$with(LinearAlgebra)$

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(1)**

$A := \langle\langle 0|1|0\rangle, \langle 1|0|2\rangle, \langle 0|-1|0\rangle\rangle;$

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \tag{2}$$

$Eigenvectors(A);$

$$\begin{bmatrix} 0 \\ \mathrm{I} \\ -\mathrm{I} \end{bmatrix}, \begin{bmatrix} -2 & -1 & -1 \\ 0 & -\mathrm{I} & \mathrm{I} \\ 1 & 1 & 1 \end{bmatrix} \tag{3}$$

$h1 := \langle -2, 0, 1\rangle;$

$$h1 := \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

$h2 := \langle -1, -I, 1\rangle;$

$$h2 := \begin{bmatrix} -1 \\ -\mathrm{I} \\ 1 \end{bmatrix} \tag{5}$$

$h3 := \langle -1, I, 1 \rangle;$

$$h3 := \begin{bmatrix} -1 \\ \mathrm{I} \\ 1 \end{bmatrix} \tag{6}$$

$lambda1 := 0;$

$$\lambda 1 := 0 \tag{7}$$

$lambda2 := I;$

$$\lambda 2 := \mathrm{I} \tag{8}$$

$lambda3 := -I;$

$$\lambda 3 := -\mathrm{I} \tag{9}$$

$Phi1 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Re}(h2), \cos(\mathrm{Im}(lambda2) \cdot t))$
$\quad - VectorScalarMultiply(\mathrm{Im}(h2), \sin(\mathrm{Im}(lambda2) \cdot t)), \exp(\mathrm{Re}(lambda2) \cdot t)) :$
$Phi2 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Im}(h2), \cos(\mathrm{Im}(lambda2) \cdot t))$
$\quad + VectorScalarMultiply(\mathrm{Re}(h2), \sin(\mathrm{Im}(lambda2) \cdot t)), \exp(\mathrm{Re}(lambda2) \cdot t)) :$

$Phi3 := VectorScalarMultiply\left(h1, \mathrm{e}^{lambda1 \cdot t}\right) :$
$\mathrm{Phi} := \langle Phi1 | Phi2 | Phi3 \rangle;$

$$\Phi := \begin{bmatrix} -\cos(t) & -\sin(t) & -2 \\ \sin(t) & -\cos(t) & 0 \\ \cos(t) & \sin(t) & 1 \end{bmatrix} \tag{10}$$

$Phi\_inv := MatrixInverse(\mathrm{Phi}) :$
$Phi\_inv0 := subs([t=0], Phi\_inv) :$

$A\_imprel\_w := MatrixMatrixMultiply(\mathrm{Phi}, Phi\_inv0)$

$$A\_imprel\_w := \begin{bmatrix} -\cos(t) + 2 & \sin(t) & -2\cos(t) + 2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t) - 1 & -\sin(t) & 2\cos(t) - 1 \end{bmatrix} \tag{11}$$

A B :

$A\_rel\_w := A\_imprel\_w$

$$A\_rel\_w := \begin{bmatrix} -\cos(t) + 2 & \sin(t) & -2\cos(t) + 2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t) - 1 & -\sin(t) & 2\cos(t) - 1 \end{bmatrix} \tag{12}$$

$K := \langle \langle ZeroMatrix(3, 3) | A \rangle, \langle -A | IdentityMatrix(3, 3) \rangle \rangle$

$$K := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$vec\_upr := \langle \langle 0, 0, 0 \rangle, \langle v1, v2, v3 \rangle \rangle :$

$vec := LinearSolve(K, vec\_upr, method = 'LU') :$

$v := \langle v1, v2, v3 \rangle :$
$T1 := \langle \langle 0 | -1 | 0 \rangle, \langle 1 | 0 | 2 \rangle, \langle 0 | 0 | 0 \rangle \rangle :$
$T2 := \langle \langle 2 | 0 | 2 \rangle, \langle 0 | 0 | 0 \rangle, \langle -1 | 0 | -1 \rangle \rangle :$

