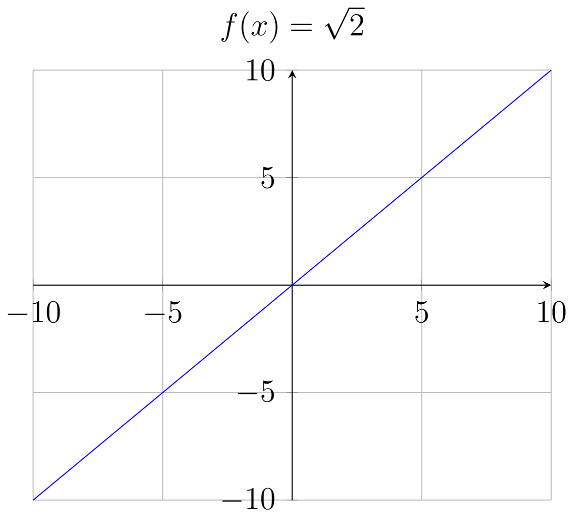
Reference Book

Zane Helton

Contents

1 12 Basic Functions

1.1 Linear Function

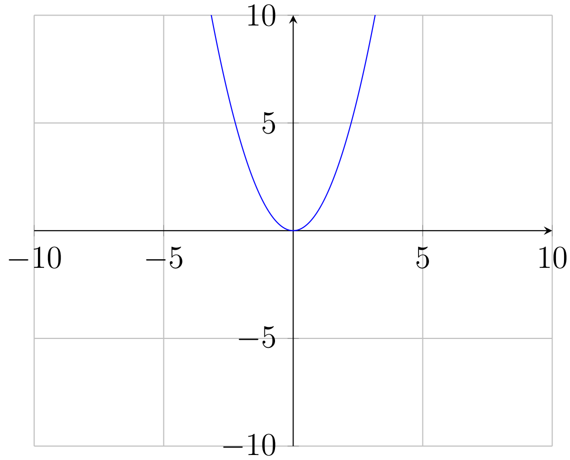


Properties

Domain Range	$(-\infty, \infty)$ $(-\infty, \infty)$
Continuous	Yes
Symmetry	Odd
Increasing	$(-\infty, \infty)$
Bounded	No
Extrema	None
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$

1.2 Quadratic Function

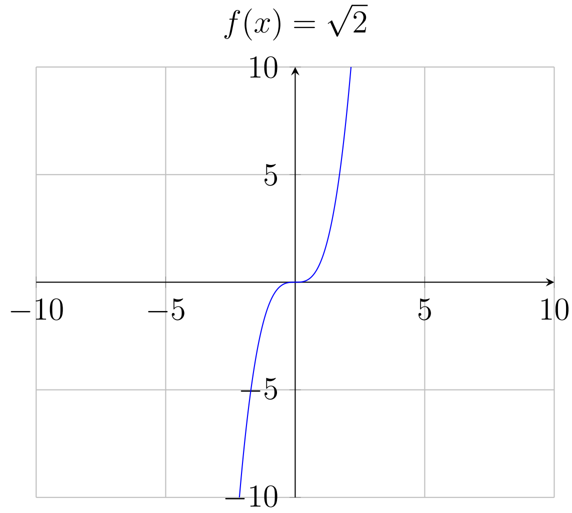
$f(x) = \sqrt{2}$



Properties

Domain Range	$(-\infty, \infty)$ $(0, \infty)$
Continuous	Yes
Symmetry	Even
Increasing Decreasing	$[0, \infty)$ $(-\infty, 0]$
Bounded	Below
Extrema	Minimum: $(0, 0)$
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$

1.3 Cubic Function

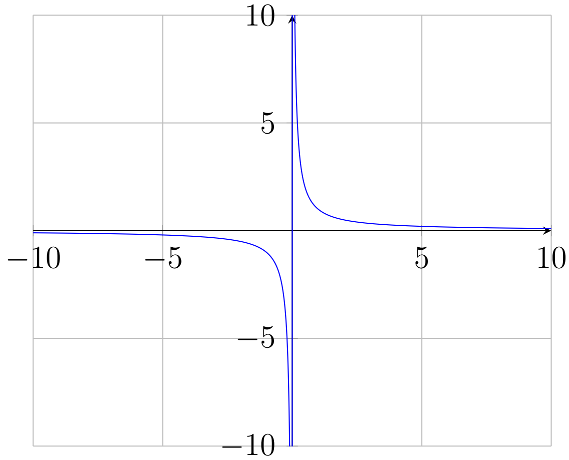


Properties

Domain Range	$(-\infty, \infty)$ $(-\infty, \infty)$
Continuous	Yes
Symmetry	Odd
Increasing	$(-\infty, \infty)$
Bounded	No
Extrema	None
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$

1.4 Reciprocal Function

$f(x) = \sqrt{2}$

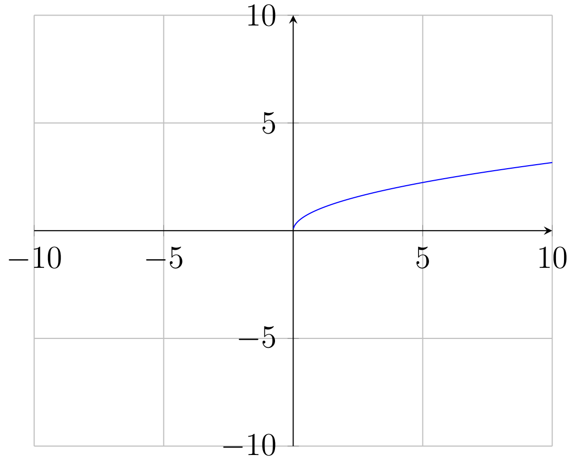


Properties

Domain Range	$(-\infty, 0) \bigcup (0, \infty)$ $(-\infty, 0) \bigcup (0, \infty)$
Continuous	Infinite discontinuity
Symmetry	Odd
Decreasing	$(-\infty, 0) \bigcup (0, \infty)$
Bounded	No
Extrema	None
Asymptotes	$y = 0$ $x = 0$
Limits	$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = 0$

1.5 Square Root Function

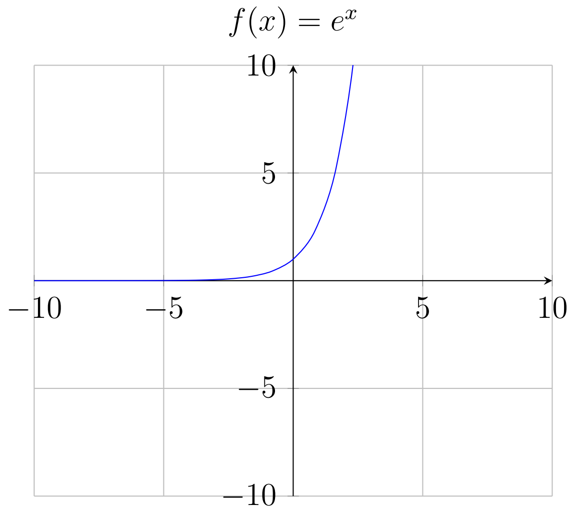
$f(x) = \sqrt{2}$



Properties

Domain Range	$[0, \infty)$ $[0, \infty)$
Continuous	Yes
Symmetry	None
Increasing	$[0, \infty)$
Bounded	No
Extrema	Minimum: $(0, 0)$
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = DNE$ $\lim_{x\to\infty} f(x) = \infty$

1.6 Exponential Function

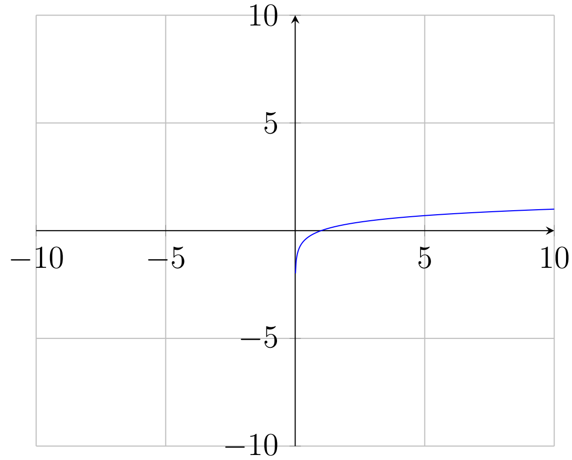


Properties

Domain Range	$(-\infty, \infty)$ $(0, \infty)$
Continuous	Yes
Symmetry	None
Increasing	$(-\infty, \infty)$
Bounded	Below
Extrema	None
Asymptotes	$y = 0$
Limits	$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = \infty$

1.7 Logarithmic Function

$f(x) = \log x$

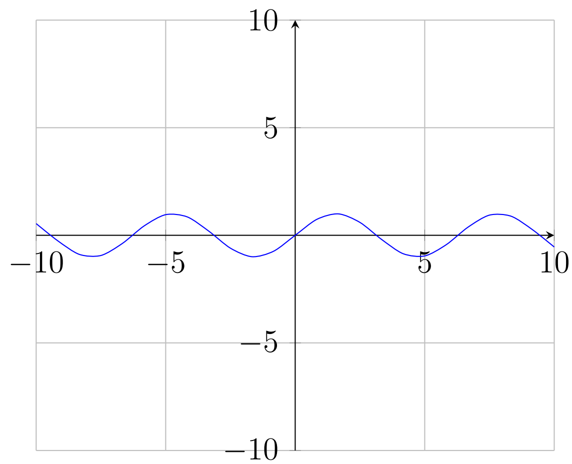


Properties

Domain Range	$(0, \infty)$ $(-\infty, \infty)$
Continuous	Yes
Symmetry	None
Increasing	$(0, \infty)$
Bounded	No
Extrema	None
Asymptotes	$x = 0$
Limits	$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$

