University of Technology - HCMC
Faculty of Applied Mathematics

Calculus 1 Project
Semester 2 (2021 - 2022)

Instruction

The students work in a group. Using Matlab or Python to solve the following problems and write a report.

The report must include 3 parts:

i) The theory and algorithm (your understanding);

ii) The Matlab or Python commands (explain the core steps);

iii) The results and conclusion.

I will inform the exact time (the last week of semester) to submit the report via email.

Project 1.

Problem 1. Given the parametric equations

$$\begin{cases} x(t) = -2t^2 + 1, \\ y(t) = t^3 - 4t - 2 \end{cases}$$

Find a intersection of the curve and the tangents at that point. Draw the figure.

Problem 2. Study the Simpson rule to approximate the definite integral of a function and its error. Using that method to approximate the following integral and estimate the error).

a) $\int\limits_1^2 1/x dx$, divide $[1, 2]$ into 10 and 20 subsintervals.

b) $\int\limits_0^2 e^{-x^2} dx$, divide $[0, 2]$ into 10 and 20 subsintervals.

Problem 3. Study the Euler method to approximate the solution of first order differential equations. The world's population in 1990 was about 5 billion, and data show that birth rates range from 35 to 40 per thousand per year and death rates from 15 to 20. Take this to imply a net annual growth rate of 20 per thousand. One model for world population assumes constant per capita growth, with a per capita growth rate of $20/1000 = 0.02$.

a) Write a differential equation for P that expresses this assumption. Use P to denote the world population, measured in billions.

b) According to the differential equation in (a), at what rate (in billions of persons per year) was the world population growing in 1990?

c) By applying Euler's method to this model, using the initial value of 5 billion in 1990, estimate the world population in the years 1980, 2000, 2040, and 2230. Present a table of successive approximations that stabilizes with one decimal place of accuracy (in billions). What step size did you have to use to obtain this accuracy?

Project 2.

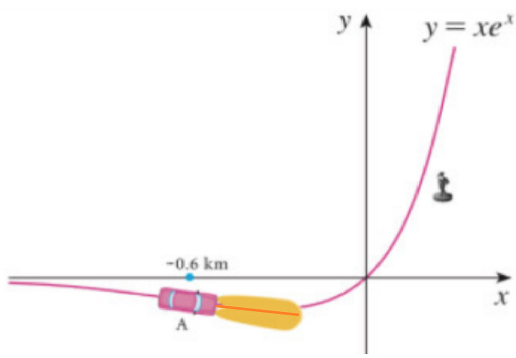
Problem 1. Given the parametric equations

$$\begin{cases} x(t) = t^3 - 4t - 2, \\ y(t) = -2t^2 + 1 \end{cases}$$

Find a intersection of the curve and the tangents at that point. Draw the figure.

Problem 2. A car moves along a path of street with the shape is the graph of the function $y = xe^x$. At each position, the light ray from the car is considered to be tangent to the street. There is a statue at the coordinates $(0.35, 0.37)$ (km). Assume that the car moves at a constant velocity 60 km/h. How long does the car take in moving from A, that has x-coordinate $= -0.6$ to B, where the light ray spot to the statue.

The trapezoidal formula is a using to approximate the integral of a function. Study the formula and use it to approximate the path AB



Problem 3. (a) Study the Euler method to approximate the solution of first order differential equations. Program a calculator or computer to use Euler's method to compute $y(1)$, where $y(x)$ is the solution of the initial-value problem

$$\frac{dy}{dx} + 3x^2y = 6x^2, \quad y(0) = 3$$

(i) h=1 (ii) h=0.1 (iii) h = 0.01 (iv) h =0.001

(b) Verify that $y = 2 + e^{-x^3}$ is the exact solution of the differential equation. (c) Find the errors in using Euler's method to compute $y(1)$ with the step sizes in part (a). Draw the figure in describing the exact solution and approximated solution in (a).