Tableau form

$$\begin{array}{c|ccc|ccccc}
\bar{c}_0 & 0 & \dots & 0 & \bar{c}_{m+1} & \dots & \bar{c}_h & \dots & \bar{c}_n \\ \hline
\bar{b}_1 & 1 & 0 \dots & 0 & & & & & \\
\vdots & 0 & \ddots & \vdots & & & & & \\
\bar{b}_t & \vdots & 1 & \vdots & \bar{a}_{t,m+1} & \dots & \bar{a}_{t,h} & \dots & \bar{a}_{t,n} \\
\vdots & & \ddots & 0 & & & & & \\
b_m & 0 & \dots\ 0 & 1 & & & & &
\end{array}$$

$\bar{c}_F \geq 0$

If $\bar{b} = B^{-1}b \geq 0 \Rightarrow$ stop

otherwise $\bar{b}_t < 0 \Rightarrow$ pivot on $\bar{a}_{th} < 0$

"x_h enters the basis" $\rightarrow$ $\tilde{c}_j = c_j - \left(\bar{c}_h / \bar{a}_{th}\right) \bar{a}_{tj} \geq 0 \ , \ \forall j$

$$\Updownarrow$$

$$\overset{+}{\bar{c}_j} \geq \frac{\overset{+}{\bar{c}_h}}{\underset{-}{\bar{a}_{th}}} \bar{a}_{tj}$$

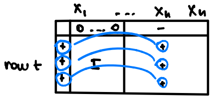
$\nearrow$ $\bar{a}_{t,j} \geq 0 \rightarrow$ no limit

$\searrow$ $\bar{a}_{t,j} < 0 \rightarrow$ limit

If $\bar{a}_{t,j} < 0$: $\bar{c}_j \geq \left(\bar{c}_h / |a_{th}|\right) |a_{tj}| \Rightarrow \dfrac{\bar{c}_h}{|\bar{a}_{th}|} \leq \dfrac{\bar{c}_j}{|a_{tj}|}$

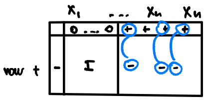
Choose $h = \operatorname{argmin} \left\{ \dfrac{|\bar{c}_j|}{|\bar{a}_{tj}|} : a_{tj} < 0 \right\}$

Primal simplex



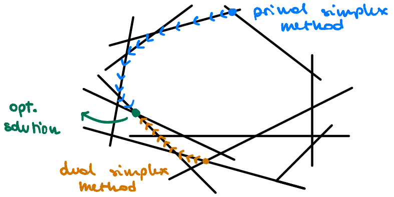
$t = \operatorname{argmin} \left\{ \dfrac{\bar{b}_i}{\bar{a}_{ih}} : \bar{a}_{ih} > 0 \right\}$

Dual simplex



$h = \operatorname{argmin} \left\{ \left| \dfrac{\bar{c}_j}{\bar{a}_{tj}} \right| : \bar{a}_{tj} < 0 \right\}$

New objective function is now worse than before



The dual simplex method is way faster than the primal simplex method

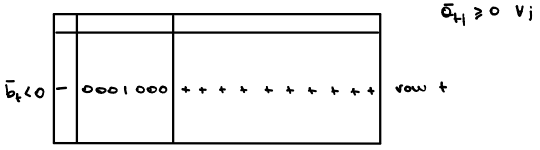
We can apply the bland's rule to the dual simplex method to be sure it doesn't loop and that it reaches the optimal solution.

Initialization

$\bar{c}_0^T \geq 0^T$: find a basis B : $\bar{c}^T = c^T - c_B^T B^{-1} A \geq 0^T$

If the initialization is problematic, apply the primal simplex method

Problematic case :



$$\underset{-}{\bar{b}_t} = \sum_{j=1}^{n} \underset{+}{\bar{a}_{tj}} \, \underset{+}{x_j}$$

↳ Infeasible (the primal problem)

↳ Refers to x, not to μ, so ↖

Es :

$$\begin{cases} \min \ 3x_1 + 4x_2 + 5x_3 \\ 2x_1 + 2x_2 + x_3 - x_4 = 6 \\ x_1 + 2x_2 + 3x_3 - x_5 = 5 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

enters

		x_1	x_2	x_3	x_4	x_5
	0	3	4	5	0	0
leaves → $x_4 =$	-6	$\boxed{-2}$	-2	-1	1	0
$x_5 =$	-5	-1	-2	-3	0	1

bland's rule

row t

↓

Infeasible, but the dual simplex is in canonical form

⇒

enters

		x_1	x_2	x_3	x_4	x_5
	-9	0	1	7/2	3/2	0
$x_1 =$	3	1	1	1/2	-1/2	0
$x_5 =$	-2	0	$\boxed{-1}$	-5/2	-1/2	1

row t

leaves

⇓

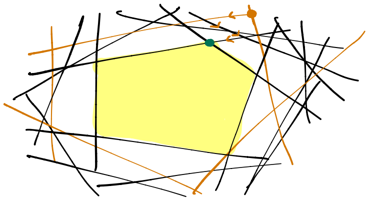
-11	0	0	1	1	1
1	1	0	-2	-1	1
2	0	1	5/2	1/2	-1

→ $\bar{c}^T \geq 0$

→ STOP

$\bar{b} \geq 0$

Row / Constraints generation



I start by considering a problem with a few variables and find an optimal solution

↳ is it feasible? Yes → finished

No ↓

Add constraints and repeat

This metod is the row/constraints generation and works really well with the dual simplex method

Es :

$$\begin{cases} \min \ -x_1 - 4x_2 \\ x_1 + x_2 \leq 2 \\ x_1 + 3x_2 \leq 3 \\ x_2 \leq 2/3 \end{cases}$$

→ mini tableau :

0	-1	-4	0
2	1	1	1

⇓

infeasible ←

8	3	0	4	0	0
2	1	1	1	0	0
3	1	3	0	1	0
2/3	0	1	0	0	1

⇗

→ $\bar{c}^T \geq 0^T$

8	3	0	4	0	0
2	1	1	1	0	0
-3	2	0	-3	1	0
$-\frac{4}{3}$	-1	0	-1	0	1

⇒ …

↳ Perfect tableau for dual simplex method