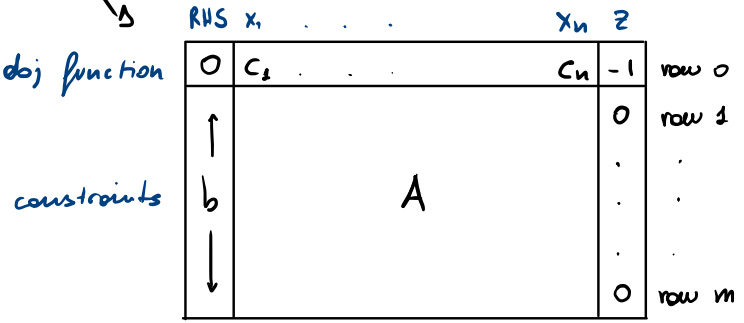
Tableau Format

We can convert our original system into a tableau form:

$$\begin{cases} \min\ c^T x = z \\ Ax = b \end{cases} \Rightarrow \quad c^T x - z = 0$$

↳ Right-hand side



We can further rewrite it to have the canonical tableau

$$Ax = b \Rightarrow B^{-1}b = I x_B + B^{-1}F x_F$$

$$c^T x = z \Rightarrow -c_B^T B^{-1} b = 0^T x_B + (c_F^T - c_B^T B^{-1} F) x_F - z$$

$-c_B^T B^{-1} b$

	$0 \dots 0$	$\leftarrow \bar{c}_F^T \rightarrow$	-1
$B^{-1}b \rightarrow \bar{b}$	I	$\bar{F} = B^{-1}F$	0 $\vdots$ 0

- remember $\bar{b}$ must be ≥ 0 at each iteration
- "x_h enters the basis" means that the vector $(a_{1h}, \dots, a_{mh})$ must become $(0 \dots 0 1 0 \dots 0)$ ↳ index h

$-c_B^T B^{-1} b$

	$0 \dots 0$	$\dots \bar{c}_h \dots$	-1
$\bar{b}_1$ $\vdots$ $\bar{b}_t$ $\vdots$ $\bar{b}_m$	I	$\bar{a}_{1h}$ $\vdots$ $\bar{a}_{th}$ $\vdots$ $\bar{a}_{mh}$	0 $\vdots$ 0

→ row $t = \operatorname{argmin} \left\{ \frac{\bar{b}_i}{\bar{a}_{ih}} : a_{ih} < 0 \right\}$

pivot element

ES)

$$\begin{cases} \min\ t = -x_1 - x_2 \\ 6x_1 + 4x_2 + x_3 = 24 \\ 3x_1 - 2x_2 + x_4 = 6 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases} \Rightarrow$$

	x_1	x_2	x_3	x_4	
$(-z =)$	0	-1	-1	0	0	-1
$(x_3 =)$	24	6	4	1	0	0
$(x_4 =)$	6	3	-2	0	1	0

(already a canonical tableau)

both $\bar{c}_1$ and $\bar{c}_2 < 0$, I can choose: x_1 enters the basis

$$\vartheta_1 = \frac{24}{6} = 4 \qquad \vartheta_2 = 2$$

$t = \operatorname{argmin}\{\dots\} = 2$

$x_{B[2]}$ leaves the basis

↳ x_4

combination of the rows to get the values of the variable entering the basis right.

	(B) x_1	(N) x_2	(B) x_3	(N) x_4	
$-z =$	2	0	$-5/3$	0	$1/3$	-1
$x_3 =$	12	0	8	1	-2	0
$x_1 =$	2	1	$-2/3$	0	$1/3$	0

↳ last column can be forgotten since it's always $[-1, 0, 0 \dots 0]$

↳ lin. comb. of rows to get 1 on the variable entering the basis (x_1)

new row 0 = previous row 0 + α (new pivot row)

↳ chosen s.t. $\bar{c}_i$ corresponding to the variable entering the basis is 0. ($\bar{c}_4$ in this case)

How to choose the variable x_h entering the base

1) $h = \min \{ i \mid \bar{c}_i < 0 \}$
2) $h = \operatorname{argmax} \{ |\bar{c}_i| \mid \bar{c}_i < 0 \}$
3) $h = \operatorname{argmax} \{ |\bar{c}_i| \vartheta_i \mid \bar{c}_i < 0 \}$
4) $h = \text{random} \{ i \mid \bar{c}_i < 0 \}$