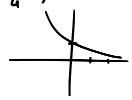
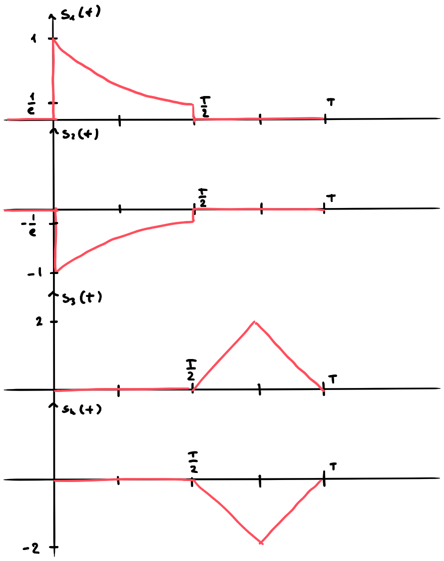
88)

$$s_1(t) = e^{-\frac{2t}{T}} \operatorname{rect}\left(\frac{t-\frac{T}{4}}{\frac{T}{2}}\right) \qquad s_3(t) = 2\operatorname{triang}\left(\frac{t-\frac{3}{4}T}{\frac{T}{4}}\right)$$

$$s_2(t) = -e^{-\frac{2t}{T}} \operatorname{rect}\left(\frac{t-\frac{T}{4}}{\frac{T}{2}}\right) \qquad s_4(t) = -2\operatorname{triang}\left(\frac{t-\frac{3}{4}T}{\frac{T}{4}}\right)$$





$s_1(t)$ e $s_2(t)$ sono ortogonali a $s_3(t)$ e $s_4(t)$, in quanto sono definiti in intervalli di tempo diversi.
$s_1(t)$ e $s_2(t)$, $s_3(t)$ e $s_4(t)$ non sono ortogonali tra di loro, ma antipodali.

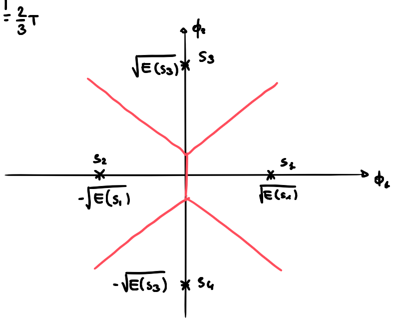
$$\phi_1 = \frac{s_1}{\sqrt{E(s_1)}}$$

$$E(s_1) = \int_0^{T/2} \left(e^{-\frac{2t}{T}}\right)^2 dt = \int_0^{\frac{T}{2}} e^{-\frac{4}{T}t}\, dt = -\frac{T}{4} e^{-\frac{4}{T}t}\Big|_0^{T/2} = -\frac{T}{4}\left(e^{-2}-1\right) = \frac{T}{4}\left(1-e^{-2}\right)$$

In quanto $s_1 = -s_2$, non lo considero per la costruzione della base

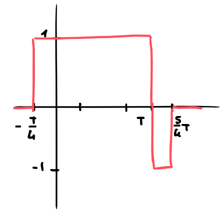
$\phi_2 = \frac{s_3(t)}{\sqrt{E(s_3)}}$ → prendo direttamente s_3 normalizzato in quanto è già ortogonale a ϕ_1 $(\langle s_3, \phi_1 \rangle = 0)$

$$E(s_3) = \frac{4\frac{T}{2}}{3} = \frac{2}{3}T$$

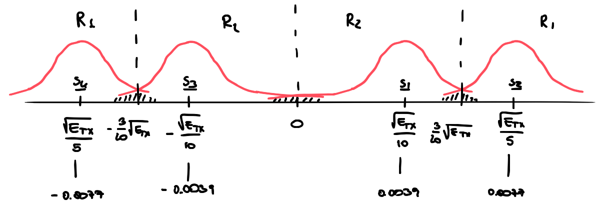


$$P(E) \leq 3Q\left(\frac{d_{min}}{2\sigma_I}\right) \leq 3Q\left(\frac{2\sqrt{E(s_1)}}{2\sigma_I}\right) \leq 3Q\left(\frac{\sqrt{E(s_1)}}{\sigma_I}\right)$$

