Compare Discrete/Continuous

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2022-02-23

Introduction

Calculation of probabilities is different depending on the nature of the random variable. In the following I will attempt to show you the main differences. These relate chiefly to the endpoints and the definitions of the probability function and probability density function.

Examples

Suppose

$$X \sim Bin(n = 10, p = 1/3)$$

and

$$Y \sim N(\mu = 5, \sigma = 2)$$

Notice that X is a **discrete** random variable and Y is a **continuous** random variable.

Probabilities

Upper tail – compare!

$P(X \geq 6)$ Notice that X is **discrete** and end points matter!!

```
1-pbinom(5,size = 10, prob = 1/3)
```

```
## [1] 0.07656353
```

$P(Y \geq 6)$ Notice that Y is **continuous** – end points are not so sensitive

```
1-pnorm(6,5,2)
```

```
## [1] 0.3085375
```

Lower tail

$P(X < 8)$

Notice that because X is discrete, the fact that X is < 8 means it has to be ≤ 7 there are no discrete values like $X = 7.89$. Therefore:

```
pbinom(7,size = 10, prob = 1/3)
```

```
## [1] 0.996596
```

$P(Y < 8)$

Since $Y < 8$ and Y is continuous, values of Y that satisfy the inequality would include $Y = 7.89$ for example. Therefore:

```
pnorm(8,5,2)
```

```
## [1] 0.9331928
```

Interval – compare!!

$P(2 \leq X \leq 7)$

```
probd <- pbinom(7,size = 10, prob = 1/3) - pbinom(1,size = 10, prob = 1/3)
probd
```

```
## [1] 0.8925469
```

