# Polynomial Models

Consider a regression problem.

linear in the parameters

Can we as hypothesis set the set of polynomials of degree $r$ with the tools we have already developed for linear regression?

Assume $\mathcal{X} = \mathbb{R}$

polynomial of degree $r$: $\sum_{i=0}^{r} w_i \cdot x^i = w_0 \cdot 1 + w_1 \cdot x + w_2 x^2 + \ldots + w_r x^r$

Given $x \in \mathbb{R}$, compute the following vector: (feature expansion)

$\vec{x}' = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ \vdots \\ x^r \end{bmatrix}$, $\vec{w} = [w_0, w_1, \ldots, w_r]^T \Rightarrow \langle \vec{w}, \vec{x}' \rangle = w_0 \cdot 1 + w_1 x + \ldots + w_r x^r$

$\Rightarrow$ the hypothesis class of linear models for $\vec{x}'$ corresponds to the hypothesis class of polynomials of degree $r$ for $x$.

Consider $X = \mathbb{R}^d$, $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$. You can use the following

feature expansion:

$$\vec{x}' = [1, x_1, x_2, \dots, x_d, x_1^2, x_2^2, \dots, x_d^2, \dots, x_1^r, x_2^r, \dots, x_d^r]^T$$

$\Rightarrow$ use linear models for $\vec{x}'$.

Different feature expansion:

$\vec{x} \in \mathbb{R}^3$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $r = 2$

$$\vec{x}' = [1, x_1, x_2, x_3, x_1^2, x_2^2, x_3^2, x_1 \cdot x_2, x_1 \cdot x_3, x_2 \cdot x_3]$$

$\Rightarrow$ build a linear model for $\vec{x}'$

# Feature normalization

Given the training set, we have "normalized" each feature $x_i$, $i = 1, \dots, d$ so that:

- the average of each feature across the training set is 0
- the standard deviation of each feature is 1

Data normalization is important:

- stability of the computation
- interpretability of linear models (weight is high $\Rightarrow$ feature is important)

If you build a model using normalized data

$\Rightarrow$ the same normalization function must be applied to the data on which you make predictions.