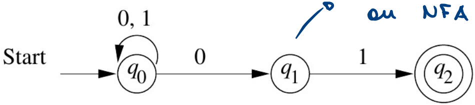
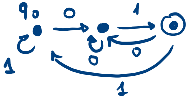
Example

states lose meaning in an NFA



↳ $L = \{w01 \mid w \in \{0,1\}^*\}$



(DFA)

Computation of $\hat{\delta}(q_0, 00101)$

- $\hat{\delta}(q_0, \epsilon) = \{q_0\}$
- $\hat{\delta}(q_0, 0) = \delta(q_0, 0) = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 00) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 001) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$
- $\hat{\delta}(q_0, 0010) = \delta(q_0, 0) \cup \delta(q_2, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 00101) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$

Accepted language for NFA

The **accepted** language for an NFA A is

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \cap F \neq \emptyset\}$$

if one of the states at the end is a final state, the string is in the language

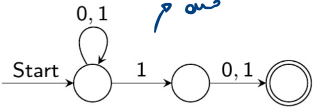
In words, $L(A)$ is the set of all strings $w \in \Sigma^*$ such that $\hat{\delta}(q_0, w)$ contains **at least one** final state. This amounts to say that at least one computation for w leads to acceptance

Equivalence for DFA and NFA

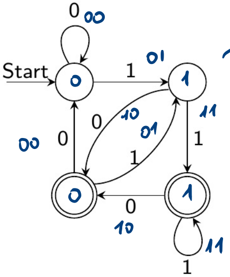
NFAs are **easier** than DFAs to “program”, since nondeterminism makes it possible to simplify the structure of the automaton

Example : compare NFA and DFA accepting strings in $\{0, 1\}^*$ with penultimate symbol 1

easy to understand and to ideate



$L = \{ w1a \mid w \in \{0,1\}^*, a \in \{0,1\} \}$



it's a mess just to understand

With an increase in the distance between 1 and the end of the string, the gap gets exponentially larger

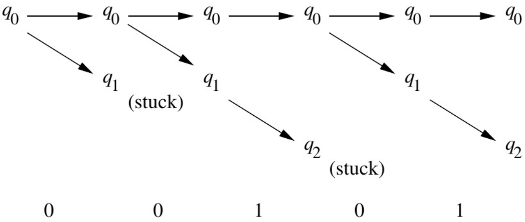
Equivalence for DFA and NFA

Quite surprisingly, for every NFA N there exists some DFA D such that $L(D) = L(N)$. The proof involves the **subset construction**

Idea : build a state in D for every state set representing a "configuration" in a computation of N. The collection of all configurations is still a **finite set**

2^Q = class of all possible subsets of Q

$X \in 2^Q$ iff $X \subseteq Q$



Equivalence for DFA and NFA

Given an NFA

$$N = (Q_N, \Sigma, \delta_N, q_0, F_N)$$

the subset construction produces a DFA

$$D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$$

such that $L(D) = L(N)$

Equivalence for DFA and NFA

Subset construction :

- $Q_D = \{S \mid S \subseteq Q_N\}$
- $F_D = \{S \subseteq Q_N \mid S \cap F_N \neq \emptyset\}$
- For every $S \subseteq Q_N$ and $a \in \Sigma$,

$$\delta_D(S, a) = \bigcup_{p \in S} \delta_N(p, a)$$

Note : $|Q_D| = 2^{|Q_N|}$. Nonetheless, the large majority of states in Q_D turn out to be **garbage**, that is, they cannot be reached from the initial state

Test

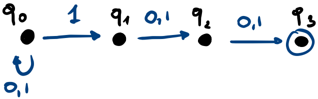
Consider the following language over $\Sigma = \{0, 1\}$

$$L = \{w \mid w = x1ab,\ x \in \Sigma^*,\ a, b \in \Sigma\}$$

Informally, L is the set of all strings with 1 as **third to last** symbol

Specify a NFA A such that $L(A) = L$

NFA :

