

10 Popular Evolutionary Algorithm Variants

Popular Evolutionary Algorithm Variants

- we're going to take a tour of these main EA variants:
 - Genetic Algorithms (GA and SGA)
 - Evolution Strategies (ES)
 - Evolutionary Programming (EP)
 - Genetic Programming (GP)
- and later on in the course we'll look more recent variants, such as:
 - Particle Swarm Optimisation (PSO)
 - Ant Colony Optimisation (ACO)

Genetic Algorithms: Quick Overview

- developed in 1960s by John Holland
- initially conceived as a means of studying adaptive behaviour
- typically applied to:
 - discrete function optimization
 - benchmarking new algorithms
 - straightforward problems that allow binary representation
- have been lots of variants developed since
 - as we've seen
- so Holland's GA is now known as the **Simple Genetic Algorithm (SGA)**

Simple Genetic Algorithm (SGA): Technical Summary Tableau

representation	bit-strings
recombination	1-point crossover
mutation	bit flip
parent selection	fitness proportional – implemented by Roulette Wheel
survivor selection	generational

SGA Reproduction Cycle

- **select parents** for the mating pool
 - (size of mating pool = population size)
- **shuffle** the mating pool
- **apply crossover** for each consecutive pair with probability p_c , otherwise copy parents
- **apply mutation** for each offspring (bit-flip with probability p_m independently for each bit)
- **replace** the whole population with the resulting offspring

SGA Example

from Goldberg 1989:

- simple problem: max x^2 over $\{0, 1, \dots, 31\}$
- GA approach:
 - representation: binary code, e.g., $01101 \leftrightarrow 13$
 - population size: 4
 - 1-point crossover, bitwise mutation
 - roulette wheel selection
 - random initialisation
- let's see one generational cycle done by hand...

X² Example: Selection

String no.	Initial population	x Value	Fitness $f(x) = x^2$	$Prob_i$	Expected count	Actual count
1	0 1 1 0 1	13	169	0.14	0.58	1
2	1 1 0 0 0	24	576	0.49	1.97	2
3	0 1 0 0 0	8	64	0.06	0.22	0
4	1 0 0 1 1	19	361	0.31	1.23	1
Sum			1170	1.00	4.00	4
Average			293	0.25	1.00	1
Max			576	0.49	1.97	2

X² Example: Crossover

String no.	Mating pool	Crossover point	Offspring after xover	x Value	Fitness $f(x) = x^2$
1	0 1 1 0 1	4	0 1 1 0 0	12	144
2	1 1 0 0 0	4	1 1 0 0 1	25	625
2	1 1 0 0 0	2	1 1 0 1 1	27	729
4	1 0 0 1 1	2	1 0 0 0 0	16	256
Sum					1754
Average					439
Max					729

X^2 Example: Mutation

String no.	Offspring after xover	Offspring after mutation	x Value	Fitness $f(x) = x^2$
1	0 1 1 0 0	1 1 1 0 0	26	676
2	1 1 0 0 1	1 1 0 0 1	25	625
2	1 1 0 1 1	1 1 0 1 1	27	729
4	1 0 0 0 0	1 0 1 0 0	18	324
Sum				2354
Average				588.5
Max				729

The Simple GA

- has been subject of many (early) studies
- still often used as benchmark for novel GAs
- shows many shortcomings, such as:
 - representation is too restrictive
 - mutation and crossover operators are only applicable for bit-string and integer representations
 - selection mechanism is sensitive for converging populations with close fitness values
 - generational population model can be improved with explicit survivor selection

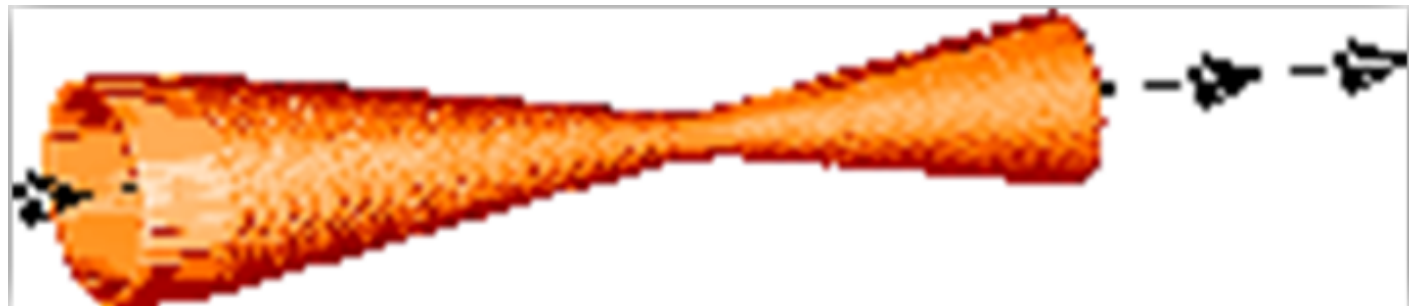
Evolution Strategies: Quick Overview

- developed by Rechenberg & Schwefel
- typically applied to:
 - numerical optimisation
- attributed features:
 - fast
 - good optimizer for real-valued optimisation
- special:
 - self-adaptation of mutation parameters is standard

An Historical Example: The Jet Nozzle Experiment

- task:
 - optimise the shape of a jet nozzle
- approach:
 - random mutations to shape, with selection

before:



after:



