```
ewith(LinearAlgebra)
```

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
  BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
  ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
  CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
  CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant,
  Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers,
  Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm,
  FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations,
  GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix,
  GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
  HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
  IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct,
  LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2,
  MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply,
  MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply,
  MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace,
  OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition,
  RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm,
  Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply,
  ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm,
  StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve,
  ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix,
  VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply,
  ZeroMatrix, ZeroVector, Zip ]                                                    (1)
```

```
A := <<0|1|0>, <1 |0|2>, <0|-1|0>>;
```

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \tag{2}$$

```
Eigenvectors(A);
```

$$\begin{bmatrix} 0 \\ I \\ -I \end{bmatrix}, \begin{bmatrix} -2 & -1 & -1 \\ 0 & -I & I \\ 1 & 1 & 1 \end{bmatrix} \tag{3}$$

```
h1 := <-2, 0, 1>;
```

$$h1 := \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

```
h2 := <-1, -I, 1>;
```

$$h2 := \begin{bmatrix} -1 \\ -I \\ 1 \end{bmatrix} \tag{5}$$

$h3 := \langle -1, I, 1 \rangle;$

$$h3 := \begin{bmatrix} -1 \\ I \\ 1 \end{bmatrix} \tag{6}$$

$lambda1 := 0;$

$$\lambda 1 := 0 \tag{7}$$

$lambda2 := I;$

$$\lambda 2 := I \tag{8}$$

$lambda3 := -I;$

$$\lambda 3 := -I \tag{9}$$

$Phi1 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Re}(h2), \cos(\mathrm{Im}(lambda2)\cdot t))$
$\quad - VectorScalarMultiply(\mathrm{Im}(h2), \sin(\mathrm{Im}(lambda2)\cdot t)), \exp(\mathrm{Re}(lambda2)\cdot t)) :$
$Phi2 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Im}(h2), \cos(\mathrm{Im}(lambda2)\cdot t))$
$\quad + VectorScalarMultiply(\mathrm{Re}(h2), \sin(\mathrm{Im}(lambda2)\cdot t)), \exp(\mathrm{Re}(lambda2)\cdot t)) :$

$Phi3 := VectorScalarMultiply(h1, e^{lambda1\cdot t}) :$
$\mathrm{Phi} := \langle Phi1 | Phi2 | Phi3 \rangle;$

$$\Phi := \begin{bmatrix} -\cos(t) & -\sin(t) & -2 \\ \sin(t) & -\cos(t) & 0 \\ \cos(t) & \sin(t) & 1 \end{bmatrix} \tag{10}$$

$Phi\_inv := MatrixInverse(\mathrm{Phi}) :$
$Phi\_inv0 := subs([t=0], Phi\_inv);$

$$Phi\_inv0 := \begin{bmatrix} \frac{\cos(0)}{\sin(0)^2+\cos(0)^2} & \frac{\sin(0)}{\sin(0)^2+\cos(0)^2} & \frac{2\cos(0)}{\sin(0)^2+\cos(0)^2} \\ \frac{\sin(0)}{\sin(0)^2+\cos(0)^2} & -\frac{\cos(0)}{\sin(0)^2+\cos(0)^2} & \frac{2\sin(0)}{\sin(0)^2+\cos(0)^2} \\ -1 & 0 & -1 \end{bmatrix} \tag{11}$$

A B :

$A\_imp\_w := MatrixMatrixMultiply(\mathrm{Phi}, Phi\_inv0)$

$$A\_imp\_w := \begin{bmatrix} -\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t)-1 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \tag{12}$$

$B\_imp\_w := A\_imp\_w$

$$B\_imp\_w := \begin{bmatrix} -\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t)-1 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad (13)$$

Если допустить, что в задаче спутника управление по первой координате (Δr) не производится ( ) , B _ i m p _ w :

$B\_imp\_w := SubMatrix(A\_imp\_w, [1..3], [2, 3])$

$$B\_imp\_w := \begin{bmatrix} \sin(t) & -2\cos(t)+2 \\ \cos(t) & 2\sin(t) \\ -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad (14)$$

A B :

$A\_rel\_w := A\_imp\_w$

$$A\_rel\_w := \begin{bmatrix} -\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t)-1 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad (15)$$

$K := \langle \langle ZeroMatrix(3, 3) | A \rangle, \langle -A | IdentityMatrix(3, 3) \rangle \rangle$

$$K := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$vec\_upr := \langle \langle 0, 0, 0 \rangle, \langle v1, v2, v3 \rangle \rangle :$

$vec := LinearSolve(K, vec\_upr, method = 'LU')$

$$vec := \begin{bmatrix} -v2 - 2\,\_t0_{1,1} \\ v1 + 2\,v3 \\ \_t0_{1,1} \\ 2\,v3 + 2\,v1 \\ 0 \\ -v3 - v1 \end{bmatrix} \tag{17}$$

$v := \langle v1, v2, v3 \rangle$

$$v := \begin{bmatrix} v1 \\ v2 \\ v3 \end{bmatrix} \tag{18}$$

$T1 := \langle \langle 0|-1|\,0\rangle, \langle 1|\,0|\,2\rangle, \langle 0|\,0|0\rangle \rangle :$
$T2 := \langle \langle 2|0|2\rangle, \langle 0|0|0\rangle, \langle -1|0|-1\rangle \rangle :$

$a := MatrixVectorMultiply(T1, v)$

$$a := \begin{bmatrix} -v2 \\ v1 + 2\,v3 \\ 0 \end{bmatrix} \tag{19}$$

$b := MatrixVectorMultiply(T2, v)$

$$b := \begin{bmatrix} 2\,v3 + 2\,v1 \\ 0 \\ -v3 - v1 \end{bmatrix} \tag{20}$$

, :

$B\_rel := MatrixMatrixMultiply(MatrixMatrixMultiply(-\mathrm{Phi}, Phi\_inv0), T1) + T1 + MatrixScalarMultiply(T2, t)$

$$B\_rel := \begin{bmatrix} -\sin(t) + 2\,t & -\cos(t) + 1 & -2\sin(t) + 2\,t \\ -\cos(t) + 1 & \sin(t) & -2\cos(t) + 2 \\ \sin(t) - t & \cos(t) - 1 & 2\sin(t) - t \end{bmatrix} \tag{21}$$

, - , :

$B\_rel\_w := SubMatrix(B\_rel, [1..3], [2, 3])$

$$B\_rel\_w := \begin{bmatrix} -\cos(t) + 1 & -2\sin(t) + 2\,t \\ \sin(t) & -2\cos(t) + 2 \\ \cos(t) - 1 & 2\sin(t) - t \end{bmatrix} \tag{22}$$

,