89)



$$E(h_{TX}) = \frac{3}{2}T$$



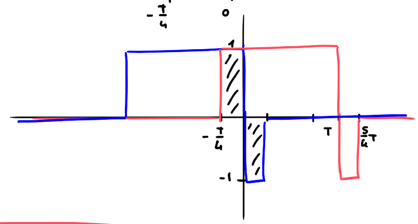
$$P(E) = \sum_{n=1}^{4} P(E \mid a_0 = n) P_{a_0}(n) = \frac{1}{2} P(E \mid a_0 = 1) + \frac{1}{2} P(E \mid a_0 = 2)$$

$$P(E \mid a_0 = 1) = Q\left(\frac{\sqrt{E_{TX}}}{20\sigma_I}\right)$$

$$P(E \mid a_0 = 2) = Q\left(\frac{\sqrt{E_{TX}}}{20\sigma_I}\right) + Q\left(\frac{\sqrt{E_{TX}}}{10\sigma_I}\right)$$

$$P(E) = Q\left(\frac{\sqrt{E_{TX}}}{20\sigma_I}\right) + \frac{1}{2} Q\left(\frac{\sqrt{E_{TX}}}{10\sigma_I}\right)$$

$$\langle h_{TX}(t-T), h_{TX}(t) \rangle = \int_{-\frac{T}{4}}^{0} dt + \int_0^{\frac{T}{4}} (-1)\, dt = 0$$ → no interferenza intersimbolica



Holder ?? p.ti 3.4

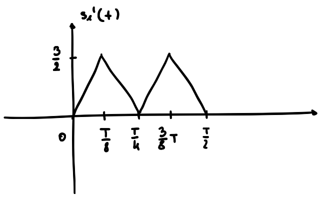
10)

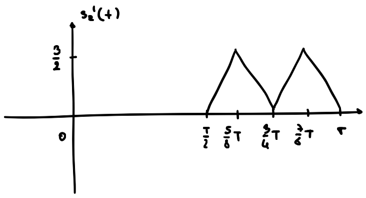
$$s_1(t) = \frac{3}{2}\,\mathrm{triang}\left(\frac{t-\frac{T}{8}}{\frac{T}{8}}\right) \qquad s_3(t) = -s_1(t)$$

$$s_2(t) = \frac{3}{2}\,\mathrm{triang}\left(\frac{t-\frac{5}{8}T}{\frac{T}{8}}\right) \qquad s_4(t) = -s_2(t)$$

$T = 1\,ms$

$$r(t) = s_{TX}(t) + s_{TX}\left(t-\frac{T}{4}\right) + w(t)$$



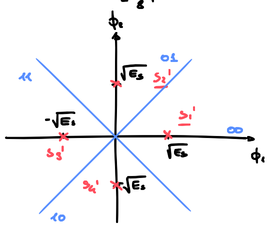


$s_3'(t) = -s_1'(t)$

$s_4'(t) = -s_2'(t)$

$$\phi_1(t) = \frac{s_1'(t)}{\sqrt{E(s_1')}} \qquad \phi_2(t) = \frac{s_2'(t)}{\sqrt{E(s_2')}}$$

$$E(s_1') = 2\,\frac{\left(\frac{3}{2}\right)^2\frac{T}{4}}{3} = \frac{3}{8}T \qquad E(s_2') = 2\,\frac{\left(\frac{3}{2}\right)^2\frac{T}{4}}{3} = \frac{3}{8}T$$



1) La modulazione in esame è di classe bi-ortogonale

2) Il criterio di decisione migliore è quello a minima distanza in quanto ci troviamo in un canale AWGN e i simboli sono equiprobabili