Evolution Strategies: Technical Summary Tableau

representation	real-valued vectors
recombination	discrete or intermediary
mutation	Gaussian (Normal) perturbation
parent selection	uniform random
survivor selection	(μ, λ) or $(\mu + \lambda)$

Evolution Strategies: (1+1) Example

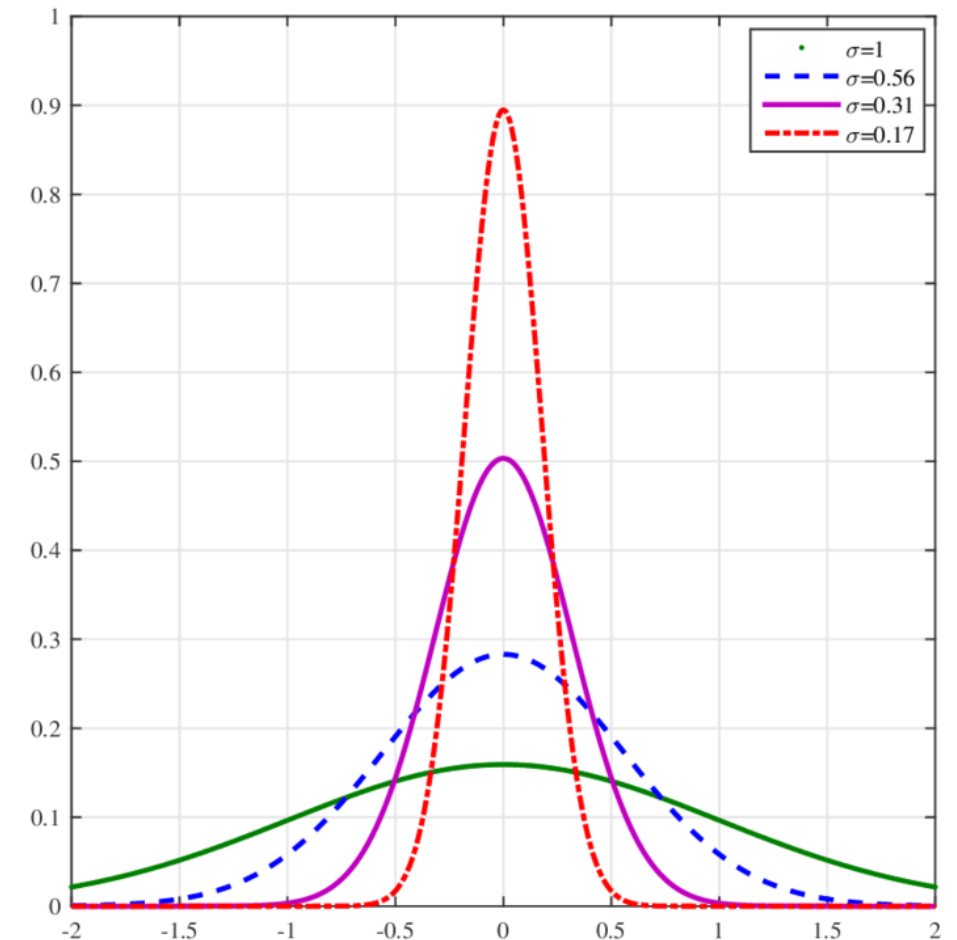
- a very simple early version
- task: maximise $f : \mathbb{R}^n \rightarrow \mathbb{R}$
- algorithm is a 'two-membered ES':
 - uses vectors from \mathbb{R}^n as genotypes
 - population size of 1
 - only uses mutation, creating one child
 - 'greedy' selection
 - accept offspring if fitter than parent
- (1,1) version always replaces the parent with the child

Evolution Strategies: Representation

- chromosomes consist of three parts:
 - object variables: x_1, \dots, x_n
 - strategy parameters:
 - mutation step sizes: $\sigma_1, \dots, \sigma_n$
 - rotation angles: $\alpha_1, \dots, \alpha_n$ (for correlated mutation)
- not every component is always present
- full size genotype: $\langle x_1, \dots, x_n, \sigma_1, \dots, \sigma_n, \alpha_1, \dots, \alpha_k \rangle$
where $k = n(n-1)/2$ (number of i, j pairs)

Evolution Strategies: Example: Mutation Mechanism

- random numbers drawn from normal distribution $N(0, \sigma)$
 - where 0 is the mean
 - and σ is the mutation step size
- σ is varied using the **1/5 success rule**:
- for some k, every k generations, the step-size should:
 - increase if 'too many' steps are successful ($>20\%$)
 - because this indicates that the search is too local
 - decrease if 'too few' steps are successful ($<20\%$)
 - because this indicates that the step size is 'too large'
 - and stay the same if exactly 20% of steps are successful



Evolution Strategies: Recombination

- creates one child
- acts per gene position by either:
 - averaging the parental values, or
 - selecting one of the parental values
- derives values from two or more parents, either:
 - uses the same two parents for all positions
 - selects two different parents for each position

Evolution Strategies: Names of Recombinations

	two fixed parents	two parents selected for each i
$z_i = (x_i + y_i) / 2$	local intermediary	global intermediary
z_i is x_i or y_i chosen randomly	local discrete	global discrete

Evolution Strategies: Parent Selection

- parents are selected by uniform random distribution
- so ES parent selection is unbiased
- every individual has the same probability of being selected

