Chapter Outline

Developing Linear Optimization Models

Decision Variables

Objective Function

Constraints

Softwater Optimization Model

OM Applications of Linear Optimization

OM Spotlight: Land
Management at the National
Forest Service

Production Scheduling

Blending Problems

Transportation Problems

A Linear Programming Model for Golden Beverages

A Linear Programming Model for Crashing Decisions

Using Excel Solver

Modeling and Solving the Transportation Problem on a Spreadsheet

Solved Problems

Key Terms and Concepts

Questions for Review and Discussion

Problems and Activities

Cases

Haller's Pub & Brewery Holcomb Candle

Endnotes

SUPPLEMENTARY CHAPTER C

Modeling Using Linear Programming

Learning Objectives

- To recognize decision variables, the objective function, and constraints in formulating linear optimization models.
- To identify potential applications of linear optimization and gain experience in modeling OM applications.
- To be able to use Excel Solver to solve linear optimization models on spreadsheets.

- Haller's Pub & Brewery is a small restaurant and microbrewery that makes six types of special beers, each having a unique taste and color. Jeremy Haller, one of the family owners who oversees the brewery operations, has become worried about increasing costs of grains and hops that are the principal ingredients and the difficulty they seem to be having in making the right product mix to meet demand and using the ingredients that are purchased under contract in the commodities market. Haller's buys six different types of grains and four different types of hops. Each of the beers needs different amounts of brewing time and is produced in 30-keg (4,350-pint) batches. While the average customer demand is 55 kegs per week, the demand varies by type. In a meeting with the other owners, Jeremy stated that Haller's has not been able to plan effectively to meet the expected demand. "I know there must be a better way of making our brewing decisions to improve our profitability."
- GE Capital provides credit card services for a consumer credit business that exceeds \$12 billion in total outstanding dollars. Managing delinquent accounts is a problem of paramount importance to the profitability of the company. Different collection strategies may be used for delinquent accounts, including mailed letters, interactive taped telephone messages, live telephone calls, and legal procedures. Accounts are categorized by outstanding balance and expected payment performance, which is based on factors like customer demographics and payment history. GE uses a multiple time period linear programming model to determine the most effective collection strategy to apply to its various categories of delinquent accounts. This approach has reduced annual losses due to delinquency by \$37 million.¹

Quantitative models that seek to maximize or minimize some objective function while satisfying a set of constraints are called optimization models. Quantitative models that seek to maximize or minimize some objective function while satisfying a set of constraints are called optimization models. An important category of optimization models is linear programming (LP) models, which are used widely for many types of operations design and planning problems that involve allocating limited resources among competing alternatives, as well as for many distribution and supply chain management design and operations. The term programming is used because these models find the best "program," or course of action, to follow. For Haller's Pub & Brewery, an LP model can be developed to find the best mix of products to meet demand and effectively use available resources (see the case at the end of this supplement). A manufacturer might use an LP model to develop a production schedule and an inventory policy that will satisfy sales demand in future periods while minimizing production and inventory costs, or to find the best distribution plan for shipping goods from warehouses to customers. Services use LP to schedule their staff, locate service facilities, minimize the distance traveled by delivery trucks and school buses, and, in the case of GE Capital, design collection strategies to reduce monetary losses. We first introduce the basic concepts of optimization modeling, then present some OM applications of LP, and finally discuss the use of Excel Solver to solve LP models.

DEVELOPING LINEAR OPTIMIZATION MODELS

To introduce the basic concepts of optimization modeling, we will use a simple production-planning problem. Softwater, Inc. manufactures and sells a variety of chemical products used in purifying and softening water. One of its products is a pellet that is produced and sold in 40- and 80-pound bags. A common production line packages both products, although the fill rate is slower for 80-pound bags. Softwater is currently planning its production schedule and wants to develop a linear programming model that will assist in its production-planning effort.

The company has orders for 20,000 pounds over the next week. Currently, it has 4,000 pounds in inventory. Thus, it should plan on an aggregate production of at least 16,000 pounds. Softwater has a sufficient supply of pellets to meet this demand but has limited amounts of packaging materials available, as well as a limited amount of time on the packaging line. The problem is to determine how many 40- and 80-pound bags to produce in order to maximize profit, given limited materials and time on the packaging line.

Decision Variables

To develop a model of this problem, we start by defining the decision variables. A **decision variable** is a controllable input variable that represents the key decisions a manager must make to achieve an objective. For the Softwater problem, the manager needs to determine the number of 40- and 80-pound bags. We denote these by the variables x_1 and x_2 , respectively:

 x_1 = number of 40-pound bags produced x_2 = number of 80-pound bags produced

Objective Function

Every linear programming problem has a specific objective. For Softwater, Inc., the objective is to maximize profit. If the company makes \$2 for every 40-pound bag produced, it will make $2x_1$ dollars if it produces x_1 40-pound bags. Similarly, if it makes \$4 for every 80-pound bag produced, it will make $4x_2$ dollars if it produces x_2 80-pound bags. Denoting total profit by the symbol z, and dropping the dollar signs for convenience, we have

Total profit =
$$z = 2x_1 + 4x_2$$
 (C.1)

Note that total profit is a function of the decision variables; thus, we refer to $2x_1 + 4x_2$ as the objective function. The constant terms in the objective function are called **objective function coefficients**. Thus, in this example, the objective function coefficients are \$2 (associated with x_1) and \$4 (associated with x_2). Softwater, Inc. must determine the values for the variables x_1 and x_2 that will yield the highest possible value of z. Using max as an abbreviation for maximize, the objective is written as

$$\operatorname{Max} z = 2x_1 + 4x_2$$

Any particular combination of decision variables is referred to as a solution. Suppose that Softwater decided to produce 200 40-pound bags and 300 80-pound bags. With Equation (C.1), the profit would be

$$z = 2(200) + 4(300)$$

= $400 + 1,200$
= \$1,600

Learning Objective

To formulate a linear optimization model by defining decision variables, an objective function, and constraints.

A decision variable is a controllable input variable that represents the key decisions a manager must make to achieve an objective.

The constant terms in the objective function are called objective function coefficients.

Any particular combination of decision variables is referred to as a solution.

What if Softwater decided on a different production schedule, such as 400 40-pound bags and 400 80-pound bags? In that case, the profit would be

$$z = 2(400) + 4(400)$$

= 800 + 1,600
= \$2,400

Certainly the latter production schedule is preferable in terms of the stated objective of maximizing profit. However, it may not be possible for Softwater to produce that many bags. For instance, there might not be enough materials or enough time available on the packaging line to produce those quantities. Solutions that satisfy all constraints are referred to as feasible solutions.

For this example, we seek a feasible solution that maximizes profit. Any feasible solution that optimizes the objective function is called an optimal solution. At this point, however, we have no idea what the optimal solution is because we have not developed a procedure for identifying feasible solutions. This requires us to first identify all the constraints of the problem in a mathematical expression. A constraint is some limitation or requirement that must be satisfied by the solution.

Solutions that satisfy all constraints are referred to as feasible solutions.

Any feasible solution that optimizes the objective function is called an optimal solution.

A constraint is some limitation or requirement that must be satisfied by the solution.

Constraints

Every 40- and 80-pound bag produced must go through the packaging line. In a normal workweek, this line operates 1,500 minutes. The 40-pound bags, for which the line was designed, each require 1.2 minutes of packaging time; the 80-pound bags require 3 minutes per bag. The total packaging time required to produce x_1 40-pound bags is $1.2x_1$, and the time required to produce x_2 80-pound bags is $3x_2$. Thus, the total packaging time required for the production of x_1 40-pound bags and x_2 80-pound bags is given by

$$1.2x_1 + 3x_2$$

Because only 1,500 minutes of packaging time are available, it follows that the production combination we select must satisfy the constraint

$$1.2x_1 + 3x_2 \le 1,500 \tag{C.2}$$

This constraint expresses the requirement that the total packaging time used cannot exceed the amount available.

Softwater has 6,000 square feet of packaging materials available; each 40-pound bag requires 6 square feet and each 80-pound bag requires 10 square feet of these materials. Since the amount of packaging materials used cannot exceed what is available, we obtain a second constraint for the LP problem:

$$6x_1 + 10x_2 \le 6{,}000 \tag{C.3}$$

The aggregate production requirement is for the production of 16,000 pounds of softening pellets per week. Because the small bags contain 40 pounds of pellets and the large bags contain 80 pounds, we must impose this aggregate-demand constraint:

$$40x_1 + 80x_2 \le 16,000 \tag{C.4}$$

Finally, we must prevent the decision variables x_1 and x_2 from having negative values. Thus, the two constraints

$$x_1 \text{ and } x_2 \ge 0$$
 (C.5)

must be added. These constraints are referred to as the **nonnegativity constraints**. Nonnegativity constraints are a general feature of all LP problems and are written in this abbreviated form:

$$x_1, x_2 \ge 0$$

Softwater Optimization Model

The mathematical statement of the Softwater problem is now complete. The complete mathematical model for the Softwater problem follows:

$$\operatorname{Max} z = 2x_1 + 4x_2 \quad (\operatorname{profit})$$

subject to

$$1.2x_1 + 3x_2 \le 1,500$$
 (packaging line)
 $6x_1 + 10x_2 \le 6,000$ (materials availability)
 $40x_1 + 80x_2 \ge 16,000$ (aggregate production)
 $x_1, x_2 = 0$ (nonnegativity)

This mathematical model of the Softwater problem is a *linear program*. A function in which each variable appears in a separate term and is raised to the first power is called a linear function. The objective function, $(2x_1 + 4x_2)$, is linear, since each decision variable appears in a separate term and has an exponent of 1. If the objective function were $2x^2_1 + 4\sqrt{x_2}$, it would not be a linear function and we would not have a linear program. The amount of packaging time required is also a linear function of the decision variables for the same reasons. Similarly, the functions on the left side of all the constraint inequalities (the constraint functions) are linear functions.

