. .   8 - 4 0 4 - 1 7

№ 14.

:

:

$with(LinearAlgebra):$

$A := \left\langle \langle 2|0|0|0\rangle, \left\langle 0\left|\frac{1}{2}\right|0\middle|0\right\rangle, \left\langle 0\middle|0\middle|\frac{\sqrt{2}}{2}\middle|-\frac{\sqrt{2}}{2}\right\rangle, \left\langle 0\middle|0\middle|\frac{\sqrt{2}}{2}\middle|\frac{\sqrt{2}}{2}\right\rangle\right\rangle$

$$A := \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{1}$$

:

$EV := Eigenvectors(A)$

$$EV := \begin{bmatrix} \frac{\sqrt{2}}{2} + \frac{I\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} - \frac{I\sqrt{2}}{2} \\ \frac{1}{2} \\ 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ I & -I & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

```
h1 := Column(EV[2], 1) :
h2 := Column(EV[2], 2) :
h3 := Column(EV[2], 3) :
h4 := Column(EV[2], 4) :
lambda1 := EV[1](1) :
lambda2 := EV[1](2) :
lambda3 := EV[1](3) :
lambda4 := EV[1](4) :

MatrixPhi := proc(h1, h2, h3, h4, lambda1, lambda2, lambda3, lambda4)
local step, Phi1, Phi2, Phi3, Phi4, Phi;
step := 1;
if Im(lambda1) = 0 then
```

```
   Phi1 ≔ VectorScalarMultiply(h1, exp(lambda1 * t));
   step ≔ 2;
else
   Phi1 ≔ VectorScalarMultiply(VectorScalarMultiply(Re(h1), cos(Im(lambda1) * t))
     - VectorScalarMultiply(Im(h1), sin(Im(lambda1) * t)), exp(Re(lambda1) * t));
   Phi2 ≔ VectorScalarMultiply(VectorScalarMultiply(Im(h1), cos(Im(lambda1) * t))
     + VectorScalarMultiply(Re(h1), sin(Im(lambda1) * t)), exp(Re(lambda1) * t));
   step ≔ 3;
end if;

if step = 2 then
   if Im(lambda2) = 0 then Phi2 ≔ VectorScalarMultiply(h2, exp(lambda2 * t)); step ≔ 3;
    else Phi2 ≔ VectorScalarMultiply(VectorScalarMultiply(Re(h2), cos(Im(lambda2) * t))
     - VectorScalarMultiply(Im(h2), sin(Im(lambda2) * t)), exp(Re(lambda2) * t)); Phi3 ≔
      VectorScalarMultiply(VectorScalarMultiply(Im(h2), cos(Im(lambda2) * t))
     + VectorScalarMultiply(Re(h2), sin(Im(lambda2) * t)), exp(Re(lambda2) * t)); step ≔ 4; end
     if; end if;
if step = 3 then
   if Im(lambda3) = 0 then Phi3 ≔ VectorScalarMultiply(h3, exp(lambda3 * t)); step ≔ 4;
   else Phi3 ≔ VectorScalarMultiply(VectorScalarMultiply(Re(h3), cos(Im(lambda3) * t))
     - VectorScalarMultiply(Im(h3), sin(Im(lambda3) * t)), exp(Re(lambda3) * t)); Phi4 ≔
      VectorScalarMultiply(VectorScalarMultiply(Re(h4), cos(Im(lambda4) * t))
     - VectorScalarMultiply(Im(h4), sin(Im(lambda4) * t)), exp(Re(lambda4) * t)); step ≔ 5; end
     if;
end if;



if step = 4 then
   if Im(lambda4) = 0 then Phi4 ≔ VectorScalarMultiply(h4, exp(lambda4 * t)); step ≔ 5;
     else step ≔ some_error_happened; end if;
end if;

Phi ≔ ⟨Phi1 | Phi2 | Phi3 | Phi4⟩;


 end proc
MatrixPhi := proc(h1, h2, h3, h4, λ1, λ2, λ3, λ4)                                    (3)
    local step, Φ1, Φ2, Φ3, Φ4, Φ;
    step := 1;
    if ℑ(λ1) = 0 then
        Φ1 := LinearAlgebra:-VectorScalarMultiply(h1, exp(λ1 * t)); step := 2
    else
        Φ1 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
        VectorScalarMultiply(ℜ(h1), cos(ℑ(λ1) * t)) − LinearAlgebra:-
        VectorScalarMultiply(ℑ(h1), sin(ℑ(λ1) * t)), exp(ℜ(λ1) * t));
```

```
        Φ2 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
        VectorScalarMultiply(ℑ(h1), cos(ℑ(λ1) * t)) + LinearAlgebra:-
        VectorScalarMultiply(ℜ(h1), sin(ℑ(λ1) * t)), exp(ℜ(λ1) * t));
        step := 3
    end if;
    if step = 2 then
        if ℑ(λ2) = 0 then
            Φ2 := LinearAlgebra:-VectorScalarMultiply(h2, exp(λ2 * t)); step := 3
        else
            Φ2 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
            VectorScalarMultiply(ℜ(h2), cos(ℑ(λ2) * t)) − LinearAlgebra:-
            VectorScalarMultiply(ℑ(h2), sin(ℑ(λ2) * t)), exp(ℜ(λ2) * t));
            Φ3 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
            VectorScalarMultiply(ℑ(h2), cos(ℑ(λ2) * t)) + LinearAlgebra:-
            VectorScalarMultiply(ℜ(h2), sin(ℑ(λ2) * t)), exp(ℜ(λ2) * t));
            step := 4
        end if
    end if;
    if step = 3 then
        if ℑ(λ3) = 0 then
            Φ3 := LinearAlgebra:-VectorScalarMultiply(h3, exp(λ3 * t)); step := 4
        else
            Φ3 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
            VectorScalarMultiply(ℜ(h3), cos(ℑ(λ3) * t)) − LinearAlgebra:-
            VectorScalarMultiply(ℑ(h3), sin(ℑ(λ3) * t)), exp(ℜ(λ3) * t));
            Φ4 := LinearAlgebra:-VectorScalarMultiply(LinearAlgebra:-
            VectorScalarMultiply(ℜ(h4), cos(ℑ(λ4) * t)) − LinearAlgebra:-
            VectorScalarMultiply(ℑ(h4), sin(ℑ(λ4) * t)), exp(ℜ(λ4) * t));
            step := 5
        end if
    end if;
    if step = 4 then
        if ℑ(λ4) = 0 then
            Φ4 := LinearAlgebra:-VectorScalarMultiply(h4, exp(λ4 * t)); step := 5
        else
            step := some_error_happened
```

```
        end if
    end if;
    Φ := < Φ1|Φ2|Φ3|Φ4 >
end proc
```

