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# MIP formulations for delete-free AI planning

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## Abstract

We compare current state-of-the-art M.I.P. formulations to solve the delete-free relaxation in cost-optimal planning, integrating different preprocessing techniques proposed in different publications and devising primal heuristics to furnish a warm-start to the M.I.P. solver.

We, then, explored a novel way to model acyclicity by computing violated landmarks and adding them as constraints: computing all landmarks a priori is intractable, since the number of potential landmarks grows exponentially with respect to the problem size, making any brute-force approach computationally infeasible; instead we compute violated landmarks iteratively, looking for the minimal amount necessary to guarantee both feasibility and optimality of the final solution.

To conclude our work, we compared the current state-of-the-art with our formulation, showing that it can compete both in space and time efficiency.



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# Chapter 1

## Introduction

AI Planning, a sub-area of Artificial Intelligence, has the objective of finding a sequence of actions, in a declaratively described system, to reach its goals while optimizing overall performance measures. Automated planners find the action to apply in each given state out of the set of possible actions for that state.

Fast Downward [Hel06] is a classical planning system based on heuristic search, which is leading the state-of-the-art in classical planning. One such heuristics that can be used to efficiently explore the search space, is the *delete-free relaxation* of the original problem, denoted as  $h^+$ .

### 1.1 Delete-free relaxation

The value of  $h^+$  for a given planning problem is the optimal cost of the corresponding delete-relaxed planning problem, obtained from the original problem by removing delete effects from all actions.

The value of  $h^+$  is a lower bound of the optimal cost of the original problem, making it an admissible heuristic, and has been shown that having  $h^+$  can significantly improve the efficiency of optimal planning [BH09]. Computing  $h^+$ , however, is NP-equivalent [Byl94] and is also hard to approximate [BH09].

Methods for computing  $h^+$  can output optimal plans for a given delete-relaxed planning problem, equivalently important to find efficiently, since there exist delete-relaxed planning tasks that are of interest for the planning community. Moreover, optimal plans for classical planning problems, can be produced by iteratively solving and reformulating delete-relaxed tasks [Has12]. This can be done by repeatedly finding optimal plans for a delete-relaxed version of the original planning problem, and reducing the relaxation by reformulating the problem whenever a plan has been found for the delete-relaxed task, that is not a valid plan for the original problem.

We will first investigate on preprocessing techniques proposed by [IF15] and [HST12], aiming to simplify the problem by taking advantage of the less restrictive structure of the delete-relaxed task, to then explore the use of primal heuristics, to provide a warm-start

to our MIP solver, combining a greedy algorithm with concepts used to compute  $h^{add}$  [BG01].

Finally we will implement the models proposed by [IF15] and [FR22], which use different ways of modeling acyclicity in the model, respectively time labeling and vertex elimination, and compare them to our technique (of modeling acyclicity) of iteratively compiling violated disjunctive action landmarks [HST12], [BC11] and subtour elimination constraints (S.E.C.), adding those to the base model as cuts.

## 1.2 Preliminary definitions

There are multiple description languages for planning tasks: STRIPS, ADL, SAS<sup>+</sup> and PDDL; we will lean on the STRIPS formalism [FN71].

A *STRIPS planning task* is a 5-tuple  $\Pi = (P, A, I, G, cost)$ :  $P$  is a finite set of boolean variables (or *atomic propositions*, *atoms* for short);  $A$  is a finite set of *actions*: each action is a triple  $(pre(a), add(a), del(a))$ , where  $pre(a), add(a), del(a) \subseteq P$  denote the set of *preconditions*, *positive effects* and *negative effects* of  $a$ ;  $I, G \subseteq P$  are respectively the *initial state* and *goal states*;  $cost : A \rightarrow \mathbb{N}$  is a cost function that maps each action to a non-negative integer.