3) Utilizzo la mappatura di Gray per avere tra zone adiacenti, un solo bit di differenza

4) La modulazione è quella di una 4-QAM a meno di una rotazione con energia media $E_s = E(s_1)$

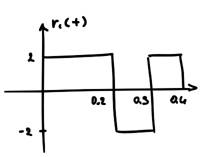
$$P(E) = 1 - \left(1 - Q\left(\sqrt{\frac{E_s}{2\sigma_z^2}}\right)\right)^2 \left(\cdot 4 \cdot \frac{1}{4}\right) = 0.16$$

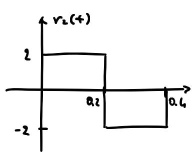
↳ simboli equiprobabili

5) L'energia di s_1' e s_3' diminuisce, facendo avvicinare i punti sulla costellazione. Si avrà quindi un aumento della probabilità d'errore

6) Il criterio da usare ora è il criterio MAP; la costellazione avrà regioni di decisioni diverse, facendo ingrandire le regioni di s_3 e s_4 verso s_1 e s_2

11)



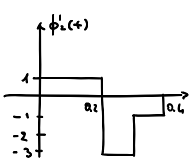


$E(s_1) = E(s_2) = 0.02$

$$\phi_1(t) = \frac{r_1(t)}{\sqrt{E(r_1)}} = 25\,r_1(t) \qquad E(r_1) = 4 \cdot \frac{4}{10}\ (= E(r_2)) = \frac{8}{5} \cdot 10^{-3}$$

$$\langle \phi_1(t); r_2(t) \rangle = 25 \langle r_1(t); r_2(t) \rangle = 25 \cdot \frac{4}{5} = 0.02$$

$$\phi_2'(t) = r_2(t) - \langle \phi_1(t); r_2(t) \rangle \phi_1(t) = r_2(t) - 0.5\,r_1(t)$$

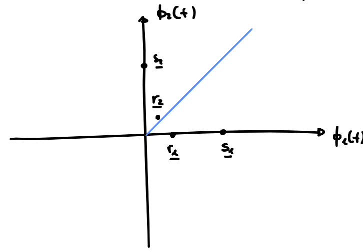


$$\phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E(\phi_2')}}$$

$$E(\phi_2') = \left(1 \cdot 0.2 + 9 \cdot 0.1 + 1 \cdot 0.1\right) \cdot 10^{-3} = 1.2 \cdot 10^{-3}$$

$$r_{1,1} = \langle r_1(t); \phi_1(t) \rangle = \frac{1}{25} \qquad r_{1,2} = 0$$

$$r_{2,1} = \langle r_2(t); \phi_1(t) \rangle = 0.02 \qquad r_{2,2} = \langle r_2(t); \phi_2(t) \rangle = \sqrt{E_{r_2}^2 - r_{2,1}^2} = 0.035$$



Il simbolo associato a $\underline{r_1}$ è $a_0 = 1$ (usiamo il criterio MD)

Il simbolo associato a $\underline{r_2}$ è $a_0 = 2$

$$P(bit) = 25 \cdot 10^{-3} = Q\left(\frac{d}{2\sigma_z}\right) = Q\left(\frac{\sqrt{2}\sqrt{E_s}}{2\sigma_z}\right) = Q\left(\sqrt{\frac{E_s}{2\sigma_z^2}}\right)$$

$\sigma_z^2 = 0.0026$

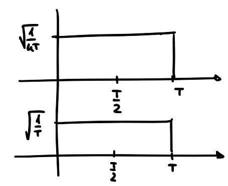
4-QAM con $E_s = E(s_1) = E(s_2)$

$$P(E) = 1 - \left(1 - Q\left(\sqrt{\frac{E_s}{2\sigma_z^2}}\right)\right)^2 = 0.21$$

13)

$$s_1(t) = \sqrt{\frac{1}{4T}}\,\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$

$$s_2(t) = \sqrt{\frac{1}{T}}\,\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$