```
with(Optimization)
    [ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]
```

(23)

```
ZLP := proc(X_vec, bx)
local lambdas, sumlam, n, constraint, solved, bds, lam;
lambdas := Vector(numelems(X_vec), symbol=λ) :
bds := Vector( ) :
for lam in lambdas do
 Append(bds, 0 ≤ lam) :
 end do:
 bds := convert(bds, set) :
sumlam := add(lambdas(n), n=1..numelems(X_vec)) :
 constraint := simplify(VectorMatrixMultiply(Transpose(lambdas), X_vec)) :
 bds := bds union {constraint(1)=bx[1], constraint(2)=bx[2], constraint(3)=bx[3]} :
  solved := LPSolve(sumlam, bds, assume=nonnegative)
 end proc
```

(24)

```
ZLP := proc(X_vec, bx)
    local lambdas, sumlam, n, constraint, solved, bds, lam;
    lambdas := Vector(numelems(X_vec), symbol=λ);
    bds := Vector( );
    for lam in lambdas do ArrayTools:-Append(bds, 0 <= lam) end do;
    bds := convert(bds, set);
    sumlam := add(lambdas(n), n=1..numelems(X_vec));
    constraint := simplify(LinearAlgebra:-VectorMatrixMultiply(LinearAlgebra:-
    Transpose(lambdas), X_vec));
    bds := bds union {constraint(1)=bx[1], constraint(2)=bx[2], constraint(3)=bx[3]};
    solved := Optimization:-LPSolve(sumlam, bds, assume=nonnegative)
end proc
```

```
ZLP_u := proc(X_vec, u_vec, bx)
local lambdas, sumlam, n, constraint, solved, bds, lam, mus, summu, i;
lambdas := Vector(numelems(X_vec), symbol=λ) :
 mus := Vector(numelems(u_vec), symbol=u) :
bds := Vector( ) :
 bds := convert(bds, set) :
sumlam := add(lambdas(n), n=1..numelems(X_vec)) :
 summu := add(mus(i), i=1..numelems(u_vec)) :
 constraint := simplify(VectorMatrixMultiply( Transpose(lambdas), X_vec)
     - VectorMatrixMultiply(Transpose(mus), u_vec)) :
 bds := bds union {constraint(1)=bx[1], constraint(2)=bx[2], constraint(3)=bx[3], summu=1} :
  solved := LPSolve(sumlam, bds, assume=nonnegative)
 end proc
```

```
ZLP_u := proc(X_vec, u_vec, bx)                                                    (25)
    local lambdas, sumlam, n, constraint, solved, bds, lam, mus, summu, i;
    lambdas := Vector(numelems(X_vec), symbol=λ);
    mus := Vector(numelems(u_vec), symbol=u);
    bds := Vector( );
    bds := convert(bds, set);
    sumlam := add(lambdas(n), n=1..numelems(X_vec));
    summu := add(mus(i), i=1..numelems(u_vec));
    constraint := simplify(LinearAlgebra:-VectorMatrixMultiply(LinearAlgebra:-
    Transpose(lambdas), X_vec) − LinearAlgebra:-VectorMatrixMultiply(LinearAlgebra:-
    Transpose(mus), u_vec));
    bds := bds union {constraint(1)=bx[1], constraint(2)=bx[2], constraint(3)=bx[3],
    summu=1};
    solved := Optimization:-LPSolve(sumlam, bds, assume=nonnegative)
end proc
```

Управление радиальной и трансверсальной скоростями осуществляется в пределах значений [-α, α]

, :

```
U_w := Vector( )
```

$$U\_w := [\quad]$$ **(26)**

```
U_w(1) := ⟨α, α⟩ :
U_w(2) := ⟨-α, α⟩ :
U_w(3) := ⟨α, -α⟩ :
U_w(4) := ⟨-α, -α⟩ :
```

$\mathbb{R}^3$:

```
t1 := time[real]( )
```

$$t1 := 2405.095$$ **(27)**

```
U_w(1)
```

$$\begin{bmatrix} \alpha \\ \alpha \end{bmatrix}$$ **(28)**

```
U_imp = Vector( ) :
for i from 1 to 4 do U_imp(i) := MatrixVectorMultiply(B_imp_w, U_w(i)) end do:
U_imp_set := {U_imp(1), U_imp(2), U_imp(3), U_imp(4)}
```

$$U\_imp\_set := \left\{ \begin{bmatrix} \sin(t)\,\alpha + (-2\cos(t)+2)\,\alpha \\ \cos(t)\,\alpha + 2\sin(t)\,\alpha \\ -\sin(t)\,\alpha + (2\cos(t)-1)\,\alpha \end{bmatrix}, \begin{bmatrix} -\sin(t)\,\alpha + (-2\cos(t)+2)\,\alpha \\ -\cos(t)\,\alpha + 2\sin(t)\,\alpha \\ \sin(t)\,\alpha + (2\cos(t)-1)\,\alpha \end{bmatrix}, \right. \tag{29}$$
$$\left. \begin{bmatrix} -\sin(t)\,\alpha - (-2\cos(t)+2)\,\alpha \\ -\cos(t)\,\alpha - 2\sin(t)\,\alpha \\ \sin(t)\,\alpha - (2\cos(t)-1)\,\alpha \end{bmatrix}, \begin{bmatrix} \sin(t)\,\alpha - (-2\cos(t)+2)\,\alpha \\ \cos(t)\,\alpha - 2\sin(t)\,\alpha \\ -\sin(t)\,\alpha - (2\cos(t)-1)\,\alpha \end{bmatrix} \right\}$$

$subs(t=\Delta t, U\_imp\_set)$

$$\left\{ \begin{bmatrix} \sin(\Delta t)\,\alpha - (-2\cos(\Delta t)+2)\,\alpha \\ \cos(\Delta t)\,\alpha - 2\sin(\Delta t)\,\alpha \\ -\sin(\Delta t)\,\alpha - (2\cos(\Delta t)-1)\,\alpha \end{bmatrix}, \begin{bmatrix} -\sin(\Delta t)\,\alpha - (-2\cos(\Delta t)+2)\,\alpha \\ -\cos(\Delta t)\,\alpha - 2\sin(\Delta t)\,\alpha \\ \sin(\Delta t)\,\alpha - (2\cos(\Delta t)-1)\,\alpha \end{bmatrix}, \right. \tag{30}$$
$$\left. \begin{bmatrix} \sin(\Delta t)\,\alpha + (-2\cos(\Delta t)+2)\,\alpha \\ \cos(\Delta t)\,\alpha + 2\sin(\Delta t)\,\alpha \\ -\sin(\Delta t)\,\alpha + (2\cos(\Delta t)-1)\,\alpha \end{bmatrix}, \begin{bmatrix} -\sin(\Delta t)\,\alpha + (-2\cos(\Delta t)+2)\,\alpha \\ -\cos(\Delta t)\,\alpha + 2\sin(\Delta t)\,\alpha \\ \sin(\Delta t)\,\alpha + (2\cos(\Delta t)-1)\,\alpha \end{bmatrix} \right\}$$

```
X_1_vec_imp := Vector( ) :
for i from 1 to 4 do
X_1_vec_imp(i) := MatrixMatrixMultiply( -MatrixInverse(A_imp_w), U_imp(i) )
end do:
```

```
X_1_set_imp := {simplify(X_1_vec_imp(1)), simplify( X_1_vec_imp(2)),
    simplify( X_1_vec_imp(3)), simplify(X_1_vec_imp(4))}
```

$$X\_1\_set\_imp := \left\{ \begin{bmatrix} 0 \\ -\alpha \\ -\alpha \end{bmatrix}, \begin{bmatrix} 0 \\ \alpha \\ -\alpha \end{bmatrix}, \begin{bmatrix} 0 \\ -\alpha \\ \alpha \end{bmatrix}, \begin{bmatrix} 0 \\ \alpha \\ \alpha \end{bmatrix} \right\} \tag{31}$$

```
X1_vec_imp_sub := simplify(subs(t=0.25, alpha=0.0035, X_1_vec_imp)) :

bx1_imp := [-0.00377869564857395, -0.00391093582674638, 0.0141511512373208 ]:
```

X (2)

```
X_2_vec_temp := Vector( ) :

for i from 1 to 4 do
X_2_vec_temp(i) := MatrixMatrixMultiply( -MatrixMatrixMultiply(MatrixInverse(A_imp_w),
    MatrixInverse(A_imp_w)), U_imp(i) )
end do:
```

```
X_2_set_temp := {simplify(X_2_vec_temp(1)), simplify(X_2_vec_temp(2)),
     simplify(X_2_vec_temp(3)), simplify(X_2_vec_temp(4))} :

with(ArrayTools) :
X_2_vec_imp := Vector( ) :

for xi_1 in X_1_set_imp do
 for xi_2 in X_2_set_temp do
 Append(X_2_vec_imp, simplify(VectorAdd(xi_1, xi_2)))
 end do
 end do