Our task now is to find the product mix (that is, the combination of x_1 and x_2) that satisfies all the constraints and, at the same time, yields a value for the objective function that is greater than or equal to the value given by any other feasible solution. Once this is done, we will have found the optimal solution to the problem. We will see how to solve this a bit later.

A function in which each variable appears in a separate term and is raised to the first power is called a linear function.

OM APPLICATIONS OF LINEAR OPTIMIZATION

In this section we present some typical examples of linear optimization models in operations management. Many more common OM problems can be modeled as linear programs (see the OM Spotlight: Land Management at the National Forest Service); we encourage you to take a look at the journal *Interfaces*, published by The Institute for Operations Research and Management Sciences (INFORMS), which publishes many practical applications of quantitative methods in OM.

Production Scheduling

One of the most important applications of linear programming is for multiperiod planning, such as production scheduling. Let us consider the case of the Bollinger Electronics Company, which produces two electronic components for an airplane engine manufacturer. The engine manufacturer notifies the Bollinger sales office each quarter as to its monthly requirements for components during each of the next three months. The order shown in Exhibit C.1 has just been received for the next three-month period.

One feasible schedule would be to produce at a constant rate for all three months. This would set monthly production quotas at 3,000 units per month for

	April	May	June
Component 322A	1,000	3,000	5,000
Component 802B	1,000	500	3,000

Learning Objective

To identify potential applications of linear programming and to gain experience in formulating different types of models for OM applications.

Exhibit C.1Three-Month Demand Schedule for Bollinger Electronics
Company



OM SPOTLIGHT

Land Management at the National Forest Service²

The National Forest Service is responsible for the management of over 191 million acres of national forest in 154 designated national forests.

The National Forest Management Act of 1976 mandated the development of a comprehensive plan to guide the management of each national forest. Decisions need to be made regarding the use of land in each of the national forests. The overall management objective is "to provide multiple use and the sustained yield of goods and services from the National Forest System in a way that maximizes long-term net public benefits in an environmentally sound manner." Linear programming models are used to decide the number of acres of various types in each forest to be used for the many possible usage strategies. For example, the decision variables of the model are variables such as the number of acres in a particular forest to be used for timber, the number of acres

to be used for recreational purposes of various types, and the number of acres to be left undisturbed.

The model has as its objective the maximization of the net discounted value of the forest over the planning time horizon. Constraints include the availability of land of different types, bounds on the amount of land dedicated to certain uses, and resources available to manage land under different strategies (for example, recreational areas must be staffed; harvesting the land requires a budget for labor, the transport of goods, and replanting). These LP planning models are quite large. Some have over 6,000 constraints and over 120,000 decision variables! The data requirements for these models are quite intense, requiring the work of literally hundreds of agricultural resource specialists to estimate reasonable values for the inputs to the model. These specialists are also used to ensure that the solutions make sense operationally.

component 322A and 1,500 units per month for component 802B. Although this schedule would be appealing to the production department, it may be undesirable from a total-cost point of view because it ignores inventory costs. For instance, producing 3,000 units of component 322A in April will result in an end-of-month inventory level of 2,000 units for both April and May.

A policy of producing the amount demanded each month would eliminate the inventory holding-cost problem; however, the wide monthly fluctuations in production levels might cause some serious production problems and costs. For example, production capacity would have to be available to meet the total 8,000-unit peak demand in June. This might require substantial labor adjustments, which in turn could lead to increased employee turnover and training problems. Thus, it appears that the best production schedule will be one that is a compromise between the two alternatives.

The production manager must identify and consider costs for production, storage, and production-rate changes. In the remainder of this section, we show how an LP model of the Bollinger Electronics production and inventory process can be formulated to account for these costs in order to minimize the total cost.

We will use a double-subscript notation for the decision variables in the problem. We let the first subscript indicate the product number and the second subscript the month. Thus, in general, x_{im} denotes the production volume in units for product i in month m. Here i = 1, 2, and m = 1, 2, 3; i = 1 refers to component 322A, i = 2 to component 802B, m = 1 to April, m = 2 to May, and m = 3 to June.

If component 322A costs \$20 per unit to produce and component 802B costs \$10 per unit to produce, the production-cost part of the objective function is

$$20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23}$$

Note that in this problem the production cost per unit is the same each month, and thus we need not include production costs in the objective function; that is, no mat-

ter what production schedule is selected, the total production costs will remain the same. In cases where the cost per unit may change each month, the variable production costs per unit per month must be included in the objective function. We have elected to include them so that the value of the objective function will include all the relevant costs associated with the problem. To incorporate the relevant inventory costs into the model, we let I_{im} denote the inventory level for product i at the end of month m. Bollinger has determined that on a monthly basis, inventory-holding costs are 1.5 percent of the cost of the product—that is, $0.015 \times \$20 = \0.30 per unit for component 322A, and $0.015 \times \$10 = \0.15 per unit for component 802B. We assume that monthly ending inventories are acceptable approximations of the average inventory levels throughout the month. Given this assumption, the inventory-holding cost portion of the objective function can be written as

$$0.30I_{11} + 0.30I_{12} + 0.30I_{13} + 0.15I_{21} + 0.15I_{22} + 0.15I_{23}$$

To incorporate the costs due to fluctuations in production levels from month to month, we need to define these additional decision variables:

 R_m = increase in the total production level during month m compared with month m-1

 D_m = decrease in the total production level during month m compared with month m-1

After estimating the effects of employee layoffs, turnovers, reassignment training costs, and other costs associated with fluctuating production levels, Bollinger estimates that the cost associated with increasing the production level for any given month is \$0.50 per unit increase. A similar cost associated with decreasing the production level for any given month is \$0.20 per unit. Thus, the third portion of the objective function, for production-fluctuation costs, can be written as

$$0.50R_1 + 0.50R_2 + 0.50R_3 + 0.20D_1 + 0.20D_2 + 0.20D_3$$

By combining all three costs, we obtain the complete objective function:

$$20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23} 0.30I_{11} + 0.30I_{12} + 0.30I_{13} + 0.15I_{21} + 0.15I_{22} + 0.15I_{23} 0.50R_1 + 0.50R_2 + 0.50R_3 + 0.20D_1 + 0.20D_2 + 0.20D_3$$

Now let us consider the constraints. First we must guarantee that the schedule meets customer demand. Since the units shipped can come from the current month's production or from inventory carried over from previous periods, we have these basic requirements:

```
(Ending inventory from previous month) + (Current production) - (This month's demand)
```

The difference between the left side and the right side of the equation will be the amount of ending inventory at the end of this month. Thus, the demand requirement takes the following form:

```
(Ending inventory from previous month) + (Current production) - (Ending inventory for this month) = (This month's demand)
```

Suppose the inventories at the beginning of the three-month scheduling period were 500 units for component 322A and 200 units for component 802B. Recalling that the demand for both products in the first month (April) was 1,000 units, we see that the constraints for meeting demand in the first month become

$$500 + x_{11} - I_{11} = 1,000$$
 and $200 + x_{21} - I_{21} = 1,000$

Moving the constants to the right side of the equation, we have

$$x_{11} - I_{11} = 500$$
 and $x_{21} - I_{21} = 800$

Similarly, we need demand constraints for both products in the second and third months. These can be written as follows:

Month 2:
$$I_{11} + x_{12} - I_{12} = 3,000$$

 $I_{21} + x_{22} - I_{22} = 500$
Month 3: $I_{12} + x_{13} - I_{13} = 5,000$
 $I_{22} + x_{23} - I_{23} = 3,000$

If the company specifies a minimum inventory level at the end of the three-month period of at least 400 units of component 322A and at least 200 units of component 802B, we can add the constraints

$$I_{13} \ge 400$$
 and $I_{23} \ge 200$

Let us suppose we have the additional information available on production, labor, and storage capacity given in Exhibit C.2. Machine, labor, and storage space requirements are given in Exhibit C.3. To reflect these limitations, the following constraints are necessary:

Machine capacity:

$$0.10x_{11} + 0.08x_{21} \le 400 \text{ (month 1)}$$

$$0.10x_{12} + 0.08x_{22} \le 500 \text{ (month 2)}$$

$$0.10x_{13} + 0.08x_{23} \le 600 \text{ (month 3)}$$

Labor capacity:

$$0.05x_{11} + 0.07x_{21} \le 300 \text{ (month 1)}$$

$$0.05x_{12} + 0.07x_{22} \le 300 \text{ (month 2)}$$

$$0.05x_{13} + 0.07x_{23} \le 300 \text{ (month 3)}$$

Storage capacity:

$$2I_{11} + 3I_{21} \le 10,000 \text{ (month 1)}$$

$$2I_{12} + 3I_{22} \le 10,000 \text{ (month 2)}$$

$$2I_{13} + 3I_{23} \le 10,000 \text{ (month 3)}$$

We must also guarantee that R_m and D_m will reflect the increase or decrease in the total production level for month m. Suppose the production levels for March, the month before the start of the current scheduling period, were 1,500 units of component 322A and 1,000 units of component 802B for a total production level

Exhibit C.2

Machine, Labor, and Storage Capacities

	Machine Capacity	Labor Capacity	Storage Capacity
	(hours)	(hours)	(square feet)
April	400	300	10,000
May	500	300	10,000
June	600	300	10,000

Exhibit C.3

Machine, Labor, and Storage Requirements for Components 322A and 802B

	Machine	Labor	Storage
	(hours/unit)	(hours/unit)	(sq. ft./unit)
Component 322A	0.10	0.05	2 3
Component 802B	0.08	0.07	

of 1,500 + 1,000 = 2,500 units. We can find the amount of the change in production for April from the relationship

Using the April production decision variables, x_{11} and x_{21} , and the March production of 2,500 units, we can rewrite this relationship as

$$x_{11} + x_{21} - 2,500 = \text{Change}$$

Note that the change can be positive or negative, reflecting an increase or decrease in the total production level. With this rewritten relationship, we can use the increase in production variable for April, R_1 , and the decrease in production variable for April, D_1 , to specify the constraint for the change in total production for the month of April:

$$x_{11} + x_{21} - 2,500 = R_1 - D_1$$

Of course, we cannot have an increase in production and a decrease in production during the same one-month period; thus, either R_1 or D_1 will be zero. If April requires 3,000 units of production, we will have $R_1 = 500$ and $D_1 = 0$. If April requires 2,200 units of production, we will have $R_1 = 0$ and $D_1 = 300$. This way of denoting the change in production level as the difference between two nonnegative variables, R_1 and D_1 , permits both positive and negative changes in the total production level. If a single variable—say, cm—had been used to represent the change in production level, only positive changes would be permitted—because of the nonnegativity requirement.

