

The success of a **machine learning algorithm** highly depends on the quality of the data fed into the model. Real-world data is often dirty containing outliers, missing values, wrong data types, irrelevant features, or non-standardized data. The presence of any of these will prevent the machine learning model to properly learn. For this reason, transforming raw data into a useful format is an essential stage in the machine learning process. One technique you will come across multiple times when pre-processing data is **normalization**.

**Data Normalization** is a common practice in machine learning which consists of transforming **numeric columns** to a **common scale**. In machine learning, some feature values differ from others multiple times. The features with higher values will dominate the leaning process. However, it does not mean those variables are more important to predict the outcome of the model. **Data normalization** transforms multiscaled data to the same scale. After normalization, all variables have a **similar influence** on the model, improving the stability and performance of the learning algorithm.

There are multiple **normalization techniques** in statistics. In this article, we will cover the most important ones:

1. **The maximum absolute scaling**

## 2. The min-max feature scaling

## 3. The z-score method

## 4. The robust scaling

Besides, we will explain how to implement them

with **Pandas** and **Scikit-Learn**.

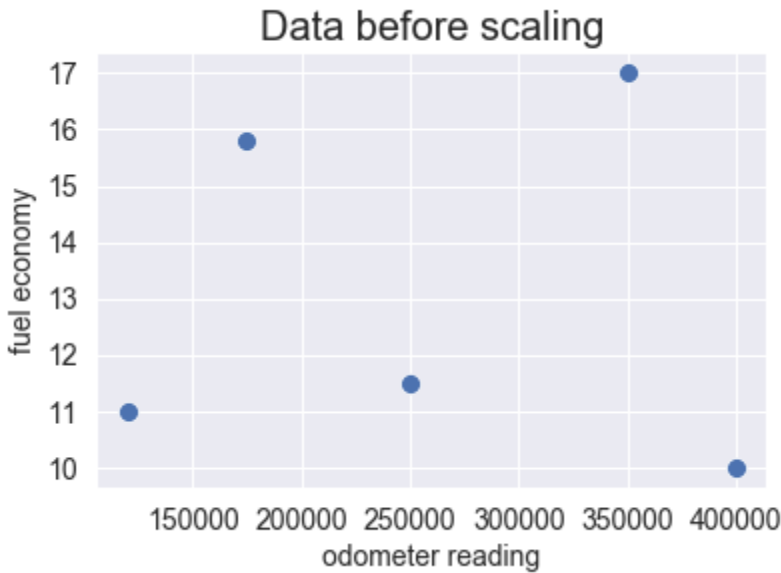
The following data frame contains the inputs (independent variables) of a **multiple regression model** for predicting the price of a second-hand car: (1) the odometer reading (km) and (2) the fuel economy (km/l). we use a small data set for learning purposes. However, in the real world, the data sets employed will be much larger.

```
import pandas as pd

# data frame containing the odometer reading (km) and the fuel economy (km/l)
# of second-hand cars
df_cars = pd.DataFrame([[120000, 11], [250000, 11.5], [175000, 15.8],
                        [350000, 17], [400000, 10]],
                        columns=['odometer_reading', 'fuel_economy'])

df_cars
```

	odometer_reading	fuel_economy
0	120000	11.0
1	250000	11.5
2	175000	15.8
3	350000	17.0
4	400000	10.0



As you can observe, the odometer reading ranges from 120000 to 400000, while the fuel economy ranges from 10 to 17.

The **multiple linear regression** model will weight the odometer reading variable more heavily than the fuel economy attribute due to its higher values. However, it does not mean that the odometer reading attribute is more important as a predictor. To solve this problem, we have to **normalize** the values of both variables.