1.8 Sine Function

$f(x) = \sin(x)$

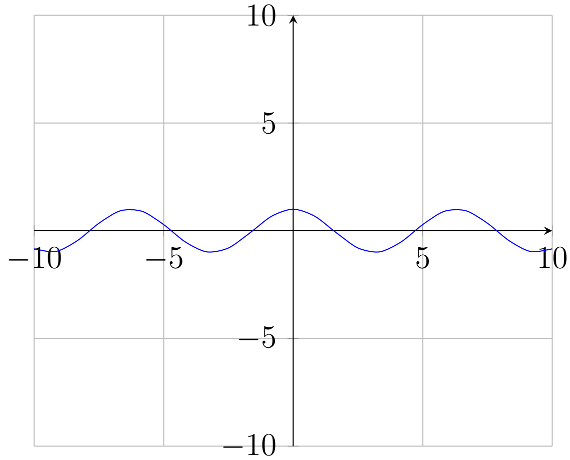


Properties

Domain Range	$(-\infty, \infty)$ $[-1, 1]$
Continuous	Yes
Symmetry	Odd
Increasing Decreasing	$[-\frac{\pi}{2}, \frac{\pi}{2}]$ $[\frac{\pi}{2}, \frac{3\pi}{2}]$
Bounded	Above and Below
Extrema	Minimum: -1 Maximum: 1
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = DNE$ $\lim_{x\to\infty} f(x) = DNE$

1.9 Cosine Function

$f(x) = \cos(x)$

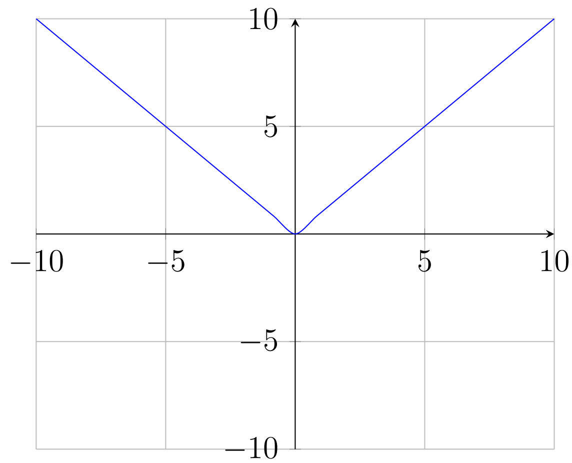


Properties

Domain Range	$(-\infty, \infty)$ $[-1, 1)$
Continuous	Yes
Symmetry	Even
Increasing Decreasing	$[-\pi, 0]$ $[0, \pi]$
Bounded	Above and Below
Extrema	Minimum: -1 Maximum: 1
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = DNE$ $\lim_{x\to\infty} f(x) = DNE$

1.10 Absolute Value Function

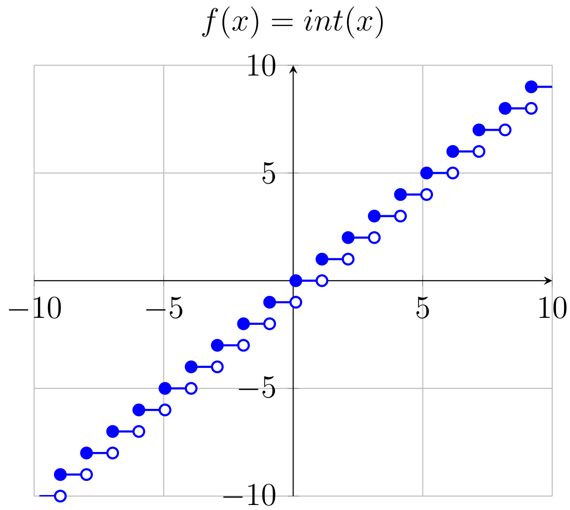
$f(x) = |x|$



Properties

Domain Range	$(-\infty, \infty)$ $[0, \infty)$
Continuous	Yes
Symmetry	Even
Increasing Decreasing	$[0, \infty)$ $(-\infty, 0]$
Bounded	Below
Extrema	Minimum: $(0, 0)$
Asymptotes	None
Limits	$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$

1.11 Greatest Integer Function

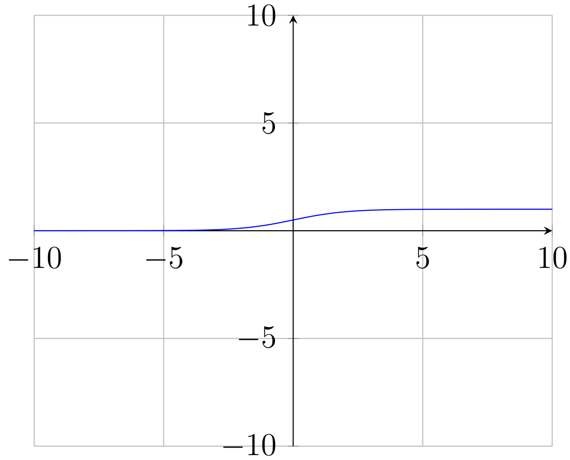


Properties

Domain Range	$(-\infty, \infty)$ $(0, \infty)$
Continuous	Yes
Symmetry	None
Increasing	$(-\infty, \infty)$
Bounded	Below
Extrema	None
Asymptotes	$y = 0$
Limits	$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = \infty$

1.12 Logistic Function

$$f(x) = \frac{1}{1+e^{-x}}$$

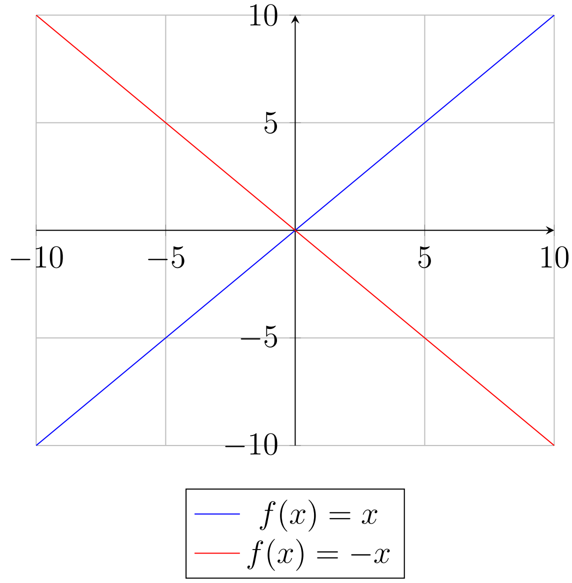


Properties

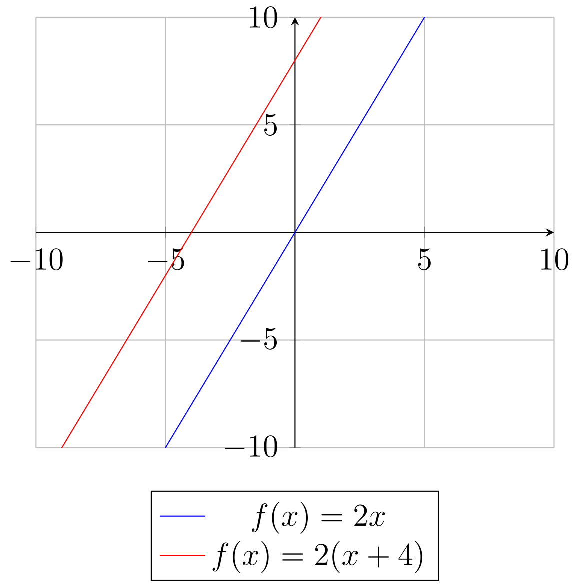
Domain Range	$(-\infty, \infty)$ $(0, 1)$
Continuous	Yes
Symmetry	None
Increasing	$(-\infty, \infty)$
Bounded	Above and Below
Extrema	None
Asymptotes	$y = 0$ $y = 1$
Limits	$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = 1$

2 Graphical Transformations

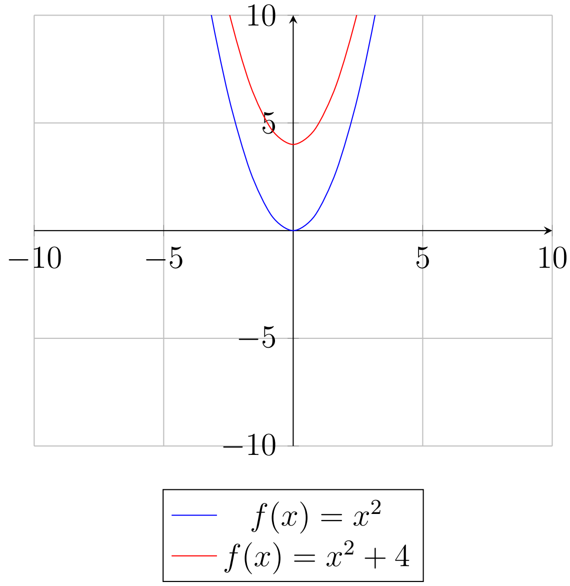
2.1 Flipping the y-values over the x-axis and the y-axis



