The Cost of Concreteness: The Effect of Nonessential Information on Analogical Transfer

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Most theories of analogical transfer focus on similarities between the learning and transfer domains, where transfer is more likely between domains that share common surface features, similar elements, or common interpretations of structure. We suggest that characteristics of the learning instantiation alone can give rise to different levels of transfer. We propose that concreteness of the learning instantiation can hinder analogical transfer of well-defined structured concepts, such as mathematical concepts. We operationalize the term concreteness as the amount of information communicated through a specific instantiation of a concept. The 5 reported experiments with undergraduate students tested the hypothesis by presenting participants with the concept of a commutative mathematical group of order 3. The experiments varied the level of concreteness of the training instantiation and measured transfer of learning to a new instantiation. The results support the hypothesis, demonstrating better transfer from more generic instantiations (i.e., ones that communicate minimal extraneous information) than from more concrete instantiations. Specifically, concreteness was found to create an obstacle to successful structural alignment across domains, whereas generic instantiations led to spontaneous structural alignment. These findings have important implications for the theory of learning and transfer and practical implications for the design of educational material. Although some concreteness may activate prior knowledge and perhaps offer a leg up in the learning process, this benefit may come at the cost of transfer.

Keywords: transfer, analogy, concreteness, learning

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A primary goal of education is transfer of knowledge from the learning context to future novel situations. A particular kind of transfer that may underlie aspects of mathematical reasoning is analogical transfer, which is the application of a learned relation or relational structure to a new situation. For example, having learned that populations grow exponentially in the absence of barriers, one may recognize that the same is true of the epidemic spread of an infectious disease. As such, analogical transfer can facilitate understanding, allow for inferences, and promote problem solving in unfamiliar situations. However, successful analogical transfer for abstract concepts, such as mathematical concepts, is often difficult to achieve (e.g., Gick & Holyoak, 1980, 1983;

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Goswami, 1991; Novick, 1988; Reed, Dempster, & Ettinger, 1985; Reed, Ernst, & Banerji, 1974; Simon & Reed, 1976).

The past several decades have produced a vast amount of research on analogical transfer with a consensus on several aspects. First, analogical transfer can be affected by superficial features of the initially learned domain (e.g., Kotovsky, Hayes, & Simon, 1985; Zhang & Norman, 1994). Second, transfer is more likely to occur between situations that share some form of similarity, beyond the relevant analogous relations, than between those that do not (e.g., Bassok, 1996; Holyoak & Koh, 1987; Ross, 1987, 1989). Third, alignment of common structure across the learned and novel domains is a necessary component of successful analogical transfer (Gentner, 1983, 1988; Gentner & Holyoak, 1997; Holyoak & Thagard, 1989). Finally, learning multiple instantiations of a given concept can result in schema induction that leads to successful transfer (Gick & Holyoak, 1983; Novick & Holyoak, 1991), particularly when learners directly compare learned instantiations (Catrambone & Holyoak, 1989; Gick & Holyoak, 1983; Kotovsky & Gentner, 1996; Loewenstein, Thompson, & Gentner, 2003; Ross & Kennedy, 1990).

More recent research suggests that another factor may also affect transfer. College students who learned a single symbolic instantiation of a novel mathematical concept successfully transferred their knowledge to a novel isomorph, whereas those who learned one or more concrete instantiations of the concept could not (Kaminski, Sloutsky, & Heckler, 2008). These findings suggest that concreteness of the initially learned instantiation can hinder subsequent transfer compared with more symbolic instan-

tiations. However, there are questions that remain unanswered. What aspects of concrete instantiations affect transfer? And why?

For the present purposes, we define *concreteness* of an instantiation of an abstract concept as the amount of information communicated by that particular instantiation. We also refer to instantiations that communicate minimal information beyond the relevant relations as *generic* because they are not deeply tied to specific information-rich contexts. We elaborate on these constructs in the next sections.

According to our account (Kaminski et al., 2008), transfer from the concrete instantiations failed because these were more familiar to the learner and thus communicated more nonessential information than a less informative, more generic instantiation. This familiarity may have hindered structural alignment between the studied and novel domains. In contrast, generic instantiations have minimal extraneous information and are more alignable to novel isomorphs; therefore, they result in successful transfer.

In contrast to our proposal, it could be argued that the observed differential transfer between the concrete and generic instantiations did not stem from differential concreteness, but from differential similarity between the learned and novel instantiations. It is well established that transfer to similar instances of the same concept is more likely to occur than transfer to dissimilar instances (Holyoak & Koh, 1987; Holyoak & Thagard, 1997; Ross, 1987, 1989). Although the generic and concrete instantiations used in the Kaminski et al. (2008) study appear to be equally dissimilar to the transfer domain in their surface features, there may be more subtle characteristics of these domains that create greater similarity between the generic and the transfer domains than between the concrete and transfer domains. If this account is true, then differences in transfer may have stemmed from differences in similarity between the studied domains and the novel transfer domain.

The purpose of the present research was to test the hypothesis that concreteness of the initially learned instantiation hinders analogical transfer across instances of a well-defined abstract concept by creating an obstacle to structural alignment. We test the *concreteness* account and contrast it against the *similarity* account to explain transfer differences between concrete and generic domains.

Similarity Account

There is considerable evidence that transfer between instances of a concept that share surface features is more likely than between instances that do not share surface features (Holyoak & Koh, 1987; Holyoak & Thagard, 1997; Ross, 1987, 1989). High surface similarity between the source and the target domains can facilitate spontaneous retrieval of prior knowledge (Gentner, Rattermann, & Forbus, 1993). For example, recall of a learned solution to mathematical story problems is greater between problems that have similar storylines than between problems with dissimilar storylines (Ross, 1987, 1989). Surface similarity can also affect transfer by affecting the process of structural alignment. Because similar elements are easier to align than dissimilar elements (Gentner, 1983, 1988; Gentner & Markman, 1997; Markman & Gentner, 1993a, 1993b), structural alignment is facilitated when similar elements play identical structural roles across the learning and transfer domains. As a result, transfer is more successful when similar elements hold analogous roles in both domains (Ross,

1987, 1989). However, when similar elements hold different structural roles across domains, learners tend to misalign structure by matching common elements and consequently appropriate transfer fails (Ross, 1987, 1989).

Surface features are an important aspect of similarity, but other aspects of instances can also contribute to similarity (see Goldstone & Son, 2005a, for a discussion). One characteristic in particular is common relational structure (Falkenhainer, Forbus, & Gentner, 1989; Goldstone, 1994; Markman & Gentner, 1993a, 1993b). Students often interpret structure through the context in which it is presented (see Bassok, 1996, 2003, for summaries). For example, given a situation involving 12 tulips and 3 vases, students are likely to divide 12 by 3 instead of performing another arithmetic operation because a group of flowers is typically divided between vases. The downside of this approach is that differences in context could be mistakenly interpreted as differences in relevant structure. For example, students who learned solutions to permutation problems in a semantically asymmetric context (e.g., a teacher assigning prizes to students) successfully transferred solution strategies to novel, analogous, asymmetric problems, but failed to do so to symmetric problems (e.g., a teacher assigning students to students; Bassok, Wu, & Olseth, 1995). Similarly, students often fail to transfer solution strategies when the variables of the source and target domains differ in continuity (i.e., continuous vs. discrete). When source and target domains differ in aspects such as symmetry or continuity, learners can form mismatching interpretations of structure, which makes structural alignment across domains difficult, creating an obstacle to transfer (Bassok & Olseth, 1995). Therefore, a similarity account of transfer can accommodate not only similarity based on surface features, but also similarity based on extraneous structural characteristics of the contexts.

Concreteness Account

Defining Concreteness

In everyday life, the term *concrete* is used in contrast to *abstract* often to differentiate what can and cannot be directly experienced by our senses. For example, there would be little disagreement that the concept *cat* is more concrete than the concept *infinity*. This is a comparison of the concreteness of two different concepts. Concreteness can also be compared between instantiations of the same concept; there also would be little disagreement that a real cat is a more concrete instantiation of a cat than a schematic outline. For the purposes of this article, we are concerned not with the concreteness of concepts, but with the concreteness of instantiations of abstract concepts.