Using the same approach in May and June (always subtracting the previous month's total production from the current month's total production) yields these constraints for the second and third months of the scheduling period:

$$(x_{12} + x_{22}) - (x_{11} + x_{21}) = R_2 - D_2$$

 $(x_{13} + x_{23}) - (x_{12} + x_{22}) = R_3 - D_3$

Placing the variables on the left side and the constants on the right side, we have the complete set of what are commonly referred to as *production-smoothing constraints*:

The initially rather small, two-product, three-month scheduling problem has now developed into an 18-variable, 20-constraint LP problem. Note that this problem involves only one machine process, one type of labor, and one storage area. Actual production scheduling problems typically involve several machine types, several labor grades, and/or several storage areas. A typical application might involve developing a production schedule for 100 products over a 12-month horizon—a problem that could have more than 1,000 variables and constraints.

Blending Problems

Blending problems arise whenever a manager must decide how to combine two or more ingredients in order to produce one or more products. These types of problems occur frequently in the petroleum industry (such as blending crude oil to produce different octane gasolines) and the chemical industry (such as blending chemicals to produce fertilizers, weed killers, and so on). In these applications, managers must decide how much of each resource to purchase in order to satisfy product specifications and product demands at minimum cost.

To illustrate a blending problem, consider the Grand Strand Oil Company, which produces regular-grade and premium-grade gasoline products by blending three petroleum components. The gasolines are sold at different prices, and the petroleum components have different costs. The firm wants to determine how to blend the three components into the two products in such a way as to maximize profits.

Data available show that the regular-grade gasoline can be sold for \$1.20 per gallon and the premium-grade gasoline for \$1.40 per gallon. For the current production-planning period, Grand Strand can obtain the three components at the costs and in the quantities shown in Exhibit C.4. The product specifications for the regular and premium gasolines, shown in Exhibit C.5, restrict the amounts of each component that can be used in each gasoline product. Current commitments to distributors require Grand Strand to produce at least 10,000 gallons of regular-grade gasoline.

The Grand Strand blending problem is to determine how many gallons of each component should be used in the regular blend and in the premium blend. The optimal solution should maximize the firm's profit, subject to the constraints on the available petroleum supplies shown in Exhibit C.4, the product specifications shown in Exhibit C.5, and the requirement for at least 10,000 gallons of regular-grade gasoline. We can use double-subscript notation to denote the decision variables for the problem. Let

 x_{ii} = gallons of component *i* used in gasoline *j*

where

i = 1, 2, or 3 for component 1, 2, or 3

and

j = r if regular or = p if premium

The six decision variables become

 x_{1r} = gallons of component 1 in regular gasoline x_{2r} = gallons of component 2 in regular gasoline x_{3r} = gallons of component 3 in regular gasoline x_{1p} = gallons of component 1 in premium gasoline x_{2p} = gallons of component 2 in premium gasoline x_{3p} = gallons of component 3 in premium gasoline gasoline

Exhibit C.4Petroleum Component Cost and Supply

Petroleum Component	Cost/Gallon	Amount Available
Component 1	\$0.50	5,000 gallons
Component 2	\$0.60	10,000 gallons
Component 3	\$0.82	10,000 gallons

Exhibit C.5Component Specifications for Grand Strand's Products

Product	Specifications
Regular gasoline	At most 30% Component 1 At least 40% Component 2 At most 20% Component 3
Premium gasoline	At least 25% Component 1 At most 40% Component 2 At least 30% Component 3

With the notation for the six decision variables just defined, the total gallons of each type of gasoline produced can be expressed as the sum of the gallons of the blended components:

Regular gasoline =
$$x_{1r} + x_{2r} + x_{3r}$$

Premium gasoline = $x_{1p} + x_{2p} + x_{3p}$

Similarly, the total gallons of each component used can be expressed as these sums:

Component
$$1 = x_{1r} + x_{1p}$$

Component $2 = x_{2r} + x_{2p}$
Component $3 = x_{3r} + x_{3p}$

The objective function of maximizing the profit contribution can be developed by identifying the difference between the total revenue from the two products and the total cost of the three components. By multiplying the \$2.20 per gallon price by the total gallons of regular gasoline, the \$2.40 per gallon price by the total gallons of premium gasoline, and the component cost per gallon figures in Exhibit C.5 by the total gallons of each component used, we find the objective function to be

Max
$$2.20(x_{1r} + x_{2r} + x_{3r}) + 2.40(x_{1p} + x_{2p} + x_{3p}) - 1.00(x_{1r} + x_{1p}) - 1.20(x_{2r} + x_{2p}) - 1.64(x_{3r} + x_{3p})$$

By combining terms, we can then write the objective function as

$$\text{Max } 1.20x_{1r} + 1.00x_{2r} + 0.56x_{3r} + 1.40x_{1p} + 1.20x_{2p} + 0.76x_{3p}$$

Limitations on the availability of the three components can be expressed by these three constraints:

$$x_{1r} + x_{1p} \le 5,000$$
 (component 1)
 $x_{2r} + x_{2p} \le 10,000$ (component 2)
 $x_{3r} + x_{3p} \le 10,000$ (component 3)

Six constraints are now required to meet the product specifications stated in Exhibit C.5. The first specification states that component 1 must account for at most 30 percent of the total gallons of regular gasoline produced. That is,

$$x_{1r}/(x_{1r} + x_{21} + x_{3r}) \le 0.30$$
 or $x_{1r} \le 0.30(x_{1r} + x_{2r} + x_{3r})$

When this constraint is rewritten with the variables on the left side of the equation and a constant on the right side, the first product-specification constraint becomes

$$0.70x_{1r} - 0.30x_{2r} - 0.30x_{3r} \le 0$$

The second product specification listed in Exhibit C.5 can be written as

$$x_{2r} \ge 0.40 (x_{1r} + x_{2r} + x_{3r})$$

and thus

$$-0.40x_{1r} + 0.60x_{2r} - 0.40x_{3r} \ge 0$$

Similarly, the four additional blending specifications shown in Exhibit C.5 can be written as follows:

$$\begin{array}{l} -0.20x_{1r} - 0.20x_{2r} + 0.80x_{3r} \leq 0 \\ 0.75x_{1p} - 0.25x_{2p} - 0.25x_{3p} \leq 0 \\ -0.40x_{1p} + 0.60x_{2p} - 0.40x_{3p} \leq 0 \\ -0.30x_{1p} - 0.30x_{2p} + 0.70x_{3p} \leq 0 \end{array}$$

The constraint for at least 10,000 gallons of the regular-grade gasoline is written

$$x_{1r} + x_{2r} + x_{3r} \ge 10,000$$

Thus, the complete LP model with six decision variables and 10 constraints can be written as follows:

Max
$$1.20x_{1r} + 1.00x_{2r} + 0.56x_{3r} + 1.40x_{1p} + 1.20x_{2p} + 0.76x_{3p}$$
 subject to

$$x_{1r} + x_{1p} \le 5,000$$
 (component 1)
 $x_{2r} + x_{2p} \le 10,000$ (component 2)
 $x_{3r} + x_{3p} \le 10,000$ (component 3)
 $0.70x_{1r} - 0.30x_{2r} - 0.30x_{3r} \le 0$
 $-0.40x_{1r} + 0.60x_{2r} - 0.40x_{3r} \ge 0$
 $-0.20x_{1r} - 0.20x_{2r} + 0.80x_{3r} \le 0$
 $0.75x_{1p} - 0.25x_{2p} - 0.25x_{3p} \le 0$
 $-0.40x_{1p} + 0.60x_{2p} - 0.40x_{3p} \le 0$
 $-0.30x_{1p} - 0.30x_{2p} + 0.70x_{3p} \le 0$
 $x_{1r} + x_{2r} + x_{3r} \ge 10,000$
 $x_{1r}, x_{2r}, x_{3r}, x_{1p}, x_{2p}, x_{3p} \ge 0$

Transportation Problems

The transportation problem is a special type of linear planning the distribution of goods and services from several supply points to several demand locations.

program that arises in

The transportation problem is a special type of linear program that arises in planning the distribution of goods and services from several supply points to several demand locations. Usually the quantity of goods available at each supply location (origin) is limited, and a specified quantity of goods is needed at each demand location (destination). With a variety of shipping routes and differing transportation costs for the routes, the objective is to determine how many units should be shipped from each origin to each destination so that all destination demands are satisfied with a minimum total transportation cost. The transportation problem was introduced in Chapter 8 on designing supply and value chains. Here we show how to model this problem and later solve it with Microsoft Excel.

Let us consider the problem faced by Foster Generators, Inc. Currently, Foster has two plants: one in Cleveland, Ohio and one in Bedford, Indiana. Generators produced at the plants (origins) are shipped to distribution centers (destinations) in Boston, Chicago, St. Louis, and Lexington, Kentucky. Recently, an increase in demand has caused Foster to consider adding a new plant to provide expanded production capacity. After some study, the location alternatives for the new plant have been narrowed to York, Pennsylvania and Clarksville, Tennessee.