X_2_set_imp := convert(X_2_vec_imp, set)
```

$$X\_2\_set\_imp := \left\{ \begin{bmatrix} \alpha(\sin(t)+2\cos(t)-2) \\ -\alpha(1+\cos(t)-2\sin(t)) \\ -\alpha(\sin(t)+2\cos(t)-2) \end{bmatrix}, \begin{bmatrix} \alpha(-\sin(t)+2\cos(t)-2) \\ \alpha(-1+\cos(t)+2\sin(t)) \\ -\alpha(-\sin(t)+2\cos(t)-2) \end{bmatrix}, \right. \tag{32}$$

$$\begin{bmatrix} \alpha(-\sin(t)+2\cos(t)-2) \\ \alpha(1+\cos(t)+2\sin(t)) \\ -\alpha(-\sin(t)+2\cos(t)-2) \end{bmatrix}, \begin{bmatrix} \alpha(\sin(t)+2\cos(t)-2) \\ -\alpha(-1+\cos(t)-2\sin(t)) \\ -\alpha(\sin(t)+2\cos(t)-2) \end{bmatrix},$$

$$\begin{bmatrix} -\alpha(\sin(t)+2\cos(t)-2) \\ \alpha(1+\cos(t)-2\sin(t)) \\ \alpha(\sin(t)+2\cos(t)) \end{bmatrix}, \begin{bmatrix} -\alpha(\sin(t)+2\cos(t)-2) \\ \alpha(-1+\cos(t)-2\sin(t)) \\ \alpha(\sin(t)+2\cos(t)) \end{bmatrix},$$

$$\begin{bmatrix} -\alpha(-\sin(t)+2\cos(t)-2) \\ -\alpha(-1+\cos(t)+2\sin(t)) \\ \alpha(-\sin(t)+2\cos(t)) \end{bmatrix}, \begin{bmatrix} \alpha(\sin(t)+2\cos(t)-2) \\ -\alpha(1+\cos(t)-2\sin(t)) \\ -\alpha(\sin(t)+2\cos(t)) \end{bmatrix},$$

$$\begin{bmatrix} -\alpha(\sin(t)+2\cos(t)-2) \\ \alpha(1+\cos(t)-2\sin(t)) \\ \alpha(\sin(t)+2\cos(t)-2) \end{bmatrix}, \begin{bmatrix} \alpha(-\sin(t)+2\cos(t)-2) \\ \alpha(1+\cos(t)+2\sin(t)) \\ -\alpha(-\sin(t)+2\cos(t)) \end{bmatrix},$$

$$\begin{bmatrix} -\alpha(-\sin(t)+2\cos(t)-2) \\ -\alpha(-1+\cos(t)+2\sin(t)) \\ \alpha(-\sin(t)+2\cos(t)-2) \end{bmatrix}, \left. \begin{bmatrix} \alpha(\sin(t)+2\cos(t)-2) \\ -\alpha(-1+\cos(t)-2\sin(t)) \\ -\alpha(\sin(t)+2\cos(t)) \end{bmatrix}, \right.$$

$$\left[\begin{array}{c} -\alpha\,(\sin(t)+2\cos(t)-2) \\ \alpha\,(-1+\cos(t)-2\sin(t)) \\ \alpha\,(\sin(t)+2\cos(t)-2) \end{array}\right], \left[\begin{array}{c} \alpha\,(-\sin(t)+2\cos(t)-2) \\ \alpha\,(-1+\cos(t)+2\sin(t)) \\ -\alpha\,(-\sin(t)+2\cos(t)) \end{array}\right],$$
$$\left[\begin{array}{c} -\alpha\,(-\sin(t)+2\cos(t)-2) \\ -\alpha\,(1+\cos(t)+2\sin(t)) \\ \alpha\,(-\sin(t)+2\cos(t)-2) \end{array}\right], \left[\begin{array}{c} -\alpha\,(-\sin(t)+2\cos(t)-2) \\ -\alpha\,(1+\cos(t)+2\sin(t)) \\ \alpha\,(-\sin(t)+2\cos(t)) \end{array}\right]\Bigg\}$$

```
X2_vec_imp_sub := simplify(subs(t = 0.25, alpha = 0.0035, X_2_vec_imp)) :
for i from 1 to 16 do
  X2_vec_imp_sub(i) := convert(X2_vec_imp_sub(i), list) :
end do:

X2_list_imp_sub := convert(X2_vec_imp_sub, list) :


with(ComputationalGeometry)
```

[*ClosestPointPair*, *ConvexHull*, *DelaunayTriangulation*, *FindLineIntersection*, *MultiSegmentIntersect*, *PointInCircle*, *PointOnSegment*, *PointOrientation*, *PolygonTriangulation*, *SegmentsIntersect*, *VoronoiDiagram*] **(33)**

```
hi := ConvexHull(X2_list_imp_sub)
```

hi := [[1, 4, 3], [4, 1, 2], [10, 4, 2], [4, 10, 12], [14, 10, 2], [6, 14, 2], [4, 11, 3], [11, 4, 12], [1, 5, 2], [5, 6, 2], [16, 11, 12], [11, 16, 15], [10, 16, 12], [14, 16, 10], [11, 7, 3], [7, 11, 15], [7, 1, 3], [7, 5, 1], [16, 13, 15], [13, 16, 14], [13, 7, 15], [7, 13, 5], [13, 14, 6], [5, 13, 6]] **(34)**

```
hivec := convert(hi, Vector) :
hiset := convert(hivec, set)
```

hiset := {1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16} **(35)**

, 5 1 0

```
for i from 16 to 1 by -1 do
 if not(member(i, hiset))
 then Remove(X2_vec_imp_sub, i);
 end if
 end do

X2_vec_imp_sub :


for i from 1 to 14 do
  X2_vec_imp_sub(i) := convert(X2_vec_imp_sub(i), list) :
 end do:
```

```
X2_list_imp_sub := convert(X2_vec_imp_sub, list) :

numelems(X2_list_imp_sub)
```

$$14 \tag{36}$$

```
bx2_imp := [ -0.00377869564857395, -0.00391093582674638, 0.0141511512373208 ] :

X_2_vec_imp_sub2 := Vector( )
```

$$X\_2\_vec\_imp\_sub2 := [\ \ ] \tag{37}$$

```
for x in X2_list_imp_sub do
  Append(X_2_vec_imp_sub2, convert(x, Vector)) :
end do:

numelems(X_2_vec_imp_sub2)
```

$$14 \tag{38}$$

```
X_2_zlp := ZLP(X_2_vec_imp_sub2, bx2_imp)
```

$$X\_2\_zlp := \left[4.85469872695601, \left[\lambda_1 = 0., \lambda_2 = 1.45071845963574, \lambda_3 = 0., \lambda_4 = 0.471482437863550, \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 2.93249782945672, \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0.\right]\right] \tag{39}$$

```
numelems(X2_list_imp_sub)
```

$$14 \tag{40}$$

X (3) :

```
X_3_vec_temp_imp := Vector( ) :

for i to 14 do
  X_3_vec_temp_imp(i) := MatrixMatrixMultiply(MatrixInverse(A_imp_w),
    X_2_vec_imp_sub2(i)) :
end do:

X_1_vec_imp_sub := simplify(subs(t = 0.25, alpha = 0.0035, X_1_vec_imp)) :

X_3_vec_temp_imp_sub := simplify(subs(t = 0.25, alpha = 0.0035, X_3_vec_temp_imp)) :