We propose an operational definition of *concreteness* of an instantiation of an abstract concept as the amount of communicated information. By communicated information, we mean the information activated in the mind of the observer. Furthermore, we suggest that concreteness has multiple dimensions, including a perceptual dimension and a conceptual dimension. As such, instantiations can communicate both perceptual and conceptual information. To illustrate this point, consider the different instantiations of the concept of *individual* presented in Figure 1. A black circle communicates little conceptual or perceptual information aside from the presence of an individual entity and its dimensions

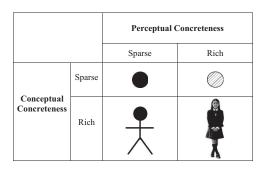


Figure 1. Examples of instantiations of the concept of *individual* that differ in the amount of communicated information in the perceptual dimension and the conceptual dimension.

and color. The striped circle (with different colored stripes) communicates this information as well as additional perceptual information of colors, patterns, and so forth. Compared with the black circle, the stick figure communicates more conceptual information, namely that the figure likely represents a human being. Finally, the image of the schoolgirl communicates substantially more information than the stick figure. Not only does this image communicate considerable perceptual information (e.g., color, shape, etc.), it also activates conceptual information. For example, this is a young girl who appears to be dressed in a school uniform; she likely has an Asian parent. Much more information would be activated if the image were of someone familiar. If the intention of any instantiation is to act as a symbol for a larger set, then the increase in communicated information likely constrains the set of potential referents. For example, a circle can easily represent any single entity, yet it would be odd for a stick figure to represent something other than a person, such as a computer. Similarly, a picture of a schoolgirl is not a good symbol for a businessman, whereas a stick figure makes a good symbol for any person.

We define instantiations that communicate minimal extraneous details, beyond the defining structural information, as *generic* because they do not activate information that ties them to specific contexts. Instantiations that communicate more extraneous information are defined as *concrete*. Therefore, *concrete* and *generic* are not a dichotomy; rather, they lie on a continuum over which the amount of communicated information varies. Specifically, for a given concept, instantiation *A* is more concrete than instantiation *B* if *A* communicates more information to an observer than *B*.

Just as instantiations of everyday concepts can vary in concreteness, instantiations of mathematical concepts, as well as any well-defined abstract concepts, can vary in concreteness. For example, concrete instantiations of mathematics include contextualized, real-world examples as well as physical manipulatives and visual representations that might be presented to elementary school students. Generic instantiations are any that minimize extraneous detail, such as traditional symbolic notation. Moreover, because mathematical concepts are well defined (unlike many everyday concepts; see Solomon, Medin, & Lynch, 1999), for any particular instantiation, there would likely be universal agreement among experts as to what constitutes defining structural information versus unnecessary information associated with the instantiation. For example, a *fraction* is defined as a number that can be expressed in the form a/b, where a and b are integers and $b \neq 0$. This

definition, like statements of any well-defined structured concept, communicates the essential structural properties of the concept. To add additional information creates a specific instantiation. For example, *a* could represent the number of products sold and *b* could represent the number of products manufactured. When more extraneous information is added (e.g., *a* represents the number of a laptop computers sold during 2011 and *b* represents the total number of all computers manufactured in 2011), a more concrete instantiation is created. Therefore, the concept is an abstract entity, and any type of expression or embodiment of the concept is an instantiation. In this sense, very generic instantiations, such as standard symbolic mathematical notation, would be as close to a zero-concreteness point as would be possible to communicate.

Furthermore, instantiations are often represented using verbal descriptions and symbols that vary in the amount of perceptual and conceptual information they communicate. Conceptual information can be increased by describing familiar situations or adding detail. Perceptual information can be increased by adding visual information or increasing the perceptual richness of the symbols. For example, representing fractions through a story of equally sharing pizza with visual images of proportions of pizza is a more concrete instantiation than representing fractions simply as monochromatic sectors of a circle.

It is important to note that concreteness is a subjective construct; the amount of information communicated by a particular instantiation depends on an individual's prior knowledge of this instantiation and the symbols present. For example, the simple symbols \bigoplus , \Im , and \square communicate conceptual information only if they are previously known. Likewise, numerals, operation signs, and other mathematical symbols also communicate conceptual information if they have been learned. Similar to other subjective measures, as opposed to objective measures, concreteness can be derived from human judgment.

This measurement for our stimuli was performed in two calibration experiments. In addition to calibrating the stimuli, in Experiment A-1 (see online Supplemental Materials), we also included objects (instantiations of person, cat, and pie) that varied in the amount of perceptual detail. For each object, participants rated either the amount of communicated information or the level of concreteness. Higher ratings for both concreteness and information were given to the more perceptually detailed objects than to the less detailed objects. The correlation between information ratings and concreteness ratings was high (r=.84), which supports our contention that the objects that are typically labeled more concrete also communicate more information than those that are considered more abstract.

Concreteness and Transfer

Recent research has suggested that learning a concrete instantiation of an abstract concept may hinder subsequent analogical transfer. Undergraduate students who learned a novel mathematical concept through a generic instantiation were more likely to transfer structural knowledge to a novel isomorph than students who learned the concept through one or more concrete instantiations (Kaminski et al., 2008).

There is other evidence for an advantage of generic instantiations over concrete instantiations. Concreteness in the form of perceptual richness of a learned instantiation can affect subsequent transfer (Goldstone & Sakamoto, 2003; Kaminski & Sloutsky, 2012; Sloutsky, Kaminski, & Heckler, 2005) such that successful transfer is more likely to occur from instantiations that are more perceptually sparse than instantiations that are more perceptually rich. For example, when undergraduate students were presented with a novel mathematical concept, learning the concept instantiated with simple monochromatic objects facilitated subsequently learning the same concept instantiated with colorful, moving objects. However, the reverse was not the case. Learning the concept instantiated with colorful, moving objects failed to facilitate learning the same concept instantiated with simple monochromatic objects (Sloutsky et al., 2005). It has also been shown that when undergraduate students learned the principle of competitive specialization (which explains how individual agents self-organize without a central plan) through a scenario of ants foraging for food, transfer was more successful when the ants and food were represented more abstractly as dots and patches than when the representations resembled ants and apples (Goldstone & Sakamoto, 2003).

The negative effect of concreteness as perceptual richness has also been demonstrated in research with young children. Preschool children have difficulty using concrete, perceptually rich objects as symbols (DeLoache, 1991, 2000). Other research has shown that 3- and 4-year-old children are more accurate at recognizing numerical equivalence between two sets of generic objects than between a set of concrete, perceptually rich objects and a set of generic objects (Mix, 1999). More recently, it was demonstrated that 4-year-olds are better able to recognize the relation of symmetry (i.e., by distinguishing ABB from ABA patterns) when comparing two instances involving simple, monochromatic geometric shapes than between two instances involving colorful, familiar objects (Son, Smith, & Goldstone, 2011). Taken together, these studies provide evidence that concreteness in the form of perceptual richness can hinder transfer of both simple relations in children and more complex relations in adults.

Goals of Present Research

The previously mentioned studies demonstrated that perceptual richness can hinder transfer of relations. We suggest that concreteness in the form of conceptual information also hinders analogical transfer (e.g., Kaminski et al., 2008) via the same basic mechanism as perceptual concreteness. Both perceptually rich and conceptually rich instantiations present more information to the learner than their more generic counterparts. This abundant nonessential information can capture the learner's attention and divert it from the relevant relational structure. When learning a conceptually rich instantiation, considerably more information is activated in the learner than if the instantiation were more generic. It may be difficult for the learner to know what information is relevant and what is not. When attempting to transfer knowledge to a novel isomorph, the learner would need to actively search through the activated information, perhaps entertaining many different hypotheses regarding the correspondence of information across domains. At the same time, more generic instantiations may activate little or no extraneous knowledge, thus reducing the need to identify and inhibit irrelevant information. Because little information besides relevant structural information is presented to the learner, structural information is a salient aspect of the representation. Therefore, we predict that instantiations that communicate more extraneous information to the learner (i.e., concrete instantiations) would hinder transfer compared with instantiations that communicate less extraneous information.

