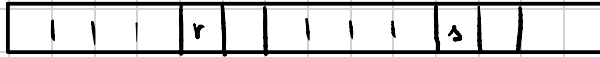
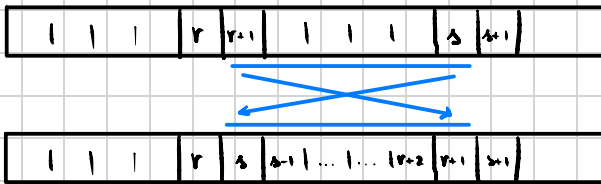


Suppose that at iteration  $t$  I find the first swap available at  $i=r, j=s$ .



This means that each pair  $(i, j), i < r \forall j$   
 and  $i=r \forall j \in [r+1, s-1]$  has already been  
 checked and didn't have a swap needed

I then reverse  $[r+1, s]$ :



Consider  $i < r$ :

- if  $j < r$  then the swap didn't  $\Rightarrow$  no new swaps change anything
- if  $j = r$ :

A swap occurs if  $C_{i,r+1} + C_{r,j+1} > C_{ij} + C_{i+1,j+1}$   
 $\Downarrow$

$$C_{i,r+1} + C_{r,s} > C_{ir} + C_{i+1,s}$$

$\hookrightarrow$  need to check again

- if  $j \in [r+1, s-1]$  I just reversed the order, so

each pair has already been checked  $\Rightarrow$  no new swap

• if  $j = s$ :

A swap occurs if  $C_{i,i+1} + C_{j,j+1} > C_{ij} + C_{i+1,j+1}$

$\Downarrow$

$$C_{i,i+1} + C_{r+1,s+1} > C_{i,r+1} + C_{i+1,s+1}$$

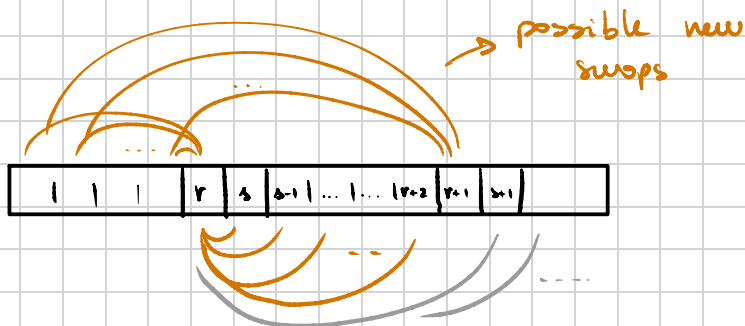
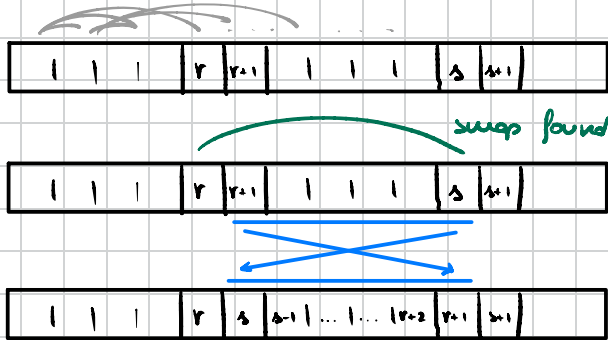
$\hookrightarrow$  Need to check again

• if  $j > s \rightarrow$  nothing changed  $\Rightarrow$  no new swaps

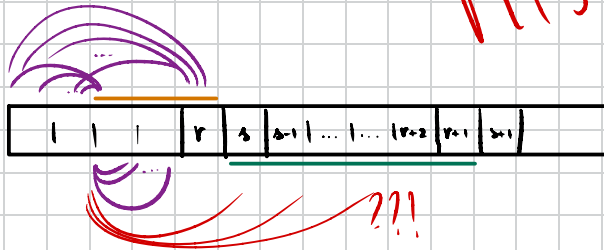
Consider  $i = r$ :

I will need to check again  $\forall j \in [i+1, s-1]$  and then continue from where I stopped.

This means:



If I find a new swap before reaching the point I was at before (one of the orange arcs):



MISSING  
ARCS?

Thus: if I find a swap in the main cycle:

- reverse
- create a queue with all arcs to check before moving on
- if an arc in the queue finds a swap:
  - reverse
  - add to the start of the queue the new arcs to check
  - repeat
- if the queue is empty, continue with the main cycle

Repeat till no swaps found.