13)

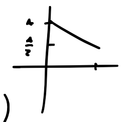
$$s_1(t) = A\,\mathrm{rect}\left(\frac{t-\frac{1}{3}T}{\frac{2}{3}T}\right) + \frac{A}{2}A\,\mathrm{rect}\left(\frac{t-\frac{1}{2}T}{\frac{1}{3}T}\right)$$

$$s_2(t) = \frac{A}{2}A\,\mathrm{rect}\left(\frac{t-\frac{1}{2}T}{\frac{1}{3}T}\right) + A\,\mathrm{rect}\left(\frac{t-\frac{4}{3}T}{\frac{2}{3}T}\right)$$

$$s_3(t) = -A\left(\frac{t}{2T}-1\right)\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$

$$s_4(t) = B\left(\frac{t}{2T}-1\right)\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$

$$-\frac{A}{2T}(t-2T)$$



14)

$$E_1 = A^2\cdot\frac{2}{3}T + \left(\frac{1}{2}A\right)^2\cdot\frac{1}{3}T = A^2\left(\frac{2}{3}+\frac{1}{20}\right)T = \frac{9}{20}A^2T$$

$$E_2 = E_1$$

$$E_3 = \int_0^T\left(-\frac{A}{2T}(t-2T)\right)^2 dt = \frac{A^2}{4T^2}\int_0^T t^2-4tT+4T^2\,dt = \frac{A^2}{4T^2}\left(\frac{t^3}{3}-4T\frac{t^2}{2}+4T^2t\right)\Bigg|_0^T = \frac{A^2}{4\cancel{T^2}}\left(\frac{T^{\cancel{3}}}{3}-2T^{\cancel{3}}+4T^{\cancel{3}}\right) = \frac{7}{12}A^2T$$

$$E_4 = \frac{7}{12}B^2T$$

15)

$$E_1 = \int_{-\frac{T}{2}}^{\frac{T}{2}}(A\cos(2\pi f_0 t))^2dt = A^2\int_{-\frac{T}{2}}^{\frac{T}{2}}\cos^2(2\pi f_0 t)\,dt = \frac{A^2}{2}\int_{-\frac{T}{2}}^{\frac{T}{2}}1-\cos(4\pi f_0 t)\,dt = \frac{A^2}{2}\left(t-\frac{1}{4\pi f_0}\sin(4\pi f_0 t)\right)\Bigg|_{-T/2}^{T/2} = \frac{A^2}{2}\left(T-\frac{1}{2\pi f_0}\sin(2\pi f_0 T)\right)$$

$$E_2 = 2T$$

$$E_3 = \frac{4}{3}T$$

$$E_4 = 4+\frac{4}{9} = \frac{40}{9}$$

16)

$$E_s = E[E_x] = E_{x_1}\cdot P_y(1) + E_{x_2}\cdot(1-P_y(1)) = 4p + \frac{4}{3}(p-1)$$

17)

$$E_s = E[E_x]$$
$$= E_x(A=-3)P_A(-3) + E_x(A=-1)P_A(-1) + E_x(A=1)P_A(1) + E_x(A=3)P_A(3)$$
$$= 2(E_x(A=1)P_A(1) + E_x(A=3)P_A(3))$$
$$= \frac{1}{2}(E_x(A=1) + E_x(A=3))$$
$$= \frac{1}{2}\left(\frac{(1)^2 2}{3} + \frac{(3)^2 2}{3}\right)$$
$$= \frac{1}{2}\left(\frac{2}{3}+6\right)$$
$$= \frac{1}{3}+3$$
$$= \frac{10}{3}$$

20)

