



# **A Logical Degeneration of Vector Symbolic Architectures**

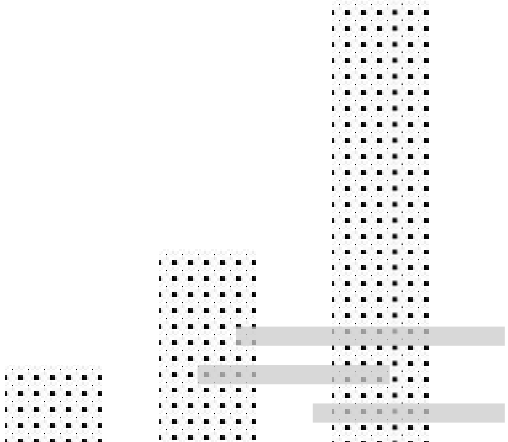
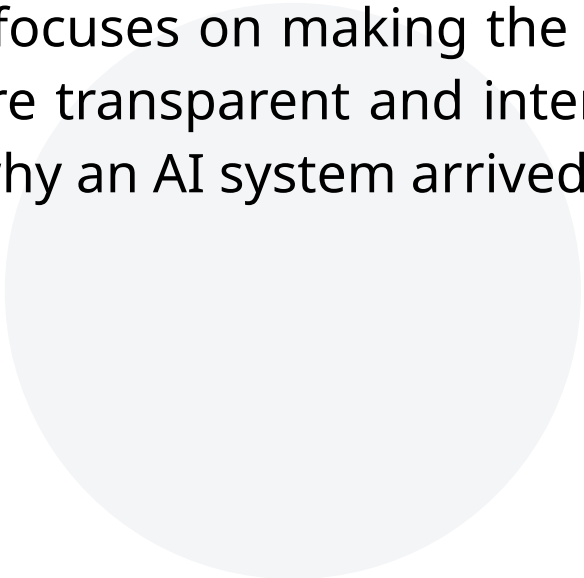
Giosuè Zanello

10/07/25



# What's XAI?

Explainable AI (XAI) refers to methods and techniques that enable humans to understand, trust, and manage the decisions made by AI systems. It focuses on making the "black box" nature of some AI models more transparent and interpretable, allowing users to see how and why an AI system arrived at a particular outcome.



# Why is it urgent nowadays?

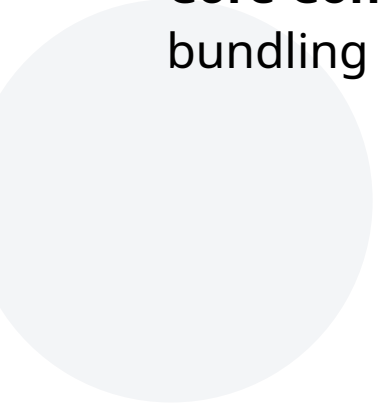
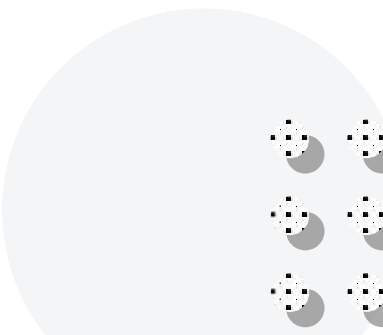
- **Increased AI Adoption:** Widespread use in critical sectors (e.g., healthcare, finance) requires transparency for trust.
- **Regulatory Demands:** Laws like the EU AI Act mandate explainability for compliance.
- **Bias and Fairness:** Opaque AI can amplify biases, needing urgent correction (see Grok recently).
- **Safety Risks:** Unexplainable errors in autonomous systems (e.g., self-driving cars) pose immediate dangers.
- **Public Trust:** Lack of understanding erodes confidence, especially in high-stakes decisions.





