Statistics of the neutrons and gamma photons emitted from a fissile sample with absorption

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Introduction

This work investigates an analytical derivation of the distribution of the number of neutrons and photons emitted by a multiplying sample. The relationship between the statistics of the generated and detected neutrons and photons is also described. The analytical model described in this paper accounts for absorption and detection, thus extending the model presented in previous studies. By using this new, improved model, one can investigate the relative feasibilities of measuring neutrons or gamma photons for the analysis of a specific fissile sample. In fact, larger mass will lead to larger self-shielding for gamma photons, whereas for neutrons a larger mass will lead to increased multiplicities due to an increased probability to induce fission for each neutron, with absorption playing a minor role. The results suggest that although photons have a larger initial (source) multiplication, neutrons might be more favourable to measure in the case of large samples because of the increasing self-shielding effect for gamma photons.

Theoretical Treatment of Neutrons

In non-destructive assay of nuclear material the statistics of the number distribution of neutrons and gamma rays emitted by fissile samples play an important role. For fissile samples the key processes regarding the distributions are:

- Spontaneous fission.
- Induced fission.
- Change of multiplicities due to:
  - Induced fission.
  - Absorption.
  - The process of detection
- Multiplicities and coincidences can give isotopic composition and mass of the sample.

With the help of master equations one can write down relationships describing the generating functions of the number distribution. For a model including absorption but not detection the equations read as:

$$h(z) = (1-p')z + p'\tilde{q}_f[h(z)], \tag{1}$$

$$H(z) = q_s[h(z)]. \tag{2}$$

Here the number distribution is taken for one initial neutron or one initial neutron event:

$$p_1(n) = \frac{1}{n!}\frac{d^n h(z)}{dz^n}\bigg|_{z=0} \quad \text{and} \quad P(n) = \frac{1}{n!}\frac{d^n H(z)}{dz^n}\bigg|_{z=0}. \tag{3}$$

Compared to factorial moments which are also calculated from probability generating functions, but at $z=1$, we can note a few differences:

- Calculated at $z=0$, more terms than for multiplicities.
- Nested functions, lower order derivatives recurring.
- Longer expressions, which on the other hand can be expressed recursively

The process of detection can be accounted for by the use of the generating function $\varepsilon(z)$ of the binary probability distribution of the number of neutrons detected per leaked neutron:

$$\varepsilon(z) = \epsilon z + (1-\epsilon). \tag{4}$$

This model accounts for detection as one stochastic variable, and is most easily thought of as a general detection efficiency for a detector surrounding the sample, such as a multiplicity counter. With some alterations one could use this model for finding statistics for other detector setups as well. Calculation of high order terms in the distribution requires high order derivations of nested implicit functions.

- Symbolic derivations.
- Using the symbolic language Mathematica.
- Reevaluations fast due to symbolic expressions.

Theoretical Treatment of Photons

The generation of gamma photons is a more intricate process since it is connected to the multiplication of neutrons and does not have any self-multiplication. When incorporating absorption there are a number of effects for the photons when looking at the main parameter of the system which is the sample mass:

- Increased mass means increased probability to induce fission.
- Absorption of neutrons lowers the number of induced fissions.
- Absorption of photons means that fewer photons escape the sample and the number of visible photons decrease.

The master equations for gamma photons generated within the sample read as follows:

$$g(z) = (1-p) + p\, r_f(z)\, q_f[g(z)], \tag{5}$$

$$G(z) = r_s(z)\, q_s[g(z)]. \tag{6}$$

For describing the process of photon detection a few changes needs to be made:

- Need to account for absorption.
- Need to account for detection.
- The form of the equations need to be kept to find the distributions easily.

All this is accomplished by the use of two extra equations:

$$l(z) = l_\gamma z + (1-l_\gamma), \tag{7}$$

$$\varepsilon_\gamma(z) = \epsilon_\gamma z + (1-\epsilon_\gamma). \tag{8}$$

Here $l(z)$ comes from a master equation describing whether a photon is absorbed or not with a special leakage probability l_γ which depends on the sample size, while $\varepsilon_\gamma(z)$ describes the detection process with the use of a detection efficiency ϵ_γ for detecting a photon. Using these in the previous equations gives us the coupled master equations used for finding the distribution of the detections statistics:

$$g_d(z) = g\big[l\{\varepsilon(z)\}\big] \quad , \quad G_d(z) = G\big[l\{\varepsilon(z)\}\big]. \tag{9}$$

The way the absorption and detection is put into the equations makes the change in factorial moments very easy, we get the leakage and detection probability raised to the same power as the order of the moment enters the expression, and lower the numerical values of the factorial moments.