The purpose of the present research was to test this prediction. If supported, the concreteness account does not undermine similarity or other factors that contribute to transfer; rather, it identifies concreteness of the source instantiation as an additional, important predictive variable for transfer.

Five experiments were conducted to investigate analogical transfer of the same mathematical concept that was used in previous research (Kaminski et al., 2008). This concept was chosen because it is a simple mathematical concept that could be instantiated in a variety of ways by fixing the defining relational structure (the component principles or rules defining the concept) and specifying the elements and contextual storyline. In Experiment 1, we considered transfer and structural alignment when structural knowledge was acquired through either a concrete instantiation or a more generic instantiation. To foreshadow, the results indicate greater transfer with the latter instantiations. The goals of Experiments 2–4 were to replicate this finding under a variety of conditions and eliminate alternative explanations of transfer differences. Finally, in Experiment 5, we considered the mechanism that may explain the reported transfer differences.

Two supplemental experiments were run to calibrate the degree of concreteness of the instantiations used in the present research. Participants were asked to rate the amount of information communicated to them by the stimuli. We measured the amount of information communicated through both elements and the systems incorporating these elements (see Experiments A-1 and A-2 in the online Supplemental Materials). The results establish that the instantiations referred to as "concrete" in the following experiments communicate more information than the instantiation referred to as "generic." Table 1 presents information ratings scores of five systems used in the experiments presented below. These scores are presented in ascending order and are referred to as concreteness ratings and used to qualitatively predict transfer from a given system.

Experiment 1A

In this experiment, participants learned a novel mathematical concept presented through either a concrete instantiation or a generic instantiation. Afterward, they were presented with a novel isomorphic instantiation of the same concept and given a series of test questions on the transfer domain. Participants were then asked to match analogous elements across the learning and transfer domains. The ability to correctly match elements was taken as a measure of accurate structural alignment of the learning and transfer domains.

Method

Participants. Sixty-one undergraduate students (32 women, 29 men, M age = 19.4 years, SD = 1.21) participated in this experiment. In this and all the reported experiments, participants were undergraduate students from The Ohio State University who received partial credit for an introductory psychology course. Also note that for all experiments presented here, none of the participants took part in more than one condition or experiment.

Table 1
Mean Ratings of Amount of Communicated Information Used as
Concreteness Ratings for Instantiations of Experiments 1–5

Instantiation	Mean rating of communicated information	
Generic Number (modulos addition) 0.1.2	4.04	
Number (modular addition) 0 1 2 Reduced concreteness	4.81 5.37	
Concrete 2	6.71	
Concrete 1	6.97	

Note. Higher numbers reflect more communicated information. The lowest possible rating was 2; the highest possible rating was 10. These values were determined in Experiments A-1 and A-2 presented in the online Supplemental Materials.

Materials and design. Participants were randomly assigned to one of three between-subjects learning conditions (baseline, generic, or concrete). The experiment included two phases: (1) training and testing in a learning domain and (2) testing in the transfer domain. The learning domains varied between participants.

The learning domains in the generic and concrete conditions were instantiations of the concept of a *commutative mathematical group of order 3*. This concept is defined by a set of principles. Specifically, a commutative mathematical group of order 3 is a set of three elements, or equivalence classes, and an associated binary operation over which the following properties hold: associativity, commutativity, existence of identity, and existence of inverses. If the operation is denoted by "+," then the following are true: The *associative property* states that for any elements x, y, z of the set, (x + y) + z = x + (y + z). The *commutative property* states that for any elements x, y of the set, x + y = y + x. Also, there is an element I in the set called the *identity element*, such that for any element x, x + I = x. Finally, for any element x, there exists an *inverse element* y, such that x + y = I.

The concept of a mathematical group can be instantiated in an unlimited number of ways. One way is through arbitrary symbols. Such a domain was constructed and described to participants as rules of a symbolic language in which combinations of two or more symbols yield a predictable resulting symbol (see Table 2). Statements were expressed as *symbol 1*, *symbol 2* \rightarrow *resulting*

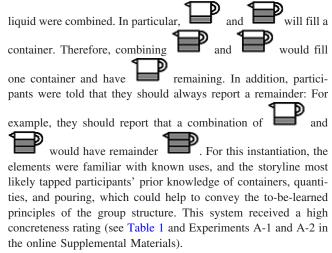
symbol. For example, , . In this case, the instantiation is generic in the sense that the symbols are not necessarily meaningful, the storyline is novel, and the rules of the language are arbitrary. Of the instantiations used in the reported experiments, this instantiation received the lowest concreteness

Supplemental Materials for how these values were determined).

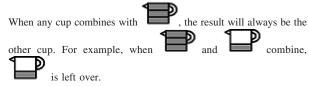
The mathematical group can also be instantiated in a more concrete manner that might facilitate learning. To construct an

rating (see Table 1 and Experiments A-1 and A-2 in the online

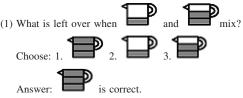
instantiation that communicates concreteness relevant to mathematical group structure, a scenario was given for which participants could draw on their everyday knowledge to determine answers to test problems. The elements were three measuring cups containing varying levels of liquid. Participants were told they needed to determine a remaining amount when various amounts of



Training and testing of the learning phase were isomorphic across the concrete and generic conditions. Training consisted of an introduction describing the cover story and explicit presentation of each of the principles in Table 2. For instance, in the concrete condition, participants were told that two cups of solution could be combined in any order and the resulting amount left over would be the same (i.e., the commutative property). The identity element was described as follows:



The remaining rules were introduced in a similar way. Fourteen questions ranging in difficulty from simple to more complex were given. Participants' responses were followed by feedback and explanations as to why a response was correct. The following are examples of questions with feedback as expressed in the concrete condition:



(2) What is the left over when these cups are mixed?

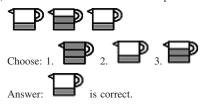


Table 2
Stimuli and Rules Across Learning Domains

Principles of commutative group:

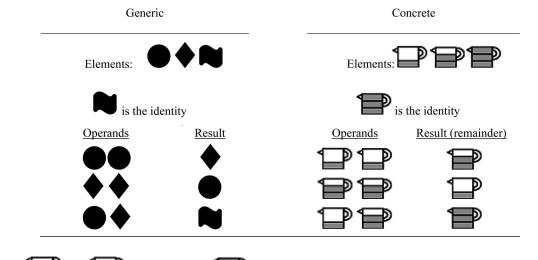
Associative: For any elements x, y, z: ((x + y) + z) = (x + (y + z))

Commutative: For any elements x, y: x + y = y + x

Identity: There is an element, I, such that for any element, x: x + I = x

Inverses: For any element, x: there exists another element, y, such that $x + y = \mathbf{I}$

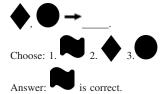
Specific elements and rules for the different instantiations:



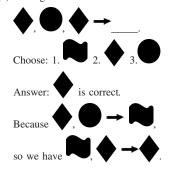


Participants in the generic condition were presented with analogous statements of the principles and were tested on exactly the same questions expressed in the generic format. The following are analogues to the above questions:

(1) What goes in the blank to make a correct statement?



(2) What goes in the blank to make a correct statement?



After training, participants were given a 24-question multiplechoice test designed to measure their ability to apply the learned rules to complex, novel problems. The following are examples of test questions in the generic condition:

(1) What can go in the blanks to make a correct statement?



(2) Find the resulting symbol:



Note that except for the manner of instantiation (i.e., concrete vs. generic), the rules, examples, and test questions were identical in both experimental conditions. The entire set of test questions for both the generic and concrete conditions is presented in the online Supplemental Materials.