# What's VSA?



- **Definition:** Vector Symbolic Architecture (VSA) integrates high-dimensional vectors with symbolic reasoning in neuro-symbolic AI, combining neural perception and symbolic logic.
  - **Core Concept:** Represents symbols as vectors, using binding (combining) and bundling (adding) for robust, distributed encoding.
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# How this work

## Binding

**Definition:** Combines two vectors using an operation (e.g., circular convolution) to create a new vector representing their relationship, a relational link

**Example:**  $C = \text{bin}(A \text{ as "Wittgenstein", } B \text{ as isAuthorOf})$

Show C: Wittgenstein isAuthorOf

**Proprieties:** they also have different mathematical proprieties which I'll list later.



## Bundling

**Definition:** Bundling superimposes multiple vectors to represent a set of attributes, using an operation like vector addition: a cluster of proprieties, not a single relation.

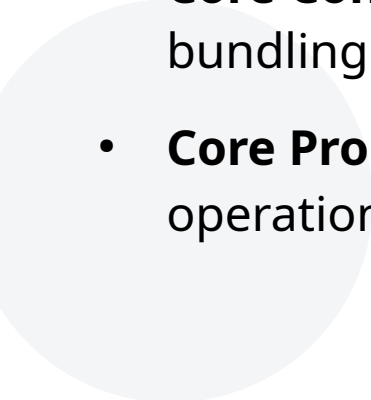

**Example:**  $D = \text{bun}(A \text{ as "Wittgenstein", } B \text{ as "Philosopher", } C \text{ as "1898"})$

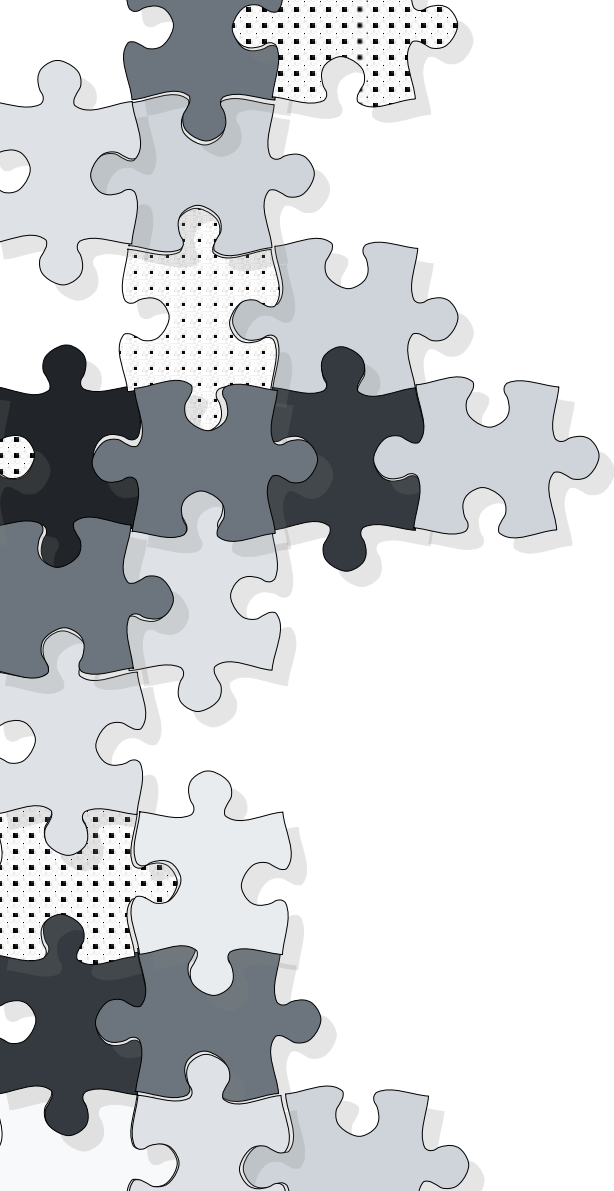
D could classificate "Wittgenstein" into "Philosopher born in the late 19th century"



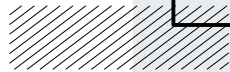
# What's VSA?



