Imperial College London

**MEng and MSc (Engineering) Examination 2023**

**Year 3 / 4**
**MSc in Advanced Aeronautical Engineering**
**MSc in Advanced Computational Methods for Aeronautics, Flow Management and Fluid-Structure Interaction**

# AERO70016/ AERO97079 Orbital Mechanics

**Coursework assignment**

**Submission Deadline:**
**Friday 17th March 2023 22:59**

The assignment accounts for 25% of the total mark of the module.
The assignment consists of ONE question with three parts.
Both a technical report (maximum 3 pages) and the code written for the assignment must be submitted.

# Instructions

## Technical report

Each part of the coursework should be answered in a technical report of 3 pages maximum. Pages exceeding the 3-page limit will not be marked. The report complements the algorithms provided in the submitted code and should contain the minimal amount of information required to answer fully each part of the assignment.

- Please include your full name and CID number on the first page of the report.
- No title page, references section, appendices shall be included in the report. There are no requirements on the font typeface and size.
- All plots in the report must be clearly labelled, and the labels must report units of measure.
- A Word template for the technical report is provided separately on Blackboard.

## Submitted code

The tasks given in the assignment are numerical in nature and will require the use of programming languages for scientific computing. Accepted programming languages are MATLAB and Python 3.

- If the code does not run, the corresponding questions will be assigned zero marks.
- It is recommended to use either MATLAB Online or the Departmental Linux environments (`spitfire.ae.ic.ac.uk`; `hurricane.ae.ic.ac.uk`; `typhoon.ae.ic.ac.uk`) to verify that your code runs correctly before submission.
- If using MATLAB, do not use any functions outside of those contained in a standard MATLAB installation.
- If using Python 3, do not use any functions outside of those in the Math, NumPy and SciPy modules.
- Please include all functions needed to run the code in a single source file.
- Adapting algorithms provided within the core reference of the course (Curtis, H.D., "Orbital Mechanics for Engineering Students", 3rd Ed., Elsevier, 2014) is encouraged.
- Some parts of the coursework require using a numerical integration method for ordinary differential equations. If using MATLAB, you can employ the `ode45` or `ode89` solvers. If using Python 3, you can employ the `scipy.integrate.solve_ivp` function within the SciPy module. In either case, please use relative and absolute tolerances of $10^{-8}$ or below for the numerical integration.

## Submission instructions

On Blackboard, please upload two files:

- One MATLAB or Python 3 source file containing the code used for calculation of the results and the generation of plots,
- One PDF file containing the technical report detailing your answers, written using either Microsoft Word or LaTeX.

The coursework question consists of three parts and is provided on the following page.

## Coursework question

The initial coordinates of the spaceship *Discovery One* in a Sun-centred inertial frame with unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are:

$$\mathbf{r}_0 = -1.05 \text{ au } \hat{\mathbf{i}}, \quad \mathbf{v}_0 = -6.1316 \text{ au/year } \hat{\mathbf{j}}.$$

The spaceship is equipped with an extremely efficient electric propulsion system which consumes a negligible amount of propellant (the mass of the spaceship can be considered constant). The propulsion system provides a tangential thrust acceleration

$$\mathbf{a}_{\mathrm{T}} = \mathrm{a}_{\mathrm{T},0} \left(\frac{1 \text{ au}}{\mathrm{r}}\right)^2 \frac{\mathbf{v}}{\mathrm{v}},$$

where $\mathrm{a}_{\mathrm{T},0} = \frac{1}{3} \cdot 10^{-4} \text{ m/s}^2$ is the thrust acceleration at 1 au, $\mathrm{r}$ is the spaceship distance from the Sun, and $\frac{\mathbf{v}}{\mathrm{v}}$ is the unit vector in the direction of the velocity. No additional perturbations act on the spaceship.

For this question, assume that the Sun's gravitational parameter is $\mu = 39.4769 \frac{\text{au}^3}{\text{year}^2}$.

Part 1 [40%]

Use Cowell's method to propagate the trajectory of the spaceship for 20 years.

i. State the equations of motion in Cartesian coordinates.
ii. Plot the trajectory of the spaceship in the Sun-centred inertial frame.
iii. Provide the final position and velocity of the spaceship.

Part 2 [40%]

Use Encke's method to propagate the trajectory of the spaceship for 20 years in the Sun-centred inertial frame.

i. Motivate the choice of the rectification timestep.
ii. Plot the position of the spacecraft at each rectification step and verify that the sequence of positions follows the trajectory calculated in Part 1 through visual inspection.

Part 3 [20%]

Plot the Keplerian trajectory corresponding to the osculating elements of the spaceship at 5 instants within the propagation interval, and state whether the semi-major axis and eccentricity are increasing or decreasing along time.