \nu_{s,1}^2\right)\mu_{r,1}\,\mathbf{M}^2 \\ + \left(\tfrac{p}{1-p}\right)^2 \left(\nu_{s,1} + \mu_{r,1}\nu_{r,2}\right)\,\mathbf{M}^3.\end{aligned} \tag{20}$$



Results

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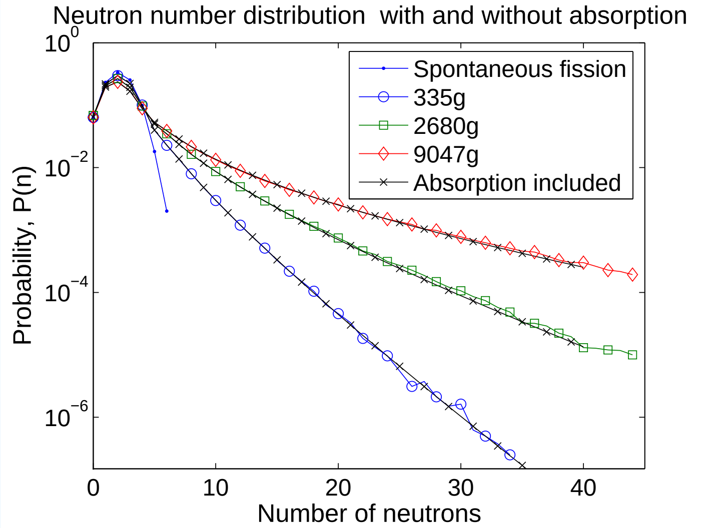
As a final step the parameters are replaced with values to get numerical expressions.

- Saves time and makes multiple evaluations easy.
- Comparisons with Monte Carlo simulations done with the MCNP-PoliMi code, from which we have taken the numerical value of p and the leakage and detection probabilities for neutrons and photons.

Effect of absorption

- Dependence on mass shown in the parameter p, and also an increased probability of absorption. 20 wt% ^{240}Pu and 80 wt% ^{239}Pu.



