CS2318

11/08/16

Number Representation

Resources

Patterson ch 3 + appendix

Britton ch3

Tarnoff free

Books on digital Logic Architecture

< subject > ppt

Sign Number Representation

1) Sign and Magnitude

2) 1's complement

3) 2's complemen

2

S+M (Sign & Magnitude) issues

1) Subtraction (addition of positive / negative)

requires a decision

subtract small from large and

set the sign

2) $(A)_{SM} - (B)_{SM} \neq (A)_{SM} + (-B)_{SM}$

$(A)_{SM} + (-A)_{SM} \neq (0)_{SM}$

$\Rightarrow$ Add / Sub / Negate

Special Hardware for Add
for Subtract

$-5 \Rightarrow 5$

3) Two representations for 0

3 ~~A~~ One's Complement (1's)

MSB denotes the sign it is not a sign bit

Agreement that numbers with MSB = 0 are positive

Given an integer A the negation of A is obtained by bitwise-NOT of the bits representing A

0000	0	1111	-0
0001	1	1110	-1
0010	2	1101	-2
0011	3	~~0~~100	-3
0100	4	1011	-4
0101	5	1010	-5
0110	6	1001	-6
0111	7	1000	-7

4 With n bits

$$\left[-2^{n-1}-1\ ,\ 2^{n-1}-1\right]$$

two representations of 0

→ 5 + -5 in One's

[111000111111 ⇒ 000111000000]

Neg 1's Pos

Find the Value

infer ⇐

~~0101~~ 0101

$$\begin{array}{r} 1010 \\ \hline 1111 \end{array}$$

~~a + (-a)~~ $(A)_{1s} + (-A)_{1s} = (0)_{1s}$

5 in Ones

$$(A)_{1s} - (B)_{1s} = (A)_{1s} + (-B)_{1s}$$

Subtraction by addition

for $A - B$ use $A + (-B)$

~~1000 0000 0110~~
~~0001~~

A 0000 0110 ~~0000 0110~~

B 0001 0011 + ~~1110 1100~~ 1's

1 00000010

0000 0110
\- 0001 0011

0000 0110
\+ 1110 1100

1111 0010
0000 1101

6 A problem is when a carry occurs

The result is off by 1

Add 1 to fix the result

carry 0 no problem

carry 1 off by 1 add 1

=> End around carry

Add carry to the result to fix it

1's

$$
\begin{array}{r} 4 \\ -\ 3 \\ \hline \end{array}
\qquad
\begin{array}{r} 0100 \\ -\ 0011 \end{array}
\qquad
\begin{array}{r} 0100 \\ +\ 1100 \\ \hline 10000 \\ \text{EOC} \\ \hline 0001 \end{array}
\qquad
\begin{array}{r} 4 \\ +\ -3 \\ \hline \end{array}
$$

7 Two's Complement

No need for EOC

Add 1 to the One's and ignore carry

Agreement MSB of 0 denotes 0 positive #

1101

1110

$(A)_{2s}$ $(-A)_{2s}$

0000
1111
1
10000

$(-A)_{2s} = \bar{A} + 1$

$\bar{A}$ bit wise negation

1000
0111
1
11000

0000 →
0001 1 1111 -1
0010 2 1110

1000?

8

0000	0		
0001	1	1111	-1
0010	2	1110	-2
0011	3	1101	-3
0100	4	1100	-4
0101	5	1011	-5
0110	6	1010	-6
0111	7	1001	-7
		1000	-8

$[-8, 7]$

n bits $[-2^{n-1}, 2^{n-1}-1]$

11 bits 2's compleme nt

$[-1024, 1023]$

-8 111000 what is it

$$\begin{array}{r} 000111 \\ 1 \\ \hline 001000 \end{array}$$ 8

9 in 2's

$$\begin{array}{lcr} + & 4 & = & 0100 \\ & 3 & & 0011 \\ \hline & & & 0111 \end{array}$$

$$\begin{array}{lcr} - & 4 & - & 0100 \\ & 3 & & 0011 \end{array} \qquad \begin{array}{r} 0100 \\ 1101 \\ \hline 1\,0001 \end{array} \quad \text{no EOC}$$

in 2's (1's) sign change denotes OVF

Tow positives generate negative result

~~P~~ Negative positive

$$\begin{array}{r} 0100 \\ 0100 \\ \hline 1000 \end{array} \quad \text{Sign charged} \Rightarrow \text{OVF}$$

$$\begin{array}{ll} + & (-6) \\ & (-7) \end{array}$$

10) -6 $0110 \Rightarrow 1001 \Rightarrow 1010$ (1's, 2's)