- **Definition:** Vector Symbolic Architecture (VSA) integrates high-dimensional vectors with symbolic reasoning in neuro-symbolic AI, combining neural perception and symbolic logic.
  - **Core Concept:** Represents symbols as vectors, using binding (combining) and bundling (adding) for robust, distributed encoding.
  - **Core Problem (in my opinion):** The sense of representation is lost in linear algebraic operations
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**//**  
The main idea is a framework  
that maps the operations of  
the VSA by symbols and  
makes them understandable  
in propositional logic. **//**





# Here's when Ludwig comes in.

Ludwig is a framework that maps distributed operations in a Vector Symbolic Architecture (VSA) to propositional logic rules. It enables transparent tracking, explanation, and verification of symbolic inference within high-dimensional vector spaces, making the system's reasoning interpretable and its errors auditable.

This occurs by a solid theory based on operation's logic properties.

We need to create logical operators for those rules;





# Bundling properties

- Aggregation of concepts (not formally composition, as will be seen in the binding)

$$R1 := ((A \wedge B) \Leftrightarrow C) \wedge (C \Leftrightarrow A) \wedge (C \Leftrightarrow B)$$

- Commutativity of operations

$$R2 := (A \wedge B) \Leftrightarrow (B \wedge A)$$

- Vectorial similarity

$$R1 := ((A \wedge B) \rightarrow C) \wedge (C \rightarrow A) \wedge (C \rightarrow B) \text{ (reformulating R1)}$$

# Binding properties

- Vectorial composition

$$R1 := ((A \wedge B) \Leftrightarrow C) \wedge (\neg(C \Leftrightarrow A) \wedge \neg(C \Leftrightarrow B))$$

- Non-commutativity

$$R2 := \neg(((A \wedge B) \Leftrightarrow C) \Leftrightarrow (B \wedge A) \Leftrightarrow C) \text{ or to try to avoid contradiction}$$

$$\neg((A \rightarrow (B \rightarrow C)) \Leftrightarrow (B \rightarrow (A \rightarrow C))), \text{ which is structural.}$$

- Symbol deduction

$$R3 := ((A \wedge C) \rightarrow B) \wedge ((B \wedge C) \rightarrow A)$$

# Second rule issue

Second rule formalizes asymmetrical inference, rejecting symmetry between A and B.

Negative condition ensures **distinct semantic levels**, not a **logical error**.

Second rule is “structurally false” but “epistemologically true,” securing model distinction. Valid degeneration of binding relies on first (new symbol generation) and third (compositive symbol deduction) rules.

01

Model acquisition (e.g. HRR) and setup: numpy, scipy, networkx, pandas, etc...

03

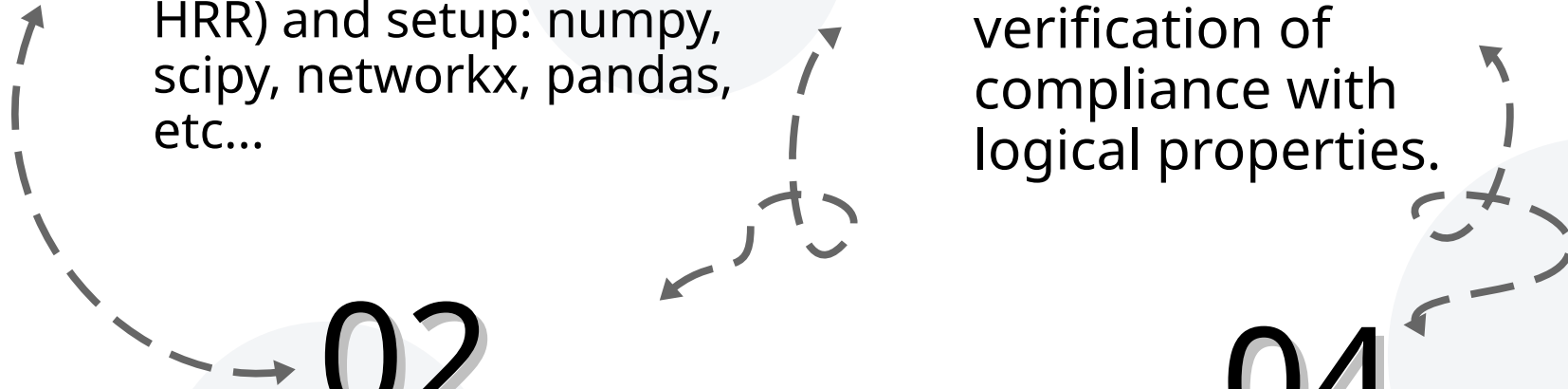
Transduction:  
verification of  
compliance with  
logical properties.