X_3_vec_imp := Vector( ) :

for xi_1 in X_1_vec_imp_sub do
  for xi_2 in X_3_vec_temp_imp_sub do Append(X_3_vec_imp, simplify(VectorAdd(xi_1, xi_2)));
  end do:
```

end do:

numelems(*X_3_vec_imp*)

56 **(41)**

X_3_list_imp := [ ]:

for *i* **to** 56 **do**
  X_3_vec_imp(*i*) := *convert*(*X_3_vec_imp*(*i*), *list*):
end do:

X_3_list_imp := *convert*(*X_3_vec_imp*, *list*):

numelems(*X_3_list_imp*)

56 **(42)**

hi2 := *ConvexHull*(*X_3_list_imp*)

hi2 := [[5, 20, 6], [10, 38, 9], [38, 40, 42], [8, 38, 10], [4, 10, 9], [4, 8, 10], [38, 37, 9], [52, 38, 42], [52, 37, 38], [38, 12, 40], [8, 12, 38], [4, 2, 8], [2, 5, 6], [5, 2, 1], [3, 4, 9], [2, 3, 1], [3, 2, 4], [5, 19, 20], [56, 54, 55], [56, 52, 42], [52, 56, 55], [40, 56, 42], [54, 56, 40], [19, 48, 20], [51, 52, 55], [52, 51, 37], [15, 5, 1], [15, 19, 5], [49, 19, 45], [19, 17, 45], [15, 17, 19], [3, 17, 1], [17, 15, 1], [47, 48, 19], [49, 47, 19], [53, 49, 55], [53, 47, 49], [54, 53, 55], [48, 53, 54], [47, 53, 48], [12, 34, 40], [34, 12, 6], [12, 2, 6], [2, 12, 8], [48, 34, 6], [20, 48, 6], [34, 48, 40], [48, 54, 40], [51, 23, 9], [37, 51, 9], [51, 49, 45], [49, 51, 55], [51, 17, 23], [17, 51, 45], [23, 17, 9], [17, 3, 9]] **(43)**

hi2vec := *convert*(*hi2*, *Vector*):

hi2set := *convert*(*hi2vec*, *set*)

hi2set := {1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 17, 19, 20, 23, 34, 37, 38, 40, 42, 45, 47, 48, 49, 51, 52, 53, 54, 55, 56} **(44)**

numelems(*hi2set*)

30 **(45)**

for *i* **from** 56 **to** 1 **by** -1 **do**
if not(*member*(*i*, *hi2set*))
then *Remove*(*X_3_vec_imp*, *i*):
end if
end do:

numelems(*X_3_vec_imp*)

30 **(46)**

for *i* **from** 1 **to** 30 **do**
  X_3_vec_imp(*i*) := *convert*(*X_3_vec_imp*(*i*), *Vector*):
end do:

numelems(*X_3_vec_imp*)

30 **(47)**

$bx3\_imp := [-0.00377869564857395, -0.00391093582674638, 0.0141511512373208]$

$$bx3\_imp := [-0.00377869564857395, -0.00391093582674638, 0.0141511512373208] \quad (48)$$

$X\_3\_imp\_zlp = ZLP(X\_3\_vec\_imp, bx3\_imp)$

$$X\_3\_imp\_zlp = \left[2.37442770563265, \left[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0.533971194955334, \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0.828536712209858, \lambda_{19} = 1.01191979846746, \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0., \lambda_{29} = 0., \lambda_{30} = 0.\right]\right] \quad (49)$$

(4)

$X\_4\_vec\_temp\_imp := Vector(\ ):$

for i **from** 1 **to** 30 **do**
$X\_4\_vec\_temp\_imp(i) := MatrixMatrixMultiply(MatrixInverse(A\_imp\_w), X\_3\_vec\_imp(i)):$
end do:

$X\_4\_vec\_temp\_imp\_sub := simplify(subs(t = 0.25, \text{alpha} = 0.0035, X\_4\_vec\_temp\_imp)):$

$X\_4\_vec\_imp := Vector(\ ):$

for xi_1 **in** $X\_1\_vec\_imp\_sub$ **do**
for xi_2 **in** $X\_4\_vec\_temp\_imp\_sub$ **do**
$Append(X\_4\_vec\_imp, simplify(VectorAdd(xi\_1, xi\_2))):$
end do:
end do:

$numelems(X\_4\_vec\_imp)$

$$120 \quad (50)$$

$X\_4\_list\_imp := [\ ]:$

for i **from** 1 **to** 120 **do**
$X\_4\_vec\_imp(i) := convert(X\_4\_vec\_imp(i), list):$
end do:

$X\_4\_list\_imp := convert(X\_4\_vec\_imp, list):$

$numelems(X\_4\_list\_imp)$

$$120 \quad (51)$$

$hi3 := ConvexHull(X\_4\_list\_imp)$

$$hi3 := [[78, 80, 90], [108, 77, 78], [23, 16, 14], [18, 9, 19], [18, 80, 78], [9, 18, 17], [23, 76, 16], [76, 23, 83], [88, 76, 83], [118, 88, 83], [79, 80, 19], [79, 76, 88], [79, 88, 90], [80, 79, 90], [107, 108, 115], [108, 107, 77], [105, 107, 115], [107, 105, 98], [80, 20, 19], [20, 18, 19], [18, 20, 80], [16, 6, 14], [9, 10, 19], [6, 10, 2], [10, 16, 19], [10, 6, 16], [8, 9, 17], [8, 3, 9], [4, 2, 9], [3, 4, 9], [2, 4, 1], [4, 3, 1], [113, 118, 83], [120, 118, 117], [2, 7, 9], [7, 10, 9], [10, 7, 2], [120, 119, 108], [108, 119, 115], [119, 120, 117], [8, 38, 3], [38, \quad (52)$$

33, 3], [33, 45, 42], [105, 45, 98], [45, 38, 98], [38, 45, 33], [111, 105, 115], [119, 111, 115], [31, 33, 42], [41, 31, 42], [43, 31, 41], [104, 112, 113], [118, 112, 117], [113, 112, 118], [112, 119, 117], [13, 44, 14], [13, 43, 44], [6, 13, 14], [13, 6, 2], [13, 2, 1], [103, 112, 104], [103, 43, 41], [114, 111, 119], [112, 114, 119], [114, 112, 111], [101, 103, 41], [112, 102, 111], [103, 102, 112], [101, 102, 103], [111, 102, 105], [102, 41, 42], [102, 101, 41], [23, 74, 83], [74, 23, 14], [77, 18, 78], [18, 77, 17], [76, 79, 16], [16, 79, 19], [77, 107, 17], [107, 47, 17], [118, 120, 88], [88, 120, 90], [120, 78, 90], [120, 108, 78], [104, 74, 14], [44, 104, 14], [74, 104, 83], [104, 113, 83], [47, 38, 17], [38, 8, 17], [107, 38, 47], [38, 107, 98], [33, 31, 3], [3, 31, 1], [31, 13, 1], [13, 31, 43], [103, 104, 44], [43, 103, 44], [45, 102, 42], [102, 45, 105]]

$hi3vec := convert(hi3, Vector)$:

$hi3set := convert(hi3vec, set)$

$$hi3set := \{1, 2, 3, 4, 6, 7, 8, 9, 10, 13, 14, 16, 17, 18, 19, 20, 23, 31, 33, 38, 41, 42, 43, 44, 45, 47, 74, 76, 77, 78, 79, 80, 83, 88, 90, 98, 101, 102, 103, 104, 105, 107, 108, 111, 112, 113, 114, 115, 117, 118, 119, 120\} \quad (53)$$

$numelems(hi3set)$

$$52 \quad (54)$$

for i **from** 120 **by** -1 **to** 1 **do**
if not$(member(i, hi3set))$
then $Remove(X\_4\_vec\_imp, i)$:
end if
end do:

$numelems(X\_4\_vec\_imp)$

$$52 \quad (55)$$

$X\_4\_list\_imp2 := convert(X\_4\_vec\_imp, list)$:

for i **from** 1 **to** $numelems(X\_4\_vec\_imp)$ **do**
$X\_4\_vec\_imp(i) := convert(X\_4\_vec\_imp(i), Vector)$:
end do:

$bxs\_imp := [-0.00377869564857395, -0.00391093582674638, 0.0141511512373208]$

$$bxs\_imp := [-0.00377869564857395, -0.00391093582674638, 0.0141511512373208] \quad (56)$$

$X\_4\_imp\_zlp := ZLP(X\_4\_vec\_imp, bxs\_imp)$

$$X\_4\_imp\_zlp := \left[1.17637795920381, \left[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0., \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0., \lambda_{29} = 0., \lambda_{30} = 0., \lambda_{31} = 0.744711249251496, \lambda_{32} = 0.305401411371101, \lambda_{33} = 0., \lambda_{34} = 0.126265298581213, \lambda_{35} = 0., \lambda_{36} = 0., \lambda_{37} = 0., \lambda_{38} = 0., \lambda_{39} = 0., \lambda_{40} = 0., \lambda_{41} = 0., \lambda_{42} = 0., \lambda_{43} = 0., \lambda_{44} = 0., \lambda_{45} = 0., \lambda_{46} = 0., \lambda_{47} = 0., \lambda_{48} = 0., \lambda_{49} = 0., \lambda_{50} = 0., \lambda_{51} = 0., \lambda_{52} = 0.\right]\right] \quad (57)$$

$X\_4\_imp\_zlp[1]$

1.17637795920381 **(58)**

(5)