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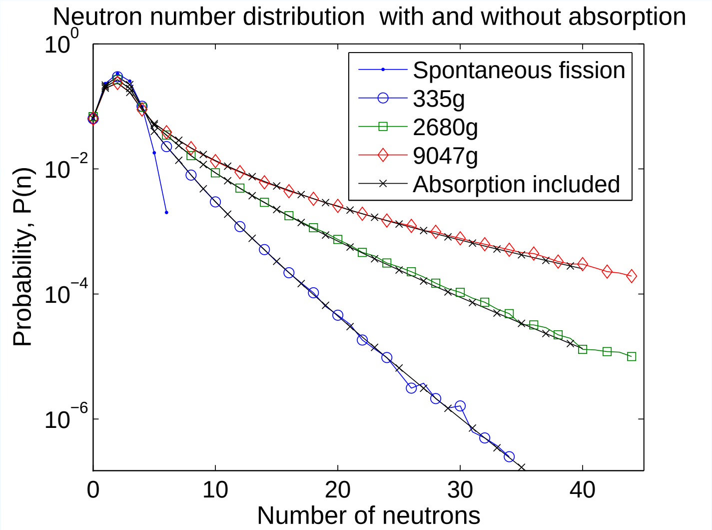
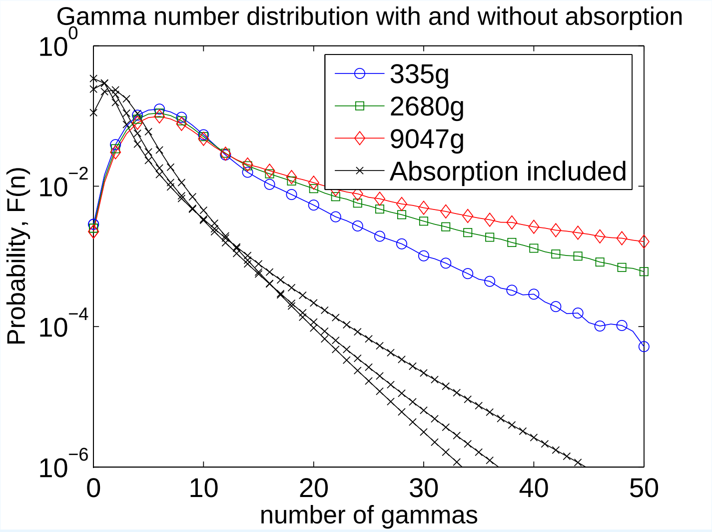
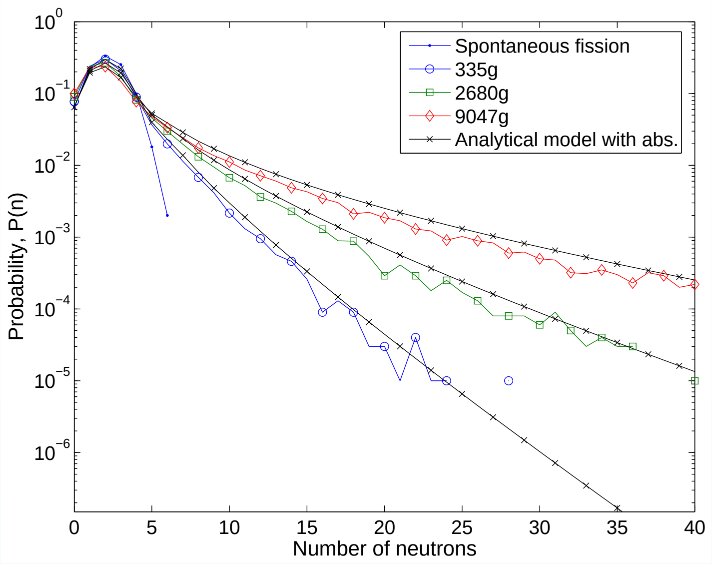
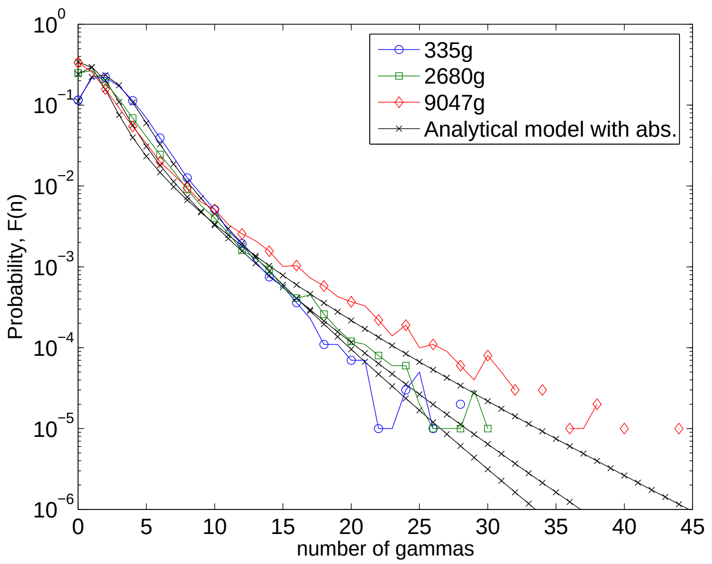




- Change compared to non-multiplying case.
- Small effect of absorption on neutrons for such a heavy element.
- Large self-shielding for photons, which are still generated with high multiplicity, but few leak out.

Comparison with Monte Carlo





- Good agreement with MCNP-PoliMi.
- For photons one have higher multiplicities per source event for *smaller* samples.

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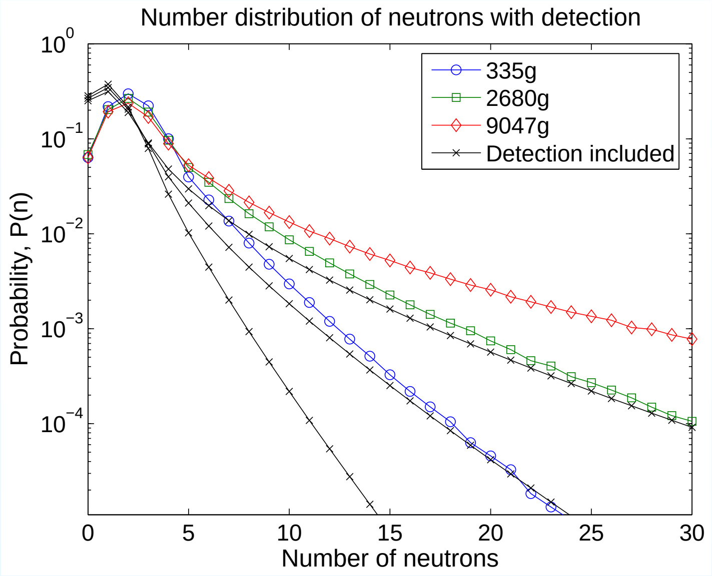