$Phi\_inv := MatrixInverse(Phi2):$
$Phi\_inv0 := simplify(subs([t=0], Phi\_inv)):$
$Phi2 := MatrixPhi(h1, h2, h3, h4, lambda1, lambda2, lambda3, lambda4)$

$$\Phi 2 := \begin{bmatrix} 0 & 0 & 0 & e^{2t} \\ 0 & 0 & e^{\frac{t}{2}} & 0 \\ -e^{\frac{\sqrt{2}\,t}{2}} \sin\left(\frac{\sqrt{2}\,t}{2}\right) & e^{\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right) & 0 & 0 \\ e^{\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right) & e^{\frac{\sqrt{2}\,t}{2}} \sin\left(\frac{\sqrt{2}\,t}{2}\right) & 0 & 0 \end{bmatrix} \tag{4}$$

A B :

$A\_imp\_w := MatrixMatrixMultiply(Phi2, Phi\_inv0)$

$$A\_imp\_w := \begin{bmatrix} e^{-t} & 0 & 0 & 0 \\ 0 & e^{-t} & 0 & 0 \\ 0 & 0 & \frac{e^{(1+\sqrt{2})t}}{2} + \frac{e^{(1-\sqrt{2})t}}{2} & \frac{e^{(1+\sqrt{2})t}\sqrt{2}}{2} - \frac{e^{(1-\sqrt{2})t}\sqrt{2}}{2} \\ 0 & 0 & \frac{e^{(1+\sqrt{2})t}\sqrt{2}}{4} - \frac{e^{(1-\sqrt{2})t}\sqrt{2}}{4} & \frac{e^{(1+\sqrt{2})t}}{2} + \frac{e^{(1-\sqrt{2})t}}{2} \end{bmatrix} \tag{5}$$

$B\_imp\_w := A\_imp\_w$

$$B\_imp\_w := \begin{bmatrix} e^{2t} & 0 & 0 & 0 \\ 0 & e^{\frac{t}{2}} & 0 & 0 \\ 0 & 0 & e^{\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right) & -e^{\frac{\sqrt{2}\,t}{2}} \sin\left(\frac{\sqrt{2}\,t}{2}\right) \\ 0 & 0 & e^{\frac{\sqrt{2}\,t}{2}} \sin\left(\frac{\sqrt{2}\,t}{2}\right) & e^{\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right) \end{bmatrix} \tag{6}$$

, .

$A := \langle\langle -1|0|0|0\rangle, \langle 0|-1|0|0\rangle, \langle 0|0|1|2\rangle, \langle 0|0|1|1\rangle\rangle$

$$A := \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{7}$$

$EV2 := Eigenvectors(A)$

$$EV2 := \begin{bmatrix} -1 \\ -1 \\ 1+\sqrt{2} \\ 1-\sqrt{2} \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{8}$$

$h1 := Column(EV2[2], 1):$
$h2 := Column(EV2[2], 2):$
$h3 := Column(EV2[2], 3):$
$h4 := Column(EV2[2], 4):$
$lambda1 := EV2[1](1):$
$lambda2 := EV2[1](2):$
$lambda3 := EV2[1](3):$
$lambda4 := EV2[1](4):$

$Phi2 := MatrixPhi(h1, h2, h3, h4, lambda1, lambda2, lambda3, lambda4)$

$$\Phi 2 := \begin{bmatrix} 0 & e^{-t} & 0 & 0 \\ e^{-t} & 0 & 0 & 0 \\ 0 & 0 & e^{(1+\sqrt{2})t}\sqrt{2} & -e^{(1-\sqrt{2})t}\sqrt{2} \\ 0 & 0 & e^{(1+\sqrt{2})t} & e^{(1-\sqrt{2})t} \end{bmatrix} \tag{9}$$