№4 - Алгоритм определения принадлежности точки $x \in \mathbb{R}^4$

$$X = conv\{x^1, ..., x^M\} \in \mathbb{R}^4$$

$$real^4 \quad \textbf{(1)}$$

M .

```
x1 := [1, 1, 1, 1]:
x2 := [-1, 1, 1, 1]:
x3 := [1,-1, 1, 1]:
x4 := [1, 1,-1, 1]:
x5 := [1, 1, 1 ,-1]:
x6 := [-1, -1, 1, 1]:
x7 := [1, -1, -1, 1]:
x8 := [1, 1, -1,-1]:
x9 := [-1,-1,-1, 1]:
x10 := [1, -1, -1, -1]:
x11 := [-1, -1, -1, -1]:
 x12 := [-1, 1, 1, -1]:
x13 := [-1 , 1, -1, 1]:
x14 := [1, -1, 1, -1]:
x15 := [-1, 1, -1, -1]:
x16 := [-1, -1, 1, -1]:
x17 := [0, 0, 0, 0]:
x18 := [0, 1, 0, 1]:
x19 := [0 , 0, 0, 1]:
```

(x 1 7 , x 1 8)

```
x_list2 := [[0, 1, 0, 1], [1, 1, 1, 1], ... ]
x_list := [x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19]
```

$$x\_list := [[1, 1, 1, 1], [-1, 1, 1, 1], [1, -1, 1, 1], [1, 1, -1, 1], [1, 1, 1, -1], [-1, -1, 1, 1], [1, -1, -1, 1], [1, 1, -1, -1], [-1, -1, -1, 1], [1, -1, -1, -1], [-1, -1, -1, -1], [-1, 1, 1, -1], [-1, 1, -1, 1], [1, -1, 1, -1], [-1, 1, -1, -1], [-1, -1, 1, -1], [0, 0, 0, 0], [0, 1, 0, 1], [0, 0, 0, 1]] \quad \textbf{(2)}$$

```
with(ComputationalGeometry) :
```

```
hull := ConvexHull(x_list)
```

$$hull := [[12, 16, 9, 11], [6, 12, 16, 9], [12, 15, 16, 11], [15, 12, 9, 11], [12, 13, 15, 9], [6, 12, 9, 2], [12, 6, 16, 2], [12, 13, 9, 2], [13, 12, 15, 2], [7, 4, 3, 1], [3, 7, 1, 2], [7, 4, 1, 2], [7, 13, 9, 4], [6, 7, 9, 3], [6, 7, 3, 2], [7, 6, 9, 2], [7, 13, 4, 2], [13, 7, 9, 2], [14, 3, 1, 2], [5, 14, 3, 1], [5, 14, 1, 2], [12, 14, 16, 5], [14, 6, 3, 2], [6, 14, 16, 2], [12, 14, 5, 2], [14, 12, 16, 2], [16, 14, 9, 11], [10, 14, 16, 11], [14, 6, 16, 9], [14, 10, 9, 11], [7, 14, 10, 9], [14, 6, \quad \textbf{(3)}$$

```
    9, 3], [7, 14, 9, 3], [14, 7, 10, 3], [4, 8, 1, 2], [8, 5, 1, 2], [13, 8, 4, 2], [8, 13, 15, 2], [8,
    12, 5, 2], [12, 8, 15, 2], [4, 8, 3, 1], [8, 5, 3, 1], [8, 7, 4, 3], [7, 8, 10, 3], [14, 8, 5, 3], [8,
    14, 10, 3], [8, 15, 9, 11], [10, 8, 9, 11], [13, 8, 15, 9], [8, 7, 10, 9], [13, 8, 9, 4], [8, 7, 9,
    4], [15, 8, 16, 11], [8, 10, 16, 11], [8, 12, 15, 16], [14, 8, 10, 16], [8, 12, 16, 5], [14, 8, 16,
    5]]
hvec := convert(hull, Vector) :

hset := convert(hvec, set)
                    hset := {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}                    (4)
     ,          (   1 7    1 8
)

     ,
        ,
.

with(ArrayTools) :
with(LinearAlgebra) :
with(Optimization) :
ZLP := proc(X_vec, bx)
local lambdas, sumlam, n, constraint, solved, bds, lam;
lambdas := Vector(numelems(X_vec), symbol = λ) :
bds := { } :
sumlam := add(lambdas(n), n = 1 ..numelems(X_vec)) :
constraint := simplify(VectorMatrixMultiply(Transpose(lambdas), X_vec)) :
bds := bds union {constraint(1) = bx[1], constraint(2) = bx[2], constraint(3) = bx[3], constraint(4)
    = bx[4]} :
solved := LPSolve(sumlam, bds, assume = nonnegative)
end proc
ZLP := proc(X_vec, bx)                                                                              (5)
    local lambdas, sumlam, n, constraint, solved, bds, lam;
    lambdas := Vector(numelems(X_vec), symbol = λ);
    bds := { };
    sumlam := add(lambdas(n), n = 1 ..numelems(X_vec));
    constraint := simplify(LinearAlgebra:-VectorMatrixMultiply(LinearAlgebra:-
    Transpose(lambdas), X_vec));
    bds := bds union {constraint(1) = bx[1], constraint(2) = bx[2], constraint(3) = bx[3],
    constraint(4) = bx[4]};
    solved := Optimization:-LPSolve(sumlam, bds, assume = nonnegative)
end proc


X_vec := Vector( ) :
for xi in x_list do
Append(X_vec, convert(xi, Vector))
end do:
```

$a := [0.34, 0.5, 0.764, 0.34]$

$$a := [0.34, 0.5, 0.764, 0.34] \quad \textbf{(6)}$$

$a\_zlp := ZLP(X\_vec, a)$

$$a\_zlp := \left[0.764000000000000, \left[\lambda_1 = 0.208000000000000, \lambda_2 = 0.212000000000000, \lambda_3 = 0.132000000000000, \lambda_4 = 0., \lambda_5 = 0.212000000000000, \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0.\right]\right] \quad \textbf{(7)}$$

0 . 7 , ,

.

$b := [-1, 2, -3, 4.12]:$

$b\_zlp := ZLP(X\_vec, b)$

$$b\_zlp := \left[4.12000000000000, \left[\lambda_1 = 0., \lambda_2 = 0.560000000000000, \lambda_3 = 0., \lambda_4 = 1.56000000000000, \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 1.06000000000000, \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0.940000000000000, \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0.\right]\right] \quad \textbf{(8)}$$

3 , ,

, . . > 1 .