

Machine Learning

Computer Engineering

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October 2nd, 2023

Machine Learning

Course + Handls: MLearn23

Distributional material: diff. anni. papers

~ 6 cars in laboratory
more 3 homeworks of 3 pts (not mandatory)
↳ 2 weeks ↳ 1h to 1h30

Exam: 30 mins multiple choice questions (part / part) } same day → no split
15/2 hrs questions and exercises → possible oral exam

Grade: 2° part grade + homeworks

25/01/24

13/02/24

02/04/24

10/09/24

"Understanding machine learning" → free to download

python (scikit-learn, numpy, ...)

jupyter lab (through Anaconda)

Homework 0 flos.vandin@unipd.it

Formal Model

A learner (us/machine) has access to:

- Domain set $X \in \mathcal{X}$: set of all possible objects to make predictions about
↳ instance space
instance (vector of features)
- Label set \mathcal{Y}
- Training data $S = ((x_1, y_1), \dots, (x_n, y_n))$ as the input
↳ finite
↳ called training set
- Output: $h: X \rightarrow \mathcal{Y}$
↳ prediction rule / predictor / hypothesis / classifier
- Data generation model: instances generated and labeled accordingly to a function
 - probability distribution that generates the instances (not known)
 - labeling function $f: X \rightarrow \mathcal{Y}$ (not known)
 - label y_i for each instance x_i
 - each point in the training set
- Measure of success: error of a classifier = probability it doesn't predict well

A Formal Model (Statistical Learning)

We have a *learner* (us, or the machine) has access to:

- ① **Domain set** \mathcal{X} : set of all possible objects to make predictions about
 - domain point $\underline{x \in \mathcal{X} = \text{instance}}$, usually represented by a vector of *features*
 - \mathcal{X} is the instance space
- ② **Label set** \mathcal{Y} : set of possible labels.
 - often two labels, e.g. $\{-1, +1\}$ or $\{0, 1\}$
- ③ **Training data** $S = ((x_1, y_1), \dots, (x_m, y_m))$: finite sequence of labeled domain points, i.e. pairs in $\mathcal{X} \times \mathcal{Y}$
 - this is the learner's **input**
 - S : *training example* or *training set*

A Formal Model

- ④ **Learner's output** h : prediction rule $h: \mathcal{X} \rightarrow \mathcal{Y}$
 - also called predictor, hypothesis, or classifier
 - $A(S)$: prediction rule produced by learning algorithm A when training set S is given to it
 - sometimes \hat{f} used instead of h
- ⑤ **Data-generation model**: instances are generated by some probability distribution and labeled according to a function
 - \mathcal{D} : probability distribution over \mathcal{X} (**NOT KNOWN TO THE LEARNER!**)
 - labeling function $f: \mathcal{X} \rightarrow \mathcal{Y}$ (**NOT KNOWN TO THE LEARNER!**)
 - label y_i of instance x_i : $y_i = f(x_i)$, for all $i = 1, \dots, m$
 - each point in training set S : first sample x_i according to \mathcal{D} , then label it as $y_i = f(x_i)$
- ⑥ **Measures of success**: error of a classifier = probability it does not predict the correct label on a random data point generate by distribution \mathcal{D}

Loss

Given domain subset $A \subset \mathcal{X}$, $\mathcal{D}(A)$ = probability of observing a point $x \in A$.

In many cases, we refer to A as event and express it using a function $\pi : \mathcal{X} \rightarrow \{0, 1\}$, that is:

$$\underline{A = \{x \in \mathcal{X} : \pi(x) = 1\}}$$

In this case we have $\mathbb{P}_{x \sim \mathcal{D}}[\pi(x)] = \mathcal{D}(A)$

Error of prediction rule $h : \mathcal{X} \rightarrow \mathcal{Y}$ is

Loss \nwarrow

Distribution \swarrow

model \swarrow

true labeling \swarrow

label predicted by h \swarrow

true label \swarrow

$$\underline{L_{\mathcal{D},f}(h) \stackrel{\text{def}}{=} \mathbb{P}_{x \sim \mathcal{D}}[h(x) \neq f(x)] \stackrel{\text{def}}{=} \mathcal{D}(\{x : h(x) \neq f(x)\})}$$

Notes:

- $L_{\mathcal{D},f}(h)$ has many different names: generalization error, true error, risk, **loss**, ...
- often f is obvious, so omitted: $L_{\mathcal{D}}(h)$

Learning Process (Simplified)

UNKNOWN TARGET FUNCTION

$$f: \mathcal{X} \rightarrow \mathcal{Y}$$

PROBABILITY DISTRIBUTION

$$\bar{\mathcal{D}} \text{ on } \mathcal{X}$$

unknown too,
but we assume
it's there

TRAINING EXAMPLES

$$(x_1, y_1), \dots, (x_m, y_m)$$

$$x_1, x_2, \dots, x_m$$

LOSS FUNCTION

$$L()$$

we want \tilde{p} to
be similar to
 $f(x)$

$$\hat{f}(x) \approx f(x)$$

LEARNING
ALGORITHM
 \mathcal{A}

FINAL HYPOTHESIS/MODEL

$$\hat{f}: \mathcal{X} \rightarrow \mathcal{Y}$$

HYPOTHESIS/MODEL SET

$$\mathcal{H}$$

(usually)
chosen from

Types of Learning

y_i are known: **training set** $(x_1, y_1), \dots, (x_m, y_m)$

➡ **supervised learning**

Training set contains only x_1, x_2, \dots, x_m

➡ **unsupervised learning**

(for supervised learning)

There can be different types of output:

- \mathcal{Y} is **discrete**
- \mathcal{Y} is **continuous**

Notes: we will see a more general learning model soon, main ideas are the same!

Types of Learning

y_i known

y_i not available

Supervised Learning

Unsupervised Learning

\mathcal{Y} is ...
Discrete
Continuous

classification

clustering

dimensionality
reduction

regression

...