$with(LinearAlgebra)$

$[\&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA\_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]$ **(1)**

$A := \langle\langle 0|1|0\rangle, \langle 1|0|2\rangle, \langle 0|-1|0\rangle\rangle;$

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \quad \textbf{(2)}$$

$Eigenvectors(A);$

$$\begin{bmatrix} \mathrm{I} \\ -\mathrm{I} \\ 0 \end{bmatrix}, \begin{bmatrix} -1 & -1 & -2 \\ -\mathrm{I} & \mathrm{I} & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \textbf{(3)}$$

$h1 := \langle -2, 0, 1\rangle;$

$$h1 := \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \quad \textbf{(4)}$$

$h2 := \langle -1, -I, 1\rangle;$

$$h2 := \begin{bmatrix} -1 \\ -\mathrm{I} \\ 1 \end{bmatrix} \quad \textbf{(5)}$$

$h3 := \langle -1, I, 1 \rangle;$

$$h3 := \begin{bmatrix} -1 \\ \mathrm{I} \\ 1 \end{bmatrix} \quad \textbf{(6)}$$

$lambda1 := 0;$

$$\lambda 1 := 0 \quad \textbf{(7)}$$

$lambda2 := I;$

$$\lambda 2 := \mathrm{I} \quad \textbf{(8)}$$

$lambda3 := -I;$

$$\lambda 3 := -\mathrm{I} \quad \textbf{(9)}$$

$Phi1 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Re}(h2), \cos(\mathrm{Im}(lambda2) \cdot t)) - VectorScalarMultiply(\mathrm{Im}(h2), \sin(\mathrm{Im}(lambda2) \cdot t)), \exp(\mathrm{Re}(lambda2) \cdot t)) :$

$Phi2 := VectorScalarMultiply(VectorScalarMultiply(\mathrm{Im}(h2), \cos(\mathrm{Im}(lambda2) \cdot t)) + VectorScalarMultiply(\mathrm{Re}(h2), \sin(\mathrm{Im}(lambda2) \cdot t)), \exp(\mathrm{Re}(lambda2) \cdot t)) :$

$Phi3 := VectorScalarMultiply\left(h1, \mathrm{e}^{lambda1 \cdot t}\right) :$

$\mathrm{Phi} := \langle Phi1 | Phi2 | Phi3 \rangle;$

$$\Phi := \begin{bmatrix} -\cos(t) & -\sin(t) & -2 \\ \sin(t) & -\cos(t) & 0 \\ \cos(t) & \sin(t) & 1 \end{bmatrix} \quad \textbf{(10)}$$

$Phi\_inv := MatrixInverse(\mathrm{Phi}) :$

$Phi\_inv0 := subs([t=0], Phi\_inv) :$

A B :

$A\_imp\_w := MatrixMatrixMultiply(\mathrm{Phi}, Phi\_inv0)$

$$A\_imp\_w := \begin{bmatrix} -\cos(t) + 2 & \sin(t) & -2\cos(t) + 2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t) - 1 & -\sin(t) & 2\cos(t) - 1 \end{bmatrix} \quad \textbf{(11)}$$

$B\_imp\_w := A\_imp\_w :$

Если допустить, что в задаче спутника управление по первой коордиенате (Δr) не производится ( ), B _ i m p _ w :

$B\_imp\_w := SubMatrix(A\_imp\_w, [1..3], [2, 3])$

$$B\_imp\_w := \begin{bmatrix} \sin(t) & -2\cos(t)+2 \\ \cos(t) & 2\sin(t) \\ -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad \textbf{(12)}$$

( ) :

:

$A\_imp\_w\_even := simplify(MatrixMatrixMultiply(A\_imp\_w, A\_imp\_w))$

$$A\_imp\_w\_even := \begin{bmatrix} -2\cos(t)^2+3 & 2\cos(t)\sin(t) & 4\sin(t)^2 \\ 2\cos(t)\sin(t) & 2\cos(t)^2-1 & 4\cos(t)\sin(t) \\ -2\sin(t)^2 & -2\cos(t)\sin(t) & 4\cos(t)^2-3 \end{bmatrix} \quad \textbf{(13)}$$

