Lecture 01
Introduction

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Game Theory 2023/24

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What is game theory?

- Game theory is a **mathematical framework** that allows to model specific type of problems
- The problems studied by game theory are called **games**

What is a game?

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- **Definition:** A game is a multi-person multi-objective problem
  - Multi-person* = There are multiple agents (players) involved in a game
  - Multi-objective = Players have, in general, different goals
- The outcome of a game depends on the choices made by all players
- The purpose of game theory is to find the "best choice" for each of them according to their objectives

* Actually, players are not necessarily people

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COMPARISON WITH OPTIMIZATION

Optimization

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- In optimization, you need to find the best solution s^* in a set of possible choices S
- The best solution maximizes a certain utility function $u(s)$

- Here we have:
  - One player: the engineer who chooses the solution
  - One objective: the utility function

Distributed optimization

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- In distributed optimization, we have N players making independent decisions:
  - Player 1 chooses the best $s_1^* \in S_1$
  - Player 2 chooses the best $s_2^* \in S_2$
  - …
  - Player N chooses the best $s_N^* \in S_N$
- They all maximize the same function $u(s_1, \dots, s_N)$
- Here we have:
  - Multiple players
  - Single objective
  - Examples?

Game theory

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- In game theory the N players make independent decisions
  - Player 1 chooses the best $s_1^* \in S_1$
  - Player 2 chooses the best $s_2^* \in S_2$
  - …
- And maximize different utility functions
  - Player 1 wants to maximize $u_1(s_1, \dots, s_N)$
  - Player 2 wants to maximize $u_2(s_1, \dots, s_N)$
  - …

Characteristics of a game

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- Multiple players
- Multiple objectives
- Typical assumptions:
  - The rules of the game are known in advance to all the players
  - Players are *rational* (i.e., they always choose the best action/strategy to maximize their utility function)
  - The outcome of the game depends on the joint decision of all players

$$u_i\underbrace{(s_1, \ldots, s_N)}$$

Predictions

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- Uncertainty in game theory: players know their own moves but can only make conjectures about other players' move
- Still, some outcomes can be predicted
  - Easier for machines and algorithms (rationality, clearly-defined utility function) , hard about humans

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BACKGROUND CONCEPTS

Monopolies and oligopolies

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- Monopoly = single entity controlling the entire market for a product or good
- Oligopoly = few entities sharing the market control for a product or good
- Examples:
  - Internet Service Providers (ISPs)
  - Software: operating systems
  - Online services: search engines, maps, social networks
- Anti-trust punishes monopolists. Why?

↳ optimize their own utility function, not the whole

Market

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- Adam Smith theory: a free market leads to the best social outcome
- Demand/offer laws, "invisible hand"
  - Competition leads to lower market price
- A rational monopolist pursues profit, not social welfare

→ supposed to be for "greater good"

- State monopoly admitted for public services and special goods (tobacco, alcohol, state lottery)

Substitute goods

- The demand for a good generally decreases with its price. Notable exceptions:
  - Giffen goods: necessity goods (e.g., bread)
  - Veblen goods: luxury goods (e.g., expensive watches)
- Two goods are substitute if one can replace the other
  - E.g., butter and margarine; coffee and tea
- Economic perspective: x and y are substitute goods if an increase in the price of x leads to an increase in the demand for y

Discount factor

- Goods do not have the same value today and tomorrow: today's value is higher
- Intuitions behind this: greed of buyers, uncertainty, deterioration, price inflation…
- The discount factor $\delta \in (0,1)$ is used to account for this phenomenon when calculating utility
- A euro is valued 1 at time 0, δ at time 1, δ^2 at time 2, …, δ^t at time t
- This can be seen as decreasing the value by a factor of $1-\delta$ each time

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DECISION PROBLEMS

Decision problem

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- A decision problem is characterized by three elements:
  - 1) Actions
  - 2) Outcomes
  - 3) Preferences
- Action a is selected from a set of possible actions A
- Action a results in a certain outcome
  - For 1-player problems actions = outcomes
- Preferences describe the relationship between different outcomes (i.e., which one is preferred)

Preferences

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- 1-player decision problem case (action = outcome)
- A **preference** is a binary relationship $\succcurlyeq$ between elements of A (set of possible actions)
- If $a, b \in A$, $a \succcurlyeq b$ means that a is preferred to b
- A preference is always reflexive, anti-symmetric
- A preference can also be:
  - **complete**, if for all $a, b \in A$ either $a \succcurlyeq b$ or $b \succcurlyeq a$
  - **transitive**, if for all $a, b, c \in A$, $a \succcurlyeq b \wedge b \succcurlyeq c \Rightarrow a \succcurlyeq c$
- A complete+transitive preference is called **rational**