```
X_5_vec_temp_imp := Vector( ) :

for i from 1 to 52 do
   X_5_vec_temp_imp(i) := MatrixMatrixMultiply(MatrixInverse(A_imp_w), X_4_vec_imp(i) ) :
 end do:

X_5_vec_temp_imp_sub := simplify(subs(t = 0.25, alpha = 0.0035, X_5_vec_temp_imp) ) :

X_5_vec_imp := Vector( ) :

for xi_1 in X_1_vec_imp_sub do
 for xi_2 in X_5_vec_temp_imp_sub do
Append(X_5_vec_imp, simplify(VectorAdd(xi_1, xi_2) ) ) :
end do:
end do:

numelems(X_5_vec_imp)
```

208 **(59)**

```
X_5_list_imp := [ ] :


for i from 1 to 208 do
   X_5_vec_imp(i) := convert(X_5_vec_imp(i), list) :
end do:

X_5_list_imp := convert(X_5_vec_imp, list) :

numelems(X_5_list_imp)
```

208 **(60)**

```
hi4 := ConvexHull(X_5_list_imp) :

hi4vec := convert(hi4, Vector) :

hi4set := convert(hi4vec, set)
```

hi4set := {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 24, 27, 28, 29, 30, 31, 32, 33, 34, 40, 53, 55, 59, 65, 70, 71, 72, 73, 74, 75, 76, 78, 81, 128, 131, 133, 134, 135, 136, 137, 138, 139, 144, 150, 154, 156, 169, 175, 176, 177, 178, 179, 180, 181, 182, 185, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208} **(61)**