In addition to these experimental learning conditions, the design also included a baseline condition in which participants were presented with an unrelated task involving simple arithmetic computations during the learning phase. In the baseline task, participants needed to compute the sums and products of small integers and answer whether the result was even, positive, or had absolute value greater than 10. The purpose of this condition was to account for any spontaneous correct performance on the transfer domain test that could be attained without explicit training on an isomorph first. Therefore, mean scores in the baseline condition would not be attributable to analogical transfer of knowledge from a previ-

Table 3
Stimuli for Transfer Domains for Experiments 1–5

Experiment 1A, 2, 4, and 5		Experiment 3	Experiment 3	
Elements:	\$2	Elements:	9	
Operands	Result	Operands	Result	
Je			(223)	
40	•			
• •				
44	•	EES EES		

ously learned isomorph. In the experimental conditions, mean transfer test scores higher than those in the baseline condition would indicate successful transfer; mean scores equivalent to those of the baseline condition would indicate an absence of transfer.

The transfer phase of the experiment followed the completion of the learning phase. The transfer domain was identical for all participants and was also a commutative group of order 3 involving three images of perceptually rich objects. It was described as a children's game in which children sequentially point to objects and "the winner" points to a final object (see Table 3). Participants were told that the correct final object was specified by the rules of the game (which were the rules of a mathematical group). Furthermore, in the two experimental learning conditions, they were told that the rules were like those of the system they had just learned. No explicit training in the transfer domain was given; instead, participants were shown a series of examples from which the rules could be deduced, such as the following.

Some children pointed to , then ; and then the winner pointed

Participants were asked to figure out the rules of the game by using their knowledge of the learned system. Then they were tested with a 24-question multiple-choice test, isomorphic to their test in the learning phase for participants in the experimental conditions, but using the elements of the transfer domain (see the online Supplemental Materials for more details). Transfer was indicated by the difference in transfer phase test scores between the experimental groups and the baseline group. Following the test, participants were asked to match analogous elements across the learning and transfer domains.

Procedure. Participants were seen individually in a laboratory on campus. All training and testing were presented on a computer. Participants proceeded at their own pace, with their responses recorded by the computer. The learning phase consisted of approx-

imately 80 slides and required approximately 15 minutes to complete. The transfer phase consisted of 48 slides and took on average 10 minutes to complete.

Participants were told that they were going to learn a new concept that would be presented to them on the computer, to read the information on the screen, and answer the multiple-choice questions presented along the way. The computer presentation first introduced the cover story. Then participants were presented with the principles of the concept. Each principle was individually shown along with an example. The presentations of the commutative, associative, and identity principles were each followed by two related questions with corrective feedback. After seeing all of the principles, participants were then given five questions with feedback. These questions were shown individually along with a summary of the principles to which the participants could refer. After responding to the questions, the rules were summarized. Then three complex problems with feedback were posed individually. Feedback included step-by-step solutions. Subsequently, the 24 test questions were presented (see the online Supplemental Materials), each on a separate slide.

Immediately after the test of the learning domain, participants proceeded to the transfer task. The cover story was introduced and participants were asked to figure out a new system (i.e., the children's game). In the experimental conditions, they were told that their knowledge of previous system could help them because the rules of the game were like the rules of the previous system they learned. Thirteen examples were shown over the course of eight slides. Participants were asked to study these examples. Afterward, 24 test questions were presented. Each question was presented individually on the screen. At the bottom of the screen, four examples were shown. The same four examples were present for each of the test questions. After proceeding through the test questions (see online Supplemental Materials), participants were asked to match analogous elements across the learning and transfer domains. For each of the three individual elements of the transfer domain, they were asked to choose which element of the previous system acted most like the given transfer element. Correct matching of elements was interpreted as evidence of structural alignment of the learning and transfer domains because alignment would result in a one-to-one correspondence of elements. For mathematical groups of order 3, there are two possible correct mappings between groups. The identity element is unique; therefore, a cor-

and and for the generic condition and and for the concrete condition. However, the mapping between the remaining two elements is not unique. Therefore, a response was considered correct if (a) the mapping was *one-to-one* and *onto* (i.e., each learning element corresponded to a single transfer element and each element of the transfer elements was used) and (b) the mapping preserved the identity element. In other words, if a participant used each of the group elements and mapped the identity element correctly, then the response was correct. Because the critical aspect was correctly choosing the identity element and most participants were expected to form mappings that were *onto*, 33% accuracy was used as a conservative measure of chance guessing from a group of partic-

To control for the possibility that potential differences in transfer stem from differential similarity of learning domains to the transfer domain based on surface features, we conducted a control experiment to compare the superficial similarity of each learning domain with the transfer domain. To measure superficial similarity, participants were given only brief descriptions of the domains (see Descriptions of Domains used in the Similarity Judgment Experiment in the Supplemental Material) and were deliberately not given any explicit training, as similarity ratings were intended to reflect superficial characteristics. A group of 40 undergraduate college students, none of whom were participants in Experiments 1-5, were asked to read descriptions of one of the learning domains and the transfer domain. The descriptions included the storyline, elements, and one example. After reading both descriptions, they were asked to rate the similarity between the two domains on a scale of 1 (completely different) to 5 (almost iden-

Half the participants rated the similarity between the concrete and transfer domains; the other half rated the similarity between the generic and transfer domains. The results of this control experiment indicated that without explicit training, both the generic and concrete domains were comparably similar to the transfer domain, independent-sample t(38) = 0.72, p = .48. The mean rating for the concrete and transfer domains was $2.95 \, (SD = 1.15)$, and the mean rating for the generic and transfer domains was $3.20 \, (SD = 1.05)$. These results suggest that differences in transfer scores are unlikely to stem from differential superficial similarity of the learning domains and the transfer domain.

Results and Discussion

Participants in both the concrete and generic conditions successfully learned the rules; mean scores were significantly above the chance score of 9 (37.5%), one-sample ts > 10.09, ps < .001, ds > 2.25. There were no differences in learning scores between the concrete and generic conditions ($M_{correct} = 79.2\%$, SD = 13.6, and $M_{correct} = 74.8\%$, SD = 16.5, respectively), independent-samples

t(39) = 0.93, p > .35. In addition, participants were accurate on the unrelated arithmetic task in the baseline condition ($M_{\rm correct} = 94.5\%$, SD = 8.6), suggesting that these participants took the task seriously.

More important, as shown in Figure 2, participants in the generic condition exhibited greater transfer than in the baseline condition (the baseline condition is represented by the dashed line); this was not the case for the concrete condition. Transfer scores across the baseline, concrete, and generic conditions were subjected to a one-way between-subjects analysis of variance (ANOVA) followed by post hoc Bonferroni tests. The analyses indicated that there was a significant effect of the training condition on transfer, F(2, 58) = 15.98, p < .001, $\eta_p^2 = .36$. In particular, transfer scores in the generic condition exceeded those in the baseline and the concrete conditions, ps < .002, d = 2.08 and d = 1.12, respectively, whereas transfer scores in the concrete condition did not exceed those in the baseline condition, p > .37, d = 0.51.

In addition, the ability to match analogous elements differed dramatically across conditions. In the generic condition, 90% of the participants correctly matched elements from the learning instantiation to the transfer instantiation, whereas only 24% of participants in the concrete condition made the correct match, not better than what is expected from chance guessing (33%). The difference across condition is clearly significant, $\chi^2(1, N=41)=18.2, p<0.001$. These findings suggest that the concreteness of the learning instantiation hindered participants' ability to align the common structure of the learning and transfer domains, which in turn led to transfer failure.

Alternatively, one could argue that the inability to transfer from the concrete instantiation was not due to concreteness per se, but to characteristics of this particular concrete instantiation that may have hindered participants' learning of the conceptual rules. Although participants in the concrete and generic conditions scored equally highly in the learning phase, it is possible that their learning was at a functional ceiling. Perhaps the generic learners actually learned more than the concrete learners, and this difference in learning drove the differences in transfer. Although we intended the concrete instantiation to be intuitively easier than the generic instantiation, it is possible that the concrete instantiation made learning more difficult. To eliminate this possibility, we ran

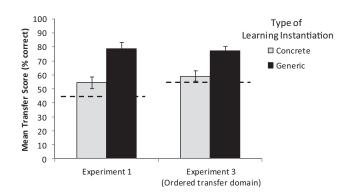


Figure 2. Mean transfer scores in Experiments 1A and 3. Error bars represent standard error of the mean. The dashed lines represent the mean transfer scores in the baseline conditions of Experiments 1 and 3.

the following control study, Experiment 1B, in which participants learned either the concrete or generic instantiation under conditions of minimal training.