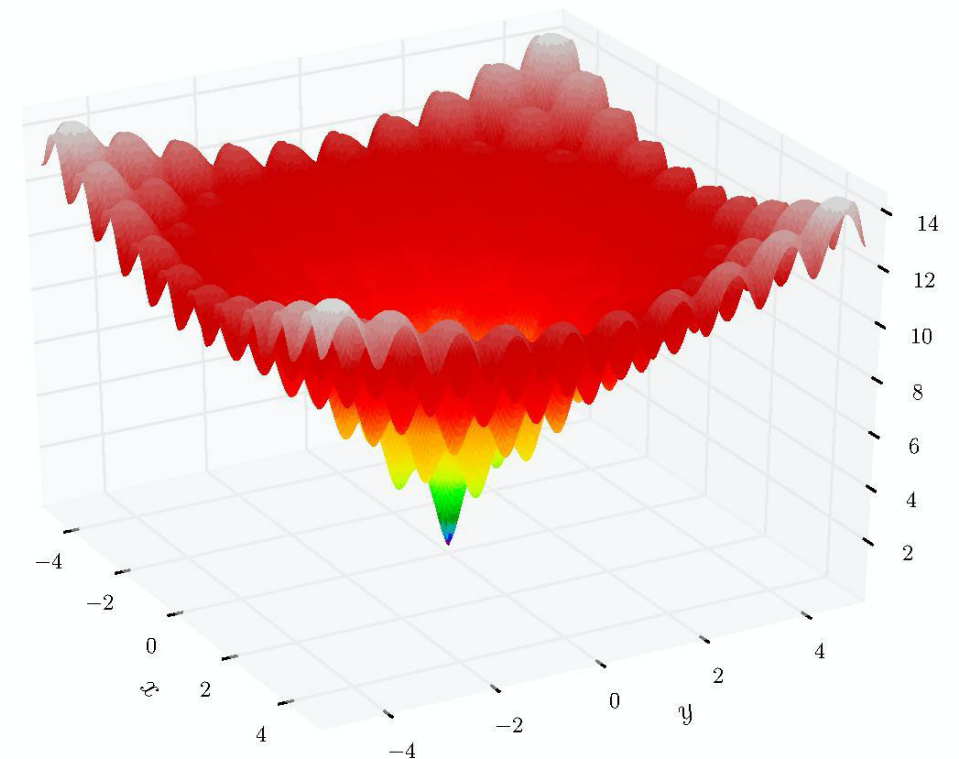
Evolution Strategies: Survivor Selection

- generally (μ, λ) is preferred to $(\mu + \lambda)$ because:
 - (μ, λ) discards all parents
 - so can in principle leave (small) local optima
 - if the fitness function changes over time, $(\mu + \lambda)$ selection preserves outdated solutions
 - so is less able to follow the moving optimum
 - $(\mu + \lambda)$ selection hinders self-adaptation, because misadapted strategy parameters may survive for a relatively large number of generations
- selective pressure in evolution strategies is very high because λ is typically much higher than μ
- typically an ES is a more aggressive optimizer than a (simple) GA

ES Example: The Ackley Function

evolution strategy:

- representation:
 - $-30 < x_i < 30$
- $(30, 200)$ survivor selection
 - $(\mu=30, \lambda=200)$
- termination:
 - after 200,000 fitness evaluations
- results:
 - average best solution is 7.48×10^{-8}
 - which is very good!



$$f(x) = -20 \cdot \exp\left(-0.2 \sqrt{\frac{1}{n}} \cdot \sum_{i=1}^n x_i^2\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$$

Evolutionary Programming: Quick Overview

- originally developed by Fogel et al in the 1960s
- why?
 - to simulate evolution as a learning process with the aim of generating artificial intelligence
- intelligence was viewed as adaptive behaviour
- so capability to predict is considered key to intelligence

Evolutionary Programming: Quick Overview

- typically applied to:
 - traditional EP: prediction by finite state machines
 - contemporary EP: (numerical) optimization
- attributed features:
 - very open framework:
 - any representation and mutation operators are OK
 - contemporary EP are 'crossbred' with ES techniques
 - consequently it's hard to say what 'standard' EP is
- special:
 - no recombination
 - self-adaptation of parameters standard (contemporary EP)