## The maximum absolute scaling

The **maximum absolute scaling** rescales each feature **between -1 and 1** by dividing every observation by its maximum absolute value.

$$x_{scaled} = \frac{x}{\max(|x|)}$$

We can apply the **maximum absolute scaling** in **Pandas** using the **.max()** and **.abs()** methods, as shown below.

```
# apply the maximum absolute scaling in Pandas using the .abs() and .max()
methods
def maximum_absolute_scaling(df):
    # copy the dataframe
    df_scaled = df.copy()
    # apply maximum absolute scaling
    for column in df_scaled.columns:
        df_scaled[column] = df_scaled[column] /
df_scaled[column].abs().max()
    return df_scaled

# call the maximum_absolute_scaling function
df_cars_scaled = maximum_absolute_scaling(df_cars)

df_cars_scaled
```

	odometer_reading	fuel_economy
0	0.3000	0.647059
1	0.6250	0.676471
2	0.4375	0.929412
3	0.8750	1.000000
4	1.0000	0.588235

Alternatively, we can use the **Scikit-learn** library to compute the **maximum absolute scaling**. First, we create an **abs\_scaler** with the **MaxAbsScaler** class. Then, we use the **fit method** to learn the required parameters for scaling the data (the **maximum absolute value** of each feature). Finally, we transform the data using those parameters.

```
from sklearn.preprocessing import MaxAbsScaler

# create an abs_scaler object
abs_scaler = MaxAbsScaler()

# calculate the maximum absolute value for scaling the data using the fit
method
abs_scaler.fit(df_cars)
```

```
# the maximum absolute values calculated by the fit method
abs_scaler.max_abs_
# array([4.0e+05, 1.7e+01])

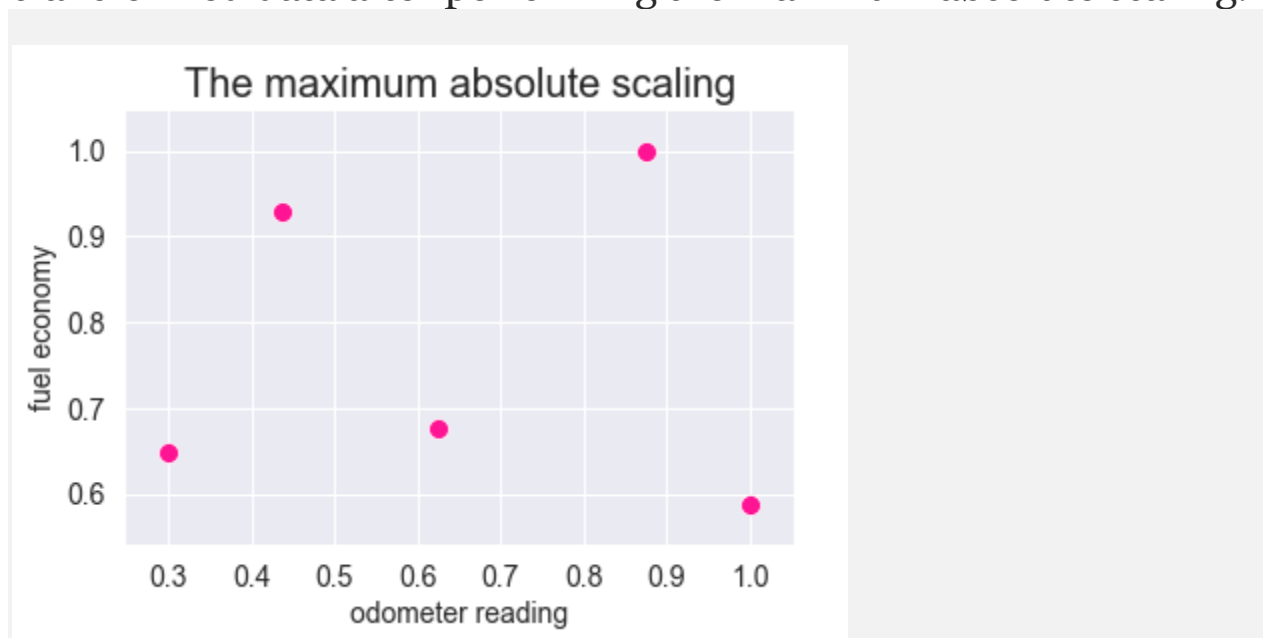
# transform the data using the parameters calculated by the fit method (the
maximum absolute values)
scaled_data = abs_scaler.transform(df_cars)

# store the results in a data frame
df_scaled = pd.DataFrame(scaled_data, columns=df_cars.columns)

# visualize the data frame
df_scaled
```

	odometer_reading	fuel_economy
0	0.3000	0.647059
1	0.6250	0.676471
2	0.4375	0.929412
3	0.8750	1.000000
4	1.0000	0.588235

As you can observe, we obtain the same results using **Pandas** and **Scikit-learn**. The following plot shows the transformed data after performing the maximum absolute scaling.