Results

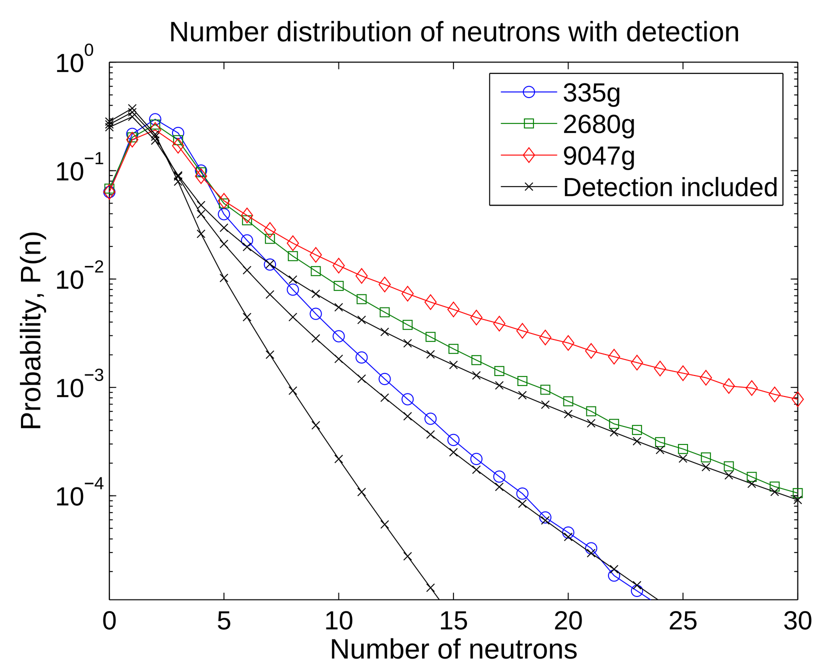

Figure 1: The statistics for neutrons when a detection efficiency of 50% is incorporated into the model. The plot shows that the statistics change and the likelihood of low detection numbers are higher compared to how many bursts there are of that multiplicity from the sample. This case is most representative for a multiplicity counter that has a high total detection efficiency. Samples contained 80 wt% Pu-239 and 20 wt% Pu-240.

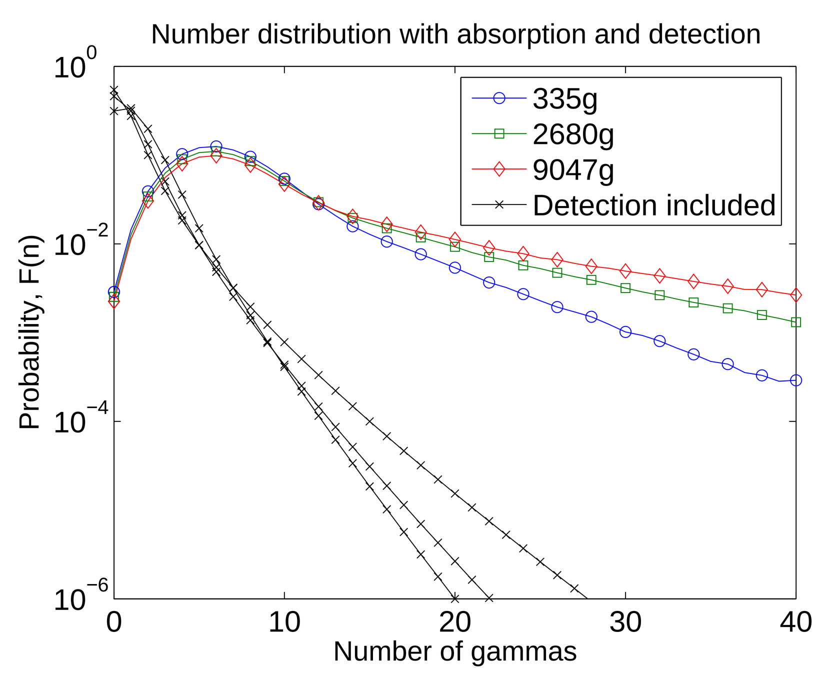

Figure 2: The statistics for photons when a detection efficiency of 50% is incorporated into the model (analytical), compared to the MCNP-PoliMi results for generated photons. Similarly to the inclusion of absorption the inclusion of the detection process lowers the probabilities of observing high multiplicities.

