Introduction to Numerical Analysis: Week 4

New format

From this week onwards the presentation of the course will be different.

- There will be no live lectures at scheduled times.
- Instead I will upload recorded lectures of the relevant theory and programming exercises well in advance.
- You will also have access to the slides and selected programmes.
- On **Wednesday's 11.00-12.00** you can get in contact with me online (Zoom link Forelæsninger v/ Mogens Bladt) for questions.

Content and exercises

- Textbook: Chapter 3.1, 3.2, 3.3, 3.4 (cursorily), 3.5 (pages 109-116 only), Python book: Chapter 6 and 7.
- Programming Training:
  1. Code Newton's method (with symbolic derivative) with a stop criteria.
  2. Code the Bisection Method with a stop criteria.
  3. Code the Secant method.
- Problem Solving:
  1. From textbook: Problems 3.1.2, 3.1.9, 3.3.4, 3.3.7.
  2. Consider the curve $\phi : t \to (t, \log(t)) \in \mathbb{R}^2$. This means that at time t, the position of the a point running on the curve is $\phi(t)$. Find the time t at which the curve is closest to the origin (i.e. to $(0, 0)$) at 5 decimal places. Use Newton's method and a calculator only!
  3. Again, using a calculator only, compute $\pi = 3.1415926...$ to a 10 decimal precision using Newton's method. (Hint: formulate the problem as a non-linear equation which has solution π)

4. Consider $f(x) = x^n$. Write down Newton's method. Realise that it converges even though $f^{(m)}(0) = 0$ for $m < n$. Get an idea about the speed of convergence towards the root by using your Newton programme and plotting. Is it linear, quadratic, something else?

5. Let $f(x)$ be a q–times continuously differentiable function with $f(r) = f^{(1)}(r) = \cdots = f^{(q-1)}(r) = 0$ and $f^{(q)}(r) \neq 0$. Then show that

$$u(x) = \frac{f(x)}{f'(x)}$$

satisfies

$$u'(r) = \lim_{x \to r} \frac{u(x)}{x - r} = \frac{1}{q} \neq 0,$$

and conclude that u must have a *simple zero* at r. (Hint: Use Taylor's theorem for f and f' about r and look at the resulting expression of f/f'). Now using u instead of f in the Newton iteration scheme, and assuming that $x_n \to r$, show that

$$x_{n+1} = x_n - \frac{u(x_n)}{u'(x_n)} \approx x_n - \frac{qf(x_n)}{f'(x_n)}.$$

As the error for u converges to zero quadratically fast, this suggests that the iteration scheme

$$x_{n+1} = x_n - \frac{qf(x_n)}{f'(x_n)}$$

also has an error which vanishes quadratically fast. This can be made precise by a further Taylor argument (see also Problem 3.2.19) but we shall not pursue the point further.