$B\_imp\_w\_even := simplify(\langle MatrixVectorMultiply(A\_imp\_w, B\_imp\_w[..,1])$
$| MatrixVectorMultiply(A\_imp\_w, B\_imp\_w[..,2]) | B\_imp\_w[..,1]\rangle)$

$$B\_imp\_w\_even := \begin{bmatrix} 2\cos(t)\sin(t) & 4\sin(t)^2 & \sin(t) \\ 2\cos(t)^2-1 & 4\cos(t)\sin(t) & \cos(t) \\ -2\cos(t)\sin(t) & 4\cos(t)^2-3 & -\sin(t) \end{bmatrix} \quad \textbf{(14)}$$

:

$A\_imp\_w\_odd := A\_imp\_w$

$$A\_imp\_w\_odd := \begin{bmatrix} -\cos(t)+2 & \sin(t) & -2\cos(t)+2 \\ \sin(t) & \cos(t) & 2\sin(t) \\ \cos(t)-1 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad \textbf{(15)}$$

$B\_imp\_w\_odd := simplify(\langle MatrixVectorMultiply(A\_imp\_w, B\_imp\_w[..,2]) | B\_imp\_w[..,1]$
$| B\_imp\_w[..,2]\rangle)$

$$B\_imp\_w\_odd := \begin{bmatrix} 4\sin(t)^2 & \sin(t) & -2\cos(t)+2 \\ 4\cos(t)\sin(t) & \cos(t) & 2\sin(t) \\ 4\cos(t)^2-3 & -\sin(t) & 2\cos(t)-1 \end{bmatrix} \quad \textbf{(16)}$$

$A\_imp\_w\_even\_sub := simplify(subs(t=0.25, \text{alpha}=0.0035, A\_imp\_w\_even))$:
$A\_imp\_w\_odd\_sub := simplify(subs(t=0.25, \text{alpha}=0.0035, A\_imp\_w\_odd))$:
$B\_imp\_w\_even\_sub := simplify(subs(t=0.25, \text{alpha}=0.0035, B\_imp\_w\_even))$:
$B\_imp\_w\_odd\_sub := simplify(subs(t=0.25, \text{alpha}=0.0035, B\_imp\_w\_odd))$:

H (k)

$t1 := time[real]()$

$$t1 := 140184.161 \quad \textbf{(17)}$$

$H\_even\_imp := \frac{1}{\alpha^2} \cdot (MatrixMatrixMultiply(Transpose(MatrixInverse(B\_imp\_w\_even)),$
$MatrixInverse(B\_imp\_w\_even))) :$

$H\_even\_imp\_sub := simplify(subs(t = 0.25, \text{alpha} = 0.0035, H\_even\_imp))$

$$H\_even\_imp\_sub := \begin{bmatrix} 3.013870878\ 10^6 & -1.037371687\ 10^6 & 448136.3063 \\ -1.037371687\ 10^6 & 388176.7448 & -156547.1146 \\ 448136.3063 & -156547.1146 & 161578.7351 \end{bmatrix} \quad \textbf{(18)}$$

$H\_odd\_imp := \frac{1}{\alpha^2} \cdot (MatrixMatrixMultiply(Transpose(MatrixInverse(B\_imp\_w\_odd)),$
$MatrixInverse(B\_imp\_w\_odd))) :$

$H\_odd\_imp\_sub := simplify(subs(t = 0.25, \text{alpha} = 0.0035, H\_odd\_imp))$

$$H\_odd\_imp\_sub := \begin{bmatrix} 4.810766597\ 10^7 & -1.247605179\ 10^7 & 2.036646112\ 10^6 \\ -1.247605179\ 10^7 & 3.289633115\ 10^6 & -562160.2626 \\ 2.036646112\ 10^6 & -562160.2626 & 161578.7351 \end{bmatrix} \quad \textbf{(19)}$$

H_wave(k)