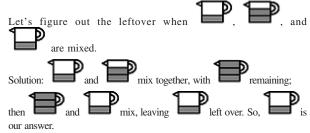
Experiment 1B

This experiment was very similar to the learning phase of Experiment 1A except that training was shortened considerably to find evidence of any possible differences in ease of learning. Participants were presented with the rules and only one additional example. Then, they were given the same 24-question multiple-choice test that was given to participants in Experiment 1A.

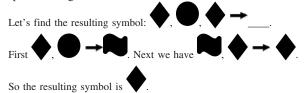
Method

Participants. Forty undergraduate students (27 women, 13 men, M age = 19.1 years, SD = 0.78) participated in this experiment.

Materials and design. As in Experiment 1A, the task was to learn the concept of a commutative mathematical group of order 3. Half of the participants learned the concrete instantiation described in Experiment 1A, and the other half learned the generic instantiation described in Experiment 1A. Training and testing were isomorphic across the two conditions. Training consisted of an introduction describing the cover story and explicit presentation of each of the rules in Table 2. In addition to stating the rules, one example was shown. This example for the concrete condition is presented below.



In the generic condition, participants were presented with the analogues to the rules and were given an example. Below is the example for the generic condition.



After training, the participants were given the same 24-question multiple-choice test described in Experiment 1A.

Procedure. The procedure was similar to that of Experiment 1A. Training and testing consisted of approximately 50 slides that were presented to individual participants via computer. The entire task took approximately 11 minutes to complete.

Results and Discussion

The results of the experiment show that participants in both conditions were able to learn. Mean scores were significantly above the chance score of 37.5%, one-sample ts(19) > 6.36, ps <

.001, ds > 1.42. However, participants in the concrete condition $(M_{\text{correct}} = 80.6\%, SD = 17.6)$ scored substantially higher than participants in the generic condition $(M_{\text{correct}} = 62.5\%, SD = 17.6)$, independent-samples t(38) = 3.26, p < .003, d = 1.06. These results suggest that the concrete instantiation used in Experiment 1A did not hinder learning; rather, it facilitated learning compared with the generic instantiation.

Therefore, the inability to transfer from the concrete instantiation demonstrated in Experiment 1A cannot be attributed to difficulty learning this domain. When participants were taken off ceiling by minimizing training, those in the concrete condition scored higher than those in the generic condition, suggesting that the concrete instantiation did not hinder learning; it elicited more efficient learning than the generic instantiation.

The purpose of the next three experiments was to further test the hypothesis that concreteness of the learning instantiation hinders transfer, while eliminating alternative explanations of the results of Experiment 1A stemming from possible similarity differences across conditions.

First, it is possible that numerical properties present in the concrete instantiation but absent in both the generic and target instantiations may have created differential similarity across conditions. In the concrete condition, the storyline of combining measuring cups of fluid communicates quantitative or numerical properties that are absent from the transfer domain; this might lead the learner to interpret the structures of the two instantiations as categorically different. For participants in the generic condition, neither the learning instantiation nor the transfer instantiation have any obvious numerical characteristics; therefore, there may be no perceived divergence in structure. If transfer difficulties stem only from mismatching structural interpretations between a numerical instantiation (used in learning) and a non-numerical instantiation (used in transfer), then transfer from a domain involving only numbers and arithmetic should be at least as poor as transfer from the concrete instantiation of Experiment 1A. The purpose of Experiments 2 was to test this possibility.

Experiment 2

This experiment considered transfer when the concept of a mathematical group was instantiated using the integers 0, 1, and 2. According to the results of the concreteness ratings, this instantiation communicated more information than the generic instantiation, but less than the concrete instantiation used in Experiment 1A (see number instantiation in Table 1 and also Experiments A-1 and A-2 of the online Supplemental Materials).

Method

Participants. Participants were 40 undergraduate students (17 women, 23 men, M age = 19.9 years, SD = 1.44).

Materials, design, and procedure. The design, materials, and procedure were similar to those of Experiment 1A. Two between-subjects conditions were included: baseline as in Experiment 1A and number. In the number condition, participants were told that they would learn a mathematical concept, addition modulo 3. In this condition, the group structure was instantiated with the integers 0, 1, and 2; 0 is the identity element and is added as in regular addition: 0 + 0 = 0, 0 + 1 = 1, and 0 + 2 = 2. Also,

1+1=2. However, a sum greater than 2 is never obtained. Instead, one would cycle back to 0, such that 1+2=0 and 2+2=1. Training and testing of this instantiation were completely isomorphic to those of the concrete and generic conditions of Experiment 1A. After this learning phase, participants were presented with the transfer instantiation. They received the same test as in the previous experiments and were asked to align analogous elements across domains.

Results and Discussion

Participants in the number condition successfully learned the rules. Learning scores were significantly above a chance score of 37.5% ($M_{\text{correct}} = 79.4\%$, SD = 13.4), one-sample t(19) = 14.0, p < .001, d = 3.12. In addition, participants were accurate on the unrelated arithmetic task in the baseline condition ($M_{\text{correct}} = 94.3\%$, SD = 6.54).

Transfer scores in the number condition were above those of the baseline group ($M_{\text{correct}} = 68.3\%$, SD = 24.0, and $M_{\text{correct}} = 41.7\%$, SD = 18.5, respectively), independent-samples t(38) = 3.93, p < .001, d = 1.28. This effect size was clearly larger than the effect size for the concrete condition in Experiment 1A (d = 0.51), which indicates that the possible numerical content of the concrete condition of Experiment 1A was not responsible for the low transfer found in Experiment 1A.

In addition, 55% of participants in the number condition correctly matched analogous elements across domains, whereas only 24% of the participants of the concrete condition of Experiment 1A did so. There was also a high correlation between matching performance and test score ($r_{\rm pb}=.87$). The mean transfer score for those who made the correct matching was 86.7% (SD=14.8), and the mean score for those who did not make the correct matching was 45.8% (SD=8.07). Those who were able to correctly match elements scored significantly higher than those who did not, independent-samples t(18)=7.43, p<.001, d=3.52.

The results of Experiment 2 indicate that transfer difficulty from the concrete domain of Experiment 1A is unlikely to stem from perceived structural differences between a numerical and a non-numerical instantiation. If this distinction between numerical and non-numerical instantiation alone created an obstacle to transfer, we would expect transfer scores in the number condition to be no better than baseline, as were the transfer scores in the concrete condition of Experiment 1A. However, this was not the case.

Although it appears that transfer is possible from a numerical instantiation to a non-numerical isomorph, we wanted to further examine the possibility that extraneous structural aspects of the concrete instantiation of Experiment 1A may have led to transfer problems. In particular, although both the concrete and generic instantiations were isomorphic to the target instantiation, the concrete instantiation communicated order information about the elements that both the generic and target domains did not. It is natural to order the concrete elements as follows:

municated order of the elements in the generic and target domains. Order is an extraneous structural aspect of the concrete instantiation. It is possible that an ordinal base domain and a nonordinal target domain led to mismatching interpretations of structure and hence a representational incongruence (similar to the symmetric/

asymmetric or discrete/continuous incongruences of the Bassok et al. studies described earlier). This incongruence may have resulted in transfer failure for participants in the concrete condition. The goal of Experiment 3 was to consider whether differential transfer stems from a representational incongruence between the base and target domains. In this experiment, we examined transfer to an ordinal target domain from the concrete and generic instantiations used in Experiment 1A. If transfer difficulties stem solely from mismatching structural interpretations and not from concreteness, then there should be more transfer in the concrete condition than in the generic condition. On the other hand, if concrete instantiations hinder transfer but generic, minimal instantiations allow for transfer, there may be better transfer in the generic condition even when the target communicates order information missing from the generic instantiation.

Experiment 3

Method

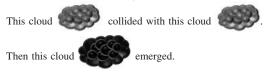
Participants. Sixty-eight undergraduate students (34 women, 34 men, M age = 19.4 years, SD = 1.75) participated in this experiment.