$a := MatrixVectorMultiply(T1, v) :$
$b := MatrixVectorMultiply(T2, v) :$

, :

$B\_rel := MatrixMatrixMultiply(MatrixMatrixMultiply(-\mathrm{Phi}, Phi\_inv0), T1) + T1 + MatrixScalarMultiply(T2, t) :$

, - , :

$B\_rel\_w := SubMatrix(B\_rel, [1..3], [2, 3])$

$$B\_rel\_w := \begin{bmatrix} -\cos(t) + 1 & -2\sin(t) + 2t \\ \sin(t) & -2\cos(t) + 2 \\ \cos(t) - 1 & 2\sin(t) - t \end{bmatrix} \tag{14}$$

( ) :

:

$A\_rel\_w\_even := simplify(MatrixMatrixMultiply(A\_rel\_w, A\_rel\_w))$

$$A\_rel\_w\_even := \begin{bmatrix} -2\cos(t)^2 + 3 & 2\cos(t)\sin(t) & 4\sin(t)^2 \\ 2\cos(t)\sin(t) & 2\cos(t)^2 - 1 & 4\cos(t)\sin(t) \\ -2\sin(t)^2 & -2\cos(t)\sin(t) & 4\cos(t)^2 - 3 \end{bmatrix} \tag{15}$$

$B\_rel\_w\_even := simplify(\langle MatrixVectorMultiply(A\_rel\_w, B\_rel\_w[.., 1]) | MatrixVectorMultiply(A\_rel\_w, B\_rel\_w[.., 2]) | B\_rel\_w[.., 1] \rangle)$

$$B\_rel\_w\_even := \begin{bmatrix} -2\cos(t)^2+\cos(t)+1 & (-4\cos(t)+2)\sin(t)+2t & -\cos(t)+1 \\ \sin(t)\ (2\cos(t)-1) & -4\cos(t)^2+2\cos(t)+2 & \sin(t) \\ 2\cos(t)^2-\cos(t)-1 & (4\cos(t)-2)\sin(t)-t & \cos(t)-1 \end{bmatrix} \quad \textbf{(16)}$$

:

$A\_rel\_w\_odd := A\_rel\_w$

$$A\_rel\_w\_odd := \begin{bmatrix} -\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t)-1 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad \textbf{(17)}$$

$B\_rel\_w\_odd := simplify(\langle MatrixVectorMultiply(A\_rel\_w, B\_rel\_w[..,2]) | B\_rel\_w[..,1] | B\_rel\_w[..,2]\rangle)$

$$B\_rel\_w\_odd := \begin{bmatrix} (-4\cos(t)+2)\sin(t)+2t & -\cos(t)+1 & -2\sin(t)+2t \\ -4\cos(t)^2+2\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ (4\cos(t)-2)\sin(t)-t & \cos(t)-1 & 2\sin(t)-t \end{bmatrix} \quad \textbf{(18)}$$

$A\_rel\_w\_even\_sub := simplify(subs(t=0.25, \mathrm{alpha}=0.0035, A\_rel\_w\_even))$

$$A\_rel\_w\_even\_sub := \begin{bmatrix} 1.122417438 & 0.4794255386 & 0.2448348763 \\ 0.4794255386 & 0.877582562 & 0.9588510772 \\ -0.1224174382 & -0.4794255386 & 0.755165124 \end{bmatrix} \quad \textbf{(19)}$$

$A\_rel\_w\_odd\_sub := simplify(subs(t=0.25, \mathrm{alpha}=0.0035, A\_rel\_w\_odd))$

$$A\_rel\_w\_odd\_sub := \begin{bmatrix} 1.031087578 & 0.2474039593 & 0.062175157 \\ 0.2474039593 & 0.9689124217 & 0.4948079186 \\ -0.0310875783 & -0.2474039593 & 0.937824843 \end{bmatrix} \quad \textbf{(20)}$$