Executing an action  $a$  from a state  $S \subseteq P$  transforms the state into  $S' := exec_a(S) := S \setminus del(a) \cup add(a)$ . Let  $\pi = (a_0, \dots, a_n)$  be a sequence of actions and  $S \subseteq P$  a state: we denote with  $S(\pi) = exec_{a_n}(\dots(exec_{a_0}(S)))$ , the state achieved through  $\pi$ , starting from  $S$ . A *solution* to a planning task is a sequence of actions that transform the system from the initial state to a state  $I(\pi) \subseteq G$ . Moreover a feasible solution, or *plan*, is a solution  $\pi = (a_0, \dots, a_n)$  that satisfies  $\forall i, pre(a_i) \subseteq I((a_0, \dots, a_{i-1}))$ , which means that each action's preconditions are achieved when using that action. Finally, the cost of an action sequence  $\pi$  is  $cost(\pi) = \sum_{a \in \pi} cost(a)$ , and our goal is to find an *optimal plan*  $\pi^*$  with minimum cost (*optimal cost*  $c^*$ ):

$$\pi^* = argmin_{\pi} \{cost(\pi), s.t. \pi \text{ is a plan for } \Pi\}, c^* = cost(\pi^*)$$

We will limit ourselves to the *delete-relaxed task*  $\Pi^+ = (P, A^+, I, G, cost)$ , where  $A^+$  is obtained by replacing, from each action, the negative effects with the empty set. Moreover, we will limit to the case in which  $I = \emptyset$  for simplicity: in a delete-relaxed task, an atom that is true in the initial state will remain true throughout the whole plan, making it useless. This won't restrict the use case of this work to instances with empty initial states, since it's easy to remove the initial state from any instance without changing its optimal plan:  $\Pi_I^+ = (P \setminus I, A_I^+, \emptyset, G \setminus I, cost)$ , where  $A_I^+ = \{(pre(a) \setminus I, add(a) \setminus I, \emptyset), \forall a \in A^+\}$ .



# Chapter 2

## Current state of the art

To build a MIP formulation that solves the delete-relaxed task  $\Pi^+$ , we identify two sections:

1. Base IP model: defining variables for atoms, actions and *first achievers* [IF15], aswell as a basic set of constraints, aiming to enforce the basic functionalities of actions;
2. Modeling acyclicity: ensuring that the solution is in fact a plan, by adding constraints that make sure that a timeline of actions can be constructed from the solution obtained by the MIP solver.

The straightforward IP models, proposed by the literature, are often intractable and not useful in practice for computing  $h^+$ ; this has lead the research community to develop preprocessing techniques to reduce the size and complexity of the IP models.

### 2.1 Base IP model

Different models have been proposed in past years; we will use the base model proposed in [FR22]:

$x_a \in \{0, 1\}$  indicates whether the action  $a$  is used in a solution  $\pi$ .

$$x_a = \begin{cases} 0 & \text{if } a \notin \pi \\ 1 & \text{if } a \in \pi \end{cases} \quad \forall a \in A^+$$

$x_p \in \{0, 1\}$  indicates whether an atom  $p$  is achieved in a solution  $\pi$ .

$$x_p = \begin{cases} 0 & \text{if } p \notin I(\pi) \\ 1 & \text{if } p \in I(\pi) \end{cases} \quad \forall p \in P$$

$x_{a,p} \in \{0, 1\}$  indicates whether  $a$  is the first achiever of  $p$ .

$$x_{a,p} = \begin{cases} 1 & \text{if } a \text{ is the first achiever of } p \\ 0 & \text{otherwise} \end{cases} \quad \forall a \in A^+, \forall p \in \text{add}(a)$$

Having defined the variables that will be used in our formulation, the model is defined as follows:

$$\left\{ \begin{array}{ll} \min \sum_{a \in A^+} x_a \cdot \text{cost}(a) & \\ (C1) \quad \sum_{a \in A_p^+} x_{a,p} = x_p & \forall p \in P, A_p^+ = \{a \in A^+, s.t. p \in \text{add}(a)\} \\ (C2) \quad \sum_{a \in A_{p,q}^+} x_{a,q} \leq x_p & \forall p, q \in P, A_{p,q}^+ = \{a \in A^+, s.t. p \in \text{pre}(a), q \in \text{add}(a)\} \\ (C3) \quad x_{a,p} \leq x_a & \forall a \in A^+, p \in \text{add}(a) \\ (C4) \quad x_p = 1 & \forall p \in G \end{array} \right.$$

(C1): An atom is achieved iff an action is its first achiever; moreover there can be only one first achiever per atom;

(C2): An action can be a first achiever only if its preconditions are achieved;

(C3): An action can be a first achiever only if it's used;

(C4): Each atom in the goal state is achieved;

Note that, since there's no upper bound on action indicator variables, 0-cost actions might be flagged as used, even if they aren't, and still be an optimal plan.