To help them decide between the two locations, managers have asked for a comparison of the projected operating costs for the two locations. As a start, they want to know the minimum shipping costs from three origins (Cleveland, Bedford, and York) to the four distribution centers if the plant is located in York. Then, finding the minimum shipping cost from the three origins including Clarksville instead of York to the four distribution centers will provide a comparison of transportation costs; this will help them determine the better location.

We will illustrate how this shipping-cost information can be obtained by solving the transportation problem with the York location alternative. (Problem 16 at the end of the chapter asks you to solve a transportation problem using the Clarksville location alternative.) Using a typical one-month planning period, the production capacities at the three plants are shown in Exhibit C.6. Forecasts of monthly demand at the four distribution centers are shown in Exhibit C.7. The transportation cost per unit for each route is shown in Exhibit C.8.

A convenient way of summarizing the transportation-problem data is with a table such as the one for the Foster Generators problem in Exhibit C.9. Note that the 12 cells in the table correspond to the 12 possible shipping routes from the

Origin	Plant	Production Capacity (units)
1 2 3	Cleveland Bedford York	5,000 6,000 2,500
		Total 13,500

Exhibit C.6Foster Generators Production Capacities

Destination	Distribution Center	Demand Forecast (units)
1 2 3 4	Boston Chicago St. Louis Lexington	6,000 4,000 2,000 1,500 Total 13,500

Exhibit C.7Foster Generators Monthly
Forecast

	Destination					
Origin	Boston	Chicago	St. Louis	Lexington		
Cleveland Bedford York	\$3 7 2	\$2 5 5	\$7 2 4	\$6 3 5		

Exhibit C.8Foster Generators
Transportation Cost per Unit

		Destir	nation		Origin
Origin	1 Boston	2 Chicago	3 St. Louis	4 Lexington	Supply
	3	2	7	6	
1. Cleveland	X ₁₁	X ₁₂	X ₁₃	X ₁₄	5,000
0. D. 16. 1	7	5	2	3	
2. Bedford	X ₂₁	X ₂₂	X ₂₃	X ₂₄	6,000
	7 2	5	4	5	0.500
3. York	X ₃₁	X ₃₂	X ₃₃	X ₃₄	2,500
Destination Demand	6,000	4,000	2,000	1,500	13,500
Cell corresponding to shipments from Bedford to Boston			Total su total de	upply and emand	

Exhibit C.9Foster Generators
Transportation Table

three origins to the four destinations. We denote the amount shipped from origin i to destination j by the variable x_{ij} . The entries in the column at the right of the table represent the supply available at each plant, and the entries at the bottom represent the demand at each distribution center. Note that total supply equals total demand. The entry in the upper-right corner of each cell represents the perunit cost of shipping over the corresponding route.

Looking across the first row of this table, we see that the amount shipped from Cleveland to all destinations must equal 5,000, or $x_{11} + x_{12} + x_{13} + x_{14} = 5,000$. Similarly, the amount shipped from Bedford to all destinations must be 6,000, or $x_{21} + x_{22} + x_{23} + x_{24} = 6,000$. Finally, the amount shipped from York to all destinations must total 2,500, or $x_{31} + x_{32} + x_{33} + x_{34} = 2,500$.

We also must ensure that each destination receives the required demand. Thus, the amount shipped from all origins to Boston must equal 6,000, or $x_{11} + x_{21} + x_{31} = 6,000$. For Chicago, St. Louis, and Lexington, we have similar constraints:

```
Chicago: x_{12} + x_{22} + x_{32} = 4,000
St. Louis: x_{13} + x_{23} + x_{33} = 2,000
Lexington: x_{14} + x_{24} + x_{34} = 1,500
```

If we ship x_{11} units from Cleveland to Boston, we incur a total shipping cost of $3x_{11}$. By summing the costs associated with each shipping route, we have the total cost expression that we want to minimize:

Total cost =
$$3x_{11} + 2x_{12} + 7x_{13} + 6x_{14} + 7x_{21} + 5x_{22} + 2x_{23} + 3x_{24} + 2x_{31} + 5x_{32} + 4x_{33} + 5x_{34}$$

By including nonnegativity restrictions, $x_{ij} \ge 0$ for all variables, we have modeled the transportation problem as a linear program.

A Linear Programming Model for Golden Beverages

In Chapter 13, we examined aggregate planning strategies and noted that linear programming can be used to find an optimal solution. In this section we illustrate how such a model can be developed for the Golden Beverages problem described in that chapter. Of course, the model is larger and more complex than the Softwater example and is representative of many practical situations that companies face.

In order to formulate the linear programming model, we define these variables, each expressed in number of barrels:

 $X_t = \text{production in period } t$

 I_t = inventory held at the end of period t

 L_t = number of lost sales incurred in period t

 O_t = amount of overtime scheduled in period t

 U_t = amount of undertime scheduled in period t

 R_t = increase in production rate from period t - 1 to period t

 D_t = decrease in production rate from period t-1 to period t

MATERIAL-BALANCE CONSTRAINT

The constraint that requires the beginning inventory plus production minus sales to equal the ending inventory is called a *material-balance equation*. In the case of Golden Beverages, it is possible for demand to exceed sales. This occurs when there is a stockout and lost sales result. Thus, in words, the material-balance equation for Golden Beverages in month t is

$$\begin{pmatrix} \text{Ending} \\ \text{inventory in} \\ \text{month } t - 1 \end{pmatrix} + \begin{pmatrix} \text{Production} \\ \text{in} \\ \text{month } t \end{pmatrix} - \begin{pmatrix} \text{Demand} \\ \text{in} \\ \text{month } t \end{pmatrix} + \begin{pmatrix} \text{Lost sales} \\ \text{in} \\ \text{month } t \end{pmatrix} = \begin{pmatrix} \text{Ending} \\ \text{inventory in} \\ \text{month } t \end{pmatrix}$$

Note that the ending inventory in month t-1 is also the beginning inventory in month t. Each term in the material-balance equation is a variable, except for demand; demand is a constant given by the demand forecast. Putting the demand constant on the right side of the equation and using our previously introduced variable definitions, we can write the material-balance equation as

$$X_t + I_{t-1} - I_t + L_t = Demand in month t$$

The beginning inventory for the aggregate-planning problem is 1,000 barrels; therefore, the material-balance equations for a 12-month planning horizon (with t=1 corresponding to January) are

$$X_1 - I_1 + 1,000 + L_1 = 1,500$$

 $X_2 - I_2 + I_1 + L_2 = 1,000$
 $X_3 - I_3 + I_2 + L_3 = 1,900$
 $X_4 - I_4 + I_3 + L_4 = 2,600$
 $X_5 - I_5 + I_4 + L_5 = 2,800$
 $X_6 - I_6 + I_5 + L_6 = 3,100$
 $X_7 - I_7 + I_6 + L_{77} = 3,200$
 $X_8 - I_8 + I_7 + L_8 = 3,000$
 $X_9 - I_9 + I_8 + L_9 = 2,000$
 $X_{10} - I_{10} + I_9 + L_{10} = 1,000$
 $X_{11} - I_{11} + I_{10} + L_{11} = 1,800$
 $X_{12} - I_{12} + I_{11} + L_{12} = 2,200$

OVERTIME/UNDERTIME CONSTRAINT

Since the normal production capacity is 2,200 barrels per month, any deviation from this amount represents overtime or undertime. The number of units produced on overtime and the number of units produced on undertime is determined by this equation for month *t*:

$$O_t - U_t = X_t - 2,200$$

Of course, we cannot have both overtime and undertime in the same month. If $X_t > 2,200$, then O_t equals the excess production over normal capacity ($X_t - 2,200$). If $X_t < 2,200$, then U_t equals the amount of undercapacity production ($2,200 - X_t$).

We need a separate overtime/undertime constraint for each month in the planning horizon. The 12 constraints needed are

$$X_1 - O_1 + U_1 = 2,200$$

 $X_2 - O_2 + U_2 = 2,200$
 $X_3 - O_3 + U_3 = 2,200$
 $X_4 - O_4 + U_4 = 2,200$
 $X_5 - O_5 + U_5 = 2,200$
 $X_6 - O_6 + U_6 = 2,200$
 $X_7 - O_7 + U_7 = 2,200$
 $X_8 - O_8 + U_8 = 2,200$
 $X_9 - O_9 + U_9 = 2,200$
 $X_{10} - O_{10} + U_{10} = 2,200$
 $X_{11} - O_{11} + U_{11} = 2,200$
 $X_{12} - O_{12} + U_{12} = 2,200$

PRODUCTION-RATE-CHANGE CONSTRAINTS

In order to determine the necessary decreases or increases in the production rate, we write a constraint of this form for each period:

$$X_t - X_{t-1} = R_t - D_t$$

Note that $X_t - X_{t-1}$ is simply the change in production rate from period t-1 to period t. If the difference between the production rate, X_t , and the rate in the previous period, X_{t-1} , is positive, then the increase, R_t , equals $X_t - X_{t-1}$ and the decrease, D_t , equals zero; otherwise, D_t equals $X_{t-1} - X_t$ and R_t equals zero to reflect the decrease in production rate. Obviously, R_t and D_t cannot both be positive. We need one production-rate-change constraint for each month in the planning horizon.