$B\_rel\_w\_even\_sub := simplify(subs(t=0.25, \mathrm{alpha}=0.0035, B\_rel\_w\_even)):$
$B\_rel\_w\_odd\_sub := simplify(subs(t=0.25, \mathrm{alpha}=0.0035, B\_rel\_w\_odd)):$

H (k)

$t1 := time[real]()$

$$t1 := 149027.141 \quad \textbf{(21)}$$

$H\_even\_rel := \frac{1}{16\cdot\alpha^2}\cdot(MatrixMatrixMultiply(Transpose(MatrixInverse(B\_rel\_w\_even)),$
$MatrixInverse(B\_rel\_w\_even))):$
$H\_even\_rel\_sub := simplify(subs(t=0.25, \mathrm{alpha}=0.0035, H\_even\_rel))$

$$H\_even\_rel\_sub := \begin{bmatrix} 2.786261413\ 10^6 & -679823.1148 & 207431.4832 \\ -679823.1148 & 200693.6715 & -57064.90546 \\ 207431.4832 & -57064.90546 & 109197.7893 \end{bmatrix} \quad \textbf{(22)}$$

$$H\_odd\_rel := \frac{1}{16\cdot\alpha^2}\cdot(MatrixMatrixMultiply(Transpose(MatrixInverse(B\_rel\_w\_odd)), MatrixInverse(B\_rel\_w\_odd))):$$

$$H\_odd\_rel\_sub := simplify(subs(t=0.25, \text{alpha}=0.0035, H\_odd\_rel))$$

$$H\_odd\_rel\_sub := \begin{bmatrix} 4.550611640\ 10^{7} & -6.462209154\ 10^{6} & -588307.0967 \\ -6.462209154\ 10^{6} & 983037.6622 & 54987.42993 \\ -588307.0967 & 54987.42993 & 67894.62390 \end{bmatrix} \tag{23}$$

H _ w a v e (k)

$$H\_1\_wave\_rel\_sub := MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(A\_rel\_w\_even\_sub), H\_even\_rel\_sub), A\_rel\_w\_even\_sub)$$

$$H\_1\_wave\_rel\_sub := \begin{bmatrix} 2.77600387343913\ 10^{6} & 659737.227295895 & 192045.168990003 \\ 659737.227295895 & 200693.683607217 & 36978.9959370014 \\ 192045.168990003 & 36978.9959370014 & 88682.6872399090 \end{bmatrix} \tag{24}$$

$$H\_2\_wave\_rel\_sub := MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub)), H\_odd\_rel\_sub), MatrixMatrixMultiply(A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub))$$

$$H\_2\_wave\_rel\_sub := \begin{bmatrix} 6.28701170407467\ 10^{7} & 3.14244358301534\ 10^{7} & 1.80756002900574\ 10^{7} \\ 3.14244358301534\ 10^{7} & 1.57472249696997\ 10^{7} & 9.05897454483837\ 10^{6} \\ 1.80756002900574\ 10^{7} & 9.05897454483837\ 10^{6} & 5.26752137049604\ 10^{6} \end{bmatrix} \tag{25}$$

$$H\_3\_wave\_rel\_sub := MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_even\_sub, A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub)), H\_even\_rel\_sub), MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_even\_sub, A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub))$$

$$H\_3\_wave\_rel\_sub := \begin{bmatrix} 5.56164149991817\ 10^{6} & 3.20943673906733\ 10^{6} & 3.53024535471699\ 10^{6} \\ 3.20943673906733\ 10^{6} & 1.88120620282137\ 10^{6} & 2.02828889054205\ 10^{6} \\ 3.53024535471699\ 10^{6} & 2.02828889054205\ 10^{6} & 2.29893290925136\ 10^{6} \end{bmatrix} \tag{26}$$

$$H\_4\_wave\_rel\_sub := MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub), A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub)), H\_odd\_rel\_sub), MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub), A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub))$$

$$H\_4\_wave\_rel\_sub := \tag{27}$$

$$\begin{bmatrix} 1.47548360332360\ 10^{8} & 8.20130067518280\ 10^{7} & 1.30165623524774\ 10^{8} \\ 8.20130067518280\ 10^{7} & 4.56019086465060\ 10^{7} & 7.23316699516892\ 10^{7} \\ 1.30165623524774\ 10^{8} & 7.23316699516892\ 10^{7} & 1.14914640828449\ 10^{8} \end{bmatrix}$$