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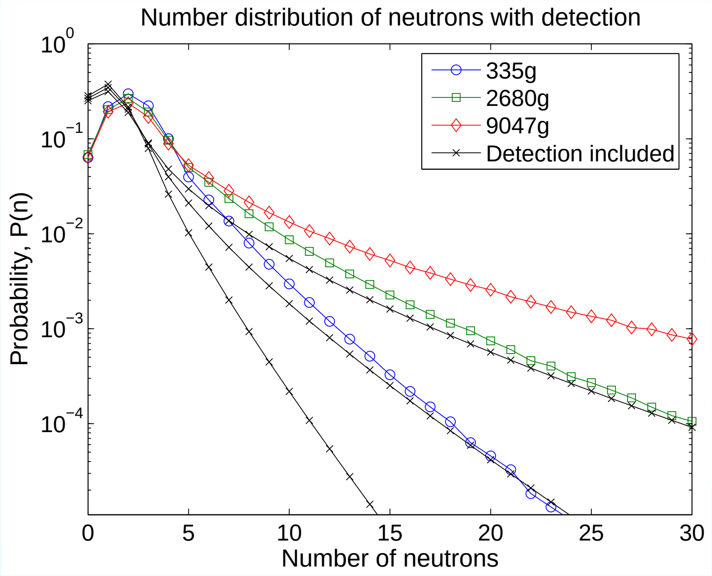

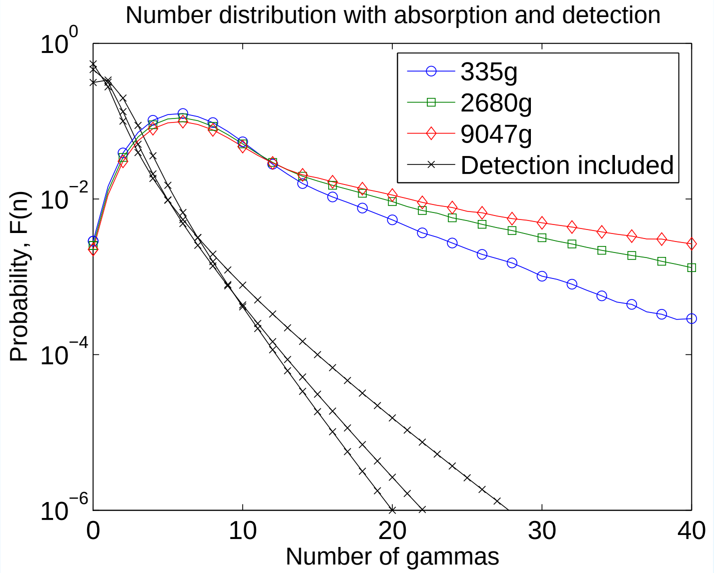

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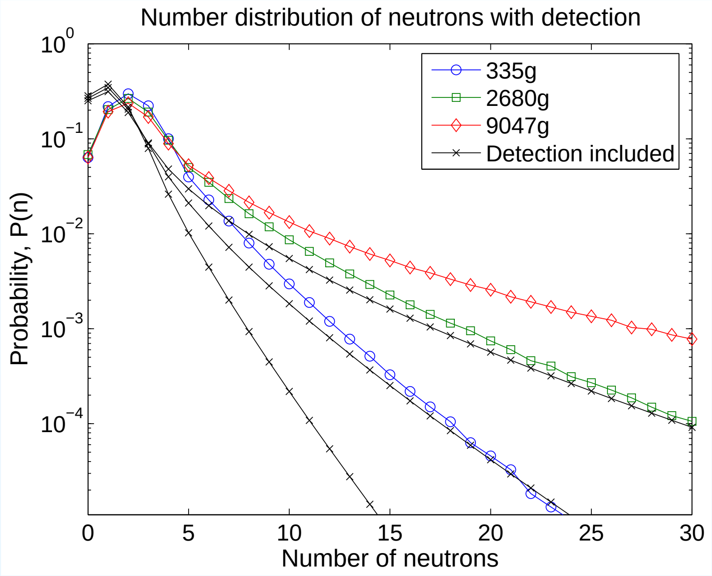