Letting X_0 denote the production rate in the last month's production, we obtain these 12 constraints:

$$\begin{array}{l} X_1 - X_0 = R_1 - D_1 \\ X_2 - X_1 = R_2 - D_2 \\ X_3 - X_2 = R_3 - D_3 \\ X_4 - X_3 = R_4 - D_4 \\ X_5 - X_4 = R_5 - D_5 \\ X_6 - X_5 = R_6 - D_6 \\ X_7 - X_6 = R_7 - D_7 \\ X_8 - X_7 = R_8 - D_8 \\ X_9 - X_8 = R_9 - D_9 \\ X_{10} - X_9 = R_{10} - D_{10} \\ X_{11} - X_{10} = R_{11} - D_{11} \\ X_{12} - X_{11} = R_{12} - D_{12} \end{array}$$

OBJECTIVE FUNCTION

A summary of the costs for the Golden Beverages aggregate-production-planning problem is

Cost Factor	Cost		
Production	\$70 per barrel		
Inventory holding	\$1.40 per barrel per month		
Lost sales	\$90 per barrel		
Overtime	\$6.50 per barrel		
Undertime	\$3 per barrel		
Production-rate change	\$5 per barrel		

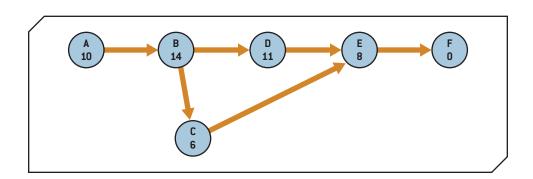
The objective function calls for minimizing total costs. It is given by

$$z = 70(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12}) + 1.4(I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 + I_9 + I_{10} + I_{11} + I_{12}) + 90(L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_9 + L_{10} + L_{11} + L_{12}) + 6.5(O_1 + O_2 + O_3 + O_4 + O_5 + O_6 + O_7 + O_8 + O_9 + O_{10} + O_{11} + O_{12}) + 3(U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7 + U_8 + U_9 + U_{10} + U_{11} + U_{12}) + 5(D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10} + D_{11} + D_{12}) + 5(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 + R_{10} + R_{11} + R_{12})$$

A Linear Programming Model for Crashing Decisions

In Chapter 18 we discussed crashing in projects. To illustrate the formulation of a linear programming model for crashing, we use the simple project network shown in Exhibit C.10. This is in activity-on-node format with normal times shown. In-

Exhibit C.10 Project Network Diagram



cluded is a dummy activity at the end of the project with a duration of 0, which is necessary for the LP model. Exhibit C.11 shows the other relevant data (refer to Chapter 18 to understand what these data mean).

To construct a linear programming model for the crashing decision, we define the following decision variables:

> x_i = start time of activity i y_i = amount of crash time used for activity i

Note that the normal-time project cost of \$2,900 (obtained by summing the column of normal costs in Exhibit C.11) does not depend on what crashing decisions we will make. As a result, we can minimize the total project cost (normal costs plus crashing costs) by minimizing the crashing costs. Thus, the linear programming objective function becomes

Min 2,000
$$y_A$$
 + 1,000 y_B + 2,500 y_C + 1,500 y_D + 500 y_E

The linear programming constraints that must be developed include those that describe the precedence relationships in the network, limit the activity crash times, and result in meeting the desired project-completion time. The constraints used to describe the precedence relationships are

$$x_i \ge x_i + t_{ni} - y_i$$

for each precedence, that is, each arc from activity i to activity j in the network. This simply states that the start time for the following activity must be at least as great as the finish time for each immediate predecessor with crashing applied. Thus, for the example, we have

$$x_{B} \ge x_{A} + 10 - y_{A}$$

 $x_{D} \ge x_{B} + 14 - y_{B}$
 $x_{C} \ge x_{B} + 14 - y_{B}$
 $x_{E} \ge x_{D} + 11 - y_{D}$
 $x_{E} \ge x_{C} + 6 - y_{C}$
 $x_{F} \ge x_{E} + 8 - y_{E}$

Alternatively, each constraint may be written as

$$x_i - (x_i + t_{ni} - y_i) \ge 0$$

The maximum allowable crash-time constraints follow; the right-hand sides are simply the difference between t_{ni} and t_{ci} :

$$y_{A} \ge 3$$

$$y_{B} \ge 4$$

$$y_{C} \ge 2$$

$$y_{D} \ge 2$$

$$y_{E} \ge 4$$

Activity	Normal Time (t _{ni})	Max. Crash Time (t _{ci})	Normal Cost (<i>C_{ni}</i>)	Crash Cost (<i>C_{ci}</i>)	K
А	10	7	\$5,000	\$11,000	\$2,000
В	14	10	\$9,000	\$13,000	\$1,000
С	6	4	\$5,000	\$10,000	\$2,500
D	11	9	\$6,000	\$9,000	\$1,500
E	8	4	\$4,000	\$6,000	\$500

Exhibit C.11Normal and Crash Data

Finally, to account for the desired project completion time of 35 days, we add the constraint

$$x_{\rm F} = 35$$

as well as nonnegativity restrictions. We will ask you to set up an Excel model and solve this as an exercise.

Learning Objective

To be able to use Excel Solver to solve linear optimization models on spreadsheets.

USING EXCEL SOLVER

In this section we illustrate how to use Excel Solver to solve linear programs. To do so, we return to the Softwater, Inc. example. The first step is to construct a spreadsheet model for the problem, such as the one shown in Exhibit C.12 for Softwater. The problem data are given in the range of cells A4:E8 in the same way that we write the mathematical model. Cells B12 and C12 provide the values of the decision variables. Cells B15:B17 provide the left-hand side values of the constraints. For example, the formula for the left side of the packaging constraint in cell B15 is =B6*B12+C6*C12. The value of the objective function, =B5*B12+C5*C12, is entered in cell B19.

To solve the problem, select the Solver option from the Tools menu in the Excel control panel. The Solver dialog box will appear as shown in Exhibit C.13. The target cell is the one that contains the objective function value. Changing cells are those that hold the decision variables. Constraints are constructed in the constraint box or edited by using the Add, Change, or Delete buttons. Excel does not assume nonnegativity; thus, this must be added to the model by checking the "Assume Nonnegative" box in the Options dialog. You should also check "Assume Linear Model" in the Options dialog. Then click on Solve in the dialog box, and Excel will indicate that an optimal solution is found as shown in Exhibit C.14. Exhibit C.15 shows the final results in the spreadsheet.

Exhibit C.12Excel Model for the Softwater, Inc. Problem

	Α	В	С	D	Е
1	Softwater, Inc. Linear	r Program			
2					
3	Problem Data				
4	Product	40-lb bag	80-lb bag		Right-hand
5	Profit/unit	\$ 2.00	\$ 4.00		side
6	Packaging line	1.2	3	<=	1,500
7	Materials availability	6	10	<=	6,000
8	Agg. production	40	80	>=	16,000
9					
10	Decision Variables				
11	Product	40-lb bag	80-lb bag		
12	Amount produced	0	0		
13					
14	Constraints				
15	Packaging line	0			
16	Materials availability	0			
17	Agg. production	0			
18					
19	Profit	\$ -			

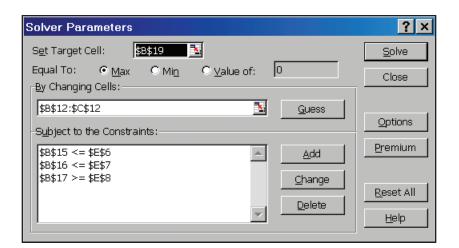


Exhibit C.13Solver Dialog Box

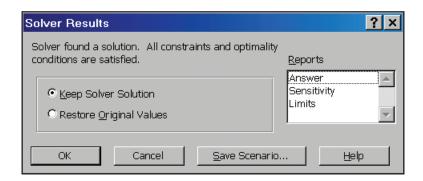


Exhibit C.14Solver Results Dialog

	А	В	С	D	Е
1	Softwater, Inc. Linear	r Program			
2					
3	Problem Data				
4	Product	40-lb bag	80-lb bag		Right-hand
5	Profit/unit	\$ 2.00	\$ 4.00		side
6	Packaging line	1.2	3	<=	1,500
-7	Materials availability	6	10	<=	6,000
8	Agg. production	40	80	>=	16,000
9					
10	Decision Variables				
11	Product	40-lb bag	80-lb bag		
12	Amount produced	500	300		
13					
14	Constraints				
15	Packaging line	1500			
16	Materials availability	6000			
17	Agg. production	44000			
18					
19	Profit	\$2,200.00			

Exhibit C.15Results of Solver Solution

After a solution is found, Solver allows you to generate three reports—an Answer Report, a Sensitivity Report, and a Limits Report—from the solution dialog box as shown in Exhibit C.14. These reports are placed on separate sheets in the Excel workbook. The Answer Report (Exhibit C.16) provides basic information about the solution. The Constraints section requires further explanation. "Cell Value" refers to the left side of the constraint if we substitute the optimal values of the decision variables:

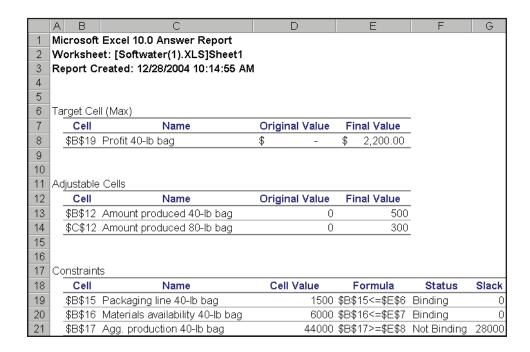
```
Packaging line: 1.2(500) + 3(300) = 1,500 minutes
Materials: 6(500) + 10(300) = 6,000 square feet
Production: 40(500) + 80(300) = 44,000 pounds
```

We see that the amount of time used on the packaging line and the amount of materials used are at their limits. We call such constraints *binding*. However, the aggregate production has exceeded its requirement by 44,000 - 16,000 = 28,000 pounds. This difference is referred to as the *slack* in the constraint. In general, slack is the absolute difference between the left and right sides of a constraint. The last two constraints in this report are nonnegativity and can be ignored.