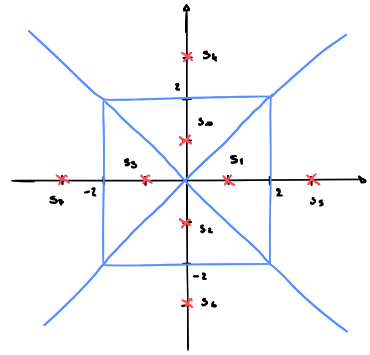
a) $x(kT) = \mathrm{rect}\left(\frac{k}{3}\right) \longrightarrow x(kT) = \begin{cases} 1 & \text{per } k\in\{-1;0;1\} \\ 0 & \text{altrove}\end{cases}$

b) $x(kT) = \frac{3}{4}\mathrm{rect}\left(\frac{kT-\frac{5}{2}}{3T}\right) \rightarrow x(kT) = \begin{cases} \frac{3}{4} & \text{per } kT\in[1;4] \\ 0 & \text{altrove}\end{cases}$

c) $x(kT) = \exp(-kT)\,1(k) \rightarrow x(kT) = \begin{cases} \exp(-kT) & \text{per } k\geq 0 \\ 0 & \text{altrove}\end{cases}$

d) $x(kT) = 3\,\mathrm{triangle}\left(\frac{k}{6}\right)+1 \rightarrow x(kT) = \begin{cases} 3-\frac{1}{2}|kT| & \text{per } k\in[-6;6] \\ 0 & \text{altrove}\end{cases}$

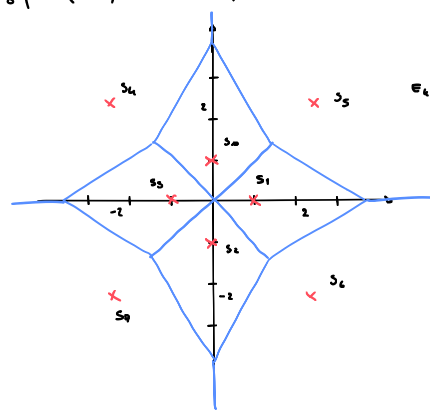
23)



P(E | s_0) è maggiore in quanto la regione di simbolo del simbolo s_0 è più piccola di quella del simbolo s_4

I simboli più probabili che vengono scelti, dopo un errore di trasmissione di s_0, sono s_1 e s_3, in quanto sono quelli più vicini a s_0

$$P(E) \geq \frac{1}{8}\sum_{m=1}^{8} Q\left(\frac{d_{min,m}}{2\sigma_I}\right) \geq \frac{1}{8}\left(4Q\left(\frac{2}{2\sigma_I}\right) + 4Q\left(\frac{\sqrt{2}}{2\sigma_I}\right)\right)$$



E_4, E_5, E_6, E_7 <u>non</u> sono <u>cambiate</u>

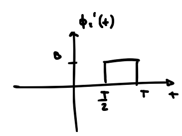
24)

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}}} = \sqrt{\frac{2}{A^2T}}\,s_1(t) \rightarrow \underline{s_1} = \left[\sqrt{\frac{A^2T}{2}};0\right]$$

$$E_{s_1} = \frac{A^2T}{2} \qquad (E_{s_2} = B^2T)$$

$$\phi_2'(t) = s_2(t) - \langle\phi_1(t);s_2(t)\rangle\,\phi_1(t)$$

$$\langle\phi_1(t);s_2(t)\rangle = \sqrt{\frac{2}{A^2T}}\,\frac{ABT}{2} = B\sqrt{\frac{T}{2}}$$

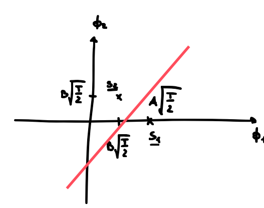


$$\phi_2'(t) = s_2(t) - \frac{B}{A}s_1(t)$$

$$\phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E_{\phi_2}}}$$

$$E_{\phi_2} = \frac{B^2T}{2}$$

$$\phi_2(t) = \sqrt{\frac{2}{B^2T}}\,s_2(t) - \sqrt{\frac{2}{A^2T}}\,s_1(t)$$

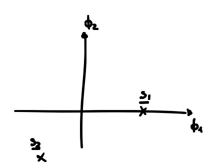


$$s_{2,1} = \langle s_2(t);\phi_1(t)\rangle = B\sqrt{\frac{T}{2}}$$

$$s_{2,2} = \sqrt{E_{s_2}-s_{2,1}^2} = \sqrt{B^2T-\frac{B^2T}{2}} = B\sqrt{\frac{T}{2}}$$

