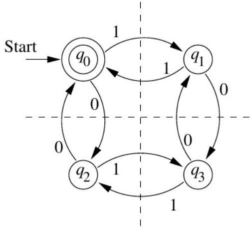
Example



start state

Is string $w = 0101$ accepted by A ?

- $\hat{\delta}(q_0, \epsilon) = q_0$ → base case
- $\hat{\delta}(q_0, 0) = \delta(\hat{\delta}(q_0, \epsilon), 0) = \delta(q_0, 0) = q_2$
- $\hat{\delta}(q_0, 01) = \delta(\hat{\delta}(q_0, 0), 1) = \delta(q_2, 1) = q_3$
- $\hat{\delta}(q_0, 010) = \delta(\hat{\delta}(q_0, 01), 0) = \delta(q_3, 0) = q_1$
- $\hat{\delta}(q_0, 0101) = \delta(\hat{\delta}(q_0, 010), 1) = \delta(q_1, 1) = q_0 \in F$

Language recognized by a DFA

The language **recognized** by DFA A is

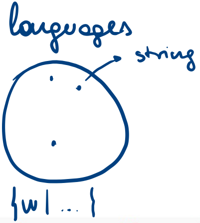
$$L(A) = \{w \mid \hat{\delta}(q_0, w) \in F\}$$

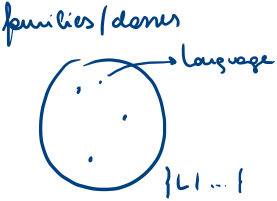
Describes the behaviour of an automaton

set of accepted strings

set of final states

The languages accepted by the class of DFAs are called **regular languages** $\{L(A) \mid A \text{ is a DFA}\}$





Notational conventions

Commonly used notation for DFAs

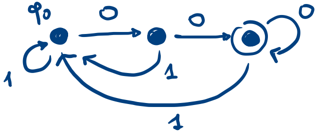
- $a, b, c, \ldots$ alphabet symbols
- u, v, w, x, y, z strings over input alphabet
- $p, q, r, s, q_0, q_1, q_2, \ldots$ states

Test

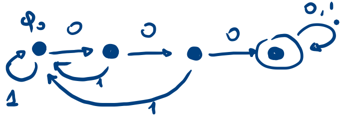
Specify DFAs for the following languages over the alphabet $\{0, 1\}$:

- set of all strings ending in 00
- set of all strings with three consecutive 0's
- set of all strings with 011 as a substring
- set of all strings that start or end (or both) with 01

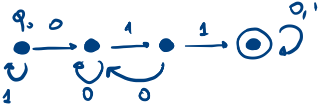
1) $\{ x00 \mid x \in \{0,1\}^* \}$



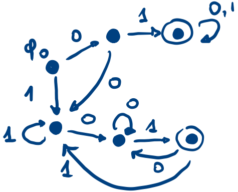
2) $\{ x000y \mid x,y \in \{0,1\}^* \}$



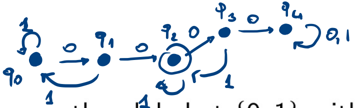
3) $\{ x011y \mid x,y \in \{0,1\}^* \}$



4) $\{ xyz \mid x,y,z \in \{0,1\}^*,\ x = 01 \lor z = 01 \}$



Exercise



Consider the language L of strings over the alphabet $\{0, 1\}$ with **exactly** one occurrence of string 00

Carry out the following points :

- draw the transition diagram of a DFA A such that $L(A) = L$
- state the meaning of each of A's states (i.e. for each state of A describe the strings leading to it)

Hint: define a "failure state" that can never reach any final state

q_0 : initial and "idle state"

q_1 : i found a single 0, still not found 00

q_2 : final state, found the first 00

q_3 : i found a single 0 after finding 00

q_4 : i found a second 00 → failure

Nondeterministic finite automata

These automata accept only regular languages

Easier to design than DFAs

Later on we will see several examples of this fact

Very useful for implementing the **search** for a pattern in a text

Nondeterministic finite automata

find branching points somewhere

A nondeterministic finite automaton can **simultaneously** be in different states

not sure which of them he is in

The automaton **accepts** if at least one final state is reached at the end of the scan of the input string

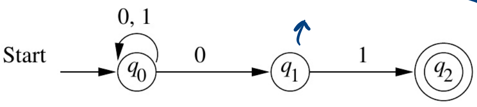
Equivalently, in a given state the automaton can **guess** which next state will lead to acceptance

This interpretation is not in the textbook

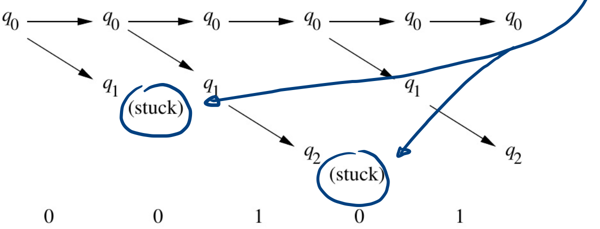
Example

Nondeterministic automaton N accepting all and only the strings ending in 01

no 0 arrow



Simultaneous computations of N on input string 00101



Nondeterministic finite automaton

A **nondeterministic finite automata** (NFA) is a 5-tuple

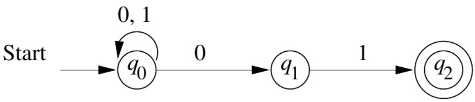
$$A = (Q, \Sigma, \delta, q_0, F)$$

where :

- Q is a **finite** set of **states**
- Σ is the **alphabet** of input symbols
- δ is a **transition** function $Q \times \Sigma \rightarrow 2^Q$, where 2^Q is the set of all subsets of Q (power set)
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is the set of final states

Example

The transition diagram



represents the nondeterministic automaton

$$A = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_2\})$$

with transition function δ

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\varnothing$	$\{q_2\}$
$\star q_2$	$\varnothing$	$\varnothing$

Extended transition function $\hat{\delta}$

it's a set

Base $\hat{\delta}(q, \epsilon) = \{q\}$

it's a set

it's a set

Induction

$$\hat{\delta}(q, xa) = \bigcup_{p \in \hat{\delta}(q,x)} \delta(p, a)$$

it's a state

it's a set

Notice the difference with the case of DFA in the induction part. Can you explain this?