# The min-max feature scaling

The **min-max approach** (often called **normalization**) rescales the feature to a **fixed range** of **[0,1]** by subtracting the minimum value of the feature and then dividing by the range.

$$x_{norm} = \frac{x - x_{min}}{x_{max} - x_{min}}$$

We can apply the **min-max scaling** in **Pandas** using the **.min()** and **.max()** methods.

```
# apply the min-max scaling in Pandas using the .min() and .max() methods
def min_max_scaling(df):
    # copy the dataframe
    df_norm = df.copy()
    # apply min-max scaling
    for column in df_norm.columns:
        df_norm[column] = (df_norm[column] - df_norm[column].min()) /
        (df_norm[column].max() - df_norm[column].min())

    return df_norm

# call the min_max_scaling function
df_cars_normalized = min_max_scaling(df_cars)

df_cars_normalized
```

	odometer_reading	fuel_economy
0	0.000000	0.142857
1	0.464286	0.214286
2	0.196429	0.828571
3	0.821429	1.000000
4	1.000000	0.000000

Alternatively, we can use the **MinMaxScaler** class available in the **Scikit-learn** library. First, we create a **scaler object**. Then,

we **fit** the scaler parameters, meaning we calculate the minimum and maximum value for each feature. Finally, we **transform** the data using those parameters.

```
from sklearn.preprocessing import MinMaxScaler

# create a scaler object
scaler = MinMaxScaler()
# fit and transform the data
df_norm = pd.DataFrame(scaler.fit_transform(df_cars),
                        columns=df_cars.columns)
```

df\_norm

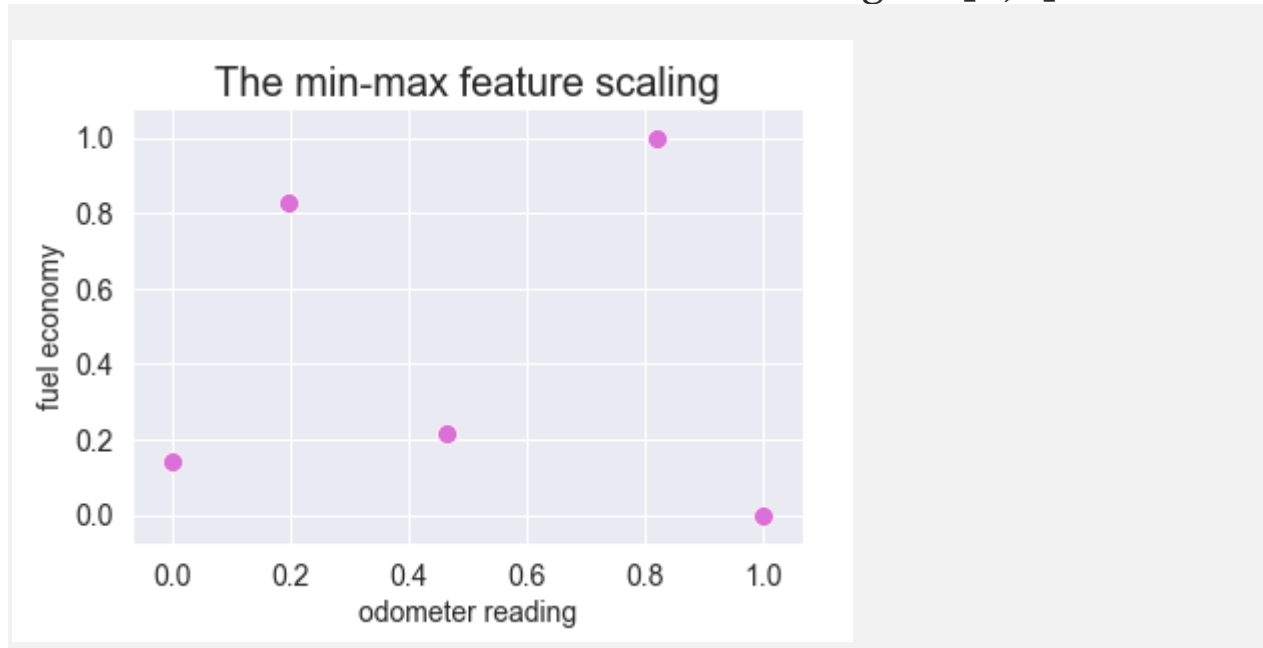
	odometer_reading	fuel_economy
0	0.000000	0.142857
1	0.464286	0.214286
2	0.196429	0.828571
3	0.821429	1.000000
4	1.000000	0.000000

