Probability

Sample space S: set of all realisation s of an experiment

(# faces of a die, # tosses of a coin, ...)

Sample point s: an element of S

Event E: subset of S

Es: two dice toss outcome

$S = \{(1,1), \dots, (6,6)\}$

$s =$ " both dice show 1 " $= (1,1)$

$E =$ " sum of outcomes is less than 6 " $= \{(1,1), (1,2), (2,1)\}$

There are 2 special events: 1) $E = S$

2) $E = S^c = \emptyset$

Power set: $A = P(S) =$ all possible events $(E \in P(S) \Leftrightarrow E \subset S)$

↳ works on finite, discrete sets.

Probability is a function $P: \mathcal{A} \to [0,1] \iff$
- $P(S) = 1$
- $0 \leq P(E) \leq 1$
- given $E_1, \dots, E_m \mid E_i \cap E_j = \emptyset \ \forall i,j$,

$$P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$$

$$\overset{1}{\overset{\shortparallel}{P(S)}} = P(S \cup \emptyset) = P(S) + P(\emptyset) \longrightarrow P(\emptyset) = 0$$

Dim:

- $P(S) = P(S \cup \emptyset) \iff S = S \cup \emptyset \iff S \cup \emptyset \subseteq S \wedge S \subseteq S \cup \emptyset$
  - $S \cup \emptyset \subseteq S$: let $x \in S \cup \emptyset$, if $x \notin S$ then $x \in \emptyset$, which is impossible so x must belong to S
  - $S \subseteq S \cup \emptyset$: let $x \in S$, then $x \in S \cup \emptyset$ since it's an union
- $P(S \cup \emptyset) = P(S) + P(\emptyset) \iff S \cap \emptyset = \emptyset \iff S \cap \emptyset \subseteq \emptyset \wedge \emptyset \subseteq S \cap \emptyset$
  - $S \cap \emptyset \subseteq \emptyset$: if it weren't, $\emptyset$ shares an element with S, which is impossible since it's empty
  - $\emptyset \subseteq S \cap \emptyset$: to not be true, $\emptyset$ must have an element, which is impossible since it's empty

Conditional probability

$$P(E_1|E_2) = P(E_1 \cap E_2) P(E_2)$$

We can use conditional probability to prove the indipendence between two events:

E_1 is indipendent from E_2 $\iff$ $P(E_1|E_2) = P(E_1)$

(${E_2}$ happening makes no effect on E_1)