$T = 1\,ms$

1) $$E_m = \frac{E(s_1) + E(s_2)}{2} = \frac{\frac{1}{4T}\cancel{T} + \frac{1}{T}\cancel{T}}{2} = \frac{5}{8}$$

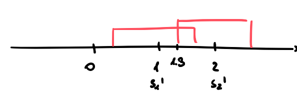
$$\phi_1(t) = 2\,s_1(t) = \frac{1}{\sqrt{T}}\,\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$

2) $$P(E) = Q\left(\frac{d}{2\sigma_z}\right) = Q\left(\frac{2-1}{2\sqrt{4 \cdot 10^{-2}}}\right) = Q(2.5) = 6.2 \cdot 10^{-3}$$

$$s_1' = \sqrt{10^{\frac{6}{10}}\,E(s_1)} = 1 \qquad s_2' = \sqrt{10^{\frac{6}{10}}\,E(s_2)} = 2$$

$P(a_0 = 1) = 0.45$

$$p_w(a) = \frac{1}{1.4}\,\mathrm{rect}\left(\frac{a}{1.4}\right)$$



Criterio MAP:

$$D(r;1) = P(r \mid a_0 = 1)\,P_{a_0}(1) = \frac{1}{1.4}\,\mathrm{rect}\left(\frac{a-1}{1.4}\right) 0.45$$

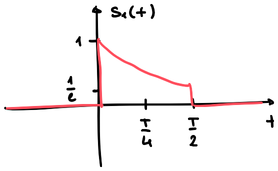
$$D(r;2) = P(r \mid a_0 = 2)\,P_{a_0}(2) = \frac{1}{1.4}\,\mathrm{rect}\left(\frac{a-2}{1.4}\right) 0.65$$

→ $D(r;2) > D(r;1)$ in $[1.3; 2.7]$

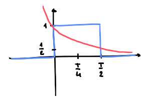
Il confine è in 1.3

$$P(E|2) = 0 \qquad P(E|1) = \int_{1.3}^{\infty} p_w(a - s_1)\,da = \int_{1.3}^{1.7} p_w(a - s_1)\,da = \frac{1}{1.4}(1.7 - 1.3) = 0.29$$

88)

1) 

$s_1(t) = e^{-\frac{2}{T}t}\, rect\left(\frac{t-\frac{T}{4}}{\frac{T}{2}}\right)$



$s_2(t)$ è l'opposto di $s_1(t)$

$s_3(t) = 2 triang\left(\frac{t-\frac{3}{4}T}{\frac{T}{4}}\right)$ $\qquad s_4(t) = -s_3(t)$

2) La modulazione è biortogonale in quanto $s_{1,2}$ e $s_{3,4}$ sono antipodali e $s_{1,3}$ e $s_{2,4}$ sono ortogonali

3) $\phi_1'(t) = s_1(t) \qquad \phi_2'(t) = s_3(t)$

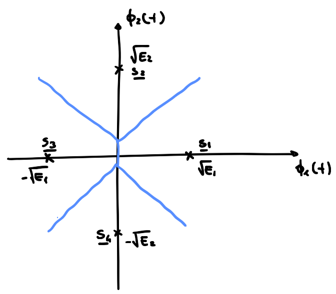
$\phi_1(t) = \frac{\phi_1'(t)}{\sqrt{E(\phi_1')}} \qquad \phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E(\phi_2')}}$

$$E(\phi_1') = \int_0^{T/2} \left(e^{-\frac{2}{T}t}\right)^2 dt = \int_0^{T/2} e^{-\frac{4}{T}t}\, dt = -\frac{T}{4} e^{-\frac{4}{T}t}\Big|_0^{T/2} = -\frac{T}{4}\left(e^{-2}-1\right)$$

$$E(\phi_2') = \frac{A^2 B}{3} = \frac{(2)^2 \frac{T}{2}}{3} = \frac{2}{3}T$$

$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \qquad \phi_2(t) = \frac{s_3(t)}{\sqrt{E_2}}$

