$$e^{\frac{e^{\theta_1}}{\theta_2} - \log(\theta_2)}$$

$$e^{\lambda - \ln\left(\frac{1}{\beta}\right)} \quad \textbf{(1)}$$

$\overset{\text{expand}}{=}$

$$\beta\, e^{\lambda} \quad \textbf{(2)}$$

$$\psi := \frac{e^{\theta_1}}{\theta_2} - \log(\theta_2)$$

$$\psi := \frac{e^{\theta_1}}{\theta_2} - \ln(\theta_2) \quad \textbf{(3)}$$

$$\psi_{d1} := \frac{\partial}{\partial \theta_1} \psi$$

$$\psi_{d1} := \frac{e^{\theta_1}}{\theta_2} \quad \textbf{(4)}$$

$$\psi_{d2} := \frac{\partial}{\partial \theta_2} \psi$$

$$\psi_{d2} := -\frac{e^{\theta_1}}{\theta_2^{\,2}} - \frac{1}{\theta_2} \quad \textbf{(5)}$$

$$\psi_{d11} := \frac{\partial}{\partial \theta_1} \psi_{d1}$$

$$\psi_{d11} := \frac{e^{\theta_1}}{\theta_2} \quad \textbf{(6)}$$

$$\psi_{d12} := \frac{\partial}{\partial \theta_2} \psi_{d1}$$

$$\psi_{d12} := -\frac{e^{\theta_1}}{\theta_2^{\,2}} \quad \textbf{(7)}$$

$$\psi_{d22} := \frac{\partial}{\partial \theta_2} \psi_{d2}$$

$$\psi_{d22} := \frac{2\,e^{\theta_1}}{\theta_2^{\,3}} + \frac{1}{\theta_2^{\,2}} \tag{8}$$

$$expand\left(subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], \psi_{d11}\right)\right)$$

$$\lambda \tag{9}$$

$$expand\left(subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], \psi_{d12}\right)\right)$$

$$-\lambda\,\beta \tag{10}$$

$$expand\left(subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], \psi_{d22}\right)\right)$$

$$2\,\lambda\,\beta^2 + \beta^2 \tag{11}$$

$$subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], \%\right)$$

$$\psi$$

$$\frac{e^{\theta_1}}{\theta_2} - \ln(\theta_2) \tag{12}$$

$$\xrightarrow{\text{differentiate w.r.t. theta\_\_2}}$$

$$-\frac{e^{\theta_1}}{\theta_2^{\,2}} - \frac{1}{\theta_2} \tag{13}$$

$$subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], \%\right)$$

$$-\beta\,\lambda - \beta \tag{14}$$

$$\frac{e^{\theta_1}}{\theta_2}$$

$$\lambda \tag{15}$$

$$-\frac{e^{\theta_1}}{\theta_2^{\,2}} - \frac{1}{\theta_2}$$

$$-\lambda\,\beta - \beta \qquad \textbf{(16)}$$

restart;

$$sx = \frac{e^{\theta_1}}{\theta_2}$$

$$sx = \frac{e^{\theta_1}}{\theta_2} \qquad \textbf{(17)}$$

$$\xrightarrow{\text{solve for theta\_\_1}}$$

$$\left[\left[\theta_1 = \ln\left(sx\,\theta_2\right)\right]\right] \qquad \textbf{(18)}$$

$$\theta_{1s} := \ln\left(sx\,\theta_2\right)$$

$$\theta_{1s} := \ln\left(sx\,\theta_2\right) \qquad \textbf{(19)}$$

$$sy = \frac{e^{\theta_{1s}}}{\theta_2^{\,2}} + \frac{1}{\theta_2}$$

$$sy = \frac{sx}{\theta_2} + \frac{1}{\theta_2} \qquad \textbf{(20)}$$

$$\xrightarrow{\text{solve for theta\_\_2}}$$

$$\left[\left[\theta_2 = \frac{sx+1}{sy}\right]\right] \qquad \textbf{(21)}$$

$$\theta_{2s} := \frac{sx+1}{sy}$$

$$\theta_{2s} := \frac{sx+1}{sy} \qquad \textbf{(22)}$$

$$\theta_{1ss} := \ln\left(sx\cdot\theta_{2s}\right)$$

$$\theta_{1ss} := \ln\left(\frac{sx\,(sx+1)}{sy}\right) \qquad \textbf{(23)}$$

$$i_n := n\cdot\begin{bmatrix} \Psi_{d11} & \Psi_{d12} \\ \Psi_{d12} & \Psi_{d22} \end{bmatrix}$$

$$i_n := \begin{bmatrix} \frac{n\,e^{\theta_1}}{\theta_2} & -\frac{n\,e^{\theta_1}}{\theta_2^2} \\ -\frac{n\,e^{\theta_1}}{\theta_2^2} & n\left(\frac{2\,e^{\theta_1}}{\theta_2^3} + \frac{1}{\theta_2^2}\right) \end{bmatrix} \tag{24}$$