$H\_0\_wave\_imp\_sub := H\_even\_imp\_sub$

$$H\_0\_wave\_imp\_sub := \begin{bmatrix} 3.013870878\ 10^6 & -1.037371687\ 10^6 & 448136.3063 \\ -1.037371687\ 10^6 & 388176.7448 & -156547.1146 \\ 448136.3063 & -156547.1146 & 161578.7351 \end{bmatrix} \quad \textbf{(20)}$$

$H\_1\_wave\_imp\_sub := MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(A\_imp\_w\_even\_sub),$
$H\_even\_imp\_sub), A\_imp\_w\_even\_sub)$

$H\_1\_wave\_imp\_sub :=$ **(21)**

$$\begin{bmatrix} 2.66735374457006\ 10^6 & 319699.376124016 & 81632.6533975776 \\ 319699.376124016 & 81632.6529730159 & 0.000115652685024412 \\ 81632.6533975775 & 0.000115652739994472 & 81632.6532745647 \end{bmatrix}$$

$H\_2\_wave\_imp\_sub :=$
$MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub$
$, A\_imp\_w\_even\_sub)), H\_odd\_imp\_sub), MatrixMatrixMultiply(A\_imp\_w\_odd\_sub,$
$A\_imp\_w\_even\_sub))$

$H\_2\_wave\_imp\_sub :=$ **(22)**

$$\begin{bmatrix} 5.61743580527194\ 10^{7} & 2.41244481364966\ 10^{7} & 1.05755276552193\ 10^{7} \\ 2.41244481364966\ 10^{7} & 1.04119464342712\ 10^{7} & 4.56527927013829\ 10^{6} \\ 1.05755276552193\ 10^{7} & 4.56527927013830\ 10^{6} & 2.05033633933653\ 10^{6} \end{bmatrix}$$

$H\_3\_wave\_imp\_sub :=$
$MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply($
$MatrixMatrixMultiply(A\_imp\_w\_even\_sub, A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub)),$
$H\_even\_imp\_sub), MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_even\_sub,$
$A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub))$

$$H\_3\_wave\_imp\_sub := \tag{23}$$
$$\begin{bmatrix} 4.83176604977843\ 10^{6} & 2.72137391369201\ 10^{6} & 2.58706279720520\ 10^{6} \\ 2.72137391369201\ 10^{6} & 1.56400929131431\ 10^{6} & 1.44985144519343\ 10^{6} \\ 2.58706279720520\ 10^{6} & 1.44985144519343\ 10^{6} & 1.44570399331187\ 10^{6} \end{bmatrix}$$

$H\_4\_wave\_imp\_sub :=$
$MatrixMatrixMultiply(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply($
$MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub, A\_imp\_w\_even\_sub),$
$A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub)), H\_odd\_imp\_sub),$
$MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub,$
$A\_imp\_w\_even\_sub), A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub))$

$$H\_4\_wave\_imp\_sub := \tag{24}$$
$$\begin{bmatrix} 1.28193106174228\ 10^{8} & 7.36345626061468\ 10^{7} & 1.02654633745983\ 10^{8} \\ 7.36345626061468\ 10^{7} & 4.23099788580496\ 10^{7} & 5.89535494081376\ 10^{7} \\ 1.02654633745983\ 10^{8} & 5.89535494081376\ 10^{7} & 8.22917679223170\ 10^{7} \end{bmatrix}$$

( ) :

1

$x\_0 := \langle -0.00377869564857395, -0.00391093582674638, 0.0141511512373208 \rangle$

$$x\_0 := \begin{bmatrix} -0.00377869564857395 \\ -0.00391093582674638 \\ 0.0141511512373208 \end{bmatrix} \tag{25}$$

$\psi\_0\_vec\_imp := \langle \psi_1, \psi_2, \psi_3 \rangle$

$$\psi\_0\_vec\_imp := \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \tag{26}$$

$eq\_1\_1\_imp := -\alpha \cdot x\_0 = \left(MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub), \psi\_0\_vec\_imp\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_imp, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub), \psi\_0\_vec\_imp\right), conjugate = false\right)\right)^{1/2}:$

