$$\begin{cases} \min & c^T x \\ & Ax \geq b \\ & x \geq 0 \end{cases} \xrightarrow{\text{Duality}} \begin{cases} \max & \nu^T b \\ & c^T \geq \nu^T A \\ & \nu \geq 0 \end{cases} = \begin{cases} -\min & (-b)^T \nu \\ & -A^T \nu \geq -c \\ & \nu \geq 0 \end{cases}$$

primal $\qquad\qquad$ dual

Duality

$$\begin{cases} -\max & y^T(-c) \\ & -b^T \geq y^T(-A^T) \\ & y \geq 0 \end{cases} = \begin{cases} \min & c^T y \\ & Ay \geq b \\ & y \geq 0 \end{cases}$$

equal to the primal problem
$y \rightleftarrows x$

What if the primal problem is infeasible/unbounded?

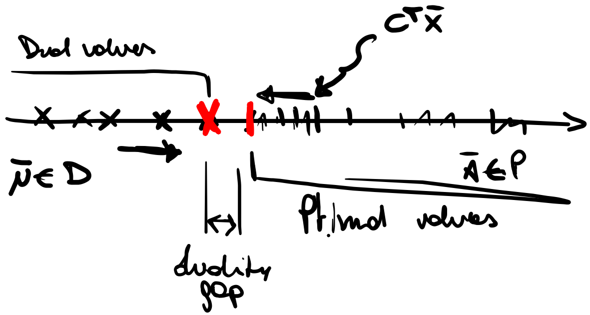
Th: Weak duality

Let $P := \{ \cancel{x} \geq 0 : Ax \geq b \} \neq \emptyset$

$D := \{ \nu \geq 0 : c^T \geq \nu^T A \} \neq \emptyset$

Then $\forall\, \bar{x} \in P, \bar{\nu} \in D, \quad \nu^T b \leq c^T \bar{x}$

Proof: $\underbrace{\bar{\mu}^T}_{\geq 0} \underbrace{b}_{b \leq A^T x} \leq \overbrace{\bar{\mu}^T A}^{\leq c^T} \underbrace{\bar{x}}_{\geq 0} \leq c^T \bar{x}$



The strong duality theorem tells us that the duality gap is 0 (in linear programming)

PRIM. \ DUAL	∃ opt	inf	unb
∃ opt	1	X	X
inf	X	2	YES
unb	X	YES	X

1) Strong duality $\min c^T \bar{x} = \max \bar{\mu}^T b$

2) Weak duality cannot be applied

→ EXAMPLE

$$\begin{cases} \min \ -4x_1 - 2x_2 \\ \begin{cases} -x_1 + x_2 \geq 2 \\ x_1 - x_2 \geq 1 \\ x_1, x_2 \geq 0 \end{cases} \end{cases} \xrightarrow{D} \begin{cases} \max \ 2u_1 + u_2 \\ \begin{cases} -u_1 + u_2 \leq -4 \\ u_1 - u_2 \leq -2 \\ u_1, u_2 \geq 0 \end{cases} \end{cases}$$

inf inf

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