The top portion of the Sensitivity Report (Exhibit C.17) tells us the ranges for which the objective-function coefficients can vary without changing the optimal values of the decision variables. They are given in the "Allowable Increase" and "Allowable Decrease" columns. Thus, the profit coefficient on 40-pound bags may vary between 1.6 and 2.4 without changing the optimal product mix. If the coefficient is changed beyond these ranges, the problem must be solved anew. The lower portion provides information about changes in the right-side values of the constraints. The shadow price is the change in the objective function value as the right side of a constraint is increased by one unit. Thus, for the packaging-line constraint, an extra minute of line availability will improve profit by 67 cents. Similarly, a reduction in line availability will reduce profit by 67 cents. This will hold for increases or decreases within the allowable ranges in the last two columns.

Finally, the Limits Report (Exhibit C.18) shows the feasible lower and upper limits on each individual decision variable if everything else is fixed. Thus, if the number of 80-pound bags is fixed at 300, the number of 40-pound bags can go as





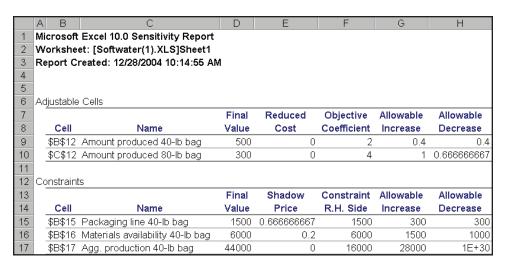


Exhibit C.17
Solver Sensitivity Report

	АВ	С	D	Е	F	G	Н		J
1	Microsoft	Excel 10.0 Limits Report							
2	Workshe	et: [Softwater(1).XLS]Sheet	1						
3	Report C	reated: 12/28/2004 10:14:55	AM						
4									
5									
6		Target							
7	Cell	Name	Value						
8	\$B\$19	Profit 40-lb bag	\$2,200.00						
9									
10									
11		Adjustable		Lo	wer	Target		Upper	Target
12	Cell	Name	Value	<u>_Li</u>	mit	Result		Limit	Result
13	\$B\$12	Amount produced 40-lb bag	500		0	1200		500	2200
14	\$C\$12	Amount produced 80-lb bag	300		0	1000		300	2200

Exhibit C.18
Solver Limits Report

low as zero or as high as 500 and maintain a feasible solution. If it drops to zero, the objective function (target result) will be \$1,200.

Modeling and Solving the Transportation Problem on a Spreadsheet

Exhibit C.19 is a spreadsheet model of the Foster Generators transportation problem along with the Solver Parameters dialog box from Excel. Cells in the range A3:F8 provide the model data. Solver works best when all the data have approximately the same magnitude. Hence, we have scaled the supplies and demands by expressing them in thousands of units so that they are similar to the unit costs. Thus, the values of the decision variables and total cost are also expressed in thousands of dollars.

In Exhibit C.19, the range B13:E15 corresponds to the decision variables (amount shipped) in the problem. The amount shipped out of each origin (Cleveland, Bedford, and York), cells F13:F15, is the sum of the changing cells for that origin. These values cannot exceed the supplies in F4:F6. For destinations (Boston, Chicago, St. Louis, and Lexington), the amount shipped into each destination is the sum of the changing cells for that destination (cells B16:E16), which must be equal to the demands in cells B8:E8. The target cell to be minimized, B18, is the total cost. The solution to the Foster Generators transportation problem is summarized in Exhibit C.20.

Exhibit C.19Foster Generators
Transportation Optimal Solution

	А	В	С	D	Е	F
1	Foster Generators					
2						
3		Boston	Chicago	St. Louis	Lexington	Supply (000)
4	Cleveland	3	2	7	6	5
5	Bedford	7	5	2	3	6
6	York	2	5	4	5	2.5
7						
8	Demand (000)	6	4	2	1.5	
9						
10	Decision variables	(000)				
11						
12		Boston	Chicago	St. Louis	Lexington	Amount Shipped
13	Cleveland	3.5	1.5	0	0	5
14	Bedford	0	2.5	2	1.5	6
15	York	2.5	0	0	0	2.5
16	Amount Shipped	6	4	2	1.5	
17						
18	Total cost (\$000)	\$39.50				

Exhibit C.20Foster Generators
Transportation Summary

Route				
From	То	Units Shipped	Unit Cost (\$)	Total Cost (\$)
Cleveland Cleveland Bedford Bedford Bedford York	Boston Chicago Chicago St. Louis Lexington Boston	3,500 1,500 2,500 2,000 1,500 2,500	3 2 5 2 3 2	10,500 3,000 12,500 4,000 4,500 5,000 39,500

SOLVED PROBLEMS

SOLVED PROBLEM #1

Par, Inc., is a small manufacturer of golf equipment and supplies. Par has been convinced by its distributor that a market exists for both a medium-priced golf bag, referred to as a *standard* model, and a high-priced golf bag, referred to as a *deluxe* model. The distributor is so confident of the market that if Par can produce the bags at a competitive price, the distributor has agreed to purchase all the bags that Par can manufacture over the next three months. A careful analysis of the manufacturing requirements resulted in Exhibit C.21.

The director of manufacturing estimates that 630 hours of cutting and dyeing time, 600 hours of sewing time, 708 hours of finishing time, and 135 hours of inspection and packaging time will be available for the production of golf bags during the next three months.

Solution:

Let x_1 be the number of standard bags to produce and x_2 be the number of deluxe bags to produce. The con-

Production Time (hours per bag) Cutting/ Inspection/ **Profit** Sewing **Product** Dyeing **Finishing Packaging** per Bag Standard 7/10 $1/_{2}$ 1/10 \$10 2/3 Deluxe 5/6 1/4 \$ 9

Exhibit C.21
Data for Solved Problem 1

straints represent the limitations on the time available in each department. The LP model is

$$\max_{7/10} 10x_1 + 9x_2$$
$$7/10x_1 + 1x_2 \le 630$$

$$1/2x_1 + 5/6x_2 \le 600$$

 $1x_1 + 2/3x_2 \le 708$
 $1/10x_1 + 1/4x_2 \le 135$
 $x_1, x_2 \ge 0$

SOLVED PROBLEM #2

Make or buy. The Carson Stapler Manufacturing Company forecasts a 5,000-unit demand for its Sure-Hold model during the next quarter. This stapler is assembled from three major components: base, staple cartridge, and handle. Until now Carson has manufactured all three components. However, the forecast of 5,000 units is a new high in sales volume, and the firm doubts that it will have sufficient production capacity to make all the components. It is considering contracting a local firm to produce at least some of the components. The production-time requirements per unit are given in Exhibit C.22 at the bottom of the page.

After considering the firm's overhead, material, and labor costs, the accounting department has determined the unit manufacturing cost for each component. These data, along with the purchase price quotations by the contracting firm are given in Exhibit C.23. Formulate a linear programming model for the make-or-buy decision for Carson that will meet the 5,000-unit demand at a minimum total cost.

Solution:

Let

 x_1 = number of units of the base manufactured

 x_2 = number of units of the cartridge manufactured

 x_3 = number of units of the handle manufactured

 x_4 = number of units of the base purchased x_5 = number of units of the cartridge purchased x_6 = number of units of the handle purchased

Min $0.75x_1 + 0.40x_2 + 1.10x_3 + 0.95x_4 + 0.55x_5 + 1.40x_6$

subject to

$$0.03x_1 + 0.02x_2 + 0.05x_3 \le 400$$
 (Dept. A)
 $0.04x_1 + 0.02x_2 + 0.04x_3 \le 400$ (Dept. B)
 $0.02x_1 + 0.03x_2 + 0.01x_3 \le 400$ (Dept. C)
 $x_1 + x_4 = 5,000$
 $x_2 + x_5 = 5,000$
 $x_3 + x_6 = 5,000$
 $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$

Exhibit C.23 Additional Data for Solved Problem 2

Component	Manufacturing Cost (\$)	Purchase Cost (\$)
Base Cartridge Handle	\$0.75 0.40 1.10	\$0.95 0.55 1.40

Department	Base	Cartridge	Handle	Total Department Time Available (hours)
A B C	0.03 0.04 0.02	0.02 0.02 0.03	0.05 0.04 0.01	400 400 400

Exhibit C.22
Data for Solved Problem 2

SOLVED PROBLEM #3

Consider the transportation problem data shown next.

	<u>Cost per Unit</u>				
	<u>Fairport</u>	<u>Mendon</u>	<u>Penfield</u>	<u>Supply</u>	
Corning	16	10	14	600	
Geneva	12	12	20	300	
Demand	300	200	300		

- a. Develop a linear program for this problem.
- b. Find the optimal solution using Excel Solver

Solution:

a. Min $16x_{11} + 10x_{12} + 14x_{13} + 12x_{14} + 12x_{15} + 20x_{16}$ subject to

Corning: $x_{11} + x_{12} + x_{13} \le 600$ Geneva: $x_{21} + x_{22} + x_{23} \le 300$ Fairport: $x_{11} + x_{21} = 300$ Mendon: $x_{21} + x_{22} = 200$ Penfield: $x_{31} + x_{32} = 300$ $x_{ij} \ge 0$

	A	В	C:	D.	E
1 2 3 4	Transportation Mod	lel.			
2					
3		Fairport	Mendon	Penfield	Supply
4	Corning	16	10	14	600
5	Geneva	12	12	20	300
6					
7	Demand (000)	300	200	300	
8					
9					
10	Decision variables				
11					
12		Fairport	Mendon	Penfield	Amount Shipped
13	Corning	0	200	300	500
14	Geneva	300	.0	- 0	300
15	Amount Shipped	300	200	300	0
16					
17					
18	Total cost (\$000)	\$9,800.00		7	

KEY TERMS AND CONCEPTS

Constraint
Decision variables
Feasible solutions
Linear functions
Linear program
Nonnegativity constraints
Objective function

Objective function coefficients
Optimal solution
Optimization models
Solution
Transportation problem
Transportation tableau

QUESTIONS FOR REVIEW AND DISCUSSION

- 1. What is an optimization model? Provide some examples of optimization scenarios in operations management.
- 2. What is an objective function?
- 3. What are the characteristics of a linear function?
- 4. Explain the difference between a feasible solution and an optimal solution.
- 5. Describe different applications of the transportation problem. That is, to what might origins and destinations correspond in different situations?