2.2 Moving y-values left and right on the x-axis

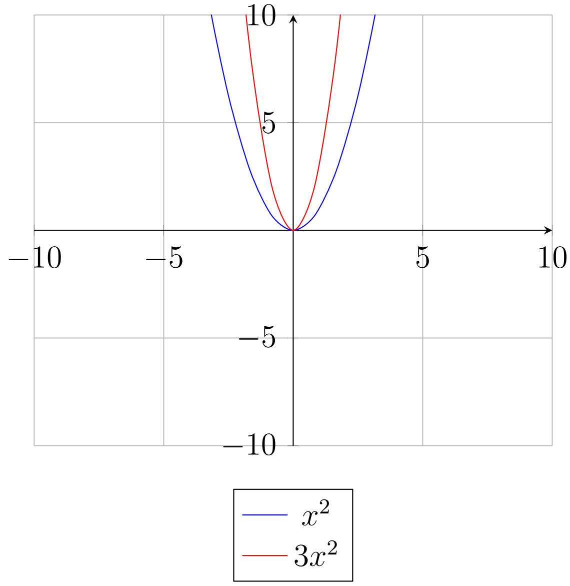


2.3 Moving the y-values up and down on the y-axis

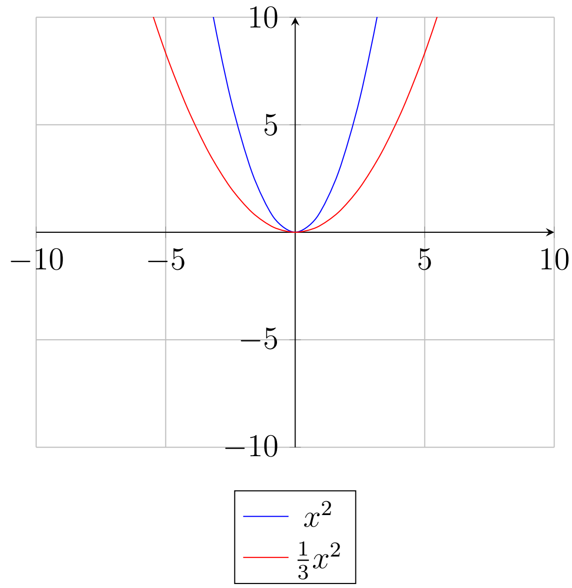


2.4 Vertical modifications to the graph

2.4.1 Stretch $f(x) \to af(x)$ when $a > 1$

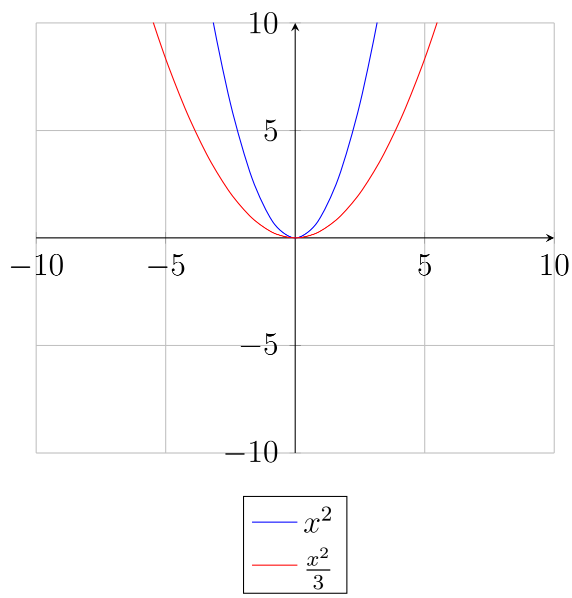


2.4.2 Shrink $f(x) \to af(x)$ when $0 < a < 1$

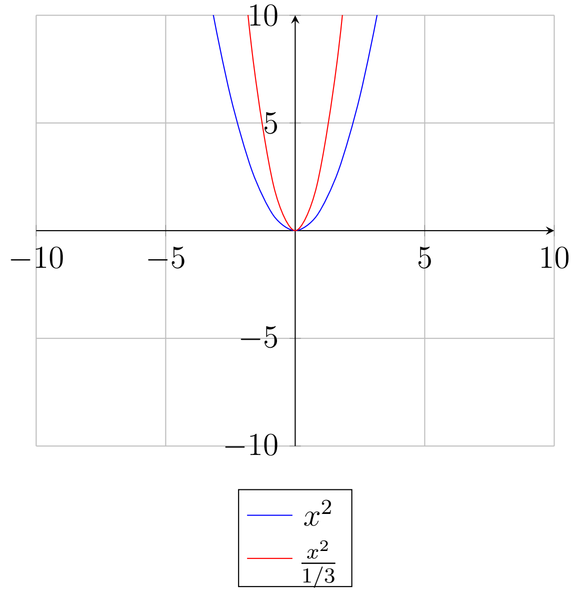


2.5 Horizontal modifications to the graph

2.5.1 Stretch $f(x) \to f(x \div c)$ when $c > 1$



2.5.2 Shrink $f(x) \to f(x \div c)$ when $0 < c < 1$



3 Function Composition and Inverse

3.1 Function Composition

Composing functions is as simple as substituting the selected function's x value with the other equation.

Example: $f(x) = 2x^2 - x$ **and** $g(x) = x^3$

Old Functions and Domains	$f(x) = 2x^2 - x$ $g(x) = x^3$	$(-\infty, \infty)$ $(-\infty, \infty)$
New Functions and Domains	$(fog)(x) = 2x^6 - x^3$ $(gof)(x) = (2x^2 - x)^3$	$(-\infty, \infty)$ $(-\infty, \infty)$

Example: $f(x) = 3x + 1$ **and** $g(x) = \frac{1}{x}$

Old Functions and Domains	$f(x) = 3x + 1$ $g(x) = \frac{1}{x}$	$(-\infty, \infty)$ $(-\infty, 0) \bigcup (0, \infty)$
New Functions and Domains	$(fog)(x) = 3(\frac{1}{3} + 1)$ $(gof)(x) = \frac{1}{3x+1}$	$(-\infty, \infty)$ $(-\infty, \infty)$

3.2 Inverse

Example: $y = 9x + 2$

To get the inverse of a function, simply replace the x and the y in the equation to get your inverse. The end result should be simplified to get y back on the other side.

Old Function and Domain	$y = 9x + 2$	$(-\infty, \infty)$
New Function and Domain	$y^{-1} = \frac{x-2}{9}$	$(-\infty, \infty)$

Example: $y = \sqrt{x^3 + 4}$

Old Function and Domain	$y = \sqrt{x^3 + 4}$	$\left[\sqrt[3]{-4}, \infty\right)$
New Function and Domain	$y^{-1} = \sqrt[3]{x^2 - 4}$	$(-\infty, \infty)$

4 Quadratic Functions

4.1 Standard Form

Standard form is beneficial for seeing transformations, and other manipulations of the graph. It's also a form that's easily factorable if needed.

$f(x) = ax^2 + bx + c$

4.2 Vertex Form

Vertex form is good for finding the vertex and axis of symmetry with ease. It's much harder to factor, but can be transformed into <u>Standard Form</u> if needed be. **Vertex:** (h, k)

$f(x) = a(x - h)^2 + k$

4.3 Converting from Standard Form to Vertex Form using Completing the Square

Starting equation	$y = 2x^2 - 4x + 5$
To complete the square, our x^2 and x terms must be isolated.	$y - 5 = 2x^2 - 4x$
The leading coefficient must be 1 to continue. Let's factor out the leading coefficient of 2.	$y - 5 = 2(x^2 - 2x)$
We need a perfect square trinomial.	$y - 5 + 2\boxed{} = 2\left(x^2 - 2x + \boxed{}\right)$
Find the perfect trinomial by taking half of the coefficient of the x term, squaring it, and putting it in the boxes.	$y - 5 + 2\boxed{1} = 2\left(x^2 - 2x + \boxed{1}\right)$
Simplify and make the right side a square expression.	$y - 3 = 2(x - 1)^2$
Isolate the y term by moving the -3 to the other side.	$y = 2(x - 1)^2 + 3$

5 Power Functions

A power function must have a degree greater than 2, it must be a monomial, and the exponents can be positive or negative.

5.1 Direct Variation

Direct variation is good for modeling equations where when one variable increases or decreases, the resulting value will do the same based on a predetermined ratio.

A couch factory makes 12 couches per hour. If the amount of hours given increases, so will the total amount of couches made.

A direct variation can only exist if the degree is positive.

Example: $c = 12h$

This example models the real world example about the couch factory. The 12 is the constant of variation, and the exponent of our h term (1) is the degree.

5.2 Indirect Variation

Indirect variation is good for modeling equations where when one variable increases or decreases, the resulting value will do the opposite based on a predetermined ration.

As the volume of a container increases, the pressure decreases.

An indirect variation can only exist if the degree is negative.

Example: $k = P/V$

This example models the real world example about the gas's pressure. The P is the pressure variable and the V is the volume variable. The constant of variation is our numerator (P) and the degree is the opposite of our denominator's exponent (-1).

6 Polynomial Functions

6.1 Multiplicity

Multiplicity is the number of times a function has a zero at a certain point. It helps us by giving us a rough sketch of what the graph will look like.

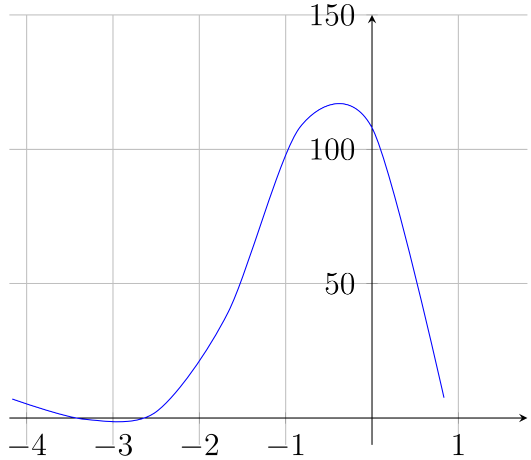
6.1.1 Guesstimating a graph from: $(x-1)^2(x+3)^3(x+4)$

The key factor in what a graph will do is based on the exponent on each term. Use the following table to determine what the graph will do:

Exponent is 1	The graph will go straight through at this point.
Exponent is 2	The graph will bounce off this point, similar to a parabola.
Exponent is 3	The graph will curve through this point, similar to how $y = x^3$ looks.

If the exponent goes higher than the table explains, if the exponent is event it follows the 2nd rule, and if the exponent is odd it follows the third rule.