Additionally, we can obtain the minimum and maximum values calculated by the **fit** function for normalizing the data with the **data\_min\_\_** and **data\_max\_\_** attributes.

```
# minimum values for normalizing the data
scaler.data_min_
# array([1.2e+05, 1.0e+01])

# maximum values for normalizing the data
scaler.data_max_
# array([4.0e+05, 1.7e+01])
```

The following plot shows the data after applying the min-max feature scaling. As you can observe, this normalization technique rescales all feature values to be within the range of [0, 1].



As you can observe, we obtain the same results using **Pandas** and **Scikit-learn**. However, if you want to perform many data transformation steps, it is recommended to use the **MinMaxScaler** as input in a **Pipeline** constructor instead of performing the normalization with **Pandas**.

Furthermore, it is important to bear in mind that the **maximum absolute scaling** and the **min-max scaling** are very sensitive to **outliers** because a single outlier can influence the minimum and maximum values and have a big effect on the results.

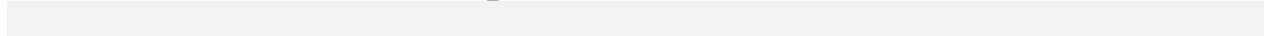


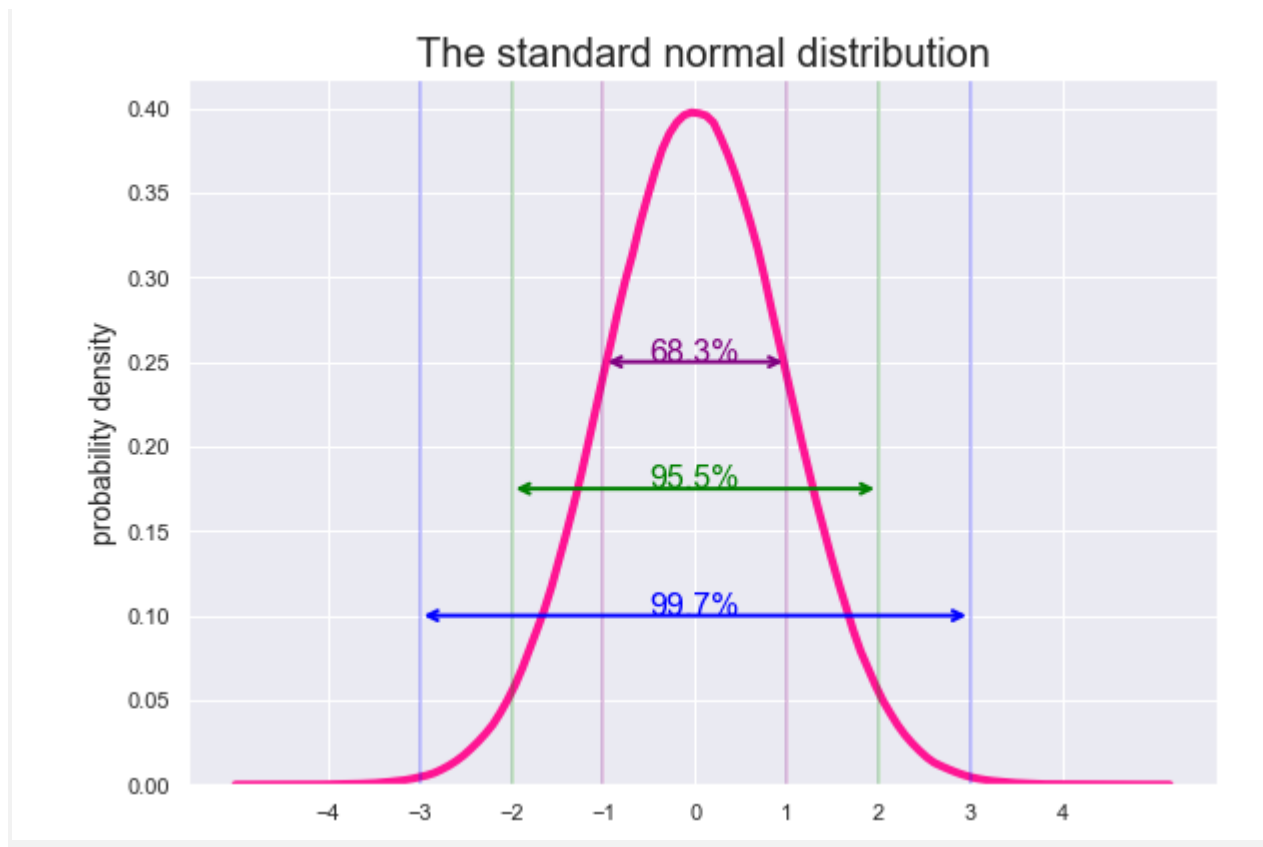
# The z-score method

The **z-score method** (often called **standardization**) transforms the data into a distribution with a **mean of 0** and a **standard deviation of 1**. Each standardized value is computed by subtracting the **mean** of the corresponding feature and then dividing by the **standard deviation**.

$$x_{std} = \frac{x - \mu}{\sigma}$$

Unlike **min-max scaling**, the **z-score** does not rescale the feature to a fixed range. The **z-score** typically ranges from **-3.00 to 3.00** (more than 99% of the data) if the input is normally distributed. However, the standardized values can also be higher or lower, as shown in the picture below.





It is important to bear in mind that **z-scores** are not necessarily normally distributed. They just scale the data and follow the same distribution as the original input. This transformed distribution has a **mean of 0** and a **standard deviation of 1** and is going to be the **standard normal distribution** (see the image above) only if the input feature follows a normal distribution.

