

Tableau format

$$X_B = B^{-1}b - B^{-1}F X_F$$

$$z = C_B^T B^{-1}b + \bar{C}^T X_B + (C_F^T - C_B^T B^{-1}F) X_F$$

Original system $\begin{cases} \min C^T X = z \\ A X = b \end{cases} \rightarrow C^T X - z = 0$ → RHS

col 0 (RHS) \rightarrow

	x_1	...	x_n	z	
0	C_1	...	C_n	-1	→ Row 0
b	A			0	1
				0	
				0	
				0	
				0	m

Canonical
Tableau

$$B^{-1}b = I X_B + B^{-1}F X_F$$

$$-C_B^T B^{-1}b = -C_B^T X_B + (C_F^T - C_B^T B^{-1}F) X_F - z$$

$-C_B^T B^{-1}b$ \rightarrow reduced cost

	x_1	...	x_m	...	x_n	z
$-C_B^T B^{-1}b$	0	0	0	...	\bar{C}_n	-1
$B^{-1}b$	I				$\bar{F} = B^{-1}F$	0
						0
						0
						0
						m

At each iteration

$$b \geq 0$$

			x_h
0	...	0	\bar{a}_{hk}
\bar{b}_h	I		\bar{a}_{hk}
...			
\bar{b}_t			
\bar{b}_m			
			\bar{a}_{mh}

\bar{a}_{mh} \rightarrow new $t = \arg \min \{ \dots \}$

\bar{a}_{mh} \rightarrow pivot element

I want to form $\bar{a}_{h1} \dots \bar{a}_{hm}$ to $(000 \dots 1, \infty)$

$\hookrightarrow h$

ES)
$$\begin{cases} \min & t = -x_1 - x_2 \\ & 6x_1 + 4x_2 + x_3 = 24 \\ & 3x_1 - 2x_2 + x_4 = 6 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{cases} \Rightarrow$$

0	-1	-1	0	0	-1
24	6	4	1	0	0
6	3	-2	0	1	0

already a canonical tableau

	x_1	x_2	x_3	x_4		
$-z =$	0	-1	-1	0	0	-1
$x_3 =$	24	6	4	1	0	0
$x_4 =$	6	3	-2	0	1	0

Sol: $x_1, x_2 = 0 \Rightarrow x_3 = 24$
 $x_4 = 6$

$\bar{c}_1 = \bar{c}_2 = -1 \Rightarrow$ I choose: x_1 enters the basis

$\theta_1 = \frac{24}{6} = 4 \quad \theta_2 = 2$

$t = \min \{ \theta_1, \theta_2 \} = 2$

x_4 leaves the basis
 $\hookrightarrow x_2$

combination of the rows
 to get the values of
 the variable entering
 the basis right.

		(0)	(1)	(0)	(1)	
		x_1	x_2	x_3	x_4	
$-z =$	2	0	$-5/3$	0	$1/3$	-1
$x_3 =$	12	0	8	1	-2	0
$x_4 =$	2	1	$-1/3$	0	$1/3$	0

last column can be
 forgotten since it's
 always $[-1, 0, 0, \dots, 0]$

lin. comb. of rows to get t on the
 variable entering the basis (x_1)

new row 0 = previous row 0 + α (new pivot row)

\hookrightarrow chosen s.t. \bar{c}_i corresponding
 to the variable entering the basis
 is 0. (c_1 in this case)

\bar{c}_i is $< 0 \Rightarrow$ opt. last found (search a new basis)

ETC ...

Choice of the entering var x_{h+1} $\bar{c}_h < 0$

1) Take the first h $\mid \bar{c}_h < 0 \rightarrow O(n)$

2) Take $h \mid h = \arg \max \{ |\bar{c}_i| \mid c_i < 0 \} \rightarrow O(n)$

3) Take $h \mid h = \arg \max \{ |\bar{c}_i| \theta_i \mid \bar{c}_i < 0 \} \rightarrow O(n \log n)$

4) Random choice $\rightarrow O(1)$