```
numelems(hi4set)
```

78 **(62)**

```
for i from 208 by -1 to 1 do
if not(member(i, hi4set) )
then Remove(X_5_vec_imp, i) :
end if
 end do:
```

$numelems(X\_5\_vec\_imp)$

$$78 \tag{63}$$

$X\_5\_list\_imp2 := convert(X\_5\_vec\_imp, list)$:

for i **from** 1 **to** $numelems(X\_5\_vec\_imp)$ **do**
$X\_5\_vec\_imp(i) := convert(X\_5\_vec\_imp(i), Vector)$:
end do:

$X\_5\_imp\_zlp := ZLP(X\_5\_vec\_imp, bxs\_imp)$

$$X\_5\_imp\_zlp := \left[0.794764920999915, \left[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0., \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0., \lambda_{29} = 0., \lambda_{30} = 0., \lambda_{31} = 0., \lambda_{32} = 0., \lambda_{33} = 0., \lambda_{34} = 0., \lambda_{35} = 0., \lambda_{36} = 0., \lambda_{37} = 0., \lambda_{38} = 0., \lambda_{39} = 0., \lambda_{40} = 0., \lambda_{41} = 0., \lambda_{42} = 0., \lambda_{43} = 0., \lambda_{44} = 0., \lambda_{45} = 0., \lambda_{46} = 0., \lambda_{47} = 0.477330021339709, \lambda_{48} = 0., \lambda_{49} = 0., \lambda_{50} = 0., \lambda_{51} = 0.0278029102934286, \lambda_{52} = 0.289631989366778, \lambda_{53} = 0., \lambda_{54} = 0., \lambda_{55} = 0., \lambda_{56} = 0., \lambda_{57} = 0., \lambda_{58} = 0., \lambda_{59} = 0., \lambda_{60} = 0., \lambda_{61} = 0., \lambda_{62} = 0., \lambda_{63} = 0., \lambda_{64} = 0., \lambda_{65} = 0., \lambda_{66} = 0., \lambda_{67} = 0., \lambda_{68} = 0., \lambda_{69} = 0., \lambda_{70} = 0., \lambda_{71} = 0., \lambda_{72} = 0., \lambda_{73} = 0., \lambda_{74} = 0., \lambda_{75} = 0., \lambda_{76} = 0., \lambda_{77} = 0., \lambda_{78} = 0.\right]\right] \tag{64}$$

$X\_5\_imp\_zlp[1]$

$$0.794764920999915 \tag{65}$$

1 ,

$U\_imp\_1 := Vector(\ )$:
for i **from** 1 **to** 4 **do**
$U\_imp\_1(i) := U\_imp(i)$
end do:

$A\_imp\_w\_sub := simplify(subs(t = 0.25, \text{alpha} = 0.0035, A\_imp\_w))$:
$bx\_imp := convert(bxs\_imp, Vector)$:
$u\_vec\_test := simplify(subs(t = 0.25, \text{alpha} = 0.0035, U\_imp\_1))$:

S n (x) 4 - (k = 1)

$s4\_x\_imp\_zlp := ZLP\_u(X\_4\_vec\_imp, u\_vec\_test, MatrixVectorMultiply(A\_imp\_w\_sub, bx\_imp))$

$$s4\_x\_imp\_zlp := \left[0.740124048004852, \left[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0., \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0.,\right.\right. \tag{66}$$

$\lambda_{29} = 0., \lambda_{30} = 0., \lambda_{31} = 0., \lambda_{32} = 0., \lambda_{33} = 0., \lambda_{34} = 0.398795814864167, \lambda_{35} = 0., \lambda_{36} = 0., \lambda_{37} = 0., \lambda_{38} = 0., \lambda_{39} = 0., \lambda_{40} = 0., \lambda_{41} = 0., \lambda_{42} = 0., \lambda_{43} = 0., \lambda_{44} = 0., \lambda_{45} = 0., \lambda_{46} = 0., \lambda_{47} = 0., \lambda_{48} = 0., \lambda_{49} = 0., \lambda_{50} = 0.0996729124553283, \lambda_{51} = 0., \lambda_{52} = 0.241655320685357, u_1 = 0., u_2 = 0., u_3 = 1.00000000000000, u_4 = 0.\big]\big]$

$u\_s4\_imp := \langle 0, 0, 1, 0 \rangle$:
$s4\_x\_imp := VectorMatrixMultiply(Transpose(u\_s4\_imp), u\_vec\_test)$

$$s4\_x\_imp := \begin{bmatrix} 0.000648300810000000 \\ 0.00165936576100000 \\ -0.00414830081000000 \end{bmatrix} \quad \textbf{(67)}$$

3.1 x(1)

$x\_star\_0 := \langle -0.00377869564857395, -0.00391093582674638, 0.0141511512373208 \rangle$:
$x\_star\_1\_imp := MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_0) + s4\_x\_imp$

$$x\_star\_1\_imp := \begin{bmatrix} -0.00333559629248136 \\ 0.00393724888307952 \\ 0.0102080518823618 \end{bmatrix} \quad \textbf{(68)}$$

$x\_star\_1\_to\_X4 = ZLP(X\_4\_vec\_imp, x\_star\_1\_imp)$

$x\_star\_1\_to\_X4 = \big[0.740124049037450, \big[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0., \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0., \lambda_{29} = 0., \lambda_{30} = 0., \lambda_{31} = 0., \lambda_{32} = 0., \lambda_{33} = 0., \lambda_{34} = 0.157140493215257, \lambda_{35} = 0.241655321717954, \lambda_{36} = 0., \lambda_{37} = 0., \lambda_{38} = 0., \lambda_{39} = 0., \lambda_{40} = 0., \lambda_{41} = 0., \lambda_{42} = 0., \lambda_{43} = 0., \lambda_{44} = 0., \lambda_{45} = 0., \lambda_{46} = 0., \lambda_{47} = 0., \lambda_{48} = 0., \lambda_{49} = 0., \lambda_{50} = 0.341328234104238, \lambda_{51} = 0., \lambda_{52} = 0.\big]\big]$ **(69)**

Sn(x) 3- (k = 2)

$s3\_x\_imp\_zlp := ZLP\_u(X\_3\_vec\_imp, u\_vec\_test, MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_1\_imp))$