Actual Graph

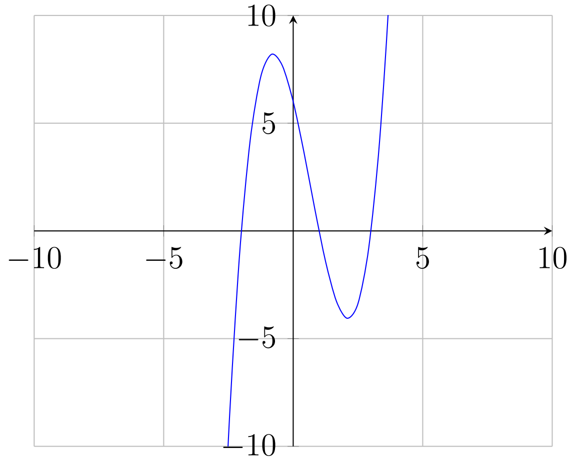


As you can see, the graph is close to our predictions, but isn't exact. Using multiplicity to guesstimate our graph is just that: Guesstimating. As you've also probably noticed, our y values are quite large, another downfall of guesstimating of graphs is we can't get y values without plugging in the x value which takes lots of effort and time.

6.2 Finding the function from zeros

We can also look at the graph and turn it into a function by finding the zeros of the function. By following our rules we can find the equation of our graph real easily.

Example Graph



By finding where $x = 0$ and knowing that all three zeros go straight through the y axis, we can solidify this information into an equation that looks something like $(x-3)(x-1)(x+2)$.

7 Long Division vs. Synthetic Division

Long division can be helpful for when synthetic division may not work, but other than that, it's very verbose and isn't the preferred method of polynomial division. Synthetic division is much more helpful (when it works) because you can get a remainder as well as an answer very quickly by simply multiplying the previous term with the current term, getting the sum, and repeating until you get to the end.

7.1 Long Divison Example

$$
\begin{array}{r|rrr}
 & & x & -\,10 \\ \hline
x+1\,) & x^2 & -\,9x & -\,10 \\
 & -\,x^2 & -\,x & \\ \hline
 & & -\,10x & -\,10 \\
 & & 10x & +\,10 \\ \hline
 & & & 0
\end{array}
$$

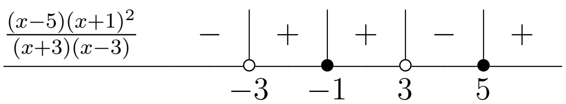
7.2 Synthetic Division Example

$$
\begin{array}{r|rrr}
 & 1 & -\,9 & -\,10 \\
-\,1 & & -\,1 & 10 \\ \hline
 & 1 & -\,10 & 0
\end{array}
$$

The last column is our remainder, and the first two can transform into our final answer: $x - 10$.

8 Solving Inequalities With Sign Charts

Example: $\frac{(x-5)(x+1)^2}{(x+3)(x-3)}$



$f(x) > 0$	$(-3,-1)\bigcup(-1,3)\bigcup(5,\infty)$
$f(x) \geq 0$	$(-3,3)\bigcup[5,\infty)$
$f(x) < 0$	$(-\infty,-3)\bigcup(3,5)$
$f(x) \leq 0$	$(-\infty,-3)\bigcup[-1]\bigcup(3,5]$

By using this sign chart, we've come up with inequalities for the function.

9 Exponential and Logarithmic Functions

9.1 Solving Exponential Functions

Exponential and Logarithmic functions are great for when the rate of change needs to change dynamically.

$$\begin{gathered}\text{Exponential: } y = ab^x \\ a = \text{initial value} \\ b = \text{growth or decay factor} \\ \text{Growth if } b > 1 \\ \text{Decay if } 0 < b < 1\end{gathered}$$

Example: Chicago has a population of 1.5 million in 2007. The population grows 2.6% every year.

The equation is structured as follows:

$$currentpoint = startingpoint \cdot (growth/decayfactor)^{yearssincestartingyear}$$

Which means our equation would look something like:

$$y = 1500000 \cdot 1.026^x$$

9.2 Solving Logarithmic Functions

Logarithmic functions can be solved by hand in certain cases, but most logistic functions will need to be solved by the calculator. All logarithmic functions can be solved very easily with your calculator by typing $log(yournumber)$. But to do it by hand, you must find an equation that'll cancel.

$$\cancel{log_{10}10^2} = 2$$

As you can see, the tens cancel out and leaves you with a 2. As this isn't usually the case, keep a calculator nearby!

9.3 Properties of Logs

9.3.1 Condensing

$$4log(xy) - 3log(yz) \rightarrow \frac{4logxy}{3logyz} \rightarrow \frac{log(xy)^4}{log(yz)^3} \rightarrow log\frac{x^4y^4}{z^3y^3} \rightarrow log\frac{x^4y}{z^3}$$

9.3.2 Expanding

$$log\frac{x^4y}{z^3} \rightarrow log\frac{x^4y^4}{z^3y^3} \rightarrow \frac{log(xy)^4}{log(yz)^3} \rightarrow \frac{4logxy}{3logyz} \rightarrow 4log(xy) - 3log(yz)$$

10 Partial Fraction Decomposition

1. Factor the denominator
2. If you have a linear factor: **Decompose:**

   $\frac{A_1}{mx+b} + \frac{A_2}{(mx+b)^2} + \ldots$
3. If you have a quadratic factor: **Decompose:**

   $\frac{B_1X+C}{ax^2+bx+c} + \frac{B_2X+C_2}{(ax^2+bx+c)^2} + \ldots$
4. Get a common denominator and solve for your variables

10.1 Decompose: $\frac{5x-1}{x^2-2x-15} = \frac{5x-1}{(x-5)(x+3)}$

$\frac{A_1}{x-5} + \frac{A_2}{x+3} = \frac{5x-1}{(x-4)(x+3)}$

$A_1(x+3) + A_2(x-5) = 5x - 1$

$A_1x + 3A_1 + A_2x - 5A_2 = 5x - 1$

$A_1 + A_2 = 5$

$3A_1 - 5A_2 = -1$

$3(3) - 5(2) = -1$

$9 - 10 = -1$

$$\boxed{\begin{matrix} A_1 = 3 \\ A_2 = 2 \end{matrix}} \rightarrow \boxed{\frac{3}{x-5} + \frac{2}{x+3}}$$

10.2 Sequences and Series

10.2.1 Sequences

	Arithmetic Sequences	**Geometric Sequences**
Recursive Formula	$a_n = a_{n-1} + d$	$a_n = a_{n-1} \cdot r$
Explicit Formula	$a_n = a_1 + d(n-1)$	$a_n = a_1 \cdot r^{n-1}$

10.3 Convergent vs. Divergent Sequences

A convergent series will be limited by a number. For example:

$$1\frac{1}{2}\frac{1}{3}\frac{1}{4}\frac{1}{5}$$

is limited to 0.
While something like:

$$5,7,9,11,13, \ldots, 2n+3$$

isn't limited (or has a limit of ∞) allowing it to continue forever.

10.4 Sum of Finite Series

Example: $29 + 24 + 19 + 14$

$$\sum_{k=1}^{4} -5k + 34 = 89$$

Example: $1 + 8 + 27 + 64 + 125$

$$\sum_{k=1}^{5} k^3 = 225$$

10.5 Partial Sum of Infinite Series

Example: $S_\infty = \frac{9}{1-r}$

$$\sum_{k=1}^{\infty} 3(\tfrac{3}{10})^{k-1}$$

11 Trigonometric Functions

11.1 Radians

Radians to Degrees: $\frac{3\pi}{2}$

$$\tfrac{3\pi}{2} \cdot \tfrac{180}{\pi} = 270°$$

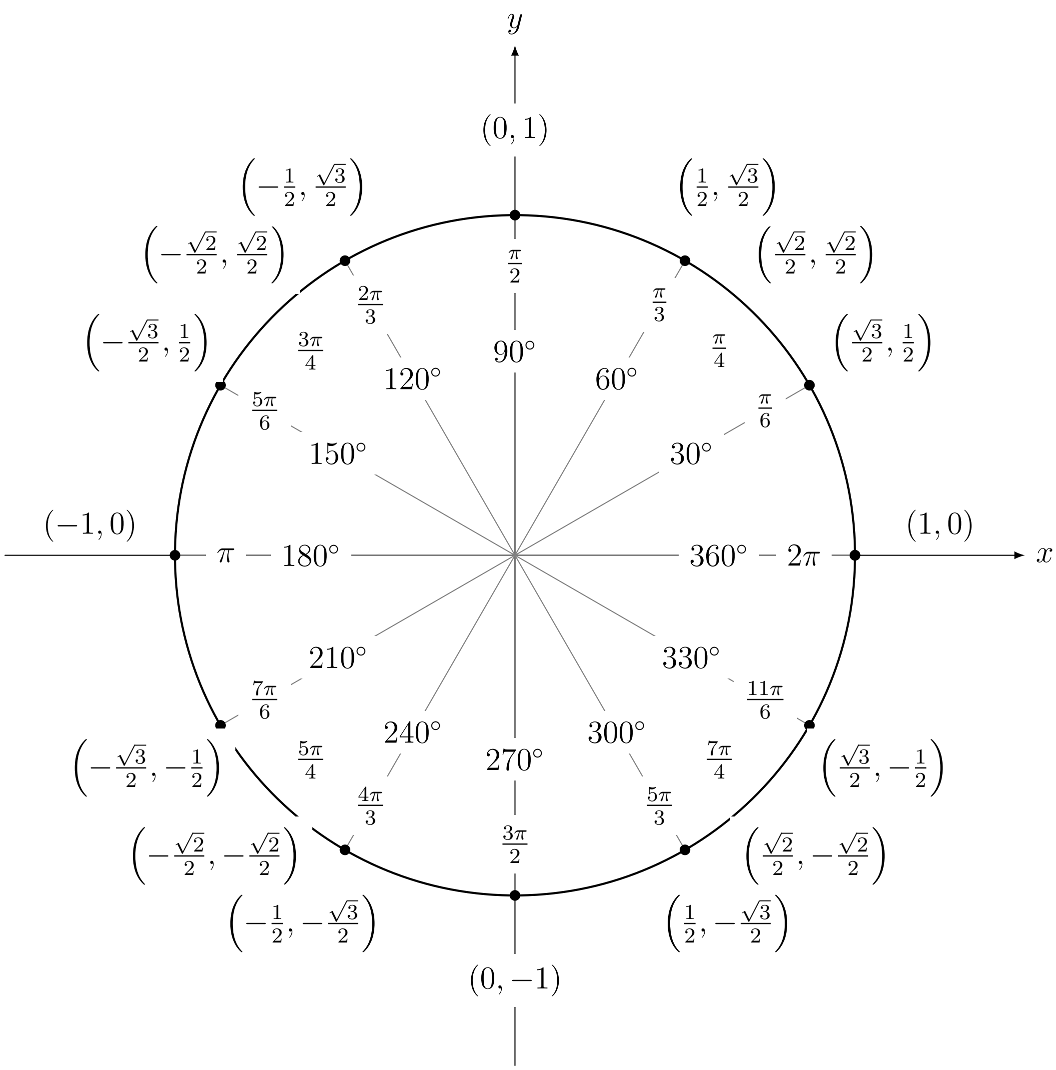
To convert radians to degrees, you multiply it by $\frac{180}{\pi}$.