- 6. Explain how to model linear programs on a spreadsheet and solve them using Microsoft Excel's Solver.
- 7. Describe how to handle the following special situations in the transportation problem:
- a. Unequal supply and demand.
- b. Maximization objective.
- c. Unacceptable transportation routes.

PROBLEMS AND ACTIVITIES

1. The Erlanger Manufacturing Company makes two products. The profit estimates are \$25 for each unit of product 1 sold and \$30 for each unit of product 2 sold. The labor-hour requirements for the products in the three production departments are shown in the following table.

	Proc	<u>duct</u>
<u>Department</u>	<u>1</u>	<u>2</u>
А	1.50	3.00
В	2.00	1.00
С	0.25	0.25

The departments' production supervisors estimate that the following number of labor-hours will be available during the next month: 450 hours in department A, 350 hours in department B, and 50 hours in department C.

- a. Develop a linear programming model to maximize profits.
- b. Find the optimal solution. How much of each product should be produced, and what is the projected profit?
- c. What are the scheduled production time and slack time in each department?
- 2. M&D Chemicals produces two products sold as raw materials to companies manufacturing bath soaps, laundry detergents, and other soap products. Based on an analysis of current inventory levels and potential demand for the coming month, M&D's managers have specified that the total production of products 1 and 2 combined must be at least 350 gallons. Also, a major customer's order for 125 gallons of product 1 must be satisfied. Product 1 requires 2 hours of processing time per gallon, and product 2 requires 1 hour; 600 hours of processing time are available in the coming month. Production costs are \$2 per gallon for product 1 and \$3 per gallon for product 2.
 - Determine the production quantities that will satisfy the specified requirements at minimum cost.

- b. What is the total product cost?
- c. Identify the amount of any surplus production.
- 3. Photo Chemicals produces two types of photographdeveloping fluids. Both products cost Photo Chemicals \$1 per gallon to produce. Based on an analysis of current inventory levels and outstanding orders for the next month, Photo Chemicals managers have specified that at least 30 gallons of product 1 and at least 20 gallons of product 2 must be produced during the next two weeks. They have also stated that an existing inventory of highly perishable raw material required in the production of both fluids must be used within the next two weeks. The current inventory of the perishable raw material is 80 pounds. Although more of this raw material can be ordered if necessary, any of the current inventory that is not used within the next two weeks will spoil-hence the management requirement that at least 80 pounds be used in the next two weeks.

Furthermore, it is known that product 1 requires 1 pound of this perishable raw material per gallon and product 2 requires 2 pounds per gallon. Since the firm's objective is to keep its production costs at the minimum possible level, the managers are looking for a minimum-cost production plan that uses all the 80 pounds of perishable raw material and provides at least 30 gallons of product 1 and at least 20 gallons of product 2. What is the minimum-cost solution?

4. Managers of High Tech Services (HTS) would like to develop a model that will help allocate technicians' time between service calls to regular-contract customers and new customers. A maximum of 80 hours of technician time is available over the two-week planning period. To satisfy cash flow requirements, at least \$800 in revenue (per technician) must be generated during the two-week period. Technician time for regular customers generates \$25 per hour. However, technician time for new customers generates an average of only \$8 per hour because in

many cases a new-customer contact does not provide billable services. To ensure that new-customer contacts are being maintained, the time technicians spend on new-customer contacts must be at least 60 percent of the time technicians spend on regularcustomer contacts. Given these revenue and policy requirements, HTS would like to determine how to allocate technicians' time between regular customers and new customers so that the total number of customers contacted during the two-week period will be maximized. Technicians require an average of 50 minutes for each regular-customer contact and 1 hour for each new-customer contact. Develop a linear programming model that will enable HTS to determine how to allocate technicians' time between regular customers and new customers.

5. Product mix. Better Products, Inc. is a small manufacturer of three products it produces on two machines. In a typical week, 40 hours of time are available on each machine. Profit contribution and production time in hours per unit are given in the following table:

		Product	
	<u>1</u>	<u>2</u>	<u>3</u>
Profit/unit	\$30	\$50	\$20
Machine 1 time/unit	0.5	2.0	0.75
Machine 2 time/unit	1.0	1.0	0.5

Two operators are required for machine 1. Thus, 2 hours of labor must be scheduled for each hour of machine 1 time. Only one operator is required for machine 2. A maximum of 100 labor-hours is available for assignment to the machines during the coming week. Other production requirements are that product 1 cannot account for more than 50 percent of the units produced and that product 3 must account for at least 20 percent of the units produced.

- a. How many units of each product should be produced to maximize the profit contribution? What is the projected weekly profit associated with your solution?
- b. How many hours of production time will be scheduled on each machine?
- 6. Hilltop Coffee manufactures a coffee product by blending three types of coffee beans. The cost per pound and the available pounds of each bean are given in the following table:

<u>Bean</u>	Cost/Pound	Available Pounds
1	\$0.50	500
2	0.70	600
3	0.45	400

Consumer tests with coffee products were used to provide quality ratings on a 0-to-100 scale, with higher ratings indicating higher quality. Product-quality standards for the blended coffee require a consumer rating for aroma to be at least 75 and a consumer rating for taste to be at least 80. The aroma and taste ratings for coffee made from 100 percent of each bean are given in the following table:

<u>Bean</u>	Aroma Rating	Taste Rating
1	75	86
2	85	88
3	60	75

It is assumed that the aroma and taste attributes of the coffee blend will be a weighted average of the attributes of the beans used in the blend.

- a. What is the minimum-cost blend of the three beans that will meet the quality standards and provide 1,000 pounds of the blended coffee product?
- b. What is the bean cost per pound of the coffee blend?
- 7. Ajax Fuels, Inc. is developing a new additive for airplane fuels. The additive is a mixture of three liquid ingredients: A, B, and C. For proper performance, the total amount of additive (amount of A + amount of B + amount of C) must be at least 10 ounces per gallon of fuel. For safety reasons, however, the amount of additive must not exceed 15 ounces per gallon of fuel. The mix or blend of the three ingredients is critical. At least one ounce of ingredient A must be used for every ounce of ingredient B. The amount of ingredient C must be greater than one-half the amount of ingredient A. If the cost per ounce for ingredients A, B, and C is \$0.10, \$0.03, and \$0.09, respectively, find the minimumcost mixture of A, B, and C for each gallon of airplane fuel.
- 8. *Production routing*. Lurix Electronics manufactures two products that can be produced on two different production lines. Both products have their lowest production costs when produced on the more modern of the two production lines. However, the modern production line does not have the capacity to handle the total production. As a result, some production must be routed to the older production line. Data for total production requirements, production-line capacities, and production costs are shown in the table at the top of page C27.

Formulate an LP model that can be used to make the production routing decision. What are the rec-

Table for Problem 8.

	<u>Production Cost/Unit</u>			
	Modern Line	Old Line	Minimum Production Requirement	
Product 1	\$3.00	\$5.00	500 units	
Product 2	\$2.50	\$4.00	700 units	
Production-line capacity	800	600		

ommended decision and the total cost? (Use notation of the form x_{11} = units of product 1 produced on line 1.)

9. The Two-Rivers Oil Company near Pittsburgh transports gasoline to its distributors by truck. The company has recently received a contract to begin supplying gasoline distributors in southern Ohio and has \$600,000 available to spend on the necessary expansion of its fleet of gasoline tank trucks. Three models of trucks are available, as shown in the table at the bottom of the page.

The company estimates that the monthly demand for the region will be 550,000 gallons of gasoline. Due to the size and speed differences of the truck models, they vary in the number of possible deliveries or round-trips per month; trip capacities are estimated at 15 per month for the Super Tanker, 20 per month for the Regular Line, and 25 per month for the Econo-Tanker. Based on maintenance and driver availability, the firm does not want to add more than 15 new vehicles to its fleet. In addition, the company wants to purchase at least three of the new Econo-Tankers to use on the short-run, low-demand routes. As a final constraint, the company does not want more than half of its purchases to be Super Tankers.

- a. If the company wants to satisfy the gasoline demand with minimal monthly operating expense, how many models of each truck should it purchase?
- b. If the company did not require at least three Econo-Tankers and allowed as many Super Tankers as needed, what would the optimal strategy be?

10. The Williams Calculator Company manufactures two models of calculators, the TW100 and the TW200. The assembly process requires three people, and the assembly times are given in the following table:

	<u>Assembler</u>				
	<u>1</u>	<u>2</u>	<u>3</u>		
TW100	4 min.	2 min.	3.5 min.		
TW200	3 min.	4 min.	3 min.		

Company policy is to balance workloads on all assembly jobs. In fact, managers want to schedule work so that no assembler will have more than 30 minutes more work per day than other assemblers. This means that in a regular 8-hour shift, all assemblers will be assigned at least 7.5 hours of work. If the firm makes a \$2.50 profit for each TW100 and a \$3.50 profit for each TW200, how many units of each calculator should be produced per day? How much time will each assembler be assigned per day?

11. An appliance store owns two warehouses and has three major regional stores. Supply, demand, and transportation costs for refrigerators are provided in the following table:

<u>Warehouse</u>	<u>A</u>	<u>B</u>	<u>C</u>	Supply
1	6	8	5	80
2	12	3	7	40
Demand	20	50	50	

- a. Set up the transportation tableau.
- b. Find an optimal solution using Excel.

Table for Problem 9.

	Capacity (gallons)	Purchase Cost (\$)	Monthly Operating Costs (\$)*
Super Tanker	5,000	\$67,000	\$550
Regular Line	2,500	55,000	425
Econo-Tanker	1,000	46,000	350

^{*}Includes depreciation

12. A product is produced at three plants and shipped to three warehouses, with transportation costs per unit as follows. Find the optimal solution.