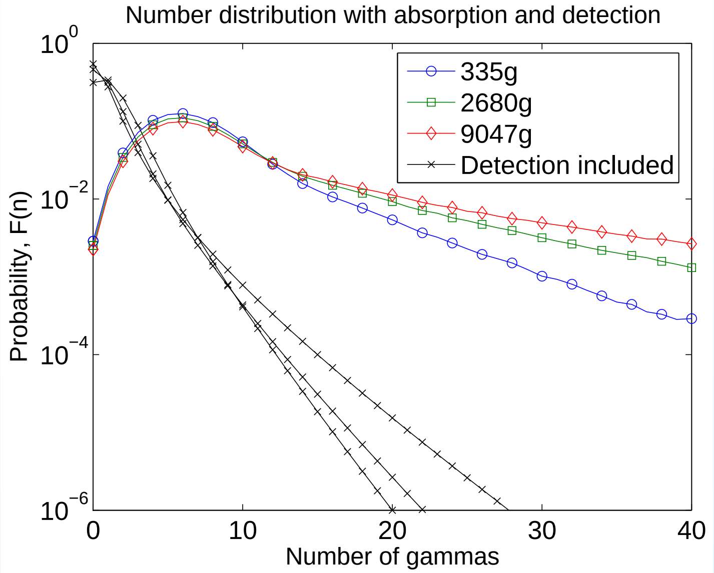

- Detection efficiencies of 50% were used here.
- The chance to detect many particles from the same source event decreases for both neutrons and photons.
- The photon multiplicities decrease so much that neutrons might be more favourable to observe even though they have lower source multiplicities.

Simulated scintillation detector results

- Organic scintillators have the advantage to be able to detect both neutrons and photons, and to discriminate between them.
- MCNP-PoliMi was also used to simulate a scintillator detector setup with six detectors:

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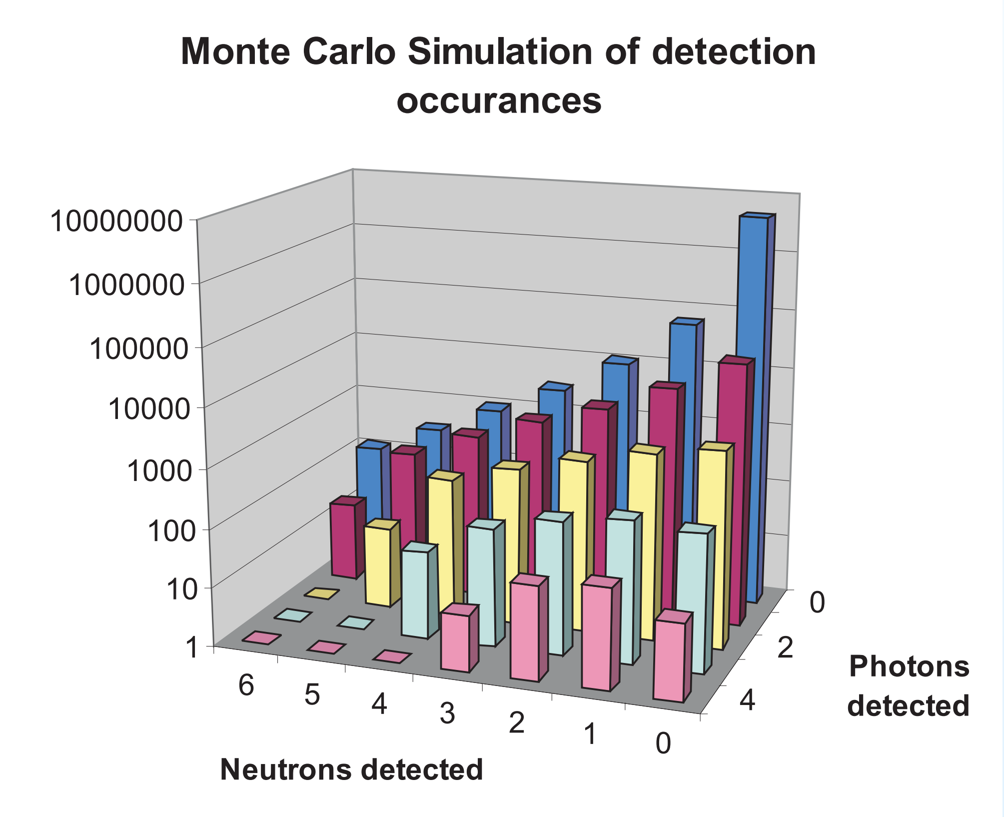

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- Extensions
  - Compare to experiments
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