Degrees to Radians: 360°

$$360 \cdot \tfrac{\pi}{180} = 2\pi$$

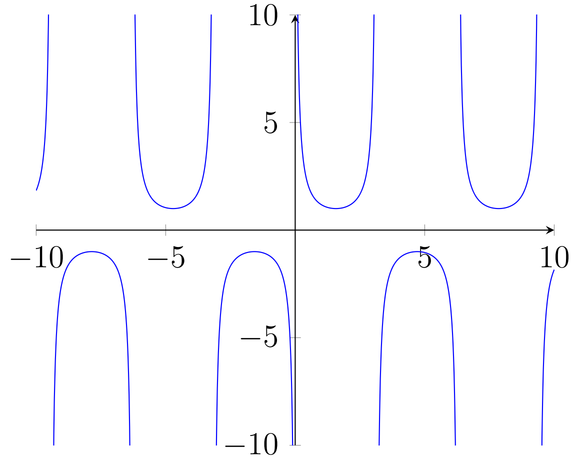
11.2 Unit Circle

The unit circle is a model which helps with calculating values of trigonometric functions by hand.

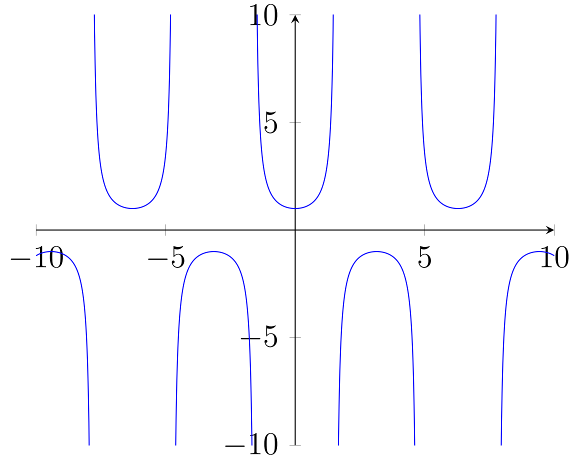


11.3 Graphs of Cosecant, Secant, and Cotangent

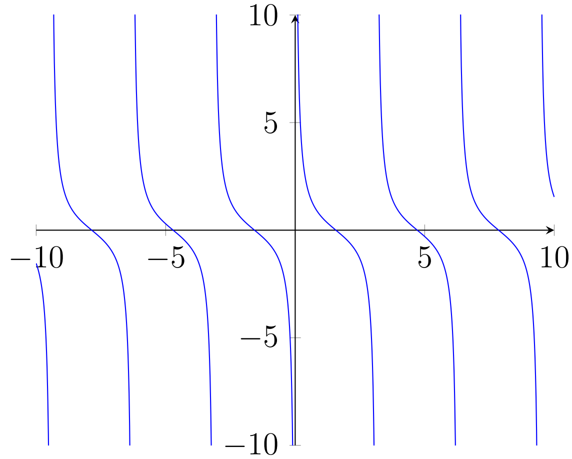
11.3.1 Cosecant



11.3.2 Secant

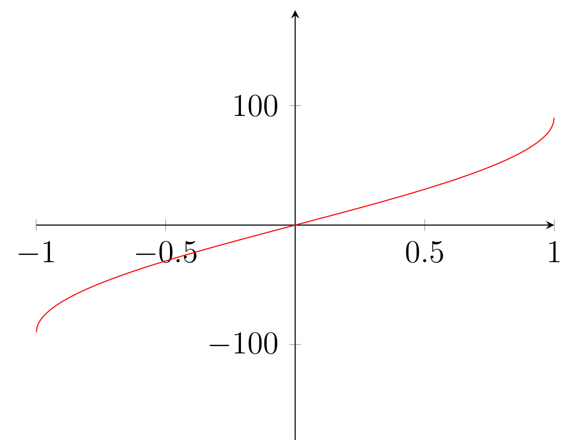


11.3.3 Cotangent

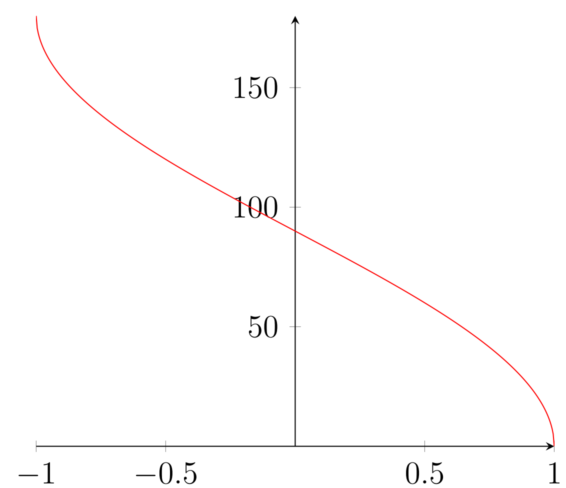


11.4 Inverse Sine, Cosine, Tangent

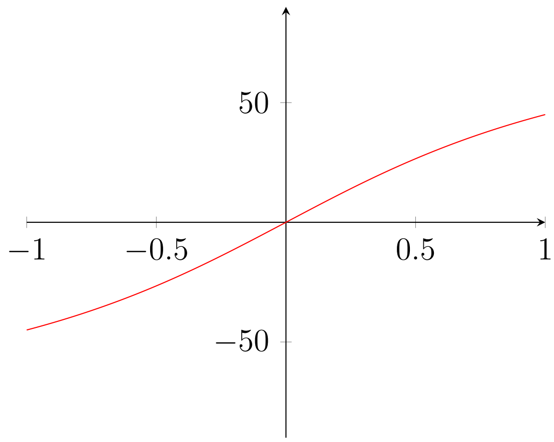
11.4.1 Inverse Sine



11.4.2 Inverse Cosine



11.4.3 Inverse Tangent



11.5 Evaluating Trigonometry Expressions Using The Unit Circle

Example: $\sin -300$

If you look at the unit circle, you'll notice that $-300°$ isn't on it. This means we need to find a coterminal angle. To find a coterminal angle, we need to add (or subtract if it's higher than 360) an entire circle. Adding $360°$ to $-300°$ results in $60°$. If we look at the sine value $(y - value)$ for $60°$ we get $\frac{\sqrt{3}}{2}$.

Example: $\cos -630$

As this is another negative angle, we need to add $360°$ to the angle. After adding $360°$ you'll see that it's still a negative angle, sometimes we need to add $360°$ more than once. So let's add it again and get $90°$. Looking at the cosine value $(xvalue)$ for $90°$ we get 0.

Example: $\tan(-135)$

Getting the coterminal angle gives us $225°$, and because we know $\tan(x) = \frac{sin(x)}{\cos(x)}$ we can see that $225° = 1$.

Example: $csc(\frac{3\pi}{2})$

Sense $\frac{3\pi}{2}$ is on our unit circle, we don't need to find any coterminal angles. Because cosecant is $\frac{1}{sin(x)}$ we take the -1 (the $sinevalueof 270°$) and we get $\frac{1}{-1}$ which can be simplified to -1.

Example: $sec(\frac{\pi}{2})$

$sec(\frac{\pi}{2})$ is on our unit circle, and because $sec(x) = \frac{1}{cos(x)}$ we know that $sec(\frac{\pi}{2}) = \frac{1}{0}$. Because anything divided by 0 can't be done, our answer is undefined.

Example: $cot(\pi)$

We know that $cot(x) = \frac{\cos(x)}{\sin x}$ so by using this function we can find that $cot(\pi) = \frac{-1}{0}$. And once again, because we know anything divided by 0 is undefined, our answer is undefined.

Example: $sin^{-1}(0)$

Any trigonometry function (except $cos(x)$, $sec(x)$, and $cos^{-1}(x)$) of 0 regardless of degrees or radians is 0. Because this is an inverse sine function, our answer is 0.

Example: $cos^{-1}(0.5)$

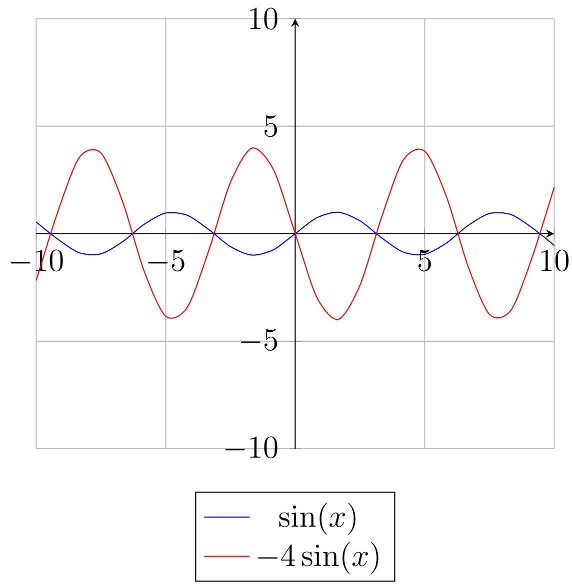
Our $x - value$ is $\frac{1}{2}$ at 60°.

