Parcial #1

a.) $d^2(x_1, x_2) = P_{x_1 - x_2} = \lim_{x \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$

Integra y pasa a binomio cuadrado perfecto

$$\frac{1}{T}\left[\int_T |x_1(t)|^2 dt - 2\int_T x_1(t) \cdot x_2(t)\, dt + \int_T |x_2(t)|^2 dt\right]$$

$$\frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 \longrightarrow P_{x_1} - \frac{2}{T}\int_1 x_1(t)\, x_2(t)\, dt + P_{x_2}$$

$$P_{x_1} = \frac{1}{T}\int_T |x_1|^2 dt$$

$$\frac{1}{T}\int_0^T |Ae^{jw_0 t}|^2 dt = \frac{A^2}{T}\int_0^T e^{jw_0 t}(e^{-jw_0 t})dt$$ complejo conjugado

$$\frac{A^2}{T}\int_0^T e^0 = \frac{A^2}{T}\int_0^T 1\, dt = \frac{A}{T}[t]_0^T$$

$$\frac{A^2}{T}[T-0] = A^2\cancel{\frac{T}{T}}$$

$$P_{x_1} = A^2$$

$$P_{x_2} = \frac{1}{T}\int_T |x_2|^2 dt$$

$$\frac{1}{T}\int_0^T |Be^{jw_0 t}|^2 dt = \frac{B^2}{T}\int_0^T e^{jw_0 t}(e^{-jw_0 t})dt$$ complejo conjugado

$$\frac{B^2}{T}\int_0^T e^0 = \frac{B^2}{T}\int_0^T 1\, dt = \frac{B}{T}[t]_0^T$$

$$\frac{B^2}{T}[T-0] = B^2\cancel{\frac{T}{T}}$$

$$P_{x_2} = B^2$$

$$\frac{2}{T}\int_T x_1(t)\, x_2(t)\, dt = \frac{2}{T}\int_T (Ae^{jw_0 t})(Be^{5jw_0 t})dt$$

$$\frac{2AB}{T}\int_T e^{6jw_0 t} dt = \frac{2AB}{T}\int_T e^{6j\frac{2\pi}{T}t} dt = \frac{2AB}{T}\int_0^T e^u \frac{T}{12j\pi} du$$

$u = 6j\frac{2\pi}{T}t$

$du = \frac{12j\pi}{T}dt$

$dt = \frac{T}{12j\pi}du$

$$\frac{2AB}{T}\frac{T}{12j\pi}\int_0^T e^u du = \frac{AB}{6j\pi}\left[e^{\cancel{T}(6j\frac{2\pi}{\cancel{T}})} \cdot e^{\cancel{T}(6j\frac{2\pi}{\cancel{T}})}\right] = \frac{AB}{6j\pi}\left[e^{12\pi j} - 1\right]$$

$$P(x_1 - x_2) = P_{x_1} - \frac{2}{T}\int_1 x_1(t)\, x_2(t)\, dt + P_{x_2}$$

$$\boxed{P(x_1 - x_2) = A^2 - \frac{AB}{6j\pi}\left[e^{12\pi j} - 1\right] + B^2}$$

b. $X(t) = 3\cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$

Se remplaza t en continuo por la discretización $t = nT_s$, con $n \in \mathbb{Z}$

$$X[t=nT_s] = 3\cos[1000\pi n T_s] + 5\sin[2000\pi n T_s] + 10\cos[11000\pi n T_s]$$

$$T_s = \frac{1}{F_s} = \frac{1}{5000}[S]$$

Se Remplaza T_s

$$X[n] = 3\cos\left[\frac{1\cancel{000}\pi n}{5\cancel{000}}\right] + 5\sin\left[\frac{2\cancel{000}\pi n}{5\cancel{000}}\right] + 10\cos\left[\frac{11\cancel{000}\pi n}{5\cancel{000}}\right]$$

$$X[n] = 3\cos[1/5\,\pi n] + 5\sin[2/5\,\pi n] + 10\cos[11/5\,\pi n]$$

$$\Omega_3 = \frac{11\pi}{5} > \pi \rightarrow \text{frecuencia copia}$$

$$\hat{\Omega}_3 = \Omega_3 - 2\pi = \frac{11\pi}{5} - 2\pi = \frac{\pi}{5} \rightarrow \text{frecuencia Original}$$

Función discretizada

$$X[n] = 3\cos[(1/5)\cdot\pi n] + 5\sin[(2/5)\cdot\pi n] + 10\cos[(11/5)\cdot\pi n]$$

$$X_{[n]} = 13\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right]$$

c. $X(t) = 20\cos(t/3) + \cos(t/4)\,[A]$

para la señal cuasiperiodica

$$\frac{w_1}{w_2} = \frac{1/3}{1/4} = \frac{4}{3}$$

$$T_1 = \frac{2\pi}{w_1} = \frac{2\pi}{1/3} = 6\pi$$

$$T_2 = \frac{2\pi}{w_2} = \frac{2\pi}{1/4} = 8\pi$$

Con $K, L \in \mathbb{Z}$. El $MCM_{(6,8)} = 24$

$$T = 24\pi$$

Se aplica cero pendiente

$$\hat{X}(t) = mX_{(t)} + b$$

$$m = \frac{\hat{X}_{max}(t) - \hat{X}_{min}(t)}{X_{max}(t) - X_{min}(t)}$$

$$b = \hat{X}_{min(t)} - mX_{min(t)}$$