We can compute the **z-score** in **Pandas** using the **.mean()** and **std()** methods.

```
# apply the z-score method in Pandas using the .mean() and .std() methods
def z_score(df):
    # copy the dataframe
    df_std = df.copy()
    # apply the z-score method
    for column in df_std.columns:
        df_std[column] = (df_std[column] - df_std[column].mean()) /
df_std[column].std()
```

```

        return df_std

# call the z_score function
df_cars_standardized = z_score(df_cars)

df_cars_standardized

```

	odometer_reading	fuel_economy
0	-1.189512	-0.659120
1	-0.077019	-0.499139
2	-0.718842	0.876693
3	0.778745	1.260647
4	1.206628	-0.979081

Alternatively, we can use the **StandardScaler** class available in the **Scikit-learn** library to perform the z-score. First, we create a **standard\_scaler** object. Then, we calculate the parameters of the transformation (in this case the **mean** and the **standard deviation**) using the **.fit()** method. Next, we call the **.transform()** method to apply the standardization to the data frame. The **.transform()** method uses the parameters generated from the **.fit()** method to perform the z-score.

```

from
sklearn.preprocessing
import StandardScaler

# create a scaler object
std_scaler = StandardScaler()
std_scaler
# fit and transform the data
df_std = pd.DataFrame(std_scaler.fit_transform(df_cars),
                      columns=df_cars.columns)

```

	df_std	
	odometer_reading	fuel_economy
0	-1.329915	-0.736918
1	-0.086110	-0.558055
2	-0.803690	0.980173
3	0.870664	1.409446
4	1.349051	-1.094646

To simplify the code, we have used the **.fit\_transform()** **method** which combines both methods (fit and transform) together.

As you can observe, the results differ from those obtained using **Pandas**. The **StandardScaler** function calculates the **population standard deviation** where the sum of squares is divided by **N** (number of values in the population).

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (y_i - \mu)^2}{N}}$$

On the contrary, the **.std()** **method** calculates the **sample standard deviation** where the denominator of the formula is **N-1** instead of **N**.