$eq\_1\_2\_imp := DotProduct\left(\psi\_0\_vec\_imp, \psi\_0\_vec\_imp, conjugate = false\right) = 1:$
$sys\_1\_imp := \{eq\_1\_1\_imp(1), eq\_1\_1\_imp(2), eq\_1\_1\_imp(3), eq\_1\_2\_imp\}:$
$list1\_imp := convert\left(fsolve\left(sys\_1\_imp, \{\alpha, \psi_1, \psi_2, \psi_3\}\right), list\right)$

$$list1\_imp := [\alpha = -0.1331549708, \psi_1 = -0.9855873959, \psi_2 = -0.1479514338, \psi_3 = 0.08202352235] \tag{27}$$

for i **from** 1 **to** 4 **do**
$list1\_imp[i] := -list1\_imp[i]$
end do:

$list1\_imp$

$$[-\alpha = 0.1331549708, -\psi_1 = 0.9855873959, -\psi_2 = 0.1479514338, -\psi_3 = -0.08202352235] \tag{28}$$

2

$eq\_2\_1\_imp := -\alpha \cdot x\_0 = \left(MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub), \psi\_0\_vec\_imp\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_imp, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub), \psi\_0\_vec\_imp\right), conjugate = false\right)\right)^{1/2} + \left(MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_imp\right)\right) \Big/ \left(DotProduct\left(\psi\_0\_vec\_imp, MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_imp\right), conjugate = false\right)\right)^{1/2}:$

$sys\_2\_imp := \{eq\_2\_1\_imp(1), eq\_2\_1\_imp(2), eq\_2\_1\_imp(3), eq\_1\_2\_imp\}:$
$list2\_imp := convert\left(fsolve\left(sys\_2\_imp, \{\alpha, \psi_1, \psi_2, \psi_3\}\right), list\right)$

$$list2\_imp := [\alpha = -0.3530107155, \psi_1 = -0.9662902694, \psi_2 = -0.2343549531, \psi_3 = 0.1065873873] \tag{29}$$

for i **from** 1 **to** 4 **do**
$list2\_imp[i] := -list2\_imp[i]$
end do:

$list2\_imp$

$$[-\alpha = 0.3530107155, -\psi_1 = 0.9662902694, -\psi_2 = 0.2343549531, -\psi_3 = -0.1065873873] \tag{30}$$

3

$eq\_3\_1\_imp := -\alpha \cdot x\_0 = \left(MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub),\right.\right.$

$\psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$
$+ (MatrixVectorMultiply(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_2\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$
$+ (MatrixVectorMultiply(MatrixInverse(H\_3\_wave\_imp\_sub), \psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_3\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$:

$sys\_3\_imp := \{eq\_3\_1\_imp(1), eq\_3\_1\_imp(2), eq\_3\_1\_imp(3), eq\_1\_2\_imp\}$:
$list3\_imp := convert(fsolve(sys\_3\_imp, \{\alpha, \psi_1, \psi_2, \psi_3\}), list)$

$$list3\_imp := [\alpha = -0.7910596623, \psi_1 = -0.8143905421, \psi_2 = -0.3730314242, \psi_3 = 0.4445397637] \quad (31)$$

4

$eq\_4\_1\_imp := -\alpha \cdot x\_0 = (MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_1\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$
$+ (MatrixVectorMultiply(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_2\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$
$+ (MatrixVectorMultiply(MatrixInverse(H\_3\_wave\_imp\_sub), \psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_3\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$
$+ (MatrixVectorMultiply(MatrixInverse(H\_4\_wave\_imp\_sub), \psi\_0\_vec\_imp))\Big/$
$(DotProduct(\psi\_0\_vec\_imp, MatrixVectorMultiply(MatrixInverse(H\_4\_wave\_imp\_sub),$
$\psi\_0\_vec\_imp), conjugate = false))^{1/2}$:

$sys\_4\_imp := \{eq\_4\_1\_imp(1), eq\_4\_1\_imp(2), eq\_4\_1\_imp(3), eq\_1\_2\_imp\}$:
$list4\_imp := convert(fsolve(sys\_4\_imp, \{\alpha, \psi_1, \psi_2, \psi_3\}), list)$

$$list4\_imp := [\alpha = -1.076923094, \psi_1 = -0.1293994658, \psi_2 = -0.2359996255, \psi_3 = 0.9630991408] \quad (32)$$

for i **from** 1 **to** 4 **do**

$list4\_imp[i] := -list4\_imp[i]$
end do:

$list4\_imp$

$$\left[-\alpha = 1.076923094,\ -\psi_1 = 0.1293994658,\ -\psi_2 = 0.2359996255,\ -\psi_3 = -0.9630991408\right] \tag{33}$$

$\psi\_0\_vec\_true := \langle 0.1293995043, 0.2359996388, -0.9630991324\rangle$

$$\psi\_0\_vec\_true := \begin{bmatrix} 0.1293995043 \\ 0.2359996388 \\ -0.9630991324 \end{bmatrix} \tag{34}$$

:

$\alpha\_true := 1.076923122$

$$\alpha\_true := 1.076923122 \tag{35}$$

$u\_star\_0 := \frac{1}{\alpha\_true} \cdot \Big(MatrixVectorMultiply\big(MatrixMatrixMultiply(A\_imp\_w\_even\_sub,$
$MatrixInverse(H\_1\_wave\_imp\_sub)), \psi\_0\_vec\_true\big)\Big) \Big/$
$\Big(DotProduct\big(\psi\_0\_vec\_true, MatrixVectorMultiply\big(MatrixInverse(H\_1\_wave\_imp\_sub),$
$\psi\_0\_vec\_true\big), conjugate = false\big)\Big)^{1/2}$

$$u\_star\_0 := \begin{bmatrix} -0.000435381745432829 \\ -0.00247791444568307 \\ -0.00271845177481970 \end{bmatrix} \tag{36}$$

$u\_star\_1 := \frac{1}{\alpha\_true}$
$\cdot \Big(MatrixVectorMultiply\big(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub, A\_imp\_w\_even\_su$
$MatrixInverse(H\_2\_wave\_imp\_sub)), \psi\_0\_vec\_true\big)\Big) \Big/$
$\Big(\big(DotProduct\big(\psi\_0\_vec\_true,$
$MatrixVectorMultiply\big(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_true\big), conjugate = false\big)\big)$
$^{1/2}\Big)$

$$u\_star\_1 := \begin{bmatrix} -0.000886877407640192 \\ -0.00400919130999021 \\ -0.00358622395847307 \end{bmatrix} \tag{37}$$

$u\_star\_2 := \frac{1}{\alpha\_true}$
$\cdot \Big(MatrixVectorMultiply\big(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_even\_$
$A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub), MatrixInverse(H\_3\_wave\_imp\_sub)), \psi\_0\_vec\_true\big)$

$) \Big/$

$\Big(\big(DotProduct\big(\psi\_0\_vec\_true,$

$MatrixVectorMultiply\big(MatrixInverse(H\_3\_wave\_imp\_sub), \psi\_0\_vec\_true \big), conjugate = false\big) \big)^{1/2} \Big)$

$$u\_star\_2 := \begin{bmatrix} -0.00173519945725429 \\ -0.00524754490734677 \\ -0.000155364888588447 \end{bmatrix} \tag{38}$$

$u\_star\_3 := \frac{1}{\alpha\_true}$

$\cdot \big(MatrixVectorMultiply\big(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixM$

$A\_imp\_w\_odd\_sub, A\_imp\_w\_even\_sub), A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub),$

$MatrixInverse(H\_4\_wave\_imp\_sub)), \psi\_0\_vec\_true) \big) \Big/$

$\big(DotProduct\big(\psi\_0\_vec\_true, MatrixVectorMultiply\big(MatrixInverse(H\_4\_wave\_imp\_sub),$

