Artificial problem

$$\begin{cases} \min z = \sum_{i=1}^{m} y_i \\ Ax + Iy = b \\ x, y \geq 0 \end{cases}$$

$y \in \mathbb{R}^m \rightarrow$ must all go to $\emptyset$ so
$\min z = 0$

If opt $w^* = 0$, then $y_1^* \dots y_m^* = 0$, then x^* is a feasible solution of the original problem

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$$\begin{cases} \min z = x_1 + x_3 \\ x_1 + 2x_2 + x_4 = 5 \\ x_2 + 2x_3 = 6 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

Σy_i

		x_1	x_2	x_3	x_4	y_1	y_2
$w =$	0	0	0	0	0	1	1
$y_1 =$	5	1	2	0	1	1	0
$y_2 =$	6	0	1	2	0	0	1

$b \geq 0$

1) $Ax + Iy = b \longrightarrow a_i^T x + y_i = b_i$

$$w = \sum_{i=1}^{m} (b_i - a_i^T x)$$

2) mini-pivot over the y_i values to bring the artificial cost to 0

3) $\begin{bmatrix} \text{new} \\ \text{row } 0 \end{bmatrix} \leftarrow \begin{bmatrix} \text{original} \\ \text{row } 0 \end{bmatrix} - \sum_{i=1}^{m} [\text{row } i]$

$$\left|\begin{array}{c|cccc:cc}
0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline
5 & 1 & 2 & 0 & 1 & 1 & 0 \\
6 & 0 & 1 & 2 & 0 & 0 & 1
\end{array}\right|
\quad \overset{(3)}{\Longrightarrow} \quad
\left|\begin{array}{c|cccc:cc}
-11 & -1 & -3 & -2 & -1 & 0 & 0 \\ \hline
5 & 1 & 2 & 0 & 1 & 1 & 0 \\
6 & 0 & 1 & 2 & 0 & 0 & 1
\end{array}\right|$$

simplex method

$-w$

$$\begin{array}{r}
\\
x_2 = \\
x_3 =
\end{array}
\left|\begin{array}{c|cccc:cc}
0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline
5/2 & 1/2 & 1 & 0 & 1/2 & 1/2 & 0 \\
7/4 & -1/4 & 0 & 1 & -1/4 & -1/4 & 1/2
\end{array}\right|$$

$\bar{b} \geq 0$

$$w^* = 0 \Rightarrow \underbrace{y_1^*, y_2^*}_{\text{non-basic}} = 0$$

$$x^* = \begin{bmatrix} 0 \\ 5/2 \\ 7/4 \\ 0 \end{bmatrix} \rightarrow \text{feasible for the original problem}$$

Phase II:

must be 0 to have a canonical form

$$\left|\begin{array}{c|cccc}
0 & 1 & 0 & 1 & 0 \\ \hline
5/2 & 1/2 & 1 & 0 & 1/2 \\
7/4 & -1/4 & 0 & 1 & -1/4
\end{array}\right| \rightarrow \text{original obj function}$$

pivot operation over x_3

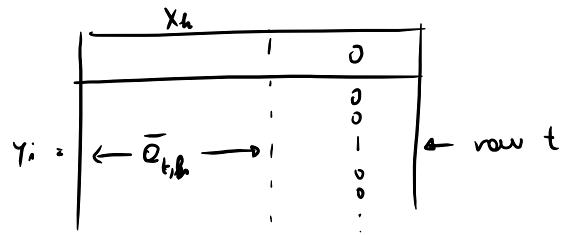
$$\left|\begin{array}{c|cccc}
-9/4 & 5/4 & 0 & 0 & 1/4 \\ \hline
5/2 & 1/2 & 1 & 0 & 1/2 \\
7/4 & -1/4 & 0 & 1 & -1/4
\end{array}\right|$$

1) What if $w^* > 0$: $\Rightarrow$ Original solution is unfeasible

$\Downarrow$

$\left(\nexists\, (x^*, y^*) \text{ opt for the artificial problem with } y^* = 0 \right)$

2) What if $w^* = 0$ and a certain y_i is basic



if $\exists\, \bar{a}_{t,h} \neq 0 \rightarrow$ pivot operation on it

$\downarrow$

otherwise $\longrightarrow$ Redundant equation row t is a linear combination of other rows

$\Downarrow$

$\text{rank}(A) < m$

$\Downarrow$

remove row t

ES1

$$\begin{cases} \min \; z = x_1 + x_3 \\ x_1 + 2x_2 \le -5 \\ x_2 + 2x_3 = 6 \\ x_1, x_2, x_3 \ge 0 \end{cases} \Rightarrow \begin{cases} \min \; z = x_1 + x_3 \\ x_1 + 2x_2 + x_4 = -5 \\ x_2 + 2x_3 = 6 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases} \Rightarrow \underline{\text{Impossible}}$$

(assume we don't realize)

		x_1	x_2	x_3	x_4	y_1	y_2
$-w =$	-11	1	1	-2	1	0	0
$y_1 =$	$+5$	-1	-2	0	-1	1	0
$y_2 =$	6	0	1	2	0	0	1

→ canonical form

I can start phase I

" x_3 enters the basis "

" y_2 leaves the basis "

		x_1	x_2	x_3	x_4	y_1	y_2
$-w =$	-5	1	2	0	1	0	1
$y_1 =$	5	-1	-2	0	-1	1	0
$x_3 =$	3	0	$1/2$	1	0	0	$1/2$

→ optimal : $w = 5$

$\Downarrow$

Unfeasible

$$\begin{cases} \min & z = x_1 + x_2 + 10x_3 \\ & x_2 + 4x_3 = 2 \\ & -2x_1 + x_2 - 6x_3 = 2 \\ & x_1, x_2, x_3 \geq 0 \end{cases} \longrightarrow \begin{array}{r|c|ccc|cc} & & x_1 & x_2 & x_3 & y_1 & y_2 \\ \hline -w = & -4 & 2 & -2 & 2 & 0 & 0 \\ \hline y_1 = & 2 & 0 & 1 & 4 & 1 & 0 \\ y_2 = & 2 & -2 & 1 & -6 & 0 & 1 \end{array}$$

"x_2 enters the basis"

"y_2 leaves the basis"

$$\begin{array}{r|c|ccc|cc} & & x_1 & x_2 & x_3 & y_1 & y_2 \\ -w = & 0 & 2 & 0 & 10 & 2 & 0 \\ \hline x_2 = & 2 & 0 & 1 & 4 & 1 & 0 \\ y_2 = & 0 & -2 & 0 & -10 & -1 & 1 \end{array}$$

$$\begin{array}{c|ccc|cc} 0 & 2 & 0 & 10 & \cancel{2} & \cancel{0} \\ \hline 2 & 0 & 1 & 4 & \cancel{1} & \cancel{0} \\ 0 & 1 & 0 & 5 & \cancel{1/2} & \cancel{1/2} \end{array}$$

change obj f.

$0 \mid 1 \quad 10$

$$\begin{array}{c|ccc} 0 & 1 & 1 & 10 \\ \hline 2 & 0 & 1 & 4 \\ 0 & 1 & 0 & 5 \end{array} \longrightarrow \text{Canonical form and } \bar{c}_N^T \geq 0$$

$\Downarrow$

Optimal solution

pivot on