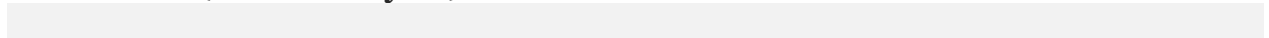
$$s = \sqrt{\frac{\sum_{i=1}^N (y_i - \bar{y})^2}{N-1}}$$

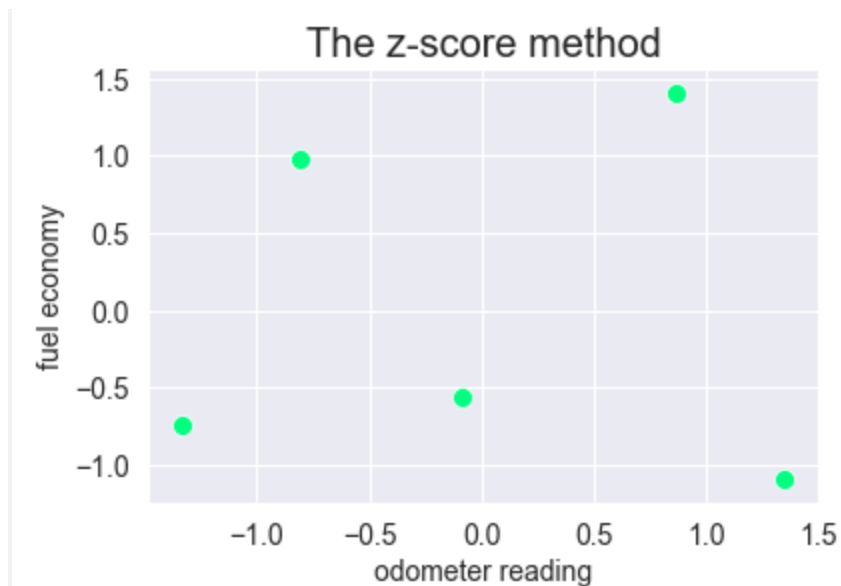
To obtain the same results with Pandas, we set the parameter **ddof** equal to 0 (default value is `ddof=1`) which represents the divisor used in the calculations (**N-ddof**).

```
#
population
standard
deviation
with
Pandas
df_cars.std(ddof=0)
```

We can obtain the parameters calculated by the **fit** function for standardizing the data with the **mean\_\_** and **scale\_\_** attributes. As you can observe, we obtain the same results in **Scikit-learn** and **Pandas** when setting the parameter **ddof equals to 0** in the **.std()** method.

The following plot shows the data after applying the z-score method which is computed using the population standard deviation (divided by N).





## The Robust Scaling

In **robust scaling**, we scale each feature of the data set by subtracting the **median** and then dividing by the **interquartile range**. The **interquartile range (IQR)** is defined as the difference between the **third and the first quartile** and represents the central 50% of the data. Mathematically the robust scaler can be expressed as:

$$x_{rs} = \frac{x_i - Q_2(x)}{Q_3(x) - Q_1(x)}$$

where **Q1(x)** is the **first quartile** of the attribute x, **Q2(x)** is the **median**, and **Q3(x)** is the **third quartile**.

This method comes in handy when working with data sets that contain many **outliers** because it uses statistics that are robust

to **outliers** (**median** and **interquartile range**), in contrast with the previous scalers, which use statistics that are highly affected by **outliers** such as the **maximum**, the **minimum**, the **mean**, and the **standard deviation**.