$\psi\_0\_vec\_true), conjugate = false) \big)^{1/2}$

$$u\_star\_3 := \begin{bmatrix} -0.00100991950767164 \\ -0.00388773325341565 \\ 0.000154963604957675 \end{bmatrix} \tag{39}$$

x

$x\_0\_star := x\_0$

$$x\_0\_star := \begin{bmatrix} -0.00377869564857395 \\ -0.00391093582674638 \\ 0.0141511512373208 \end{bmatrix} \tag{40}$$

$x\_1\_star := MatrixVectorMultiply(A\_imp\_w\_even\_sub, x\_0\_star) + u\_star\_0$

$$x\_1\_star := \begin{bmatrix} -0.00308696278676284 \\ 0.00584715988266537 \\ 0.0103055848602582 \end{bmatrix} \tag{41}$$

$x\_2\_star := MatrixVectorMultiply(A\_imp\_w\_odd\_sub, x\_1\_star) + u\_star\_1$

$$x\_2\_star := \begin{bmatrix} -0.00198244452852470 \\ 0.00599175271099335 \\ 0.00472796523683293 \end{bmatrix} \tag{42}$$

$x\_3\_star := MatrixVectorMultiply(A\_imp\_w\_even\_sub, x\_2\_star) + u\_star\_2$

$$x\_3\_star := \begin{bmatrix} 0.0000698402885985856 \\ 0.00359369281210606 \\ 0.000785116075681785 \end{bmatrix} \tag{43}$$

$x\_4\_star := MatrixVectorMultiply(A\_imp\_w\_odd\_sub, x\_3\_star) + u\_star\_3$

$$x\_4\_star := \begin{bmatrix} 4.91838022132624\ 10^{-10} \\ 7.67193365260044\ 10^{-10} \\ -3.02925714569004\ 10^{-11} \end{bmatrix} \quad \textbf{(44)}$$

:

$u\_star\_0 :=$
$\left(MatrixVectorMultiply\left(MatrixInverse(MatrixMatrixMultiply(Transpose(A\_imp\_w\_even\_sub), H\_even\_imp\_sub)), \psi\_0\_vec\_true\right)\right) / \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_1\_wave\_imp\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}\right)$

$$u\_star\_0 := \begin{bmatrix} -0.000468872668555394 \\ -0.00266852336090565 \\ -0.00292756357235815 \end{bmatrix} \quad \textbf{(45)}$$

$u\_star\_1 :=$
$\left(MatrixVectorMultiply\left(MatrixInverse(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub, A\_imp\_w\_even\_sub)), H\_odd\_imp\_sub)), \psi\_0\_vec\_true\right)\right) / \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_2\_wave\_imp\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}\right)$

$$u\_star\_1 := \begin{bmatrix} -0.000955098786671332 \\ -0.00431759082226888 \\ -0.00386208750156708 \end{bmatrix} \quad \textbf{(46)}$$

$u\_star\_2 :=$
$\left(MatrixVectorMultiply\left(MatrixInverse(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_even\_sub, A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub)), H\_even\_imp\_sub)), \psi\_0\_vec\_true\right)\right) / \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_3\_wave\_imp\_sub), \psi\_0\_vec\_true\right), conjugate = false\right)\right)^{1/2}\right)$

$$u\_star\_2 := \begin{bmatrix} -0.00186867641680721 \\ -0.00565120244447993 \\ -0.000167316040868611 \end{bmatrix} \quad \textbf{(47)}$$

$$u\_star\_3 := \left(MatrixVectorMultiply\left(MatrixInverse(MatrixMatrixMultiply(Transpose(MatrixMatrixMultiply(MatrixMatrixMultiply(MatrixMatrixMultiply(A\_imp\_w\_odd\_sub, A\_imp\_w\_even\_sub), A\_imp\_w\_odd\_sub), A\_imp\_w\_even\_sub)), H\_odd\_imp\_sub)), \psi\_0\_vec\_true \right) \right) \Big/ \left(\left(DotProduct\left(\psi\_0\_vec\_true, MatrixVectorMultiply\left(MatrixInverse(H\_4\_wave\_imp\_sub), \psi\_0\_vec\_true \right), conjugate = false \right) \right)^{1/2} \right)$$

$$u\_star\_3 := \begin{bmatrix} -0.00108760566917523 \\ -0.00418678983278976 \\ 0.000166883889249692 \end{bmatrix} \quad \textbf{(48)}$$