Example: $tan^{-1}(1)$

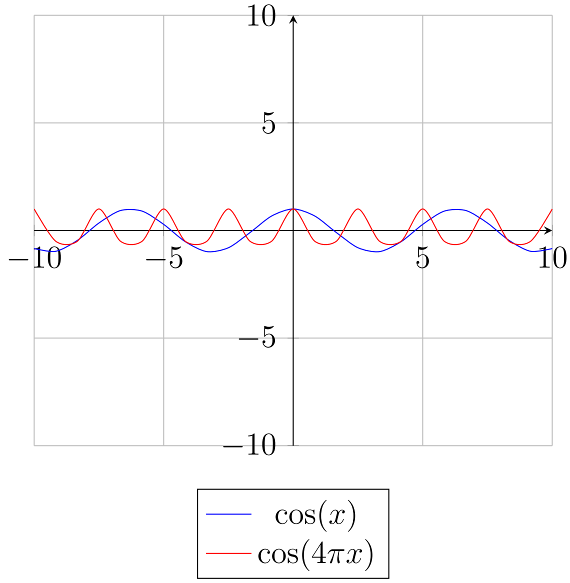
Our $\frac{y-value}{x-value}$ is 1 at 45°.

11.6 Sinusoidal Functions

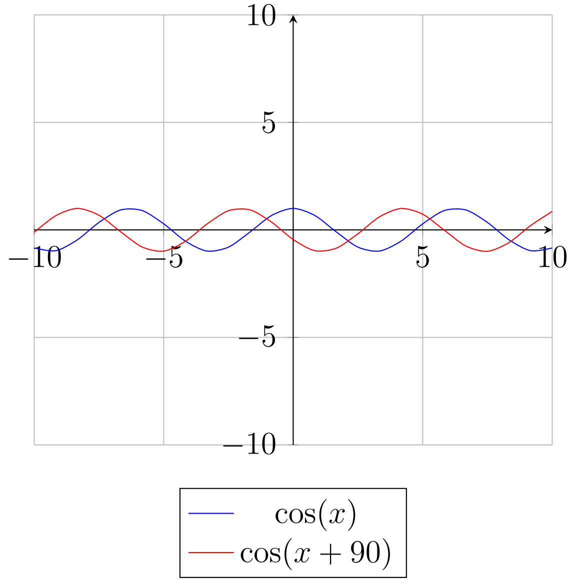
Amplitute



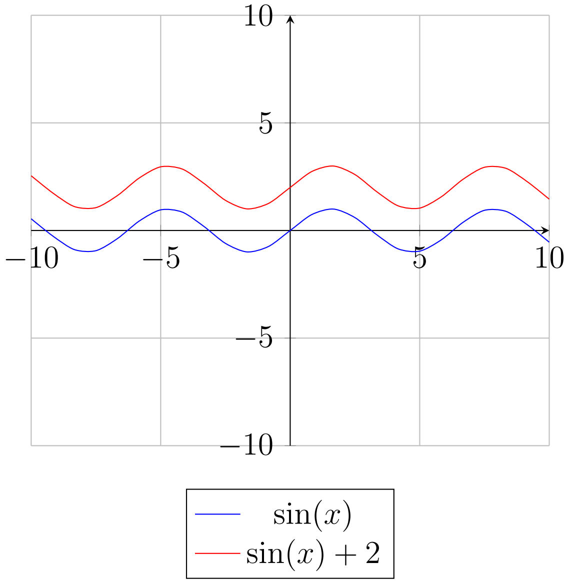
Period



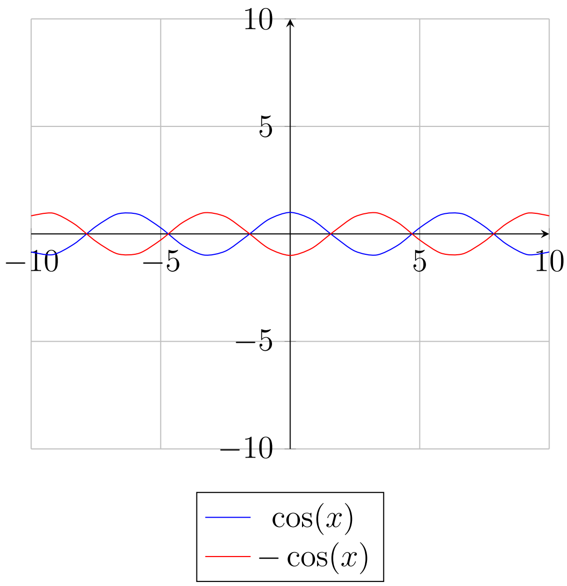
Phase Shift



Midline



Reflections



12 Analytic Trigonometry

12.1 Reciprocal Identities and Quotient Identities

Reciprocal Identities

1. $sin\theta = \frac{1}{csc\theta}$
2. $cos\theta = \frac{1}{sec\theta}$

3. $tan\theta = \frac{1}{cot\theta}$
4. $csc\theta = \frac{1}{sin\theta}$
5. $sec\theta = \frac{1}{cos\theta}$
6. $cot\theta = \frac{1}{tan\theta}$

Quotient Identities

1. $tan\theta = \frac{sin\theta}{cos\theta}$
2. $cot\theta = \frac{cos\theta}{sin\theta}$

12.2 Pythagorean Identities

1. $sin^2\theta + cos^2\theta = 1$
2. $sin^2\theta = 1 - cos^2\theta$
3. $cos^2\theta = 1 - sin^2\theta$

12.3 Double Angle Identities

1. $sin2x = 2sinxcosx$
2. $cos2x = cos^2x - sin^2x$
3. $cos2x = 2cos^2x - 1$
4. $cos2x = 1 - 2sin^2x$
5. $tan2x = \frac{2+tanx}{1-tan^2x}$

12.4 Half Angle Identities

1. $sin\frac{x}{2} = \pm\sqrt{\frac{1-cosx}{2}}$
2. $cos\frac{x}{2} = \pm\sqrt{\frac{1+cosx}{2}}$
3. $tan\frac{x}{2} = \pm\sqrt{\frac{1-cosx}{1+cosx}}$
4. $tan\frac{x}{2} = \frac{1-cosx}{sinx}$
5. $tan\frac{x}{2} = \frac{sinx}{1+cosx}$

12.5 Power-Reducing Identities

1. $sin^2x = \frac{1-cos2x}{2}$
2. $cos^2x = \frac{1+cos2x}{2}$
3. $tan^2x = \frac{1-cos2x}{1+cos2x}$

12.6 Simplifying Trig Functions

Example: $\frac{tan\theta}{sec\theta} = \frac{sin\theta}{cos\theta}$

Tip #1: When possible transform to *sin* and *cos*.

$\frac{tan\theta}{sec\theta} = \frac{\frac{sin\theta}{cos\theta}}{\frac{1}{cos\theta}}$

$\frac{sin\theta}{\cancel{cos\theta}} \cdot \frac{\cancel{cos\theta}}{1}$

$\frac{sin\theta}{1} \rightarrow sin\theta$

Example: $\frac{cosx}{1-sinx} - \frac{sinx}{cosx}$

Tip #2: Get a common denominator.

$\frac{cos^2x-sinx+sin^2x}{(1-sinx)cosx}$

$\frac{\cancel{1-sinx}}{\cancel{(1-sinx)}cosx} \rightarrow \frac{1}{cosx} = secx$

Example: $sin^3x + sinxcos^2x$

Tip #3: Where there's two terms with something in common, factor it out.

$sinx(\cancel{sin^2x + cos^2x}) \rightarrow sinx$

12.7 Solving Trig Equations

Example: $cos^3x - cosx = 0$

$cosx(-sin^2x - 1) = 0$

$cosx(-sin^2x) = 0$

$cosx = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Example: $5sin^2x + 12sinx + 7 = 0$

$5x^2 + 12x + 7 = 0$

$(5x + 7)(x + 1) = 0$

$5x + 7 = 0 \quad x + 1 = 0$

$5x + 7 = 0$

$sinx = -1$

$x = \frac{3\pi}{2}$

12.8 Proving Trig Identities

Proof.

$$
\begin{aligned}
tanx + cotx &= secxcscx \\
\frac{sinx}{cosx} + \frac{cosx}{sinx} &= \\
\frac{sin^2x + cos^2x}{sinxcosx} &= \\
\frac{1}{sinxcosx} &= \\
secxcscx &=
\end{aligned}
$$

$\square$

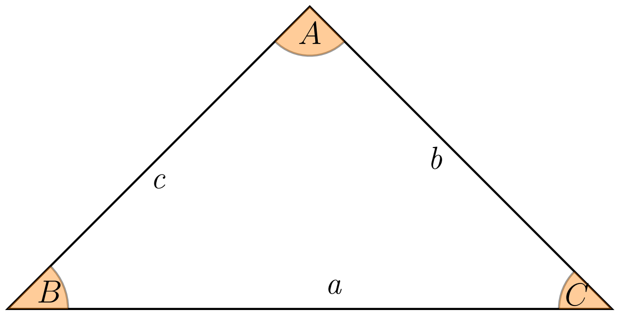
Proof.

$$
\begin{aligned}
\frac{1}{secx - 1} + \frac{1}{secx + 1} &= 2cotxcscx \\
\frac{(secx \cancel{+1}) + (secx \cancel{-1})}{(secx - 1)(secx + 1)} &= \\
\frac{2secx}{sec^2x - 1} &= \\
\frac{2secx}{tan^x} &= \\
\frac{\frac{2}{cosx}}{\frac{sin^2x}{cos^2x}} &= \\
\frac{2}{\cancel{cosx}} \cdot \frac{\cancel{cos^x}}{sin^x} \rightarrow \frac{2cosx}{sin^x} &= \\
\frac{2cosx}{sinx} \cdot \frac{1}{sinx} &= \\
2cotxcscx &=
\end{aligned}
$$