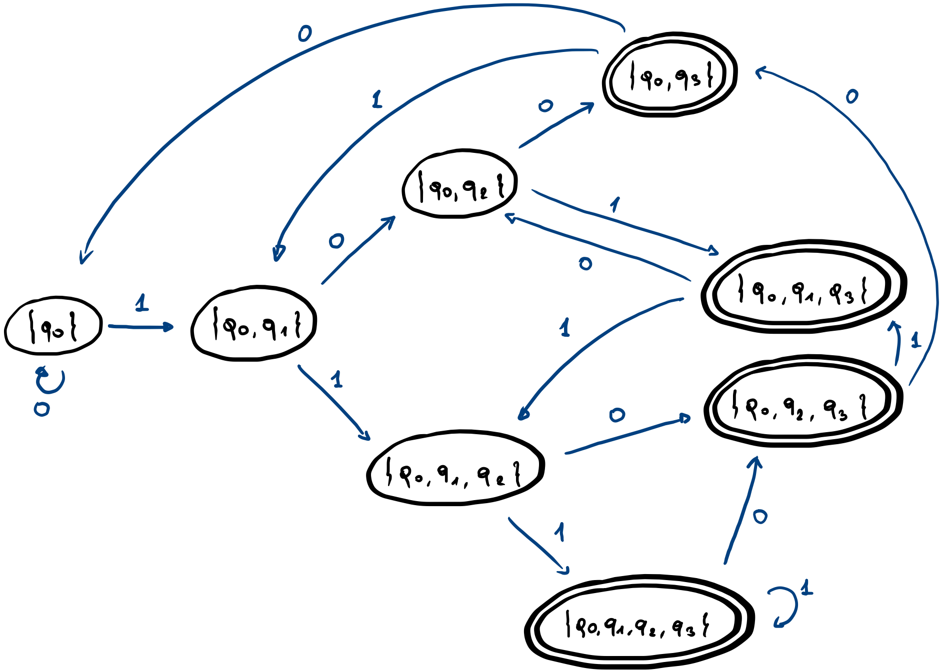


			0	1
$\rightarrow$	$\{q_0\}$	$\Rightarrow$	$\{q_0\}$	$\{q_0, q_1\}$
	$\{q_1\}$	$\Rightarrow$	$\{q_2\}$	$\{q_2\}$
	$\{q_2\}$	$\Rightarrow$	$\{q_3\}$	$\{q_3\}$
$*$	$\{q_3\}$	$\Rightarrow$	$\emptyset$	$\emptyset$
	$\{q_0, q_1\}$	$\Rightarrow$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
	$\{q_0, q_2\}$	$\Rightarrow$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
$*$	$\{q_0, q_3\}$	$\Rightarrow$	$\{q_0\}$	$\{q_0, q_1\}$

reachable

unreachable

			0	1
	$\{q_1, q_2\}$	$\Rightarrow$	$\{q_2, q_3\}$	$\{q_2, q_3\}$
$*$	$\{q_1, q_3\}$	$\Rightarrow$	$\{q_2\}$	$\{q_2\}$
$*$	$\{q_2, q_3\}$	$\Rightarrow$	$\{q_3\}$	$\{q_3\}$
	$\{q_0, q_1, q_2\}$	$\Rightarrow$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$*$	$\{q_0, q_1, q_3\}$	$\Rightarrow$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$*$	$\{q_0, q_2, q_3\}$	$\Rightarrow$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
$*$	$\{q_1, q_2, q_3\}$	$\Rightarrow$	$\{q_2, q_3\}$	$\{q_2, q_3\}$
$*$	$\{q_0, q_1, q_2, q_3\}$	$\Rightarrow$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$


{q0}
{q0,q1}
{q0,q2}
{q0,q3}
{q0,q1,q2}
{q0,q1,q3}
{q0,q2,q3}
{q0,q1,q2,q3}