02

Vectorial  
mapping: def  
LogicMap.

04

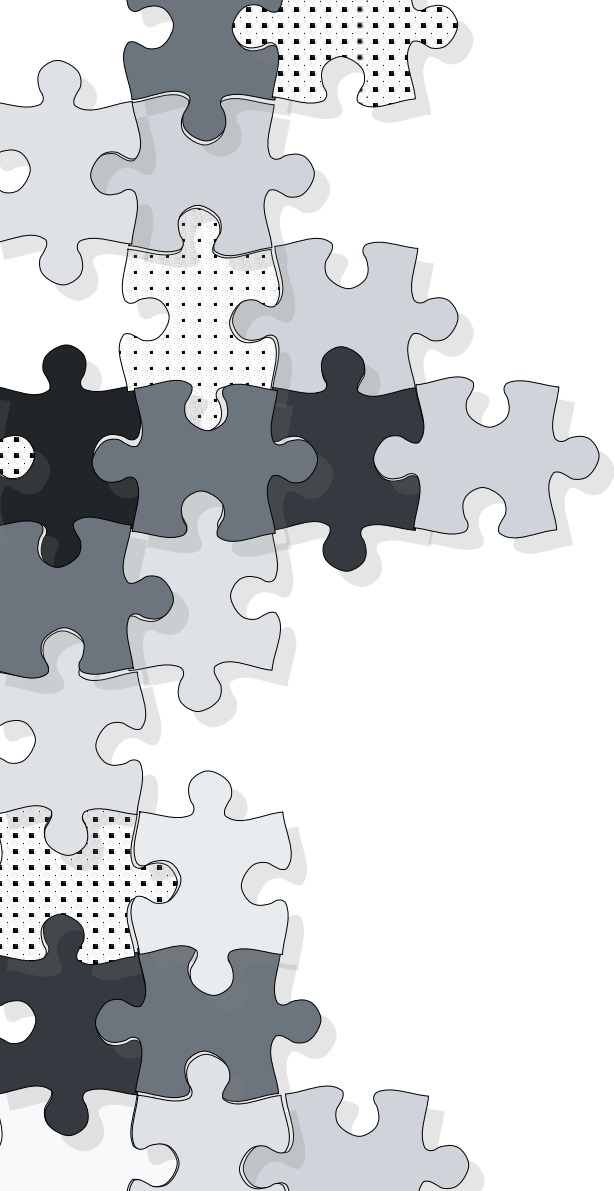
Verify: pyEDA,  
Z3 (Microsoft).



# Conclusions

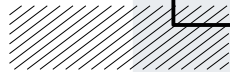
- This framework proposes a logical degeneration of VSAs, translating distributed operations into propositional logic; symbols that can be followed
- By collapsing high-dimensional spaces into verifiable symbolic rules, we offer an explainable and auditable approach to neuro-symbolic reasoning.
- Inspired by Wittgenstein's vision of **logic as the mirror of the world**, we reclaim the epistemic traceability of meaning within AI systems.





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“Logic fills the world: the limits  
of the world are also its limits.”  
— Wittgenstein, Tractatus 5.61

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