$\square$

12.9 Law of Sines

Finding missing side lengths and angle measurements in non-right Δs



$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Example: $A = 15°$ $C = 109°$ $c = 46$ **find a, B, b**

$$B = 180 - 15 - 109 = 56$$
$$\frac{sin15}{a} = \frac{sin109}{46}$$
$$11.91 = a(0.95) \to a = 12.59$$
$$\frac{sin56}{b} = \frac{sin109}{46} \to b = 40.33$$

Example: $a = 7$ $b = 6$ $A = 26.3°$ **find B, C, c**

$$\frac{sin26.3}{7} = \frac{sinB}{6}$$
$$0.36 = sinB$$
$$B = 22.3°$$
$$180 - 22.3 = 131.4°$$
$$C = 131.4°$$
$$c = 11.85$$

12.10 Law of Cosines

1. $a^2 = b^2 + c^2 - 2bccosA$
2. $b^2 = a^2 + c^2 - 2accosB$
3. $c^2 = a^2 + b^2 - 2abcosC$

Example: $a = 7$ $c = 5$ $B = 97.3°$

$b^2 = 7^2 + 5^2 - 2(7)(5)cos(97.3)$

$\sqrt{b^2} = \sqrt{82.89°}$

$b = 9.1°$

$25 = 49 + 82.81 - 2(7)(9.1)cosC$

$25 = 49 + 82.8 - 127.4cosC$

$25 = 131.81 - 127.4cosC$

$\frac{-106.81}{-127.4} = \frac{-127.4cosC}{-127.4}$

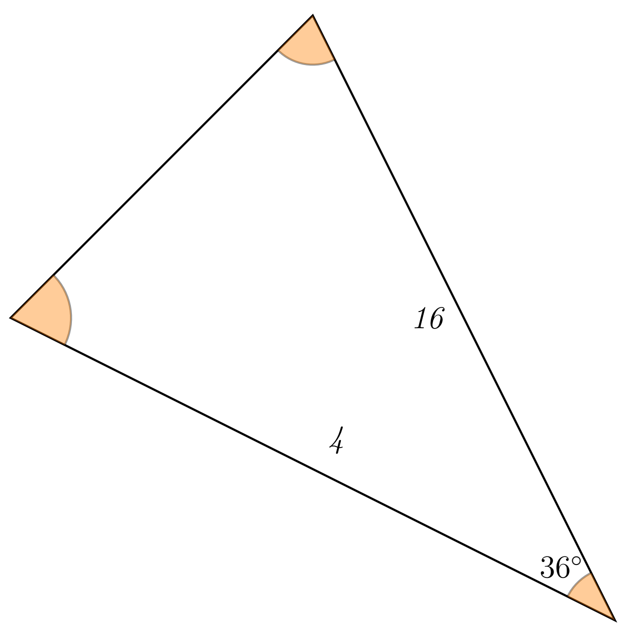
$A = 119.6$

$(0.838)^{-1} = cosC^{-1}$

$33.1° = C$

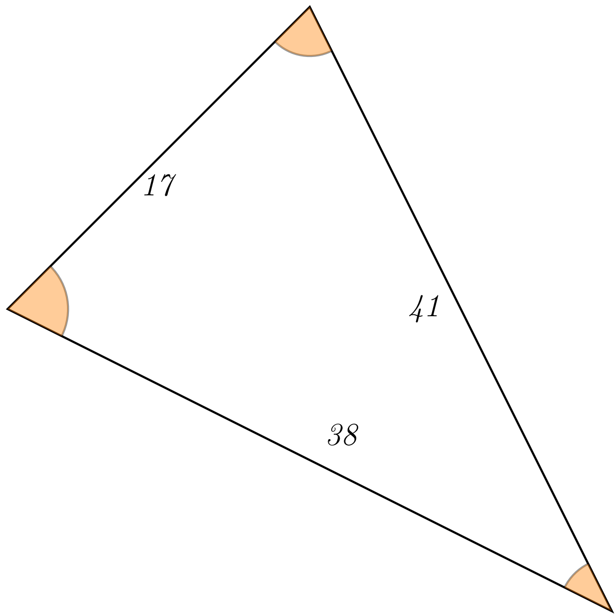
12.11 Area of a Triangle (SAS and SSS)

SAS: $\frac{1}{2}absinC$



$A = \frac{1}{2}(4)(16)sin36$
$A = 18.81\ units^2$

SSS: Heron's Formula: $A = \sqrt{(s)(s-a)(s-b)(s-c)}$



$$s = 48$$
$$A = \sqrt{(48)(48-17)(48-38)(48-41)}$$
$$A = 322.738\ units^2$$

13 Application of Trigonometry

13.1 Vectors

Vectors are directed line segments that have a direction and magnitude with an initial point and a terminal point.

Component Form of a Vector: $< x, y >$

x: Change in x values
y: Change in y values
$\vec{RS} =< 11, -2 >$

Standard Form of a Vector: $< 4, -1 >\rightarrow 4i - j$

Trig Form of Complex Numbers: $a + bi = r(cos\theta + isin\theta)$

1. $a^2 + b^2 = r^2$
2. $b = rsin\theta$

Unit Vector

A unit vector is a vector in the same direction as a given vector but with a magnitude of 1.

$$u = \frac{v}{|v|}$$

What appears to be absolute value notation is the notation for magnitude in this situation.

Dot Products of Vectors

The answer to a dot product will always be a single number.

$$u =< u_1, u_2 >$$
$$v =< v_1, v_2$$
$$uv = u_1v_1 + u_2v_2$$

Example: Find $a \cdot b$ if $a =< 4, -1 >$ and $b =< 6, 11 >$

$$ab = (4 \cdot 6) + (-1 \cdot 11)$$
$$ab = 24 - 11$$
$$ab = 13$$

Finding the Angle Between Two Vectors

To find the angle between two vectors, you need to use:

$$cos\theta = \frac{u \cdot v}{|u||v|}$$

Example: Find the angle between $u =< 9, 11 >$ and $v =< 14, 3 >$

$$u \cdot v = (9)(14) + (11)(3) = 159$$

$$|u| = \sqrt{9^2 + 11^2} = \sqrt{202}$$

$$|v| = \sqrt{14^2 + 3^2} = \sqrt{205}$$

$$cos\theta = \frac{159}{\sqrt{202} \cdot \sqrt{205}} = 38.62^\circ$$

13.2 Parametric Equation

Parametric equations use a variable (t) in order to graph equations that are not functions.

Converting from Parametric to Function Mode

This is also commonly referred to as "eliminating the parameter" because the objective is to remove the t variable from the equation.

Example: $y = 3t \quad x = t^2 - 2$

$$y = 3t \quad x = t^2 - 2$$

$$2 + \sqrt{x} = \sqrt{t^2}$$

$$t = \sqrt{x+2}$$

$$y = 3\sqrt{x+2}$$

Example: $x = 2cost \quad y = 2sint$

$$x^2 + y^2 = 4cos^2t + 4sin^2t$$

$$x^2 + y^2 = 4(cos^2t + sin^2t)$$

$$x^2 + y^2 = 4$$

Finding Restrictions on the Domain and Range

To find the restriction of a parametric equation, we can use the calculator's table feature to find the minimum and maximum of both the x values and the y values. To do so, simply switch your calculator's mode to PAR and type the x equation into $\backslash X_{1T}$, and the y into $\backslash Y_{1T}$. After the equations have been entered into their respective areas, go to the $TABLE$ and find the lowest and highest x and y values, this is your domain and range.

Projectile Motion

$$x = V_0 cos\theta t + g$$

$$y = -16t^2 + V_0 sin\theta t + h_0$$

V_0: Initial velocity
g: Gust
h_0 : Initial height
A baseball player hits a ball $5ft$ off the ground with an initial velocity of $120ft/sec$, at a $33°$angle. It heads towards a fence that is $40ft$ tall and $350ft$ away. If the gust is $6ft/sec$ with the ball, does it clear the fence?
$x = 120cos33t + 6$
$y = 16t^2 + 120sin33t + 5$

13.3 Polar Coordinates

Plotting Polar Coordinates

Plotting polar points isn't done on a standard x and y plane. Instead, you'll need a polar graph which can be visualized as a circle. To start plotting, first you'll need to find the angle on the coordinate plane. Next you'll need to find where the radius intersects the angle, and finally plot the point.

Converting Points From Polar to Rectangular and Vice Versa

To convert rectangular coordinates to polar coordinates:

$$r = (x^2 + y^2)^{\frac{1}{2}}$$

r = distance from origin to the point
x = rectangular x-coordinate
y = rectangular y-coordinate

$$\theta = atan(\tfrac{y}{x})$$

To convert polar coordinates to rectangular coordinates:

$$x = rcos(\theta)$$
$$y = rsin(\theta)$$

Converting Equations From Polar to Rectangular and Vice Versa

Example: Convert $x^2 + y^2 = 2x$ to polar form.