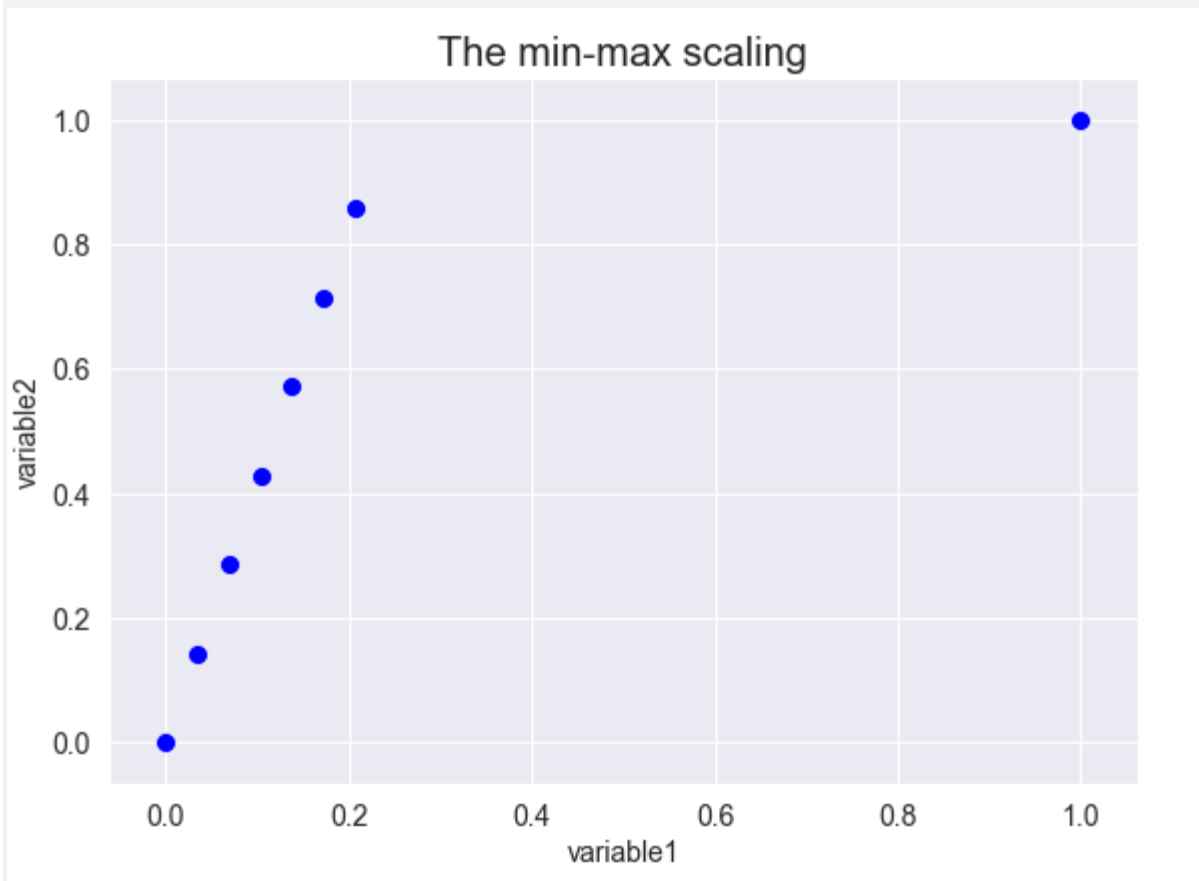
Let's see how **outliers** affect the results after scaling the data with **min-max scaling** and **robust scaling**.

The following data set contains 10 data points, one of them being an **outlier** (variable1 = 30).

	variable1	variable2
0	1	1
1	2	2
2	3	3
3	4	4
4	5	5
5	6	6
6	7	7
7	30	8

The **min-max scaling** shifts the variable 1 towards 0 due to the presence of an **outlier** as compared with variable 2 where the points are evenly distributed in a range from 0 to 1.

	variable1	variable2
0	0.000000	0.000000
1	0.034483	0.142857
2	0.068966	0.285714
3	0.103448	0.428571
4	0.137931	0.571429
5	0.172414	0.714286
6	0.206897	0.857143
7	1.000000	1.000000



