Topics for Exam in Statistical Analysis of Econometric Time Series (StatØ2) 2021/2022, Block 1, Lecturer T. Mikosch

(1) **Stationary processes** Definition of stationarity, autocovariance and autocorrelation functions, examples of stationary processes, strict stationarity, examples of strictly stationary processes, relations between stationarity and strict stationarity, Gaussian stationary time series.

(2) **Ergodic time series** Definition of ergodicity, examples of ergodic time series, the strong law of large numbers for the sample mean, sample autocovariance and sample autocorrelation functions of ergodic time series.

(3) **Mixing and strong mixing** Definitions of mixing and strong mixing, examples of mixing and strongly mixing time series, the limit of the variance $\mathrm{var}(\sqrt{n}\,\overline{X}_n)$, Ibragimov's central limit theorem (Theorem 2.40), examples of its use.

(4) **Transformations to stationarity** Returns and log-returns of financial time series, transformations in the presence of trends or seasonalities, differencing and the backshift operator.

(5) **The autocovariance and autocorrelation functions of a stationary process** Elementary properties, equivalence of symmetric non-negative definite and autocovariance functions, conditions for the existence of the inverse of the covariance matrix (Proposition 3.3)

(6) **The sample autocovariance and sample autocorrelation functions** Definition of the sample autocovariance and sample autocorrelation functions, property of non-negative definiteness, conditions for asymptotic properties (consistency, asymptotic normality), pitfalls of the sample autocorrelation function (what can go wrong when using data?)

(7) **ARMA processes** Definition of an ARMA process, conditions for causality (Theorem 4.10) and invertibility (Theorem 4.14). Causal ARMA processes have representation as linear processes driven by white noise, calculation of the coefficients of the linear process, meaning of FARIMA processes

(8) **Estimation of ARMA processes** Estimation of the autocorrelation and autocovariance functions, Bartlett's formula, construction of asymptotic confidence bands for the sample autocorrelation function of a linear process driven by white noise, the Yule-Walker estimator (derivation) and Gaussian maximum likelihood (main ideas), meaning of AIC

(9) **ARCH and GARCH I** Definition of ARCH and GARCH processes (X_n), squared ARCH and GARCH processes are ARMA processes driven by dependent white noise, conditional and unconditional moment/covariance calculations for ARCH(1) and GARCH(1,1) processes, conditional prediction intervals or conditionally predicted quantiles of X_{n+1} given $X_1, \ldots, X_n$.

(10) **ARCH and GARCH II** Conditions for stationarity and strict stationarity of ARCH(1) and GARCH(1,1) processes, ergodicity of ARCH(1) and GARCH(1,1) processes, IGARCH, Gaussian maximum likelihood estimation (main ideas), asymptotic properties of these estimators

(11) **Spectral analysis** Spectral distribution function and spectral density of a stationary process, examples (sinusoid, white noise, ARMA, GARCH, squared GARCH,...), Herglotz's theorem (spectral representation of the autocovariance function), the spectral representation of a stationary process (main ideas)

(12) **Estimation of the spectral density** Fourier series representation of the spectral density, the periodogram as estimator of the spectral density, definition and different representations. Properties of the periodogram (non-consistency, asymptotic unbiasedness), smoothing of the periodogram

(13) **Prediction of time series** The projection theorem in Hilbert space, linear h-step prediction of a stationary process, relation with Yule-Walker estimation, the sinusoid as a deterministic time series, Wold decomposition, the general prediction problem