Evolutionary Programming: Technical Summary Tableau

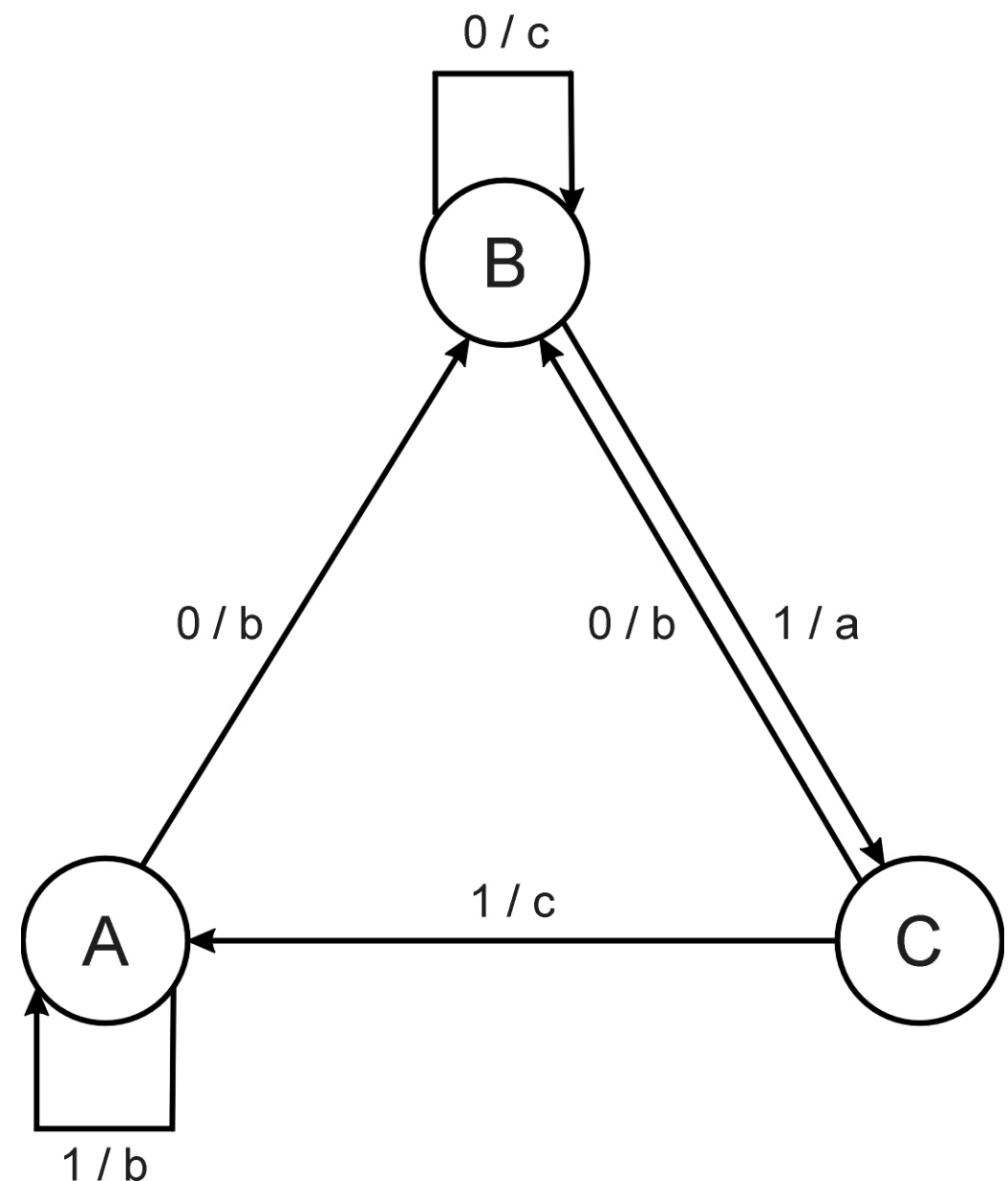
representation	real-valued vectors
recombination	none
mutation	Gaussian (Normal) perturbation
parent selection	deterministic (each parent produces one offspring)
survivor selection	probabilistic ($\mu+\mu$)

Evolutionary Programming: Prediction by Finite State Machines

- finite state machine (FSM):
 - states S
 - inputs I
 - outputs O
 - transition function $\delta : S \times I \rightarrow S \times O$
 - transforms input stream into output stream
- can be used for predictions, such as predicting the next input symbol in a sequence

Evolutionary Programming: FSM Example

- consider the FSM with:
 - $S = \{A, B, C\}$
 - $I = \{0, 1\}$
 - $O = \{a, b, c\}$
 - d , given by a diagram:

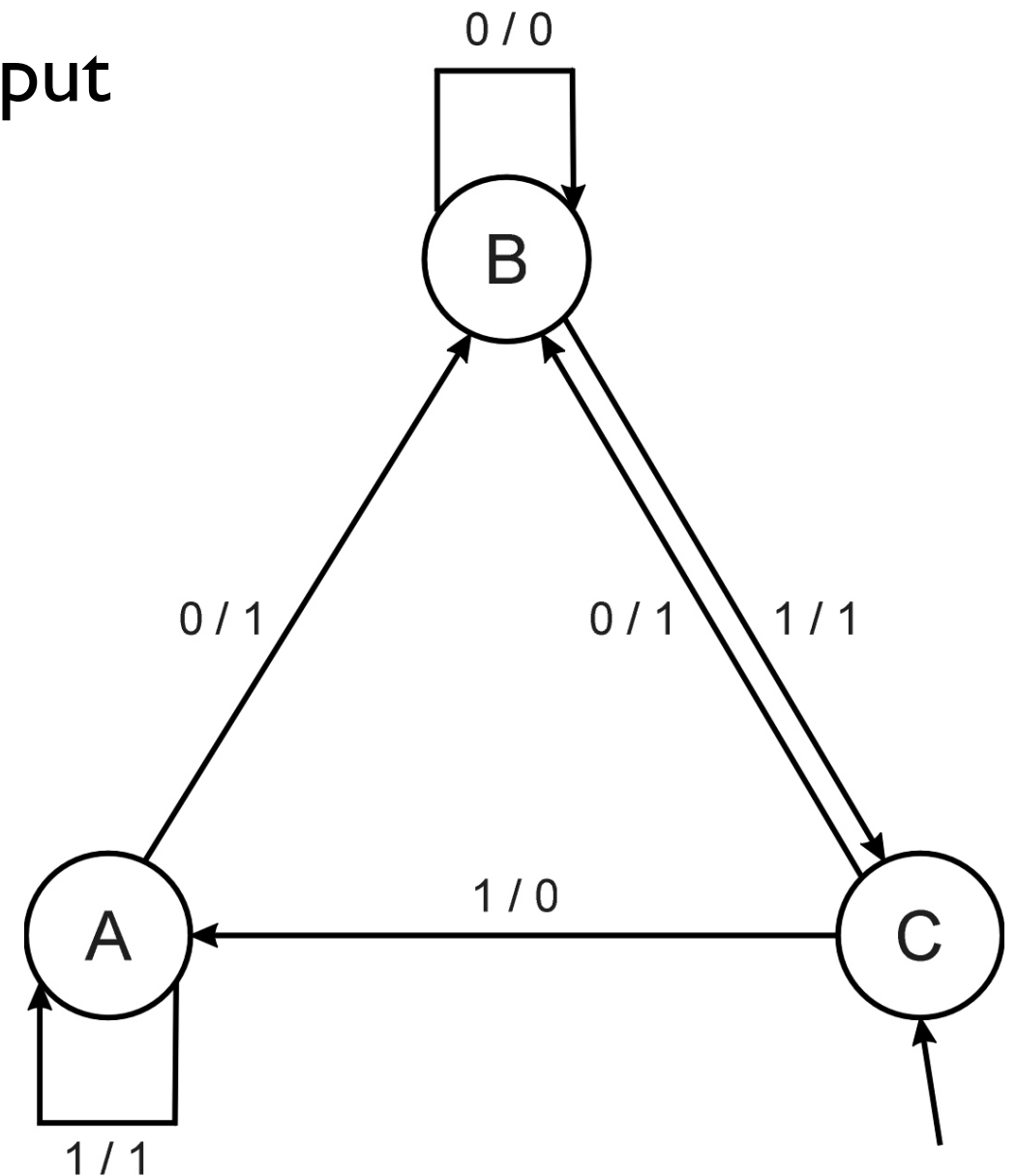


Evolutionary Programming: FSM As Predictor

- consider an FSM to predict next input
- quality: % of $\text{in}_{(i+1)} = \text{out}_i$
- given initial state C
- example:

i	1	2	3	4	5	6
input	0	1	1	1	0	1
output	1	1	0	1	1	1

quality: 60% (3 out of 5)



Evolutionary Programming: Evolving FMS to Predict Primes

- $P(n) = 1$ if n is prime, 0 otherwise
- $I = \mathbb{N} = \{1, 2, 3, \dots, n, \dots\}$
- $O = \{0, 1\}$
- correct prediction: $out_i = P(in_{i+1})$
- fitness function:
 - 1 point for correct prediction of next input
 - 0 point for incorrect prediction
 - penalty for 'too many' states

Evolutionary Programming: Evolving FMS to Predict Primes

- parent selection: each FSM is mutated once
- mutation operators (one selected randomly):
 - change an output symbol
 - change a state transition (i.e. redirect edge)
 - add a state
 - delete a state
 - change the initial state
- survivor selection: ($\mu+\mu$)
- results: overfitting, after 202 inputs best FSM had one state and both outputs were 0,
 - so it always predicted 'not prime'!
- *take away thought*: not perfect accuracy, but proof that simulated evolutionary process can create good solutions for intelligent task

Evolutionary Programming: Modern EP

- no predefined representation in general
- so no predefined mutation
 - must match representation
- often applies self-adaptation of mutation parameters

Evolutionary Programming: Representation

- for continuous parameter optimisation
- chromosomes consist of two parts:
 - object variables: x_1, \dots, x_n
 - mutation step sizes: $\sigma_1, \dots, \sigma_n$
- full size genotype: $\langle x_1, \dots, x_n, \sigma_1, \dots, \sigma_n \rangle$

Evolutionary Programming: Mutation

- genotypes: $\langle x_1, \dots, x_n, \sigma_1, \dots, \sigma_n \rangle$
- $\sigma_1' = \sigma_1 \cdot (1 + \alpha \cdot N(0, 1))$
- $x_1' = x_1 + \sigma_1' \cdot N_i(0, 1)$
- $\alpha \approx 0.2$
 - (so this is correlated stepwise mutation seen in previous slides)
- boundary rule: $\sigma' < \varepsilon \Rightarrow \sigma' = \varepsilon$
- other variants proposed and tried:
 - using variance instead of standard deviation
 - mutate σ -last
 - other distributions, such as Cauchy instead of Gaussian

Evolutionary Programming: Recombination

- none
- why?
- because EP has a different biological inspiration to GA and ES
- in EP one point in the search space stands for a species, not for an individual
- so there can be no crossover between species
- much historical debate about the benefits of 'mutation versus crossover'

Evolutionary Programming: Parent & Survivor Selection

- parent selection:
 - each individual creates one child by mutation
 - deterministic
 - not biased by fitness
- survivor selection:
 - parents and offspring populations are merged and compete in stochastic round-robin tournaments for survival

Evolutionary Programming: Evolving Checkers Players

- Fogel 2002:
- neural nets for evaluating future values of moves are evolved
- nets have fixed structure with 5046 weights
- these are evolved, plus one weight for 'kings'
- representation:
 - vector of 5046 real numbers for object variables (weights)
 - vector of 5046 real numbers for σ s
- mutation:
 - correlated stepwise, σ -first
 - plus special mechanism for the kings' weight
- population size of 15