Utility functions

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- Utility functions (also called payoff functions) are an arbitrary quantification of an outcome's goodness
- Utility functions reflect preferences
  - $u(a) \geq u(b) \Leftrightarrow a \succcurlyeq b$
- The exact formulation of u does not matter, it just needs to reflect all preferences

Rationality

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- **Definition**: Saying that players are **rational** means that they always maximize their utility function, i.e., choose the action that leads to their preferred outcome
- In other words, rational player act for their own good
- Does this mean that players always act in a "selfish" manner? (no)

Decision trees

- We can represent decision problems using tree diagrams
- **Example:** A student (S) goes to the university cafeteria for lunch. The cafeteria offers beef (B) or fish (F) as the main dish and polenta (P) or vegetables (V) as the side dish. The student prefers fish over beef. Also, being born and raised in Veneto, she clearly prefers polenta over vegetables.

Decision trees

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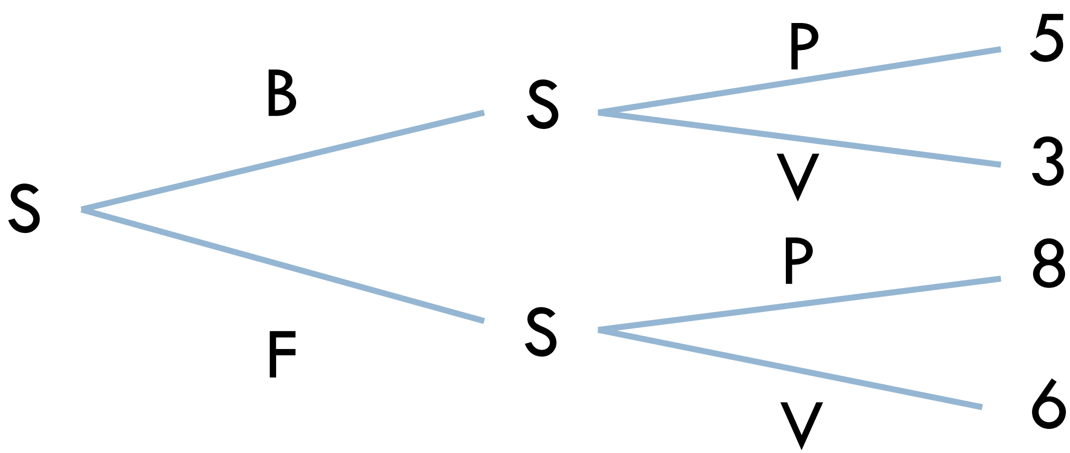
- In this problem, the player S needs to take two decisions
  - B or F?
  - P or V?
- Her preferences are F$\succcurlyeq$B and P$\succcurlyeq$V
- We can model these with a utility function (for now)
- The actual payoff values are not important for the time being, they just need to reflect preferences
  - u(B)=2, u(F)=5, u(P)=3, u(V)=1

Decision trees

- A decision tree has the following structure:
  - On nodes, we write the player who needs to act
  - On branches, we write the actions
  - On leaves, we write payoffs
    - u(B)=2, u(F)=5, u(P)=3, u(V)=1



Decision trees

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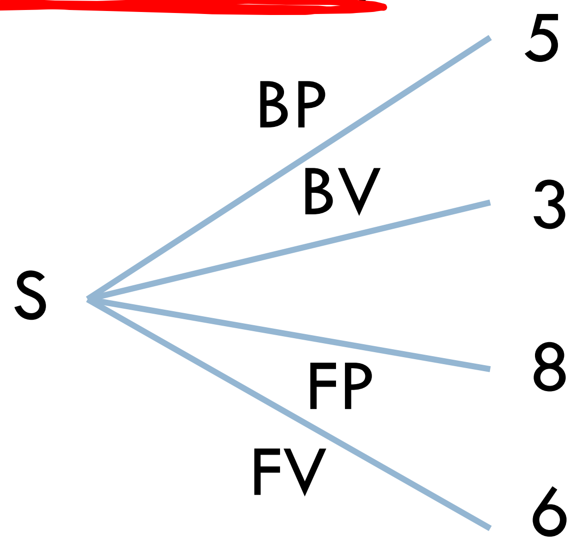
- We can always make a single-player decision tree collapse into a single layer



- This is a normal optimization problem
- Things get interesting when multiple players are involved