( ) :

1

$x\_0 := \langle -0.00377869564857395, -0.00391093582674638, 0.0141511512373208 \rangle$

$$x\_0 := \begin{bmatrix} -0.00377869564857395 \\ -0.00391093582674638 \\ 0.0141511512373208 \end{bmatrix} \quad \textbf{(28)}$$

$\psi\_0\_vec\_rel := \langle \psi_1, \psi_2, \psi_3 \rangle$

$$\psi\_0\_vec\_rel := \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \quad \textbf{(29)}$$

$eq\_1\_1\_rel := -\alpha \cdot x\_0 = (MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel)) / (DotProduct(\psi\_0\_vec\_rel, MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel), conjugate = false))^{1/2}:$

$eq\_1\_2\_rel := DotProduct(\psi\_0\_vec\_rel, \psi\_0\_vec\_rel, conjugate = false) = 1:$
$sys\_1\_rel := \{eq\_1\_1\_rel(1), eq\_1\_1\_rel(2), eq\_1\_1\_rel(3), eq\_1\_2\_rel\}:$
$list1\_rel := convert(fsolve(sys\_1\_rel, \{\alpha, \psi_1, \psi_2, \psi_3\}), list)$

$$list1\_rel := [\alpha = -0.1344323178, \psi_1 = -0.9657526846, \psi_2 = -0.2569708925, \psi_3 = 0.03588471226] \quad \textbf{(30)}$$

for i **from** 1 **to** 4 **do**
$list1\_rel[i] := -list1\_rel[i]$
end do:

$list1\_rel$

$$[-\alpha = 0.1344323178, -\psi_1 = 0.9657526846, -\psi_2 = 0.2569708925, -\psi_3 = -0.03588471226] \quad \textbf{(31)}$$

2

$eq\_2\_1\_rel := -\alpha \cdot x\_0 = (MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel)) /$

$\left(DotProduct\left(\psi\_0\_vec\_rel, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel\right), conjugate = false\right)\right)^{1/2} + \left(MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub), \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel, MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub), \psi\_0\_vec\_rel\right), conjugate = false\right)\right)^{1/2}$:

$sys\_2\_rel := \{eq\_2\_1\_rel(1), eq\_2\_1\_rel(2), eq\_2\_1\_rel(3), eq\_1\_2\_rel\}$:
$list2\_rel := convert\left(fsolve\left(sys\_2\_rel, \{\alpha, \psi_1, \psi_2, \psi_3\}\right), list\right)$

$$list2\_rel := [\alpha = -0.3699153490, \psi_1 = -0.9444447350, \psi_2 = -0.3218639492, \psi_3 = 0.06654127103] \quad \textbf{(32)}$$

for i **from** 1 **to** 4 **do**
$list2\_rel[i] := -list2\_rel[i]$
end do:

$list2\_rel$

$$[-\alpha = 0.3699153490, -\psi_1 = 0.9444447350, -\psi_2 = 0.3218639492, -\psi_3 = -0.06654127103] \quad \textbf{(33)}$$

3

$eq\_3\_1\_rel := -\alpha \cdot x\_0 = \left(MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_rel\right), conjugate = false\right)\right)^{1/2} + \left(MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub), \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel, MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub), \psi\_0\_vec\_rel\right), conjugate = false\right)\right)^{1/2} + \left(MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_rel\_sub), \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel, MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_rel\_sub), \psi\_0\_vec\_rel\right), conjugate = false\right)\right)^{1/2}$:

$sys\_3\_rel := \{eq\_3\_1\_rel(1), eq\_3\_1\_rel(2), eq\_3\_1\_rel(3), eq\_1\_2\_rel\}$:
$list3\_rel := convert\left(fsolve\left(sys\_3\_rel, \{\alpha, \psi_1, \psi_2, \psi_3\}\right), list\right)$

$$list3\_rel := [\alpha = -0.8259339994, \psi_1 = -0.6793714921, \psi_2 = -0.4017710325, \psi_3 = 0.6140312803] \quad \textbf{(34)}$$

for i **from** 1 **to** 4 **do**
$list3\_rel[i] := -list3\_rel[i]$
end do:

$list3\_rel$

$$\left[-\alpha=0.8259339994,\ -\psi_1=0.6793714921,\ -\psi_2=0.4017710325,\ -\psi_3=-0.6140312803\right] \qquad (35)$$

4

$$eq\_4\_1\_rel := -\alpha\cdot x\_0 = \left(MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel,\ MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right),\ conjugate=false\right)\right)^{1/2}$$
$$+ \left(MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel,\ MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right),\ conjugate=false\right)\right)^{1/2}$$
$$+ \left(MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel,\ MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right),\ conjugate=false\right)\right)^{1/2}$$
$$+ \left(MatrixVectorMultiply\left(MatrixInverse(H\_4\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_rel,\ MatrixVectorMultiply\left(MatrixInverse(H\_4\_wave\_rel\_sub),\ \psi\_0\_vec\_rel\right),\ conjugate=false\right)\right)^{1/2} :$$

$sys\_4\_rel := \{eq\_4\_1\_rel(1),\ eq\_4\_1\_rel(2),\ eq\_4\_1\_rel(3),\ eq\_1\_2\_rel\}:$

$list4\_rel := convert\left(fsolve\left(sys\_4\_rel,\ \{\alpha,\ \psi_1,\ \psi_2,\ \psi_3\}\right),\ list\right)$

$$list4\_rel := \left[\alpha=-1.046093556,\ \psi_1=0.5554171309,\ \psi_2=0.2149403850,\ \psi_3=0.8033134143\right] \qquad (36)$$

```
for i from 1 to 4 do
 list4_rel[i] := -list4_rel[i]
 end do:
```

$list4\_rel$

$$\left[-\alpha=1.046093556,\ -\psi_1=-0.5554171309,\ -\psi_2=-0.2149403850,\ -\psi_3=-0.8033134143\right] \qquad (37)$$

$\psi\_0\_vec\_true := \langle -0.5554171309,\ -0.2149403850,\ -0.8033134143\rangle$

$$\psi\_0\_vec\_true := \begin{bmatrix} -0.5554171309 \\ -0.2149403850 \\ -0.8033134143 \end{bmatrix} \qquad (38)$$

:

$\alpha\_true := 1.046093556 :$

$$u\_star\_0 := \frac{1}{\alpha\_true}\cdot\left(MatrixVectorMultiply\left(MatrixMatrixMultiply(A\_rel\_w\_even\_sub,\ MatrixInverse(H\_1\_wave\_rel\_sub)),\ \psi\_0\_vec\_true\right)\right) \Big/$$

$$\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_rel\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}$$

$$u\_star\_0 := \begin{bmatrix} -0.000929651067355703 \\ -0.00423355423312759 \\ -0.00205262762800944 \end{bmatrix} \quad \textbf{(39)}$$

$$u\_star\_1 := \frac{1}{\alpha\_true} \cdot \left(MatrixVectorMultiply\left(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub), MatrixInverse(H\_2\_wave\_rel\_sub)), \psi\_0\_vec\_true\right)\right) \Big/ \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_rel\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}\right)$$

$$u\_star\_1 := \begin{bmatrix} -0.000554064874461612 \\ -0.00354109179555145 \\ -0.00376839531771916 \end{bmatrix} \quad \textbf{(40)}$$