Evolutionary Programming: Evolving Checkers Players

- selection:
 - tournament size $q = 5$
 - programs (with nets inside) play against other programs
 - no human trainer or hard-wired intelligence
- after 840 generations (6 months computing time!) the best strategy was tested against humans via Internet
- program earned 'expert class' ranking, outperforming 99.61% of all rated players

Genetic Programming: Quick Overview

“programming of computers by means of natural selection” or

“automatic evolution of computer programs”

- developed in the 1990's
- typically applied to:
 - machine learning tasks (prediction, classification...)
- attributed features:
 - competes with neural nets and alike
 - needs huge populations (thousands)
 - slow
- special:
 - non-linear chromosomes: trees, graphs
 - mutation possible but not necessary

Genetic Programming: Technical Summary Tableau

representation	tree structures
recombination	exchange of subtrees
mutation	random change in trees
parent selection	fitness proportional
survivor selection	generational replacement

Genetic Programming: Example: Credit Scoring

- bank wants to distinguish good from bad loan applicants
- model needed that matches historical data

ID	No. of children	Salary	Marital status	OK?
ID-1	2	45000	Married	0
ID-2	0	30000	Single	1
ID-3	1	40000	Divorced	1
...				

Genetic Programming: Example: Credit Scoring

- a possible model:

```
IF (NOC = 2) AND (S > 80000) THEN  
good ELSE bad
```

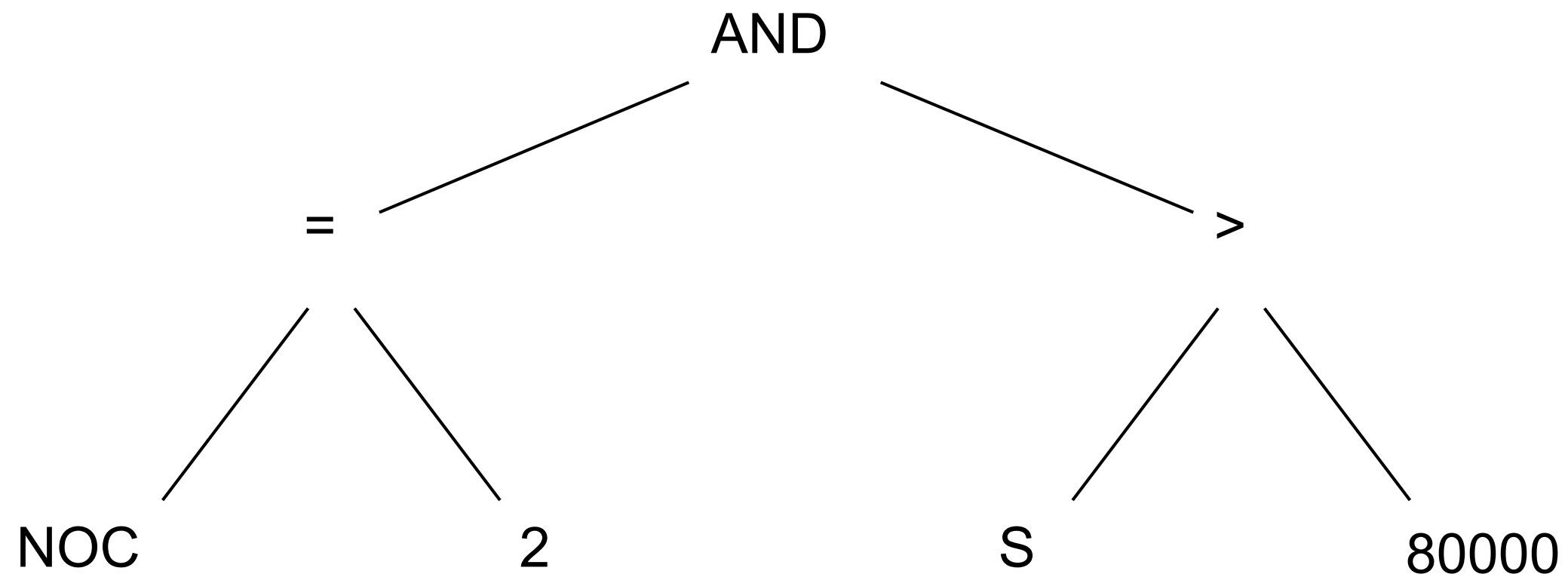
- in general we see that this takes the form:

```
IF formula THEN good ELSE bad
```

- the only unknown is the right formula, hence our search space (phenotypes) is the set of formulas
- the natural fitness of a formula is percentage of well classified (test) cases of the model it stands for

Genetic Programming: Example: Credit Scoring

IF (NOC = 2) AND (S > 80000) THEN good ELSE bad
can be represented by the following tree:



Genetic Programming: Initialisation

- most common method is **ramped half-and-half**
- given sets of functions F and terminals T
- a maximum initial depth for trees is chosen, D_{\max}
- each member of the initial population is created using one of two methods, each with equal probability:
 - **full method:**
 - each branch has depth D_{\max}
 - beginning at the root:
 - nodes at depth $d < D_{\max}$ are randomly chosen from function set F
 - nodes at depth $d = D_{\max}$ are randomly chosen from function set T
 - **grow method:**
 - each branch has depth $\leq D_{\max}$
 - beginning at the root:
 - nodes at depth $d < D_{\max}$ are randomly chosen from function set $F \cup T$
 - nodes at depth $d = D_{\max}$ are randomly chosen from function set T