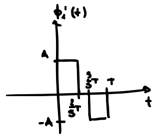
$$P(E) = Q\left(\frac{d}{2\sigma_I}\right)$$

$$E(s_2) = E(s_1) \rightarrow \frac{A^2T}{2} = B^2T \rightarrow B = \pm\frac{A}{\sqrt{2}}$$
$$\hookrightarrow B = -\frac{A}{\sqrt{2}}$$



25)

$$E_s = A^2\frac{2}{5}T + \left(\frac{A}{2}\right)^2\frac{1}{5}T$$
$$= A^2\left(\frac{2}{5}+\frac{1}{20}\right)T$$
$$= \frac{9}{20}A^2T \rightarrow A = \sqrt{\frac{10E_s}{9T}}$$



$$\phi_1'(t) = s_1(t) - s_2(t)$$

$$\phi_1(t) = \frac{\phi_1'(t)}{\sqrt{E_{\phi_1}}}$$

$$E_{\phi_1} = 2A^2\cdot\frac{2}{5}T = \frac{4}{5}A^2T \qquad \phi_1(t) = \sqrt{\frac{5}{T}}\frac{1}{2A}\left(s_1(t)-s_2(t)\right)$$

$$\phi_2'(t) = s_1(t) - \langle s_1(t)\phi_1(t)\rangle\phi_1(t)$$

$$\langle s_1(t), \phi_1(t)\rangle = \int_{-\infty}^{+\infty} s_1(t)\frac{1}{\sqrt{E_{\phi_1}}}\left(s_1(t)-s_2(t)\right)dt$$
$$= \frac{1}{\sqrt{E_{\phi_1}}}\left(E_{s_1} - \langle s_1(t); s_2(t)\rangle\right)$$

$$\langle s_1(t); s_2(t)\rangle = \left(\frac{1}{2}A\right)^2\frac{1}{5}T = \frac{A^2T}{20}$$

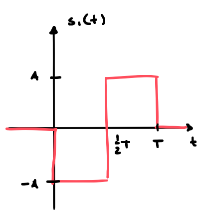
$$= \sqrt{\frac{5}{4A^2T}}\left(\frac{9}{10}A^2T - \frac{1}{10}A^2T\right)$$
$$= A\sqrt{\frac{T}{5}}$$

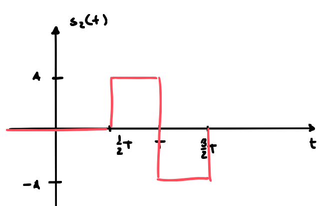
$$\phi_2'(t) = s_1(t) - \frac{1}{2}\left(s_1(t)-s_2(t)\right) = \frac{1}{2}s_1(t) + \frac{1}{2}s_2(t)$$

$$\phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E_{\phi_2'}}}$$

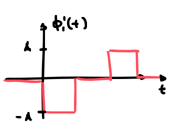
$$E(\phi_2') = \int\frac{1}{4}\left(s_1(t)+s_2(t)\right)^2dt$$
$$= \frac{1}{4}\left(E_{s_1} + E_{s_2} + 2\langle s_1(t); s_2(t)\rangle\right)$$
$$= \frac{1}{4}\left(\frac{9}{10}A^2T + \frac{A^2T}{10}\right)$$
$$= \frac{A^2T}{4}$$

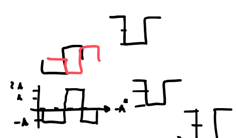
27)





$$\phi_1'(t) = s_1(t) - s_2(t)$$





$$E_{\phi_1'} = A^2T$$

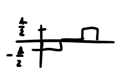
$A^2T + 2A^2T$

$3A^2T$ $\qquad \phi_1 = \frac{1}{A\sqrt{3T}}(s_1 - s_2)$

$$\phi_1(t) = \frac{1}{A\sqrt{T}}\left(s_1(t)-s_2(t)\right)$$

$$\phi_2'(t) = s_2(t) - \langle s_1(t); \phi_1(t)\rangle\phi_1(t)$$

$\frac{1}{A\sqrt{3T}}$



$$\langle s_1(t); \phi_1(t)\rangle = \frac{1}{A\sqrt{T}}\langle s_1(t); s_1(t)-s_2(t)\rangle$$
$$= \frac{1}{A\sqrt{T}}\left(E_{s_1} - \langle s_1(t); s_2(t)\rangle\right)$$