$with(linalg):$
$iinv_n := i_n^{-1}$

$$iinv_n := \begin{bmatrix} \frac{\left(2\,e^{\theta_1} + \theta_2\right)\theta_2}{n\,e^{\theta_1}\left(e^{\theta_1} + \theta_2\right)} & \frac{\theta_2^2}{n\left(e^{\theta_1} + \theta_2\right)} \\ \frac{\theta_2^2}{n\left(e^{\theta_1} + \theta_2\right)} & \frac{\theta_2^3}{n\left(e^{\theta_1} + \theta_2\right)} \end{bmatrix} \tag{25}$$

$$\phi := \begin{bmatrix} \frac{e^{\theta_1}}{\theta_2} \\ \frac{1}{\theta_2} \end{bmatrix}$$

$$\phi := \begin{bmatrix} \frac{e^{\theta_1}}{\theta_2} \\ \frac{1}{\theta_2} \end{bmatrix} \tag{26}$$

$$\phi_{d1} := \frac{\partial}{\partial \theta_1} \phi$$

$$\phi_{d1} := \begin{bmatrix} \frac{e^{\theta_1}}{\theta_2} \\ 0 \end{bmatrix} \tag{27}$$

$$\phi_{d2} := \frac{\partial}{\partial \theta_2} \phi$$

$$\phi_{d2} := \begin{bmatrix} -\frac{e^{\theta_1}}{\theta_2^2} \\ -\frac{1}{\theta_2^2} \end{bmatrix} \tag{28}$$

$$Dphi := \begin{bmatrix} \frac{e^{\theta_1}}{\theta_2} & -\frac{e^{\theta_1}}{\theta_2^2} \\ 0 & -\frac{1}{\theta_2^2} \end{bmatrix}$$

$$Dphi := \begin{bmatrix} \frac{e^{\theta_1}}{\theta_2} & -\frac{e^{\theta_1}}{\theta_2^2} \\ 0 & -\frac{1}{\theta_2^2} \end{bmatrix} \tag{29}$$

$$Vari := simplify\left(Dphi \cdot iinv_n \cdot transpose(Dphi)\right)$$

$$Vari := \begin{bmatrix} \frac{e^{\theta_1}}{n\,\theta_2} & 0 \\ 0 & \frac{1}{\theta_2\, n \left(e^{\theta_1} + \theta_2\right)} \end{bmatrix} \tag{30}$$

$$simplify\left(subs\left(\left[\theta_1 = \log\left(\frac{\lambda}{\beta}\right), \theta_2 = \frac{1}{\beta}\right], Vari\right)\right)$$

$$\begin{bmatrix} \frac{\lambda}{n} & 0 \\ 0 & \frac{\beta^2}{n\,(\lambda + 1)} \end{bmatrix} \tag{31}$$

restart;

$$sumsum := \sum_{i=1}^{n} \left(x[i] - \frac{e^{\beta}}{\beta} + \frac{e^{\beta}}{\beta^2} + \frac{1}{\beta} - y[i] \right)$$

$$sumsum := -\frac{n\, e^{\beta}}{\beta} + \frac{n\, e^{\beta}}{\beta^2} + \frac{n}{\beta} + \sum_{i=1}^{n} (x_i - y_i) \tag{32}$$

$$sumsum_2 := -\frac{n\, e^{\beta}}{\beta} + \frac{n\, e^{\beta}}{\beta^2} + \frac{n}{\beta} + sd$$

$$sumsum_2 := -\frac{n\, e^{\beta}}{\beta} + \frac{n\, e^{\beta}}{\beta^2} + \frac{n}{\beta} + sd \tag{33}$$

$$solve\left(sumsum_2 = 0, \beta \right)$$

$$RootOf\left(-e^{\_Z}\, n\, \_Z + \_Z^2\, sd + e^{\_Z}\, n + \_Z\, n \right) \tag{34}$$

$assume(\text{beta} > 0)$

$solve\left(sumsum = 0, \beta \right)$

Warning, solve may be ignoring assumptions on the input variables.

$$RootOf\left(-e^{\_Z}\, n\, \_Z + \_Z^2 \left(\sum_{i=1}^{n} x_i \right) - \_Z^2 \left(\sum_{i=1}^{n} y_i \right) + e^{\_Z}\, n + \_Z\, n \right) \tag{35}$$