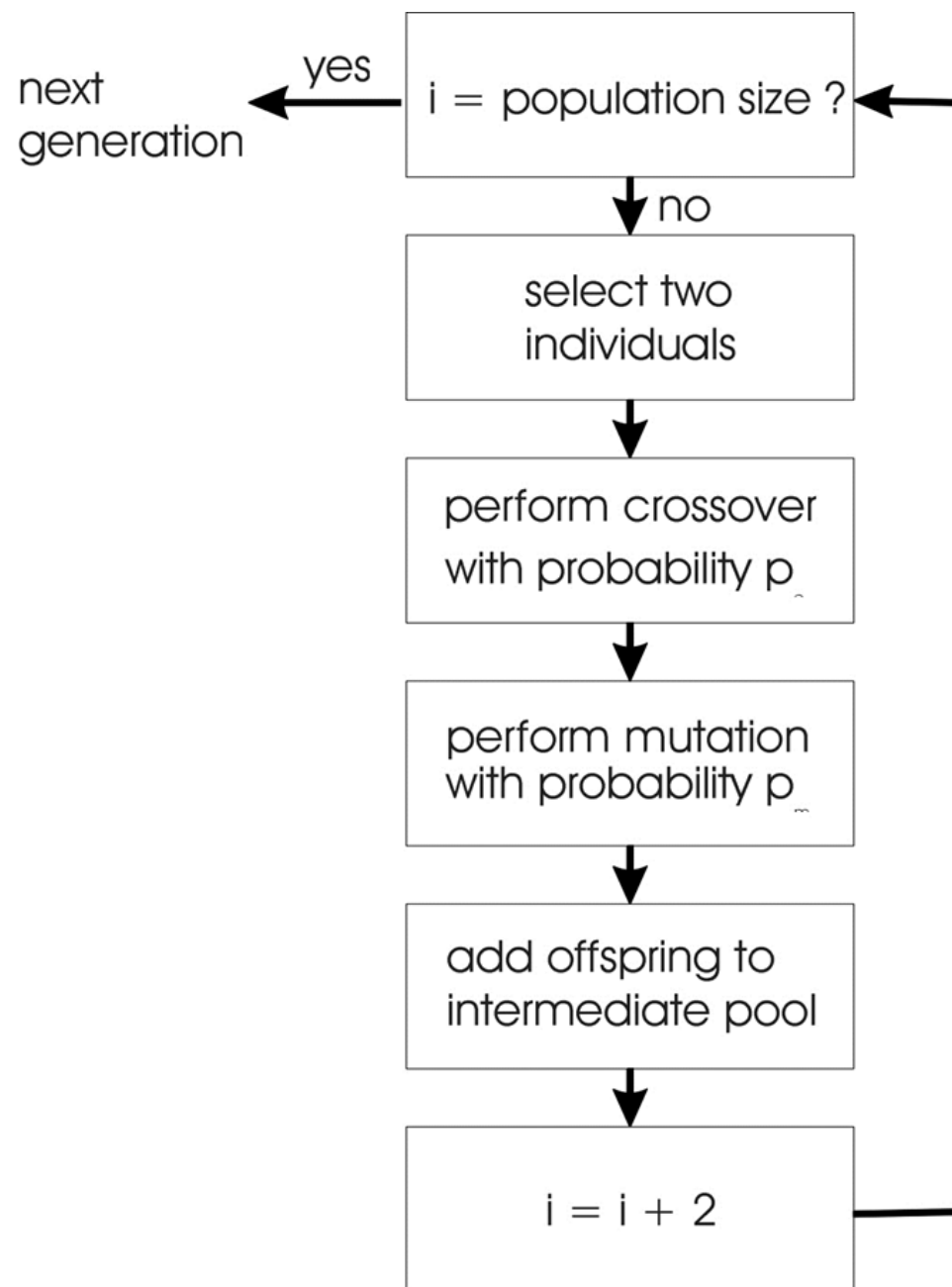
Genetic Programming: Mutation

- typically there is a low or zero mutation rate
- early on Koza advised using no mutation at all
- later Banzhaf et al advised a low rate of around 5%
- so very different from other EA variants
 - but why?
- the consensus is that crossover in GP has a large shuffling effect
- which acts as a kind of 'macromutation' operator