$$u\_star\_2 := \frac{1}{\alpha\_true} \cdot \left(MatrixVectorMultiply\left(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_rel\_w\_even\_sub, A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub), MatrixInverse(H\_3\_wave\_rel\_sub)), \psi\_0\_vec\_true\right)\right) \Big/ \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_rel\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}\right)$$

$$u\_star\_2 := \begin{bmatrix} -0.00124210734412104 \\ -0.00513342027416163 \\ -0.000581078615477272 \end{bmatrix} \quad \textbf{(41)}$$

$$u\_star\_3 := \frac{1}{\alpha\_true} \cdot \left(MatrixVectorMultiply\left(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixM\ldots A\_rel\_w\_odd\_sub, A\_rel\_w\_even\_sub), A\_rel\_w\_odd\_sub), A\_rel\_w\_even\_sub), MatrixInverse(H\_4\_wave\_rel\_sub)), \psi\_0\_vec\_true\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_4\_wave\_rel\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}$$

$$u\_star\_3 := \begin{bmatrix} -0.000576886557204929 \\ -0.00398672212055113 \\ -0.000667644202491066 \end{bmatrix} \quad \textbf{(42)}$$

$$x\_0\_star := x\_0$$

$$x\_0\_star := \begin{bmatrix} -0.00377869564857395 \\ -0.00391093582674638 \\ 0.0141511512373208 \end{bmatrix} \quad (43)$$

$x\_1\_star := MatrixVectorMultiply(A\_rel\_w\_even\_sub, x\_0\_star) + u\_star\_0$

$$x\_1\_star := \begin{bmatrix} -0.00358123210868571 \\ 0.00409152009522085 \\ 0.0109714090070685 \end{bmatrix} \quad (44)$$

$x\_2\_star := MatrixVectorMultiply(A\_rel\_w\_odd\_sub, x\_1\_star) + u\_star\_1$

$$x\_2\_star := \begin{bmatrix} -0.00255222146702335 \\ 0.00496596190037892 \\ 0.00561993817429974 \end{bmatrix} \quad (45)$$

$x\_3\_star := MatrixVectorMultiply(A\_rel\_w\_even\_sub, x\_2\_star) + u\_star\_2$

$$x\_3\_star := \begin{bmatrix} -0.000349999397871417 \\ 0.00338970491393784 \\ 0.00159453014774592 \end{bmatrix} \quad (46)$$

$x\_4\_star := MatrixVectorMultiply(A\_rel\_w\_odd\_sub, x\_3\_star) + u\_star\_3$

$$x\_4\_star := \begin{bmatrix} -9.81340236936440\ 10^{-12} \\ -1.67741133483279\ 10^{-11} \\ 9.69142301132653\ 10^{-14} \end{bmatrix} \quad (47)$$

$\psi\_1\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_rel\_w\_even)), \psi\_0\_vec\_true)))$

$$\psi\_1\_vec := \begin{bmatrix} -0.4220223931 \\ -0.3074757436 \\ -0.5365239390 \end{bmatrix} \quad (48)$$

$\psi\_2\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_rel\_w\_odd)), \psi\_1\_vec)))$

$$\psi\_2\_vec := \begin{bmatrix} -0.3423921008 \\ -0.3262452031 \\ -0.3772633546 \end{bmatrix} \quad (49)$$

$\psi\_3\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_rel\_w\_even)), \psi\_2\_vec)))$

$$\psi\_3\_vec := \begin{bmatrix} -0.1817129690 \\ -0.3030252719 \\ -0.0559050913 \end{bmatrix} \quad (50)$$

$\psi\_4\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_rel\_w\_odd)),$

$\psi\_3\_vec)))$

$$\psi\_4\_vec := \begin{bmatrix} -0.1106543792 \\ -0.2624795830 \\ 0.08621208833 \end{bmatrix} \quad \textbf{(51)}$$

$t2 := time[real]()$

$$t2 := 149033.257 \quad \textbf{(52)}$$

$total\_time := t2 - t1$

$$total\_time := 6.116 \quad \textbf{(53)}$$