Materials, design, and procedure. The design, materials, and procedure were similar to those of Experiment 1A. Participants were randomly assigned to three between-subjects conditions: concrete, generic, or baseline. As in Experiment 1A, there was a learning phase followed by a transfer phase. The learning phase was identical to that of Experiment 1A; participants were given the concrete, generic, or baseline tasks. After learning, participants were presented with a novel isomorphic instantiation of a mathematical group that was different from the one used in Experiment 1A.

The target domain was designed to communicate ordinal information. It was described as a phenomenon observed in another solar system in which atmospheric clouds collide and a reaction takes place after which one cloud emerges. There were three types

of clouds:

. The appearance of these elements suggests a natural ordering by both color saturation and size. Participants were told that the type of cloud that emerges from a collision follows specific rules (which were the rules of a mathematical group). As in Experiment 1A, in both the experimental learning conditions, participants were told that the rules were like those of the system they had just learned. No explicit training in the transfer domain was given; instead, participants were shown a series of examples from which the rules could be deduced, such as the following.



Participants were asked to figure out the rules by which types of clouds emerged by using their knowledge of the system they had learned previously. Then, they were tested with a 24-question multiple-choice test, isomorphic to their test in the learning phase, but using the elements of this transfer domain. Following the test,

participants were asked to match analogous elements across the learning and transfer domains.

To recap, if differences in extraneous structure between the learned and transfer domains hindered transfer in the concrete condition of Experiment 1A, then effects should reverse in the current experiment. Recall that in Experiment 1A, the concrete condition had ordinal information, whereas the transfer domain did not. In contrast, in the current experiment, both the concrete and the transfer domains convey ordinal information.

Results and Discussion

Participants in both the concrete and generic conditions successfully learned the rules; mean scores were significantly above the chance score of 9 (37.5%), one-sample ts(21) > 11.63, ps < .001, ds > 2.47. There were no differences in learning scores between the concrete and generic conditions ($M_{\rm correct} = 79.7\%$, SD = 10.7, and $M_{\rm correct} = 73.9\%$, SD = 14.7, respectively), independent-samples t(42) = 1.52, p > .13. In addition, participants were accurate on the unrelated arithmetic task in the baseline condition ($M_{\rm correct} = 95.2\%$, SD = 7.14).

As in Experiment 1A, participants in the generic condition exhibited greater transfer than in the baseline condition, whereas participants in the concrete condition did not. The results of a one-way between-subjects ANOVA indicate that condition had a significant effect on transfer scores, F(2, 65) = 14.30, p < .001, $\eta_p^2 = .31$. Transfer scores in the generic condition ($M_{\text{correct}} = 77.1\%$, SD =14.6) exceeded those in the baseline ($M_{\text{correct}} = 54.5\%$, SD = 11.9) and concrete ($M_{\text{correct}} = 59.1\%$, SD = 18.1) conditions, post hoc Bonferroni tests, ps < .002, d = 1.74 and d = 1.12, respectively, whereas transfer scores in the concrete condition did not exceed those in the baseline condition, p > .91, d = 0.31. In the concrete condition, transfer scores had a bimodal distribution. Approximately 77% of participants scored below 59% correct ($M_{\text{correct}} = 50.2\%$, SD =7.58), whereas the remaining minority transferred well and scored above 83% correct ($M_{\text{correct}} = 89.2\%$, SD = 4.75). Nonparametric statistics were used to account for the non-normality of the distribution. Transfer scores were higher in the generic condition than in the concrete condition, Mann–Whitney U test, U = 92.0, p < .001. Figure 2 presents test scores across condition with all participants in the concrete condition (high and low performers) presented together. The fact that some participants in the concrete condition (23%) scored well on the transfer test should not be completely surprising. Unlike Experiment 1A, the concrete and target domains shared additional structure (i.e., order); therefore, one would expect that transfer performance might be better than in situations with less structural overlap such as that of Experiment 1A.

Performance on matching analogous elements also differed across conditions. In the generic condition, 86% of participants correctly matched elements, whereas only 36% in the concrete condition made correct matches, not better than chance (33%). The difference across condition is significant, $\chi^2(1, N = 44) = 11.6$, p < .001. These results suggest that even when the base and target domains shared extraneous structural information, concreteness hindered transfer. At the same time, the generic instantiation allowed for transfer, suggesting that generic instantiations may facilitate transfer even in the presence of mismatches in extraneous structure across domains.

The results of Experiments 2 and 3 support the argument that concrete instantiations, rich in extraneous information, can hinder

transfer, whereas generic instantiations, sparse in extraneous information, can allow for successful transfer. The goal of Experiment 4 was to directly test the hypothesis that concreteness hinders transfer. If concreteness was responsible for lower transfer performance in Experiment 1A, then reducing the degree of concreteness should increase transfer scores. In Experiment 4, we used elements similar to the measuring cups of Experiment 1A, while minimizing concreteness by removing the storyline. The handles and spouts were also removed from the elements because these features are arguably part of the concreteness. Removal of the storyline and handles likely minimizes the amount of contextualized real-world information activated in the minds of participants. At the same time, the elements of Experiment 4 convey the same numerical information as those in the concrete condition of Experiment 1A. Therefore, the elements presented in the context of the structural rules are likely to activate more information than the arbitrary generic symbols. As a result, this instantiation is likely to be less concrete than the concrete instantiation but more concrete than the generic instantiation used in the previous experiments. This intuition was supported by the concreteness ratings of Experiments A-1 and A-2 of the online Supplemental Materials (see Table 1 and the online Supplemental Materials).

Experiment 4

Method

Participants. Forty undergraduate students (18 women, 22 men, M age = 19.8 years, SD = 2.21) participated in this experiment.

Materials, design, and procedure. The design, materials, and procedure were similar to those of Experiment 1A. Two between-subjects conditions were considered: baseline as in Experiment 1A and the reduced-concreteness condition. In the reduced-concreteness condition, participants learned an instantiation of a mathematical group that was similar to that of the concrete condition of Experiment 1A with one critical exception. The storyline referring to measuring cups was replaced with the storyline from the generic condition. The elements of the group

were represented as , and ; the wording of all questions and examples was exactly the same as that of the generic condition of Experiment 1A. As in the two previous experiments, there was a learning phase followed by a transfer phase, each with a 24-question multiple-choice test. Training and testing were isomorphic to those of the previous experiments.

Results and Discussion

One participant was removed from the baseline condition because of scoring more than 2.5 standard deviations from the mean of the group. The participants in the reduced-concreteness condition successfully learned the rules; scores were significantly above a chance score of 37.5%, ($M_{\text{correct}} = 82.3\%$, SD = 14.8), one-sample t(19) = 13.5, p < .001, d = 3.03. In addition, participants were accurate on the unrelated arithmetic task in the baseline condition ($M_{\text{correct}} = 92.3\%$, SD = 9.18).

Importantly, transfer scores in the reduced-concreteness condition were well above those of the baseline group ($M_{\rm correct} = 69.6\%$, SD = 24.5, and $M_{\rm correct} = 41.0\%$, SD = 12.7, respec-

tively), independent-samples t(37) = 4.53, p < .001, d = 1.49. This effect size was markedly above that in the concrete condition of Experiment 1A (d = 0.51). Therefore, a reduction in concreteness clearly resulted in increased transfer.

Similar to Experiments 1–3, the results suggest that concreteness hindered structural alignment. In the reduced-concreteness condition, 75% of participants correctly matched analogous elements across domains (recall that only 24% of the participants of the concrete condition of Experiment 1A did so). In addition, there was a significant correlation between matching ability and test score ($r_{\rm pb}=.70$). The mean transfer score for those who made the correct matching was 79.2% (SD=20.0), whereas the mean score for those who did not make the correct matching was 40.8% (SD=8.01), independent-samples t(18)=4.11, p<.002, d=2.24.