$P(2 \leq Y \leq 7)$

```
prob <- pnorm(7,5,2)-pnorm(2,5,2)
prob
```

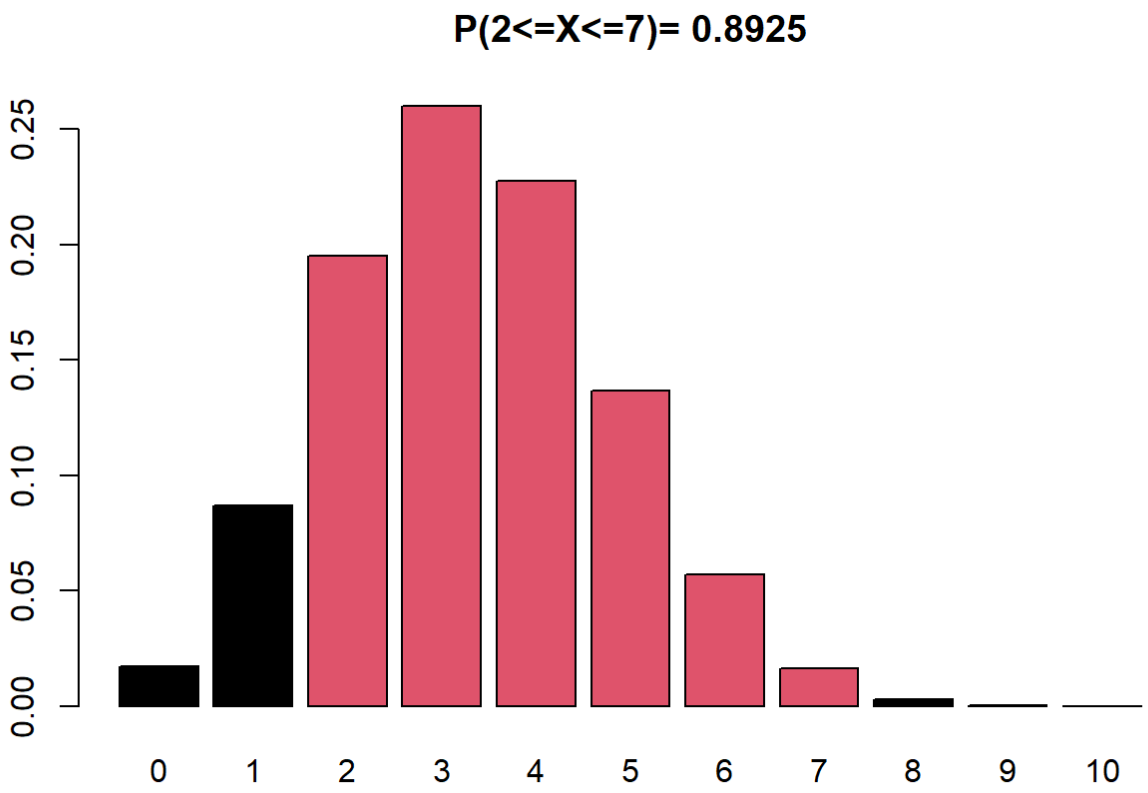
```
## [1] 0.7745375
```

Now we can compare plots

Please compare code chunks and understand the detail

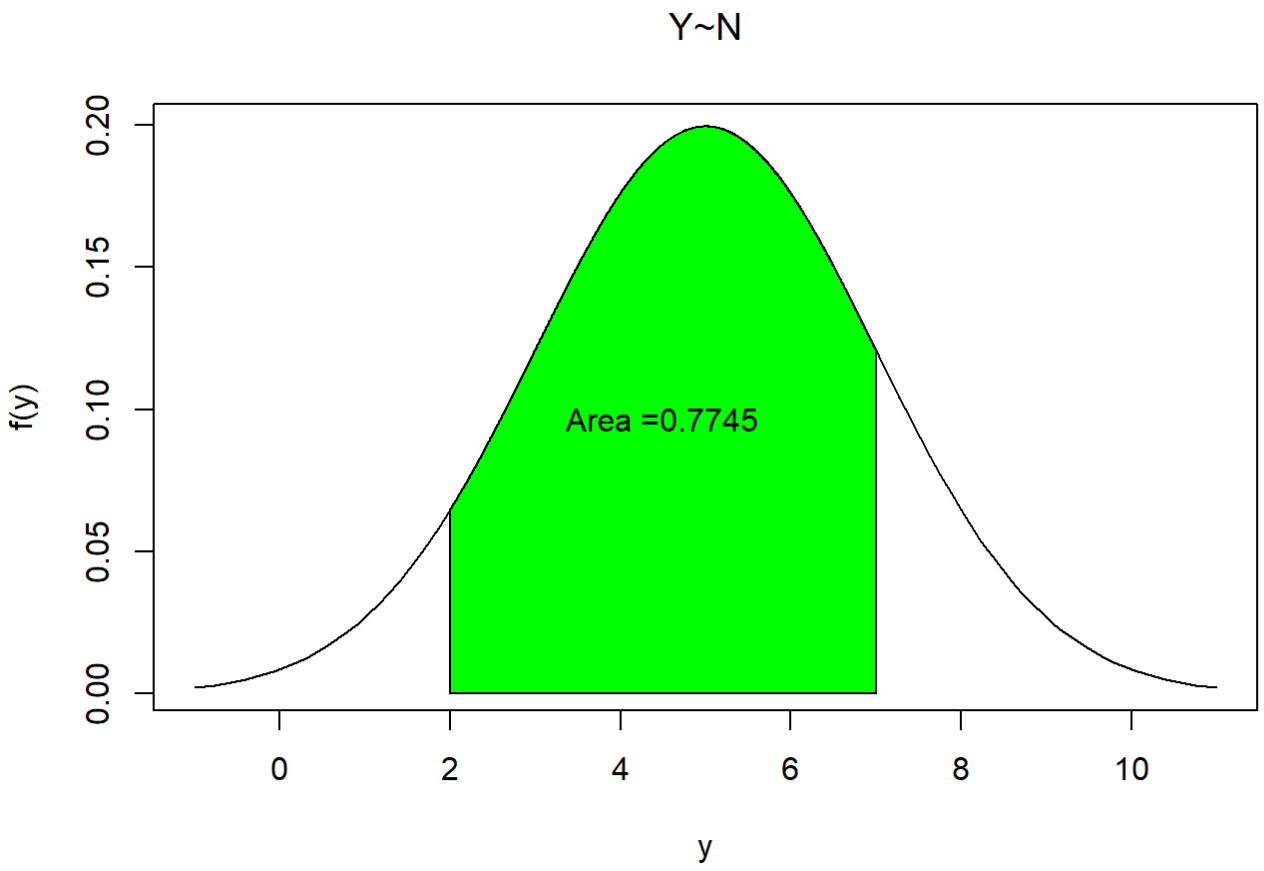
Discrete

```
x <- 0:10
probx <- dbinom(x,size = 10, prob = 1/3)
names(probx) <- x
barplot(probx, col = rep(c(1,2,1), c(2,6,3)), main = paste("P(2<=X<=7)=", round(probd,4)))
```



Continuous

```
xcurve <- seq(2,7, length=1000)
ycurve <- dnorm(xcurve, 5,2)
curve(dnorm(x,5,2), xlim = c(5-3*2, 5+3*2), main = expression(paste("Y","~","N")), xlab =
"y", ylab = "f(y)")
polygon(c(2,xcurve,7), c(0,ycurve,0), col = "green")
text((2+7)/2,dnorm(4.5,5,2)/2,paste0("Area =", round(prob,4)))
```


Y~N
f(y)
0.00
0.05
0.10
0.15
0.20
Area =0.7745
0
2
4
6
8
10
y