$+ -7$ $0111 \Rightarrow 1000 \Rightarrow 1001$

$$\begin{array}{lr} A & 1010 \\ B & 1001 \\ \hline C & 1|0011 \end{array}$$

$OVF = S_A' S_B' S_C + S_A S_B S_C'$

$\quad\;\; 0 \; 0 \; 1 \quad\;\; 1 \; 1 \; 0$

~~Add~~ Multiplication / Division

S+M is "easier" to implement

$$\begin{array}{ll} S_A & M_A \\ S_B & M_B \\ \hline S_C & M_C \end{array}$$

$S_C = S_A \oplus S_B$

$M_C = M_A \times M_B$

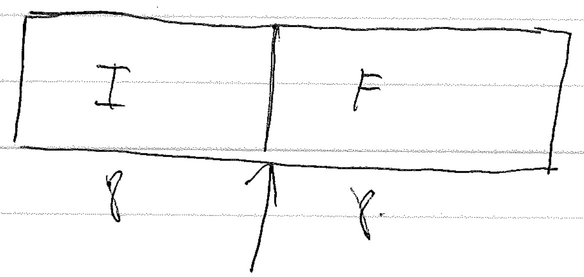
$|M_C| = |M_A| + |M_B|$

$2n \quad n \quad n$

10 Representing Real Numbers

(Rational) i/j

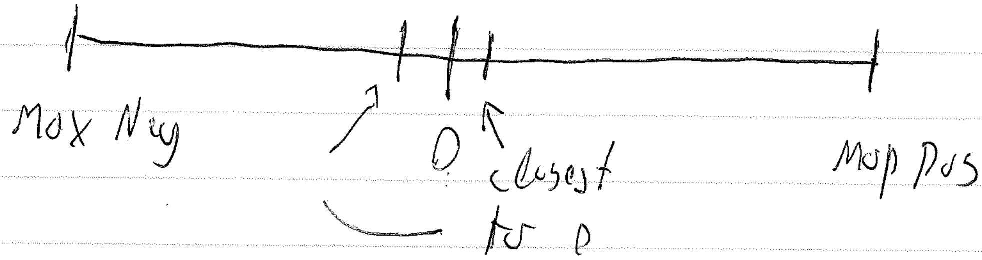
Fixed Point imaginary radix point



Register

16 bit

S+M for I US for F

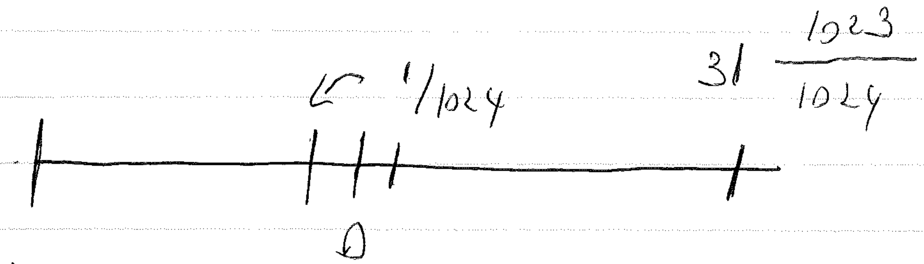


Symmetric $\frac{1}{256}$ $127\frac{255}{256}$

(-128 128)

12

I	F
6	10



$32\frac{1023}{1024}$ 32)

6 $[-2^5, 2^5-1]$

(33

16

2^{15}

$[-2^{15}, 2^{15}-1]$ as ints

as fraction

$\left[\frac{-2^{15}}{2^{15}}, \frac{2^{15}-1}{2^{15}}\right]$ $[-1,1)$