The results of Experiment 4 clearly demonstrate that both storyline and appearance of elements contribute to concreteness and subsequently affect transfer. Replacing the familiar measuring cup storyline with the more generic storyline of a symbolic language resulted in more efficient transfer (29% above the baseline, d=1.49) compared with the concrete condition of Experiment 1A (9% above the baseline, d=0.51). Therefore, simply changing the verbal description of an instantiation resulted in a substantial increase in transfer. In addition, the reduced-concrete instantiation differed from the generic condition only in the appearance of the elements and was rated more concrete (see Table 1) than the generic instantiation. This also resulted in differences in transfer: Transfer was more efficient in the generic instantiation of Experiment 1A (34% above the baseline, d=2.08) than in the reduced-concrete condition.

Taken together, the results of Experiments 1–4 suggest that transfer difficulties arise from the concreteness of the learned instantiation. Concreteness appears to hinder structural alignment between domains. Participants in the concrete conditions were consistently less likely to correctly match analogous elements of the learning and transfer domains than were participants in the generic condition. Because correct structural alignment across domains would place learning elements in correspondence with their transfer analogues, the inability to match elements implies that those learners were unable to align structure from the learning domain to the transfer domain.

Our hypothesis is that concreteness hinders transfer by hindering structural alignment between isomorphs. In most theories of analogical transfer, structural alignment is a necessary component (e.g., Gentner, 1983, 1989; Gentner & Holyoak, 1997; Gentner & Markman, 1997; Holyoak & Thagard, 1997). It has also been demonstrated that alignment is not sufficient for successful transfer (Novick & Holyoak, 1991); giving college students the mapping of analogous elements across solved mathematical word problems and novel unsolved problems does not necessarily guarantee successful transfer. If concreteness hinders structural alignment and the inability to align common structure alone is responsible for transfer failure, then assisting participants with structural alignment should increase transfer. The goal of Experiment 5 was to test this prediction.

Experiment 5

In Experiment 5, some participants were assisted with structural alignment across the learning and transfer domains and some were

not. To aid successful structural alignment, we gave participants the correct correspondence of analogous elements.

Method

Participants. One hundred undergraduate students (42 women, 58 men, M age = 19.8 years, SD = 1.89) participated in this experiment.

Materials, design, and procedure. The design and procedure were identical to those of Experiment 1A with one critical exception. Half of the participants, who learned either the concrete or generic instantiation, were given the correspondence of analogous elements across learning and transfer domains, whereas the other half were not given this correspondence. Therefore, participants were randomly assigned to one of five between-subjects conditions (baseline, generic, concrete, generic with alignment, or concrete with alignment). Participants in the generic with alignment condi-

tion were explicitly told that is like , is like

and is like Participants in the concrete with alignment condition were shown the analogous correspondences. In the generic and concrete conditions, the correspondences were not given. With the exception of stating the element correspondences, all training and testing was identical to those of Experiment 1A.

Results and Discussion

One participant in the concrete with alignment condition was removed from the analysis for scoring more than 2.5 standard deviations below the mean learning score for that condition. Participants in all conditions exhibited successful learning, with mean scores significantly above chance score of 37.5%, one-sample ts > 26.02, ps < .001, ds > 4.11. The generic and concrete conditions yielded equivalent learning scores ($M_{\text{correct}} = 85.2\%$, SD = 11.6, and $M_{\text{correct}} = 85.5\%$, SD = 9.46, respectively), t(77) = 0.110, p = .91. In addition, participants were accurate on the unrelated arithmetic task in the baseline condition ($M_{\text{correct}} = 94.3\%$, SD = 5.45).

At the same time, as shown in Figure 3, there were clear differences in transfer across conditions. These findings were

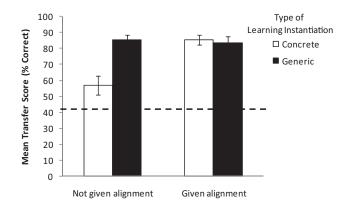


Figure 3. Mean transfer scores in Experiment 5. Error bars represent standard error of the mean. The dashed line represents the mean transfer score for the baseline condition.

supported by a 2 (instantiation: generic or concrete) × 2 (alignment: yes or no) ANOVA. The results revealed significant effects of both condition, F(1, 75) = 13.90, p < .001, $\eta_p^2 = .16$, and alignment, F(1, 75) = 14.13, p < .001, $\eta_p^2 = .16$, and a significant interaction between the two, $F(1, 75) = 18.40 p < .001, \eta_p^2 = .20.$ The interaction is of utmost importance; whereas there was a clear advantage for the generic instantiation in the no-alignment conditions, independent-samples t(38) = 4.96, p < .001, d = 1.61, the advantage disappeared in the alignment condition, t(37) = 0.482, p = .63, d = 0.16. In addition, to compare transfer with the baseline condition, we submitted transfer scores to a one-way ANOVA with condition as a factor. Condition had a significant effect on transfer, F(4, 94) = 38.29, p < .001, $\eta_p^2 = .62$, with scores above the baseline group in each of the conditions, post hoc Bonferroni tests, p < .02, d = 0.93, in the concrete condition, and ps < .001, ds > 3.66, for the other conditions. There were no differences between scores in the generic, generic alignment, and concrete alignment conditions, post hoc Bonferroni tests, ps = 1.00. Scores in each of these conditions were significantly higher than those in the concrete condition, post hoc Bonferroni test, ps < .001, ds > 1.44.

These results indicate that assisting the alignment allowed participants in the concrete condition to transfer successfully. Therefore, although mapping of structure could be spontaneous when the training instantiation is generic, it is not spontaneous when the training instantiation is concrete. Overall, results of Experiments 1–5 point to concreteness preventing the spontaneous alignment of the training and transfer domains, thus preventing the transfer of learning.

General Discussion

In the present research, we investigated analogical transfer of a well-defined abstract concept (i.e., a commutative mathematical group of order 3) and tested a hypothesis that concreteness of the learning domain hinders subsequent transfer. The results of five experiments provide consistent support for this hypothesis. The results further suggest that concreteness hinders transfer of the studied concept by creating an obstacle to structural alignment across the learned and target domains.

It is well accepted that the degree of similarity between the learned and target domains mediates the likelihood of successful analogical transfer (e.g., Holyoak & Koh, 1987; Ross, 1987, 1989). The present findings suggest that in the absence of any overt similarities between base and target, generic learning domains can outperform more concrete ones with respect to analogical transfer.

In Experiment 1A, participants who learned a concrete instantiation of a novel mathematical concept failed to transfer, whereas participants who learned a generic instantiation of the same concept successfully transferred. In addition, participants in the concrete condition, unlike those in the generic condition, were unable to match analogous elements across the learned and target instantiations, suggesting that they were unable to align common structure. Is it possible that these findings stemmed from some other properties of the stimuli? One such possibility is that concrete instantiation was less similar to the transfer domain than the generic instantiation. Subsequent experiments attempted to eliminate this and several alternative explanations.

Experiment 2 demonstrated successful transfer from an instantiation involving only numbers. These results eliminate the possibility that transfer failed from the concrete instantiation of Experiment 1A because of a mismatch between a base domain conveying numerical information and a target domain that did not. The results of Experiment 3 demonstrated that transfer from the generic instantiation to an ordered target instantiation was successful, whereas transfer from the concrete instantiation again failed even in the context of matching extraneous structure (i.e., order). The findings of this experiment are particularly important because they demonstrate the potential for structural knowledge to be transferred from a generic instantiation to novel instantiation that differs not only on irrelevant surface features but also on extraneous structural aspects. Transfer from the generic surpassed that from the concrete even when the concrete and target shared more structural aspects than were shared between the generic and target.

Experiment 4 was a direct test of the hypothesis that concreteness hinders transfer. In the experimental condition, the storyline was that of the generic instantiation of Experiment 1A and the elements were essentially the measuring cups without spouts and handles. This instantiation was rated lower in concreteness than the concrete instantiation (see Table 1). When the concreteness of an instantiation was reduced by simply changing the storyline, transfer performance improved. This is an interesting and novel finding that needs further examination in future research. In particular, more research is needed to further understand how aspects of storyline and appearance of elements contribute to concreteness and affect transfer.