<u>Warehouse</u>					
<u>Plant</u>	<u>W1</u>	<u>W2</u>	<u>W3</u>	Plant <u>Capacity</u>	
P1	20	16	24	300	
P2	10	10	Ê8	500	
P3	12	18	10	100	
Warehouse	200	400	300		
Demand					

13. Consider the following data for a transportation problem:

	Distribution Center					
<u>Plant</u>	<u>Los Angeles</u>	San Francisco	San Diego	<u>Supply</u>		
San Jose	4	10	6	100		
Las Vegas	8	16	6	300		
Tucson	14	18	10	300		
Demand	200	300	200			

- Construct the linear program to minimize total cost.
- b. Find an optimal solution.
- c. How would the optimal solution differ if we must ship 100 units on the Tucson to San Diego route? Explain how you can modify the model to incorporate this new information.
- d. Because of road construction, the Las Vegas to San Diego route is now unacceptable. Explain how you can modify the model to incorporate this new information.
- 14. Find the optimal solution to the transportation problem shown.

	<u>Customer Zone</u>						
Distribution Center	<u>1</u>	<u>2</u>	<u>3</u>	<u>Availability</u>			
А	2	8	10	50			
В	6	11	6	40			
С	12	7	9	30			
Demand	45	15	30				

15. Reconsider the Foster Generators transportation and facilities-location problem. Assume that the York plant location alternative is replaced with the Clarksville, Tennessee location. Using the 2,500-unit capacity for the Clarksville plant and the unit transportation costs shown at the top of the next column, determine the minimum-cost transportation-problem solution if the new plant is located in Clarksville.

Shipping from Clarksville to	<u>Unit Cost</u>
Boston	9
Chicago	6
St. Louis	3
Lexington	3

Compare the total transportation costs of the York and Clarksville plant locations. Which location provides for lower-cost transportation?

16. Forbelt Corporation has a one-year contract to supply motors for all refrigerators produced by the Ice Age Corporation. Ice Age manufactures the refrigerators at four locations around the country: Boston, Dallas, Los Angeles, and St. Paul. Plans call for these numbers (in thousands) of refrigerators to be produced at the four locations.

Boston	50
Dallas	70
Los Angeles	60
St. Paul	80

Forbelt has three plants that are capable of producing the motors. The plants and their production capacities (in thousands) follow:

Denver	100
Atlanta	100
Chicago	150

Because of varying production and transportation costs, the profit Forbelt earns on each lot of 1,000 units depends on which plant produced it and to which destination it was shipped. The accounting department estimates of the profit per unit (shipments are made in lots of 1,000 units) are as follows:

	Shipped to				
<u>Produced at</u>	<u>Boston</u>	<u>Dallas</u>	Los Angeles	St. Paul	
Denver	7	11	8	13	
Atlanta	20	17	12	10	
Chicago	8	18	13	16	

Given profit maximization as a criterion, Forbelt would like to determine how many motors should be produced at each plant and how many should be shipped from each plant to each destination.

17. Arnoff Enterprises manufactures the central processing unit (CPU) for a line of personal computers. The CPUs are manufactured in Seattle, Columbus, and New York and shipped to warehouses in Pittsburgh, Mobile, Denver, Los Angeles, and Washington, D.C.

for further distribution. The following data show the number of CPUs available at each plant and the number of CPUs required by each warehouse. The shipping costs (dollars per unit) are also shown.

- a. Determine the number of CPUs that should be shipped from each plant to each warehouse to minimize the total transportation cost.
- b. The Pittsburgh warehouse has just increased its order by 1,000 units, and Arnoff has authorized the Columbus plant to increase its production by 1,000 units. Do you expect this development
- to lead to an increase or a decrease in the total transportation cost? Solve for the new optimal solution
- 18. Set up an Excel model for the project-crashing problem in this supplement and find the optimal solution.
- 19. Develop and solve a linear programming model for crashing the Wildcat Software Consulting problem in Chapter 18.

Table for Problem 17b.

<u>Warehouse</u>						
<u>Plant</u>	<u>Pittsburgh</u>	<u>Mobile</u>	<u>Denver</u>	Los Angeles	<u>Washington</u>	<u>Supply</u>
Seattle	10	20	5	9	10	9,000
Columbus	2	10	8	30	6	4,000
New York	1	20	7	10	4	8,000
Demand	3,000	5,000	4,000	6,000	3,000	

CASES

HALLER'S PUB & BREWERY

Jeremy Haller of Haller's Pub & Brewery, described in the opening episode, has compiled data describing the amount of different ingredients and labor resources needed to brew the six different types of beers that the brewery makes. He also gathered financial information and estimated demand over a 26-week forecast horizon. These data are shown in Exhibit C.24. The profits for each batch of each type of beer are

Light Ale: \$3,925.78 Golden Ale: \$4,062.75 Freedom Wheat: \$3,732.34 Berry Wheat: \$3,704.49 Dark Ale: \$3,905.79 Hearty Stout: \$3,490.22

These values incorporate fixed overhead costs of \$7,500 per batch.

a. Use the data in Exhibit C.19 to validate the profit figures.

- b. How many batches should Haller plan to make of each product? Develop and solve an LP model.
- c. In the brewing business, the price of grain and hops fluctuates fairly regularly. Examine the effect of a 10 percent increase in the price of all grains and hops on the optimal solution.
- d. Customer demand for beer at Haller's, especially during holiday months and economic slowdowns, has a tendency to fluctuate just as the price of grains and hops. Examine the effect of a 10 percent decrease in overall customer demand.
- e. Due to a shrinking interest in Stout beer, Haller's would like to understand the effect on profitability of removing it from its product line. Assume that all Stout beer drinkers would be lost with the elimination of the beer (that is, they do not switch to another type of beer).

Summarize all your results in a memo to Jeremy Haller.

Exhibit C.24 Data for Haller's Pub & Brewery

	Amounts for one batch (14 Barrels-30 kegs-4350 pints) of beer							
	Light Ale (A)	Golden Ale (G)	Freedom Wheat	Berry Wheat	Dark Ale	Hearty Stout	Availability	Cost per Unit
Percent of Demand	27%	22%	19%	10%	11%	11%		
American 2-Row Grain (lb.)	525.00	400.00	375.00	350.00	450.00	375.00	30,000	\$0.35
American 6-Row Grain (lb.)		125.00	125.00	150.00	250.00	225.00	8,000	\$0.40
American Crystal Grain (lb.)	175.00					175.00	5,000	\$0.42
German Vienna Grain (lb.)	125.00		200.00	175.00	50.00		5,000	\$0.45
Flaked Barley (lb.)	75.00		150.00	150.00		75.00	5,000	\$0.47
Light Dry Malt Extract (lb.)	35.00		45.00	50.00	25.00		2,000	\$0.37
Hallertauer Hops (lb.)	4.00	3.00	2.00	2.00	8.00		500	\$0.32
Kent Goldings Hops (lb.)		1.00			4.00	4.00	500	\$0.29
Tettnanger Hops (lb.)	4.00		2.00	2.00	4.00	2.00	500	\$0.31
Brewing Labor (hr.)	70.00	72.00	81.00	83.00	75.00	96.00	4,032	\$18.00
Average # Pints per Batch	4,350	4,350	4,350	4,350	4,350	4,350		
Beer Price (per pint)	\$3.00	\$3.00	\$3.00	\$3.00	\$3.00	\$3.00		
Avg. Demand (pints/week)	2,153	1,755	1,515	798	877	877		
Avg. Demand (batches/wk)	0.495	0.403	0.348	0.183	0.202	0.202		
Avg. Demand (batches/26 wks)	12.870	10.487	9.057	4.767	5.243	5.243		

HOLCOMB CANDLE

Holcomb Candle has signed a contract with a national chain of discount department stores to supply a seasonal candle set in the checkout aisle of its 15,000 stores. Eight feet of display space has been designated for candles in each store. The different types of candles that Holcomb produces are 8-ounce jars, 4-ounce jars, 6-inch pillars, 3-inch pillars, and 4-packs of votive candles. The contract signed with the store specifies that at least 2 feet must be dedicated to 8-ounce jars, at least 2 feet to 6-

inch pillars, and at least 1 foot to votives. The number of jars shipped should be at least as many as the number of pillars shipped.

Holcomb recently bought 200,000 pounds of wax for a special price. Its inventory also includes 250,000 feet of spooled wick and 100,000 ounces of holiday fragrances. Relevant data are given in Exhibit C.25. Formulate an LP model, solve it, and explain what the solution means for the company.

Exhibit C.25 Data for Holcomb Candle

	8-oz. Jar	4-oz. Jar	6-in. Pillar	3-in. Pillar	Votive pack	Available
Wax (lb.)	0.5	0.25	0.5	0.25	0.3125	200,000
Fragrance (oz.)	0.24	0.12	0.24	0.12	0.15	100,000
Wick (ft.)	0.43	0.22	0.58	0.33	0.80	250,000
Display space (ft.)	0.48	0.24	0.23	0.23	0.26	124,000
Sales price	\$0.76	0.44	0.74	0.42	0.72	
Manufacturing cost	\$0.52	0.25	0.51	0.21	0.55	

ENDNOTES

¹ Makuch, William M., Dodge, Jeffrey L., Ecker, Joseph E., Granfors, Donna C., and Hahn, Gerald J., "Managing Consumer Credit Delinquency in the U.S. Economy: A Multi-Billion Dollar Management Science Application," *Interfaces* 22, no. 1, January–February 1992, pp. 90–109.

² Field, Richard C. "National Forest Planning is Promoting US Forest Service Acceptance of Operations Research," *Interfaces* 14, no. 5, September–October 1984, pp. 67–76.