$s3\_x\_imp\_zlp := \big[0.675997940266569, \big[\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18} = 0., \lambda_{19} = 0., \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0., \lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0.532217541582521, \lambda_{29} = 0., \lambda_{30} = 0.143780398684047, u_1 = 0., u_2 = 0., u_3$ **(70)**

$= 0.582397929665253,\ u_4 = 0.417602070334747\ ]\ ]$

$u\_s3\_imp := \langle 0, 0, 0.582397929665253, 0.417602070334747\rangle$:
$s3\_x\_imp := VectorMatrixMultiply(Transpose(u\_s3\_imp), u\_vec\_test)$

$$s3\_x\_imp := \begin{bmatrix} -0.0000749140296646964 \\ -0.00117297307196657 \\ -0.00342508597033530 \end{bmatrix} \quad (71)$$

3.1 x(2)

$x\_star\_2\_imp := MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_1\_imp) + s3\_x\_imp$

$$x\_star\_2\_imp := \begin{bmatrix} -0.00190542774119172 \\ 0.00686766145365960 \\ 0.00527788333207286 \end{bmatrix} \quad (72)$$

$x\_star\_2\_to\_X3 = ZLP(X\_3\_vec\_imp, x\_star\_2\_imp)$

$x\_star\_2\_to\_X3 = [0.675997941124306, [\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6 = 0., \lambda_7 = 0.,$ (73)
$\lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0., \lambda_{13} = 0., \lambda_{14} = 0., \lambda_{15} = 0., \lambda_{16} = 0., \lambda_{17} = 0., \lambda_{18}$
$= 0., \lambda_{19} = 9.24818971403951\ 10^{-10}, \lambda_{20} = 0., \lambda_{21} = 0., \lambda_{22} = 0., \lambda_{23} = 0., \lambda_{24} = 0., \lambda_{25} = 0.,$
$\lambda_{26} = 0., \lambda_{27} = 0., \lambda_{28} = 0.532217541944311, \lambda_{29} = 0., \lambda_{30} = 0.143780398255177\ ]\ ]$

Sn(x) 2- (k = 3)

$s2\_x\_imp\_zlp := ZLP\_u(X\_2\_vec\_imp\_sub2, u\_vec\_test, MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_2\_imp))$

$s2\_x\_imp\_zlp := [0.596095229913988, [\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 0., \lambda_6$ (74)
$= 0.0182206297924619, \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0.577874600121526,$
$\lambda_{13} = 0., \lambda_{14} = 0., u_1 = 0., u_2 = 0., u_3 = 0.216571601324376, u_4 = 0.783428398675624\ ]\ ]$

$u\_s2\_imp := \langle 0, 0, 0.216571601324376, 0.783428398675624\rangle$:
$s2\_x\_imp := VectorMatrixMultiply(Transpose(u\_s2\_imp), u\_vec\_test)$

$$s2\_x\_imp := \begin{bmatrix} -0.000708462204327943 \\ -0.00365414878800381 \\ -0.00279153779567206 \end{bmatrix} \quad (75)$$

3.1 x(3)

$x\_star\_3\_imp := MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_2\_imp) + s2\_x\_imp$

$$x\_star\_3\_imp := \begin{bmatrix} -0.000645885219480629 \\ 0.00514014180130297 \\ 0.000518340810933399 \end{bmatrix} \quad (76)$$

$x\_star\_3\_to\_X1 = ZLP(X\_2\_vec\_imp\_sub2, x\_star\_3\_imp)$

$x\_star\_3\_to\_X1 = [0.596095230946585, [\lambda_1 = 0., \lambda_2 = 0., \lambda_3 = 0., \lambda_4 = 0., \lambda_5 = 1.03259696279667\ 10^{-9}, \lambda_6 = 0.0182206287598644, \lambda_7 = 0., \lambda_8 = 0., \lambda_9 = 0., \lambda_{10} = 0., \lambda_{11} = 0., \lambda_{12} = 0.577874601154124, \lambda_{13} = 0., \lambda_{14} = 0.]]$ (77)

Sn(x) 1- (k = 4) [
]

$s1\_x\_imp\_zlp := ZLP\_u(X\_1\_vec\_imp\_sub, u\_vec\_test, MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_3\_imp))$

$s1\_x\_imp\_zlp := [0.596095227783182, [\lambda_1 = 0., \lambda_2 = 0.596095227783182, \lambda_3 = 0., \lambda_4 = 0., u_1 = 0.201952384251319, u_2 = 0.0182206317410895, u_3 = 0., u_4 = 0.779826984007591]]$ (78)

$u\_s1\_imp := \langle 0.201952384251319, 0.0182206317410895, 0, 0.779826984007591 \rangle :$

$s1\_x\_imp := VectorMatrixMultiply(Transpose(u\_s1\_imp), u\_vec\_test)$

$$s1\_x\_imp := \begin{bmatrix} -0.000637955127446362 \\ -0.00299069851274698 \\ -0.00132083376060678 \end{bmatrix} \quad (79)$$

3.1 x(4)

$x\_star\_4\_imp := MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_3\_imp) + s1\_x\_imp$

$$x\_star\_4\_imp := \begin{bmatrix} 2.38438392294316\ 10^{-13} \\ 0.00208633330506542 \\ -0.00208633329664504 \end{bmatrix} \quad (80)$$

$x\_star\_4\_to\_X1 = ZLP(X\_1\_vec\_imp\_sub, x\_star\_4\_imp)$

$x\_star\_4\_to\_X1 = [0.596095227783183, [\lambda_1 = -1.03259769146314\ 10^{-9}, \lambda_2 = 0.596095228815780, \lambda_3 = 0., \lambda_4 = 0.]]$ (81)

Sn(x) (k = 5 = Nmin)

X(Nmin) = 0, :

$s0\_x\_imp := -MatrixVectorMultiply(A\_imp\_w\_sub, x\_star\_4\_imp)$

$$s0\_x\_imp := \begin{bmatrix} -0.0003864490200652 57 \\ -0.0009891400191244 81 \\ 0.0024727823164718 6 \end{bmatrix} \quad \textbf{(82)}$$

$t2 := time[real]( )$

$$t2 := 2421.148 \quad \textbf{(83)}$$

$total\_time := t2 - t1$

$$total\_time := 16.053 \quad \textbf{(84)}$$