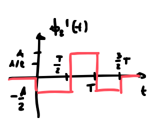
$\left(\frac{A}{2}\right)^2 +$

$$\langle s_1(t); s_2(t)\rangle = \frac{A^2T}{2} \qquad E_{s_1} = A^2T \qquad \phi_2 = \frac{1}{A\sqrt{T}}(s_1 + s_2)$$

$$= \frac{1}{A\sqrt{T}}\left(A^2T - \frac{A^2T}{2}\right)$$
$$= \frac{A^2T}{2A\sqrt{T}}$$
$$= \frac{A\sqrt{T}}{2}$$

$\frac{A\sqrt{3T}}{2}$

$$\phi_2'(t) = s_1(t) - \frac{\cancel{A\sqrt{T}}}{2}\frac{1}{\cancel{A\sqrt{T}}}\left(s_1(t)-s_2(t)\right)$$
$$= \frac{1}{2}\left(s_1(t)+s_2(t)\right)$$

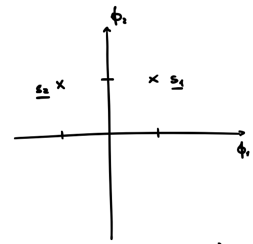


$$\phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E_{\phi_2'}}} \qquad E_{\phi_2'} = \left(-\frac{A}{2}\right)^2T + A^2\frac{T}{2} = \frac{A^2T}{4} + \frac{A^2T}{2} = \frac{3A^2T}{4}$$
$$= \sqrt{\frac{4}{3A^2T}}\frac{1}{2}\left(s_1(t)+s_2(t)\right)$$
$$= \frac{1}{A\sqrt{3T}}\left(s_1(t)+s_2(t)\right)$$

$$s_{1,1} = \frac{A\sqrt{T}}{2} \qquad s_{1,2} = \sqrt{E_1 - s_{11}^2} = \sqrt{\frac{3A^2T}{4}}$$

$$s_{2,1} = \langle s_2(t); \phi_1(t)\rangle$$
$$= \frac{1}{A\sqrt{T}}\langle s_2(t); s_1(t)-s_2(t)\rangle$$
$$= \frac{1}{A\sqrt{T}}\left(\langle s_1(t); s_2(t)\rangle - E_{s_2}\right)$$
$$= \frac{1}{A\sqrt{T}}\left(\frac{A^2T}{2} - A^2T\right)$$
$$= -\frac{A^2T}{2A\sqrt{T}}$$
$$= -\frac{A\sqrt{T}}{2}$$

$$s_{2,2} = \sqrt{E_{s_2} - s_{21}^2} = \sqrt{A^2T - \frac{A^2T}{4}} = \sqrt{\frac{3A^2T}{4}} \qquad \frac{A}{2}\sqrt{3T}$$



$$\frac{1}{A\sqrt{3T}}\left(\frac{A^2T}{2} - A^2T\right)$$
$$-\frac{A\sqrt{T}}{2A\sqrt{3T}}$$
$$-\frac{A\sqrt{T}}{2\sqrt{3}}$$

$$T_{segnale} = \frac{3}{2}T$$

$$P_{bit} = P(E) = Q\left(\sqrt{\frac{E_s(1-\rho^2)}{2\sigma_z^2}}\right)$$

$$\frac{E_s}{2\sigma_z^2} = \frac{\left(Q^{-1}(P_{bit})\right)^2}{1-\rho}$$

Con la nuova segnalazione, avremmo una segnalazione ortogonale quindi $\rho' = 0 < \rho = \frac{1}{2}$

29)

$$s_1(t) = 2\cos(2\pi f_0 t)\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$
$$s_2(t) = -s_1(t)$$
$$s_3(t) = \sin(2\pi f_0 t)\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}}} \qquad E_{s_1} = \int_0^T\left(2\cos(2\pi f_0 t)\right)^2dt$$