If we were just looking for the optimal cost  $c^*$ , this wouldn't bother us; however, since we are interested in the optimal plan  $\pi^*$ , we must acknowledge those actions by filtering the solution obtained by removing actions that aren't first achiever of any atom.

## 2.2 Modeling acyclicity

As shown both in [IF15] and [FR22], any solution  $\pi$  needs to be acyclic for it to be a plan, hence the need to add variables and constraints to force such acyclicity in the base model.

Multiple methods have been proposed: the time labeling method was first proposed in [IF15] while vertex elimination was proposed in [FR22]; the latter, however, proposed a revised time labeling method, proving that it was better than its predecessor, hence we will only mention the improved version.

### 2.2.1 Time labeling

For the time labeling method, we just need to create for each atom  $p$ , a timestamp  $t_p \in \{0, \dots, |A^+|\}$ .

The constraints to add to the model will then be:

$$(C5) \quad t_p + 1 \leq t_q + |P|(1 - x_{a,q}) \quad \forall a \in A^+, p \in \text{pre}(a), q \in \text{add}(a)$$

This constraint reads: if  $a$  is first achiever of  $q$ , then every  $p \in \text{pre}(a)$  must have a timestamp that precedes than  $q$ .

The correctness of this model is implied from its construction, having tackled directly the problem of having to construct a timeline of actions out of the obtained solution with constraint (C5).

### 2.2.2 Vertex elimination graph

To explain how to use a vertex elimination graph to model acyclicity, we first need to define a *causal relation graph* [RR22]. A causal relation graph of  $\Pi^+$  is  $G_{\Pi^+} = (P, E_{\Pi^+})$ , where  $E_{\Pi^+} = \{(p, q), \text{ s.t. } \exists a \in A^+ \text{ with } p \in \text{pre}(a), q \in \text{add}(a)\}$ . As shown in [FR22], let  $\mathcal{O}$  be an elimination ordering for  $G_{\Pi^+}$ , and  $G_{\Pi^+}^* = (P, E_{\Pi^+}^*)$  be the vertex elimination graph of  $G_{\Pi^+}$  according to  $\mathcal{O}$ . Let  $\Delta$  be the set of all triangles produced by elimination ordering  $\mathcal{O}$  for graph  $G_{\Pi^+}$ : members of  $\Delta$  are all ordered triples  $(p, q, r)$  s.t.  $(p, r)$  is added to  $E_{\Pi^+}^*$  by eliminating  $q$ .

We then create a variable for each edge  $(p, q) \in E_{\Pi^+}^*$ ,  $e_{p,q} \in \{0, 1\} \forall (p, q) \in E_{\Pi^+}^*$  and add the following constraints:

$$\begin{cases} (C6) & x_{a,q} \leq e_{p,q} & \forall a \in A^+, p \in \text{pre}(a), q \in \text{add}(a) \\ (C7) & e_{p,q} + e_{q,p} \leq 1 & \forall (p, q) \in E_{\Pi^+}^* \\ (C8) & e_{p,q} + e_{q,r} - 1 \leq e_{p,r} & \forall (p, q, r) \in \Delta \end{cases}$$

The correctness of the resulting model, composed of constraints (C1) to (C4) and (C6) to (C9), is proved in [FR22].

We will identify the two formulations respectively  $\text{TL}(\Pi^+)$  and  $\text{VE}(\Pi^+)$ .

## 2.3 Preprocessing

### 2.3.1 Landmark-based model reduction

A *landmark* is an element which needs to be used in every feasible solution [HPS04]. We define *fact landmarks* and *action landmarks*, as in [GB12]: a fact landmark of a planning task  $\Pi$  is an atom that becomes true in some state of every plan, and similarly, an action landmark of a planning task  $\Pi$  is an action that is included in every plan.