The analytical expressions derived for the number distribution were evaluated using the nuclear data taken from the Monte Carlo code MCNP-PoliMi. For neutrons, inclusion of the absorption does not affect the probabilities in a significant way compared to the non-absorbing case. This is to be expected for the samples investigated in this work since they are comprised of high-Z material. The three samples investigated were of different masses with a mix of 80 wt% Pu-239 and 20 wt% Pu-240. A much larger effect is seen when incorporating a detection efficiency. If it is low, then probabilities to see several neutrons decrease drastically even when considering high multiplicities in big samples.

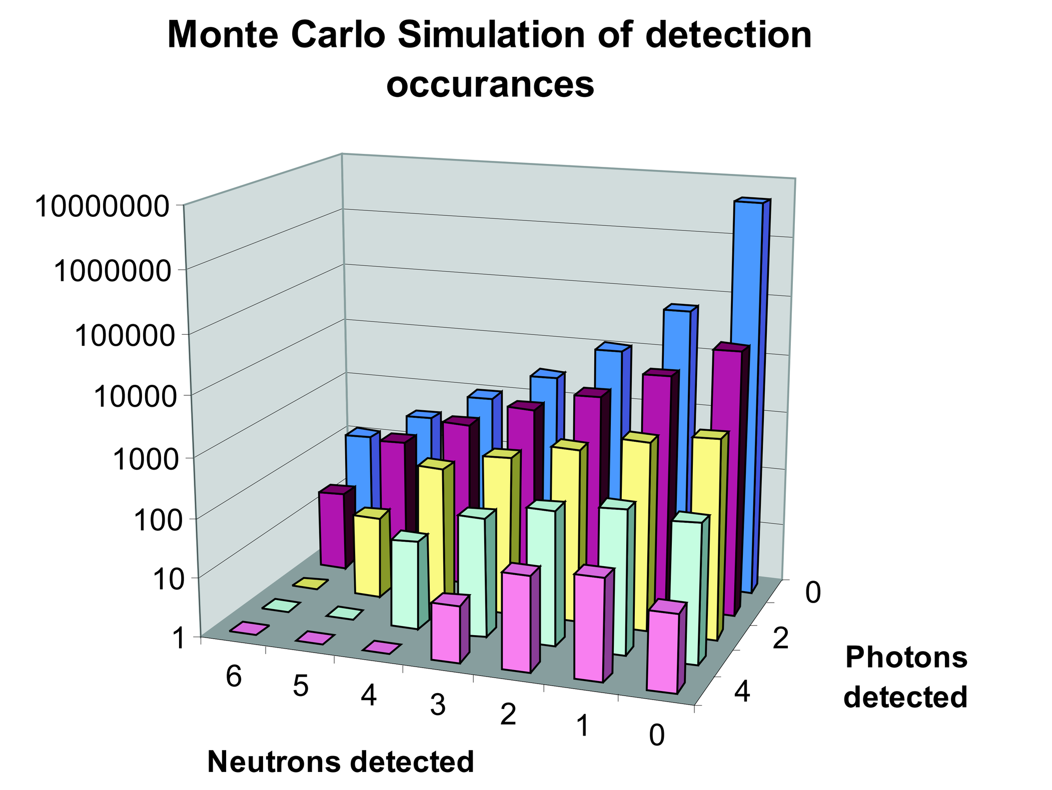

Figure 3: The detection statistics calculated for a setup of six scintillator detectors sensitive to both photons and neutrons, with detection efficiencies of 0.00546 and 0.00549 for neutrons and photons respectively. MCNP-PoliMi was used to generate these results for a 9.00 kg sample of 80 wt% Pu-239 and 20 wt% Pu-240.

For photons the effect of absorption is very big and one can see a massive effect of the self-shielding in all samples. Looking at what happens in different sample masses the following can be observed:

- Larger mass means higher numbers of neutrons and photons are generated.
- Heavier mass means more shielding for the photons.
- The combined effect is dominated by the shielding.
- Smaller samples have larger numbers of photons escaping compared to larger samples per initial source event.

The effect of detection is straight-forward and seen as an attenuation of the probabilities in much the same way as the absorption, with the change that the absorption is different for different sample masses, while the detection has the same effect for all masses, since the detection efficiency is the same.

Conclusions

We have used the symbolic computation code Mathematica to calculate high order terms of the number distribution of neutrons and photons from fissile samples with the inclusion of absorption. The results show that when absorption is accounted for, the number of photons emerging from the sample will decrease significantly, whereas the neutrons are not affected to the same extent. The multiplicities of photons leaving the sample could decrease so much that neutron multiplicities become higher. With the introduction of the detection process into the model of the leaked neutrons and photons, it is possible to simulate the detector response and find the probabilities for different multiplicities of both neutrons and photons. Using this information, one could assess different sample masses with regards to what type of emission has the higher multiplicity when using detectors for non-destructive assay of the material.