$\Sigma = \{a, b\}$

$L = \{a^j b^i \mid j \geq 1\}$

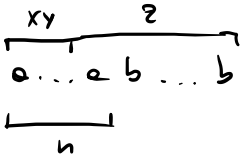
Prove with the pumping lemma that this is not a regular language

We must prove that $\exists w \mid \forall n$ the lemma doesn't hold

↓

make an argument as a function of n

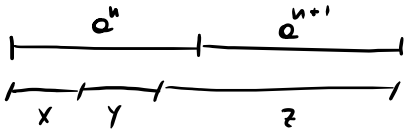
consider $a^n b^n$ →



for $k = 0$, xz has fewer a than b → L is not regular

$L_< = \{a^i b^j \mid 0 \le i < j\}$

Consider $i = n$ and $j = n+1$

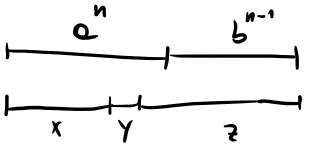


if $k > 1$: $\#(a) > n$ $\Rightarrow$ $\#(a) \ge \#(b)$ $\Rightarrow L_<$ is not a regular language

since $|y| \ge 1$ and $y = 'a'$

$L_3 = \{ a^p b^q \mid p > q \geq 0 \}$

consider $p = n$, $q = n-1$



$\#(a) = n - |y|^k = n-1$

for $k = 0$ $\#(a) \leq \#(b)$ $\rightarrow$ $\notin L_3$

since $|y| \geq 1$

$L_{\neq} = \{ a^p b^q \mid p \neq q, p,q \geq 0 \}$

Consider $L(a^* b^*)$ (regular language) $= L_> \cup L_< \cup L_=$

$L_{\neq} = L_< \cup L_> \Rightarrow L_= = L(a^* b^*) \setminus L_{\neq}$

If $L_{\neq}$ was regular, then $L_=$ is obtained by two regular languages, but $L_=$ is not regular $\Rightarrow$ $L_{\neq}$ must not be regular

Test

State whether the following claims hold true, and motivate your answer

- the intersection of a non-regular language and a finite language is always a regular language ↘ not R ∩ R ⊂ R ⇒ it's regular
- the intersection of a non-regular language L_1 and an infinite regular language L_2 is never a regular language ↘ ∅ is a regular language
- every subset of a non-regular language is a non-regular language

false since we can take the empty set, which is a regular language

Superset and subset

Assume L is a regular language. We **cannot say anything** about languages L' and L'' with $L' \subset L$ and $L'' \supset L$

More precisely

- L' could be regular or non-regular
- L'' could be regular or non-regular

Often student gets confused about this, thinking that adding strings to L makes it 'more difficult' and removing strings from L makes it 'less difficult'. But this is **not true in general**