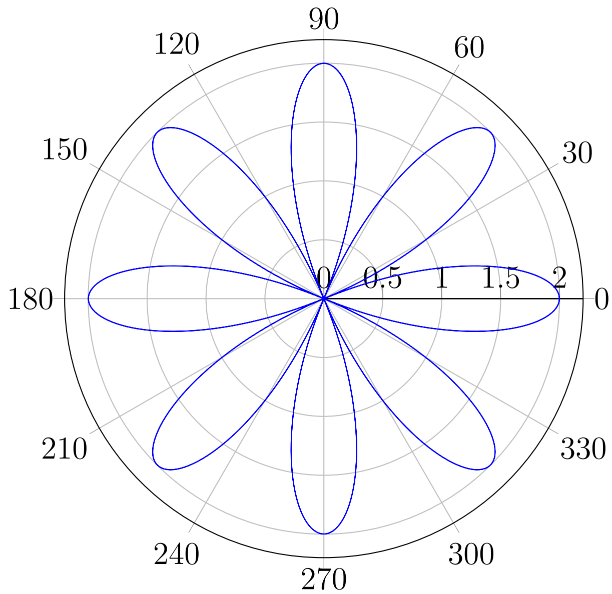
$$\begin{aligned} x^2 + y^2 &= 2x && (1)\\ r^2 &= 2(rcos\theta) && (2)\\ r^2 &= 2rcos\theta && (3)\\ r &= 2cos\theta && (4) \end{aligned}$$

Example: Convert $P(3, -5)$ to rectangular form.

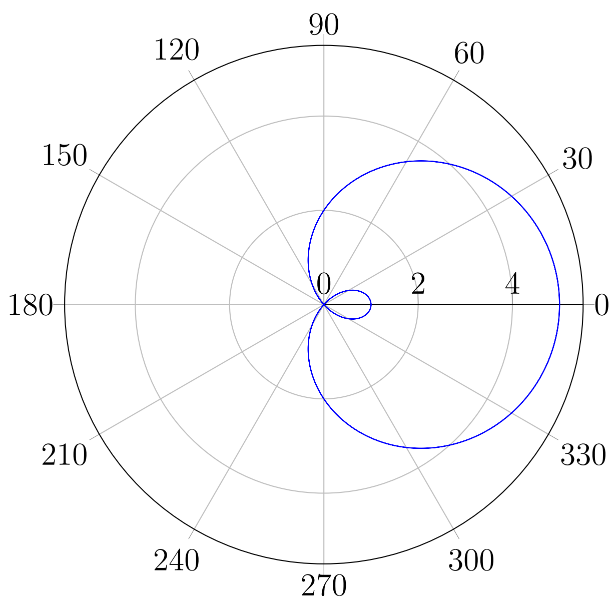
$$\begin{aligned} r &= \sqrt{x^2 + y^2} & \theta &= arctan\frac{y}{x}\\ r &= \sqrt{(3)^2 + (-5)^2} & \theta &= tan^{-1}\left(-\frac{5}{3}\right)\\ r &= \sqrt{34} & &\\ r &\approx 5.83 & \theta &\approx -1.03 \end{aligned}$$

13.4 Graphs and Equations of Polar Relations

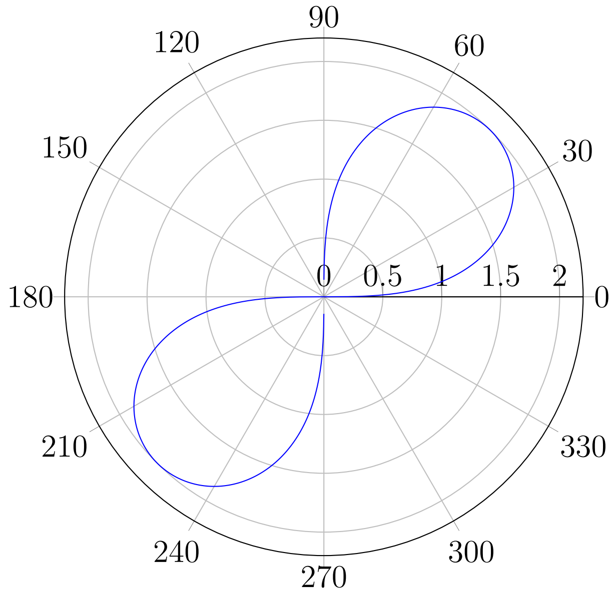
Rose Curves



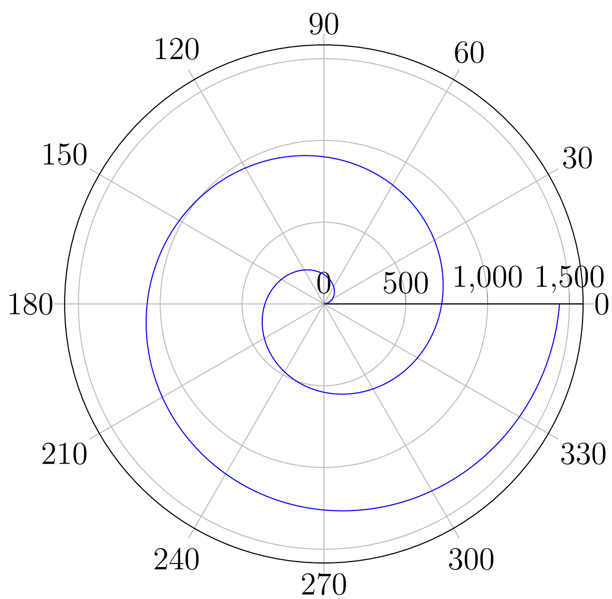
Limacon



Lemniscates

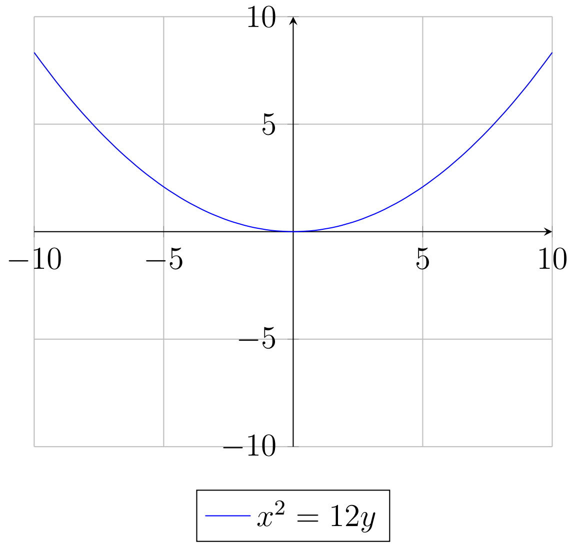


Spiral of Archimedes



14 Conic Sections

14.1 Parabolas



Vertex

The vertex is the point in which the graph switches directions or "bounces" directions. All parabolas have a vertex, and by default it's at $(0, 0)$.

Focus

The focus determines the space between the parabola on the inside. The further this point is from the axis, the more space that'll be between the parabola on the inside.

Directrix

Same distance from the vertex as the focus, this imaginary line never touches the parabola. This is because the parabola always opens in the opposite direction.

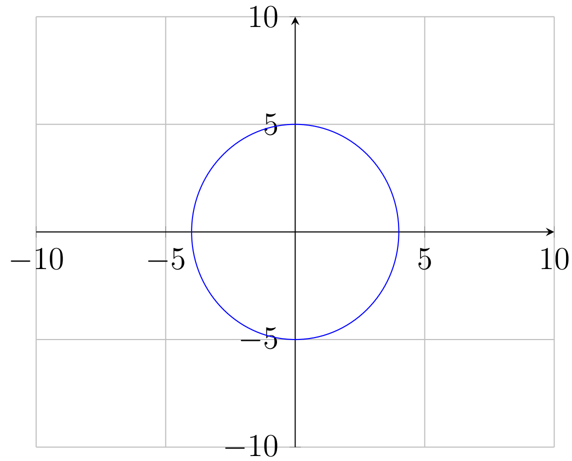
Focal Length / Width

Both the focal length and the focal width have an equation. Focal length: p and Focal width: $|4p|$. These are used to determine how much space is between the focus and a parabola point.

Direction of Opening

The direction of opening is always opposite from the directrix.

14.2 Ellipse



Center

The center of an ellipse is helpful for finding the foci and the vertices assuming you know the distance between them.

Vertices

Vertices are helpful for finding how wide / tall your ellipse. If you have both vertices, you can find the center with them.

Foci

By using the foci, you can find the center, as well as the vertex if you know the distance between them.

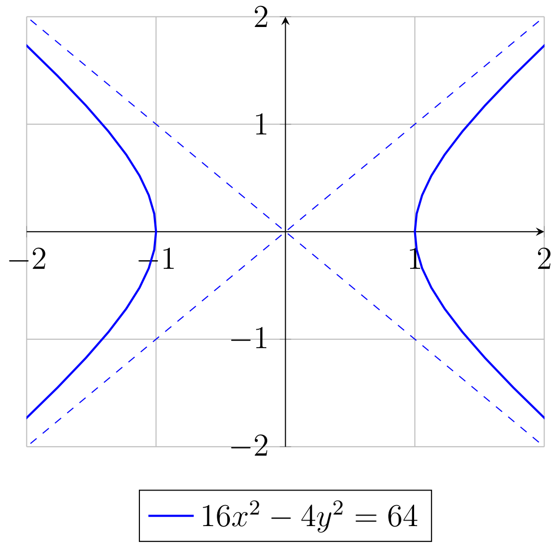
Wide vs. Tall

If the larger number is under the x in the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The ellipse will be wide, if the larger number is under the y in the equation, the equation will be tall.

14.3 Hyperbola



Center

The center can be used to find the vertices if you know the distance between them, as well as the foci.

Vertices

$(\pm a, 0)$ / $(0, \pm a)$ depending on the shape of the hyperbola. If it's aligned with the x axis, you'll use $(\pm a, 0)$, and if it's aligned with the y axis, you'll use $(0, \pm a)$.

Foci

Similar to how the focus for parabolas work, except there are two of them in a hyperbola.

Asymptotes

If the hyperbola is aligned with the x axis, you'll use $y = \pm(\frac{b}{a})x$ to find the asymptote line. If the hyperbola is aligned with the y axis, you'll use $y = \pm(\frac{a}{b})x$.

Left / Right vs. Up / Down

If the x comes first in the equation like:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

the hyperbola is aligned with the x axis and you're hyperbola will be sideways (in relation to $y = x^2$) and they'll both go in opposite directions. If the y comes first in the equation like:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

then the hyperbola goes up and down.