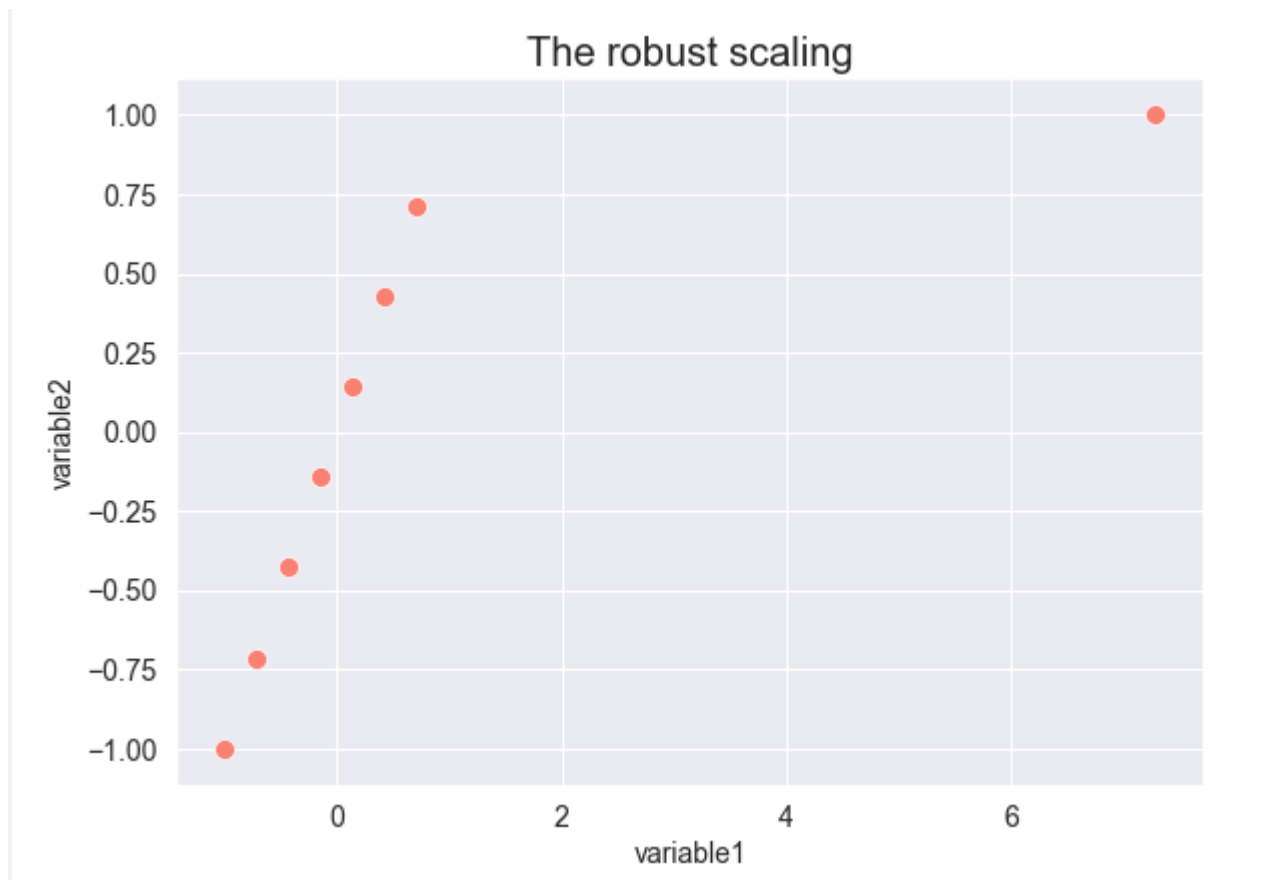
Before scaling, the first data point has a value of (1,1), both variable 1 and variable 2 have equal values. Once transformed, the



value of variable 2 is much larger than variable 1 (0.034,0.142).  
This is because variable 1 has an **outlier**.

On the contrary, if we apply **robust scaling**, both variables have the same values (-1.00,-1.00) after the transformation, because both features have the same **median** and **interquartile range**, being the **outlier** the value that is shifted.

	variable1	variable2
0	-1.000000	-1.000000
1	-0.714286	-0.714286
2	-0.428571	-0.428571
3	-0.142857	-0.142857
4	0.142857	0.142857
5	0.428571	0.428571
6	0.714286	0.714286
7	7.285714	1.000000



Now, it is time to apply the robust scaling to the cars data set 🍷

As we previously did, we can perform robust scaling using **Pandas**.

	odometer_reading	fuel_economy
0	-0.742857	-0.104167
1	0.000000	0.000000
2	-0.428571	0.895833
3	0.571429	1.145833
4	0.857143	-0.312500

The **median** is defined as the midpoint of the distribution, meaning 50% of the values of the distribution are smaller than the **median**. In **Pandas**, we can calculate it with the **.median()** or the **.quantile(0.5)** methods. The **first quartile** is the median of the lower half of the data set (25% of the values lie below the first quartile) and can be calculated with the **.quantile(0.25)** method. The **third quartile** represents the median of the upper half of the data set (75% of the values lie below the third quartile) and can be calculated with the **.quantile(0.75)** method.

As an alternative to **Pandas**, we can also perform **robust scaling** using the **Scikit-learn** library.

	odometer_reading	fuel_economy
0	-0.742857	-0.104167
1	0.000000	0.000000
2	-0.428571	0.895833
3	0.571429	1.145833
4	0.857143	-0.312500

As shown above, we obtain the same results as before 🙌

The following plot shows the results after transforming the data with robust scaling.

