Introduction to Numerical Analysis: Week 7

Content and exercises

- Textbook: Chapters 7.1 and 7.2
- Programming:
  1. Write a program that calculates the derivative of a function f using $f(x+h) - f(x-h)/(2h)$ together with its Richardson and double Richardson extrapolation. Compare error to the exact value of $f'(x)$ (choose e.g. $f(x) = \exp(x)$ and $x = 1$).
- Problem Solving:
  1. Consider the function $f(x) = \log(x+1)$. Compute (with only a calculator) the numerical derivative of f in $x = 2$ using $h = 0.1$. Employ Richardson extrapolation once and twice to improve the result. Compare to the theoretical value.
  2. Consider the integral of $f(x)$ over $[a, b]$ with n sub–divisions such that the step–length is $h = (b-a)/n$. Show that the Richardson extrapolation of the composite Trapez formula with step–length h yields Simpson's formula with step–length $h/2$.
  3. We want to calculate the natural logarithm at 2,
  $$\log(2) = \int_1^2 \frac{1}{x} dx,$$
  with a two decimal precision (i.e. error of at most 0.01) using (1) the composite Trapezoidal method and (2) the composite Simpson's method. Find the number of divisions of $[1, 2]$ in either case in order to achieve this.
  4. Consider the distribution function of the normal distribution,
  $$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$
  We want to calculate the $F(1.96)$ with a two decimal precision. Devise a method for doing so, and find the number of divisions needed using the Trapezoidal method.
- Test sheets will be published before the start of weeks 7 and 8 as a practice for the final exam. TA's will be available helping you with their solution.
- Q& A (spørgetime) will also be available on Wednesdays 11-12 in both weeks 7 and 8.