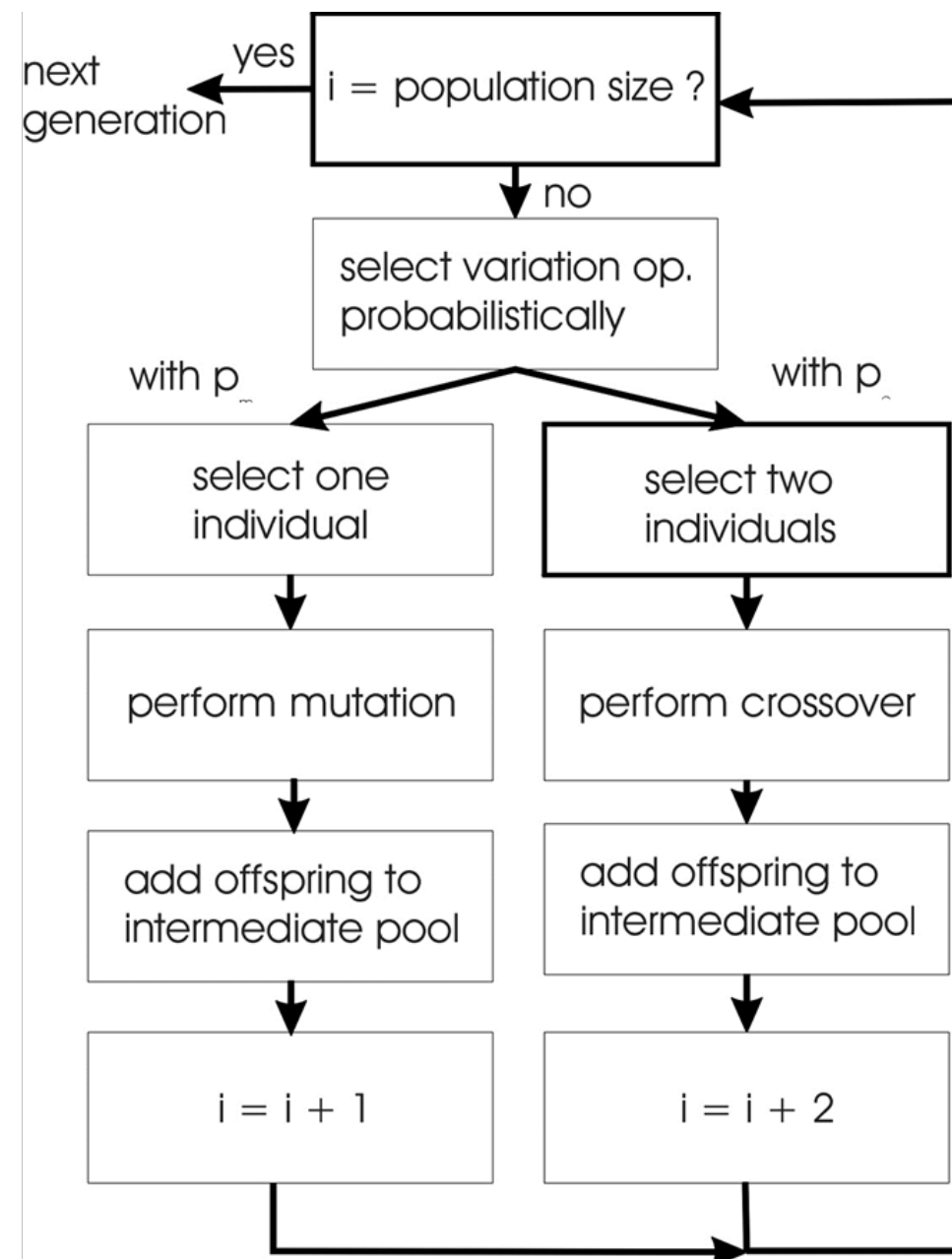
Genetic Programming: Offspring Creation Scheme

- let's compare:
 - GA scheme using crossover AND mutation
- with:
 - GP scheme using crossover OR mutation
 - (chosen probabilistically)

Genetic Programming: GA versus GP



GA flowchart



GP flowchart

Genetic Programming: Selection

- parent selection is typically fitness proportionate
- over-selection is often used with very large populations, to increase efficiency
 - rank population by fitness and divide it into two groups:
 - group 1: the best $x\%$ of population
 - group 2: the other $(100-x)\%$
 - then:
 - 80% of selection operations choose from group 1
 - 20% choose from group 2
 - for population sizes 1000, 2000, 4000, 8000, good x values are 32%, 16%, 8%, 4%
- survivor selection:
 - typically: generational scheme (none)
 - recently: steady-state is becoming popular because of the elitism that it provides

Genetic Programming: Bloat

Bloat: “survival of the fattest”

- so tree sizes in the population are increasing over time
- ongoing research and debate about the reasons
- not much insight, other than "because it's possible they can, and so without any pressure for smaller sizes, they will!"
- so needs countermeasures, such as:
 - prohibiting variation operators that would deliver 'too big' children
 - but this could inhibit the search
 - parsimony pressure: a penalty for being oversized
 - more widely used

Genetic Programming: Example: Symbolic Regression

- given some points in \mathbb{R}^n , $(x_1, y_1), \dots, (x_n, y_n)$
- find function $f(x) : \forall i = 1, \dots, n : f(x_i) = y_i$
- possible GP solution:
 - representation:
 - $F = \{+, -, /, \sin, \cos\}$
 - $T = R \cup \{x\}$
 - fitness is the error
 - all operators standard
 - population size = 1000, ramped half-and-half initialisation
 - termination: n 'hits' or 50000 fitness evaluations reached
 - (where 'hit' is if $|f(x_i) - y_i| < 0.0001$)

Reading & References

- slides based on and adapted from, Chapter 6 (and slides) of Eiben & Smith's *Introduction to Evolutionary Computing*
- the book also mentions several other variants not covered here
- see the Resources section of Brightspace for wider reading
- Genetic Algorithms:
 - Kenneth De Jong, *Genetic algorithms are NOT function optimizers*, pp 5–18 in *Foundations of Genetic Algorithms 2*, Morgan Kaufmann, San Francisco, 1993
 - D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, 1989
 - J.H. Holland, *Adaption in Natural and Artificial Systems*, MIT Press, 1992
 - M.Mitchell, *An Introduction to Genetic Algorithms*, MIT Press, 1996 (a good supplementary course text book!)

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 - D.B. ~ Fogel, *Blondie24: Playing at the Edge of AI*, Morgan Kaufmann, San Francisco, 2002

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 - W.B. Langdon and R. Poli, *Foundations of Genetic Programming*, Springer-Verlag, 2001