导数的基本公式

1. $(C)' = 0$

2. $(x)' = 1$

3. $(x^{\alpha})' = \alpha x^{\alpha-1}$

4. $(a^x)' = a^x \ln a$

5. $(e^x)' = e^x$

6. $(\log_a x)' = \dfrac{1}{x\ln a}$

7. $(\ln x)' = \dfrac{1}{x}$

8. $(\sin x)' = \cos x$

9. $(\cos x)' = -\sin x$

10. $(\tan x)' = \dfrac{1}{\cos^2 x}$

11. $(\cot x)' = -\dfrac{1}{\sin^2 x}$

12. $(arc\sin x)' = \dfrac{1}{\sqrt{1-x^2}}$

13. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$

14. $(\arctan x)' = \dfrac{1}{1+x^2}$

15. $(\mathrm{arc}\cot x)' = -\dfrac{1}{1+x^2}$

证明:$\left(x^{\alpha}\right)' = \alpha x^{\alpha-1}$

分析.$\left(x^{\alpha}\right)' = \lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^{\alpha} - x^{\alpha}}{\Delta x}$

$$= x^{\alpha} \lim_{\Delta x \to 0} \frac{(1+\frac{\Delta x}{x})^{\alpha} - 1}{\Delta x} \quad \left(\square \sim \frac{(1+\square)^{\alpha} - 1}{\alpha}\right)$$

$$= x^{\alpha} \lim_{\Delta x \to 0} \alpha \cdot \frac{\Delta x}{x} \cdot \frac{1}{\Delta x}$$

$$= \alpha x^{\alpha-1}$$

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导数的四则运算

定理. 如果$f(x), g(x)$均可导，则

$$(1)\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x);$$

$$(2)\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x);$$

$$(3)\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}\left(g(x) \neq 0\right).$$

证明: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

分析. $(f(x)g(x))' = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x+\Delta x) + f(x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$= f'(x)\lim_{\Delta x \to 0} g(x+\Delta x) + f(x)g'(x)$$

$$= f'(x)g(x) + f(x)g'(x)$$