01/21/16

CS2318

Factorial

Loop counter

Peano set of Axioms for Arithmetic

Prolog (Logic programming)

Assume Two basic concepts

For every integer there is a successor except for 1

S' if S is an ~~int~~ INT

The S' is the successor

~~prac~~ Praticaly $S' = \underbrace{S+1}$ $(S+1)$

predecessor p' if p INT

p',

2 if p is an Int the p' is
The predecessor of p
$p'' = p--$

$$\text{Padd} = a + b = \begin{cases} b = 0 & a \\ \text{o.w.} & a' + b'' \end{cases}$$

Termination condition is $b == 0$

Recursive step ~~return~~
call Padd with a', b''
Return $a' + b''$

$$a \times b = \begin{cases} b == 1 & a \\ \text{o.w.} & \end{cases}$$

$a \times b'' + a$

3

$$Pmul = \begin{cases} a & b = 1 \\ Padd(a \times b'', a) \end{cases}$$

$$Padd(a, b) = \begin{cases} b = 0 & a \\ o.w. & Padd(a', b'') \end{cases}$$

a^b

~~P(Exp)~~

$$PExp(a, b) = \begin{cases} b = 1 & a \\ o.w. \; \cancel{Pexp} & PExp(\end{cases}$$

$$a^b = a^{b-1} \times a$$

$$Pexp(a, b) = Pmul(Pexp(a, b''), a)$$

4 Collatz conjecture thc (Erdos)

Sequence

$$C(n+1) = \begin{cases} (C_n)/2 & \text{if } n \text{ even} \\ \frac{3C_n+1}{2} & \text{if } n \text{ is odd} \end{cases}$$

converging to 1

$1 \leftarrow 2 \leftarrow 4 \leftarrow 8 \leftarrow 16 \leftarrow 32$

Sieve remove elements of no interest

9 14 7 22 11 34 17

52 26 13 ...

The termination condition is not guanteed to stop

5 $\cancel{c(n+1)} =$

$$c(n+1) = \begin{cases} 1 & n = 1 \\ \text{even} & c(n)/2 \\ \text{odd} & \dfrac{c(n) \times 3 + 1}{2} \end{cases}$$

Terminate ?

Yes start to go back

No odd collatz((3n+1)/2)

~~Ye~~ No even collatz(n/2)

X, ✓ ✓ ✓ ✓

6

If n items Unique binary code (name) fa

each items $\lceil \log_2 n \rceil$

requires $\lceil \log_2 n \rceil$ bits

16 4 bits

with k bits

=> Unique (distinct) items are 2^k

Representation of INTS in

Radix (base) R

$(d_{n-1}, d_{n-2} \ldots d_1, d_0 \uparrow d_{-1}, d_{-2} \ldots d_{-m})_R$

Radix point

346.5334 assume base 6

7 $(d_{n-1} \dots d_0, d_{-m}, d_{-2} \dots d_{-m})_R$

in Base 10

$$\sum_{i=-m}^{n-1} d_i R^i$$

342.89 $\quad 2\times10^0 + 4\times10^1 + 3\times10^2$

$8\times10^{-1} + 9\times10^{-2}$

in Base R we have R digits

(0.., R-1)

$R=8 \quad 0,1,2,\dots,7$

$R \Longleftrightarrow 2, 8, 16, 10$

32

8 Base 10 to base 2

INT

Divide by 2 | Remainder

collect bits

Fraction

multiply By 2 | * collect

stop if acceptable accuracy reached

340.555

* $n \times 2$ if $(n_2 \times 2) > 1$ $1 - n \times 2$

o.w $n \times 2$

from 2 to 8 to 16

8 group into 3 bits

16 " into 4 bits

9 ^{000}11101.01111^{000} to octal

011 101 . 011 110

3 5 . 3 6

0001 1101 . 0111 1000

0x 1D.78 extra zeros

not a part of the actual

binary number

16 -> 2 take the binary

8 -> 2 representation

of each digit

ABCD 1234

1010 1011 1100 1101 0001

0010 0011 0100

10

Binary Coded Decimals

Use 4 bits per digit

0 0000
1 0001
⋮
9 1001

X 1010

9+5 = 14

9 1001 BCD
5 0101 BCD
1110
110 adding 6
000 1 0100

will fix the result

9 7 3
3 4 2

0011 0100 0010