It could be argued that the difficulty transferring from the concrete instantiation of Experiment 1A may stem from the possibility that concreteness hindered learning and in turn hindered transfer. It has been demonstrated that irrelevant information can hinder learning (e.g., Mayer, Griffith, Jurkowitz, & Rothman, 2008; Sanchez & Wiley, 2006). However, in Experiment 1A, participants in both the concrete and generic conditions demonstrated comparable mastery of the learning domain. Furthermore, when only minimal training was given in Experiment 1B, participants in the concrete condition scored higher than those in the generic condition, suggesting that this concrete instantiation did not hinder but rather facilitated learning. Therefore, differential transfer cannot be attributed to differential levels of learning. This point extends the findings of the present research beyond those of prior research that demonstrated a negative effect of concreteness on transfer (Goldstone & Sakamoto, 2003; Sloutsky et al., 2005). The concrete instantiations used in previous work hindered learning as well as transfer.

It could also be argued that the concrete instantiation provided a scaffold from which participants did not genuinely acquire the relational knowledge that defines the mathematical concept. Overscaffolding through the use of structured, concrete learning materials can lead to transfer failure (Martin & Schwartz, 2005). However, in Experiment 5 when participants were assisted with structural alignment, those who learned the concrete instantiation transferred as well as those who learned the generic instantiation. If relational structure was not learned, it is highly unlikely that the mere aligning of analogous elements in the transfer phase would result in successful performance on complex questions in the novel domain. It is important to note that the manipulation of assisting structural alignment took place only during the transfer phase.

Therefore, transfer difficulties from the concrete instantiation likely stem from difficulties with structural alignment involving the concrete instantiation and not from difficulties with learning. This point is not to suggest that success on this transfer task implies that participants in both the generic and concrete conditions formed structurally identical representations. Rather, we suggest that participants in the concrete condition acquired sufficient structural knowledge to transfer this knowledge to a novel isomorph when they were assisted with alignment. It is possible that there are limitations to participants' representations of structural knowledge that would hinder their ability to problem solve in nonisomorphic situations, but most participants in both conditions acquired structural knowledge sufficient enough to answer questions about a novel analogous instantiation.

The results of Experiments 1–5 point to an important regularity: When structural knowledge is acquired through a concrete instantiation, successful analogical transfer is less likely than when knowledge is acquired through a more generic instantiation. The correlation of transfer scores and correct matching of analogous elements across learning and transfer domains suggests that concreteness hindered learners' ability to align common structure, which in turn created an obstacle to transfer.

Concreteness and Theories of Analogical Transfer

The reported findings reveal an important theoretical aspect of analogical transfer that supplements existing transfer theories. With regard to complex knowledge such as mathematical knowledge or problem-solving strategies, prevailing theories predict transfer based on characteristics of both the learning and transfer domains. Specifically, transfer is more likely between domains that share common features or contexts (Holyoak & Koh, 1987), similar elements (Ross, 1987, 1989), or common interpretations of structure (Bassok & Olseth, 1995; Bassok, Wu, & Olseth, 1995). We have identified a specific, measurable characteristic of the learning instantiation that affects the likelihood of transfer. Generic instantiations of an abstract concept result in markedly better analogical transfer than concrete instantiations. Concreteness of the learning instantiations used in Experiments 1, 2, and 4, as measured by reported informativeness, appears to be correlated highly with transfer performance in the target domain, accounting for 94% of variance in transfer scores (see Figure 4) and 89% of

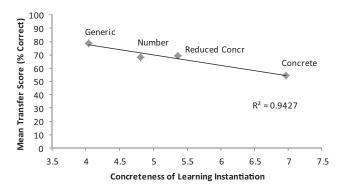


Figure 4. Transfer as a function of concreteness for Experiments 1A, 2, and 4. Note that concreteness of the learning instantiation is the mean rating of communicated information for each domain.

the variance in effect size $(R^2 = .89)$.

The proposition that concreteness of the learning instantiation hinders analogical transfer may allow us to predict the likelihood of transfer to an arbitrary instantiation given a choice of possible learning instantiations. Therefore, given a set of potential learning instantiations and an unspecified transfer domain, it is possible to predict the candidate most likely to yield successful transfer by examining the amount of communicated information. For other theories, transfer predictions are dependent on both the learning and transfer instantiations. The present finding is not only theoretically novel, it has practical importance as potential transfer domains are not necessarily known.

In addition to contributing to theories of analogical transfer, the present findings are related to work that has investigated effects of extraneous information on learning. For example, when seductive details (i.e., interesting, yet irrelevant information) were included in computer presentations designed to teach aspects of human digestion or viral infections, undergraduate students performed worse on subsequent tests of problem solving than students who were not presented with the seductive details (Mayer et al., 2008; see also Harp & Mayer, 1998). These studies suggest that irrelevant aspects of the learning instantiation can hinder learning and problem solving within that domain. Our findings demonstrate that extraneous, nonessential information in the learning instantiation that does not hurt learning within that domain does hurt transfer to a novel domain.

Broader Implications

These findings could also have important implications for educational design. Because abstract concepts such as mathematical and scientific concepts are often difficult to acquire, concrete instantiations that tap prior knowledge may appear appealing for teaching because they may facilitate initial learning. In addition, concrete instantiations may be more engaging for the learner. However, engagement and ease of initial learning do not translate into successful transfer from concrete material. In contrast, assisting learners with structural alignment did result in transfer success in our studies. However, outside of controlled learning situations such as classrooms, it is unlikely that structures can be aligned for the learner. In most unfamiliar real-world situations, it is not known a priori what structural knowledge can be applied. Furthermore, prior research has demonstrated that aligning two concrete instantiations during learning did not result in successful transfer to a third instantiation (Kaminski et al., 2008).

We posit that more generic instantiations result in more transfer than more concrete instantiations, but some may argue that there must be some constraint in this proposition on the degree of desired abstraction (i.e., minimization of extraneous information). First, to reiterate, we posit a generic advantage for analogical transfer of well-defined structural concepts such as mathematical concepts. We are not necessarily suggesting that the same advantage will always hold for simpler, very familiar relations. Comprehension of analogies and metaphors are evidence that people can transfer simple relations across concrete domains. Even young children can transfer familiar, simple relations to novel situations (e.g., Gentner, 1977). Our study and proposition are concerned with transfer of well-defined, complex concepts that are typically comprised of systems of relations. Furthermore, although we ex-

pect these findings to generalize to other mathematical concepts (and to other well-defined structured abstract concepts), the reported study examines only a particular concept, the commutative group of order 3. Therefore additional research is needed to investigate the generality of reported findings.

With regard to possible constraints on abstraction, it is possible that generic instantiations (which are often expressed through symbols) may be difficult to learn. Zhang and Norman (1995) suggested that a symbolic representation (e.g., "3" vs. "2") may put greater demand on working memory than more direct external analogue representations (e.g., ||| vs. ||). Therefore, increased demands on working memory may make some generic instantiations more difficult to learn. Although in our study the generic instantiation was as learnable as the concrete instantiation, this may not be the case if the number of elements or conceptual complexity is substantially greater. This is an important issue to consider in future research.

Therefore, although we propose that learning a generic instantiation provides a transfer advantage over learning a concrete instantiation, we acknowledge that pedagogical practice may involve balancing generic instantiations and concrete instantiations. Our caution is that grounding abstract concepts deeply in concrete contexts can limit learners' ability to apply that knowledge elsewhere. It has been shown that lessening the concreteness of the appearance of elements through the course of learning (i.e., concreteness fading) can lead to better transfer than learning either a statically concrete or statically generic depiction (Goldstone & Son, 2005b). Research on concreteness fading has investigated the effects of only the physical appearance of the learning instantiation; further research is needed to consider how and when concrete instantiations that vary in context and as well as appearance could be introduced to allow for analogical transfer.

Conclusion

The present work identifies concreteness of the learning instantiation as an important factor affecting analogical transfer. These findings demonstrate the obstacles that can arise when attempting to transfer from concrete instantiations and the potential power of learning a generic instantiation for analogical transfer. At the same time, it is important to note that although we expect that they are generalizable to analogical transfer for many well-defined abstract concepts, the present findings pertain to a single concept. Therefore, future research will need to test the generalizability of these findings to other concepts, further examine the contribution of element concreteness and storyline concreteness, and investigate transfer to nonisomorphic situations.

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