$\hookrightarrow \underline{s_1} = \sqrt{E_1}\,\underline{\phi_1} \qquad \underline{s_2} = -\sqrt{E_1}\,\underline{\phi_1} \qquad \underline{s_3} = \sqrt{E_2}\,\underline{\phi_2} \qquad \underline{s_4} = -\sqrt{E_2}\,\underline{\phi_2}$

4-5) 

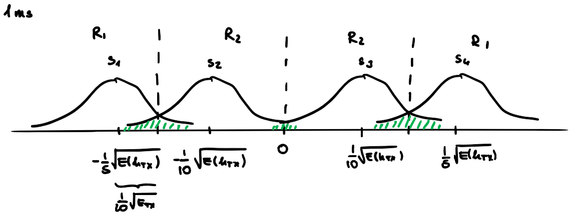
Questa è una modulazione 4-aria

$$P(E) \leq (M-1)\, Q\left(\frac{d_{min}}{2\sigma_I}\right) \leq 3Q\left(\sqrt{\frac{E_1+E_2}{4\sigma_I^2}}\right)$$

$$d_{min} = d(\underline{s_1}, \underline{s_2}) = \sqrt{E_1+E_2}$$

89)

$T = 1ms$

1) 

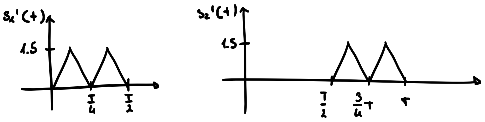
$$P(E) = \sum_m P(E \mid a_0 = m) P_{a_0}(m) = \frac{1}{2}P(E \mid a_0 = 1) + \frac{1}{2}P(E \mid a_0 = 2)$$

$$= \frac{1}{2}\left[Q\left(\frac{\frac{1}{10}\sqrt{E(h_{TX})}}{\sigma_I}\right)\right] + \frac{1}{2}\left[Q\left(\frac{\frac{1}{10}\sqrt{E(h_{TX})}}{\sigma_I}\right) + Q\left(\frac{\frac{1}{10}\sqrt{E(h_{TX})}}{\sigma_I}\right)\right]$$

$$= Q\left(\frac{\sqrt{E(h_{TX})}}{20\sigma_I}\right) + \frac{1}{2}Q\left(\frac{\sqrt{E(h_{TX})}}{10\sigma_I}\right)$$

2) $\langle h_{TX}(t)\,;\, h_{TX}(t-T)\rangle = 0 \rightarrow$ non c'è interferenza inter-simbolo.

100)

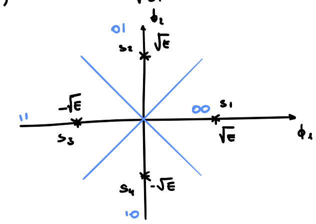
1) 

$\phi_1'(t) = s_1'(t) \qquad \phi_2'(t) = s_3'(t)$

$\phi_1(t) = \frac{\phi_1'(t)}{\sqrt{E(\phi_1')}} \qquad \phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E(\phi_2')}} \qquad E(\phi_1') = E(\phi_2')$

$$E = 2\cdot\frac{\left(\frac{3}{2}\right)^2 \frac{T}{4}}{3} = \frac{3}{8}T$$

$\phi_1(t) = \sqrt{\frac{8}{3T}}\, s_1'(t) \qquad \phi_2(t) = \sqrt{\frac{8}{3T}}\, s_2'(t)$



Si tratta di una modulazione biortogonale

2) Utilizzo il criterio MD

3) Utilizzo la mappatura di Gray per minimizzare gli errori sul bit

4) La costellazione è quella di una 4-QAM ruotata, quindi:

$$P(E) = \sum_m P(E \mid a_0 = m) P_{a_0}(m) = 1 - \left(1 - Q\left(\sqrt{\frac{E}{2\sigma_I^2}}\right)\right)^2$$

5,6) A parole