Moreover, a fact or action landmark  $l$  if a landmark for an atom  $p$  if  $l$  is a landmark for the task  $(P, A, I, \{p\}, cost)$ , and similarly,  $l$  is a landmark for an action  $a$  if  $l$  is a landmark for the task  $(P, A, I, pre(a), cost)$ . For our delete-relaxed task  $\Pi^+$ , this leads to an obvious simplification of our model:

$$x_p, x_a \equiv 1 \quad \forall p \in P, a \in A^+ \text{ that is a fact/action landmark for an atom in } G$$

We can easily find a fact landmark  $p$  or an action landmark  $a$  by checking whether  $(P, A^+ \setminus A_p^+, I, G, cost)$ , with  $A_p^+ = \{a \in A^+, s.t. p \in add(a)\}$ , or  $(P, A^+ \setminus \{a\}, I, G, cost)$  respectively, are infeasible tasks; however, since this algorithm will be accompanied by other, more powerful, preprocessing algorithms, we opted for the efficient extraction method, which however doesn't compute all landmarks, proposed in [IF15]:

---

**Algorithm 1** Efficient landmark extraction algorithm

---

```

 $L[p] \leftarrow P$  for each  $p \in P$       #  $L[p]$  stores candidates of fact landmarks for  $p \in P$ 
 $S \leftarrow \emptyset$ 
for  $a \in A^+$  do
    insert  $a$  into a FIFO queue  $Q$  if  $pre(a) \subseteq S$ 
end for
while  $Q \neq \emptyset$  do
    retrieve an action  $a$  from  $Q$ 
    for  $p \in add(a)$  do
         $S \leftarrow S \cup \{p\}$ 
         $X \leftarrow L[p] \cap (add(a) \cup \bigcup_{p' \in pre(a)} L[p'])$ 
        if  $L[p] \neq X$  then
             $L[p] \leftarrow X$ 
            for  $a' \in \{a \in A^+, s.t. p \in pre(a)\}$  do
                insert  $a'$  into  $Q$  if  $pre(a') \subseteq S$  and  $a' \notin Q$ 
            end for
        end if
    end for
end while
# Now  $L[p]$  contains the set of fact landmarks for  $p \in P$ 

```

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### 2.3.2 First achievers filtering

### 2.3.3 Relevance analysis

### 2.3.4 Dominated actions extraction

### 2.3.5 Inverse actions extraction

By applying preprocessing to  $TL(\Pi^+)$  and  $VE(\Pi^+)$ , we obtain  $TL^e(\Pi^+)$  and  $VE^e(\Pi^+)$  respectively.

# Chapter 3

## Primal heuristics

TODO ...

### 3.1 Greedy heuristic

TODO ...

### 3.2 H-add heuristic

TODO ...

### 3.3 Combining the greedy and hadd heuristic

TODO ...

The use of the simple or hadd-enhanced greedy heuristics as warm-start to our formulations, will be indicated with  $_g$  and  $_h$  respectively, i.e.  $\text{VE}^e(\Pi^+)$  will either become  $\text{VE}_g^e(\Pi^+)$  or  $\text{VE}_h^e(\Pi^+)$ .



# Chapter 4

## Iterative approach to modeling acyclicity

TODO ...

### 4.1 Using landmarks to model acyclicity

TODO ...

#### 4.1.1 Minimal landmarks

TODO ...

#### 4.1.2 Complete landmarks

TODO ...

### 4.2 Using S.E.C. to model acyclicity

TODO ...

The formulation that use (both) landmarks to model acyclicity will be indicated with  $LM(\Pi^+)$ , while the formulation that use (both) landmarks aswell as S.E.C.  $LMS(\Pi^+)$ .





# Chapter 5

## Instances dataset

TODO ...



# Chapter 6

## Empirical results

TODO ...



# Chapter 7

## Future works

TODO ...



# Chapter 8

## Conclusions

TODO ...





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