:

$\psi\_0 := \psi\_0\_vec\_true :$

$\psi\_1 := MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_even\_sub)), \psi\_0) :$
$\psi\_2 := MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_odd\_sub)), \psi\_1) :$
$\psi\_3 := MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_even\_sub)), \psi\_2) :$
$\psi\_4 := MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_odd\_sub)), \psi\_3) :$

$$u\_star\_0 := \frac{MatrixVectorMultiply(MatrixInverse(H\_even\_imp\_sub), \psi\_1)}{\sqrt{DotProduct(\psi\_1, MatrixVectorMultiply(MatrixInverse(H\_even\_imp\_sub), \psi\_1))}}$$

$$u\_star\_0 := \begin{bmatrix} -0.000468872668555393 \\ -0.00266852336090565 \\ -0.00292756357235815 \end{bmatrix} \quad \textbf{(49)}$$

$$u\_star\_1 := \frac{MatrixVectorMultiply(MatrixInverse(H\_odd\_imp\_sub), \psi\_2)}{\sqrt{DotProduct(\psi\_2, MatrixVectorMultiply(MatrixInverse(H\_odd\_imp\_sub), \psi\_2))}}$$

$$u\_star\_1 := \begin{bmatrix} -0.000955098786671338 \\ -0.00431759082226891 \\ -0.00386208750156708 \end{bmatrix} \quad \textbf{(50)}$$

$$u\_star\_2 := \frac{MatrixVectorMultiply(MatrixInverse(H\_even\_imp\_sub), \psi\_3)}{\sqrt{DotProduct(\psi\_3, MatrixVectorMultiply(MatrixInverse(H\_even\_imp\_sub), \psi\_3))}}$$

$$u\_star\_2 := \begin{bmatrix} -0.00186867641680721 \\ -0.00565120244447994 \\ -0.000167316040868590 \end{bmatrix} \quad \mathbf{(51)}$$

$$u\_star\_3 := \frac{MatrixVectorMultiply(MatrixInverse(H\_odd\_imp\_sub), \psi\_4)}{\sqrt{DotProduct(\psi\_4, MatrixVectorMultiply(MatrixInverse(H\_odd\_imp\_sub), \psi\_4))}}$$

$$u\_star\_3 := \begin{bmatrix} -0.00108760566917521 \\ -0.00418678983278968 \\ 0.000166883889249581 \end{bmatrix} \quad \mathbf{(52)}$$

$$\psi\_1\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_even)), \psi\_0\_vec\_true)))$$

$$\psi\_1\_vec := \begin{bmatrix} 0.1499961347 \\ -0.3166625789 \\ -0.9219058720 \end{bmatrix} \quad \mathbf{(53)}$$

$$\psi\_2\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_odd)), \psi\_1\_vec)))$$

$$\psi\_2\_vec := \begin{bmatrix} 0.2616625480 \\ -0.5720111067 \\ -0.6985730449 \end{bmatrix} \quad \mathbf{(54)}$$

$$\psi\_3\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_even)), \psi\_2\_vec)))$$

$$\psi\_3\_vec := \begin{bmatrix} 0.6534488620 \\ -0.9623484406 \\ 0.0849995834 \end{bmatrix} \quad \mathbf{(55)}$$

$$\psi\_4\_vec := evalf(subs([t=0.25], MatrixVectorMultiply(Transpose(MatrixInverse(A\_imp\_w\_odd)), \psi\_3\_vec)))$$

$$\psi\_4\_vec := \begin{bmatrix} 0.9092093877 \\ -1.073067960 \\ 0.5965206351 \end{bmatrix} \quad \mathbf{(56)}$$

$t2 := time[real]()$

$$t2 := 140191.628 \quad \mathbf{(57)}$$

$total\_time := t2 - t1$

$$total\_time := 7.467 \quad \mathbf{(58)}$$