HCM University of Technology
Department of applied mathematics
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Team work projects
Subject: Linear Algebra

1 Topics

1) (LU factorization) Given A be a square matrix size n. Factorizing LU of A is the process of finding a lower triangular matrix L and a upper triangular matrix U such that $A = LU$. Write a code to find matrices L, U for a given A. Knowing that the **main diagonal** of L contains only number 1.

2) (Applying LU in solving a linear system) Given a square matrix A_n and a vector $b_{n\times 1}$. Using the code in previous question to find L, U. Then solving the following systems: Find y such that $Ly = b$ and find x such that $Ux = y$.

3) The following tables indicate the birth rate and survival rate of a population of woodland caribou and the number of individuals in each group of age class in 1990, respectively.

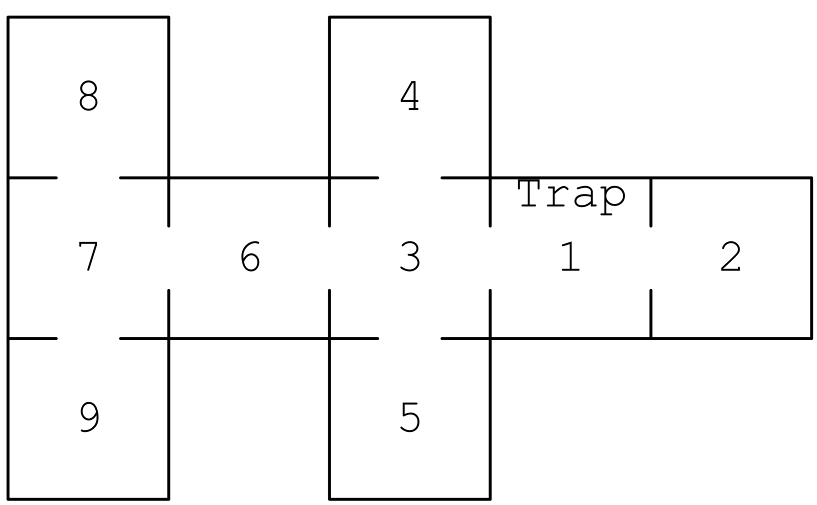
Age (years)	Birth Rate	Survival Rate
0–2	0.0	0.3
2–4	0.4	0.7
4–6	1.8	0.9
6–8	1.8	0.9
8–10	1.8	0.9
10–12	1.6	0.6
12–14	0.6	0.0

National Park, 1990

Age (years)	Number
0–2	10
2–4	2
4–6	8
6–8	5
8–10	12
10–12	0
12–14	1

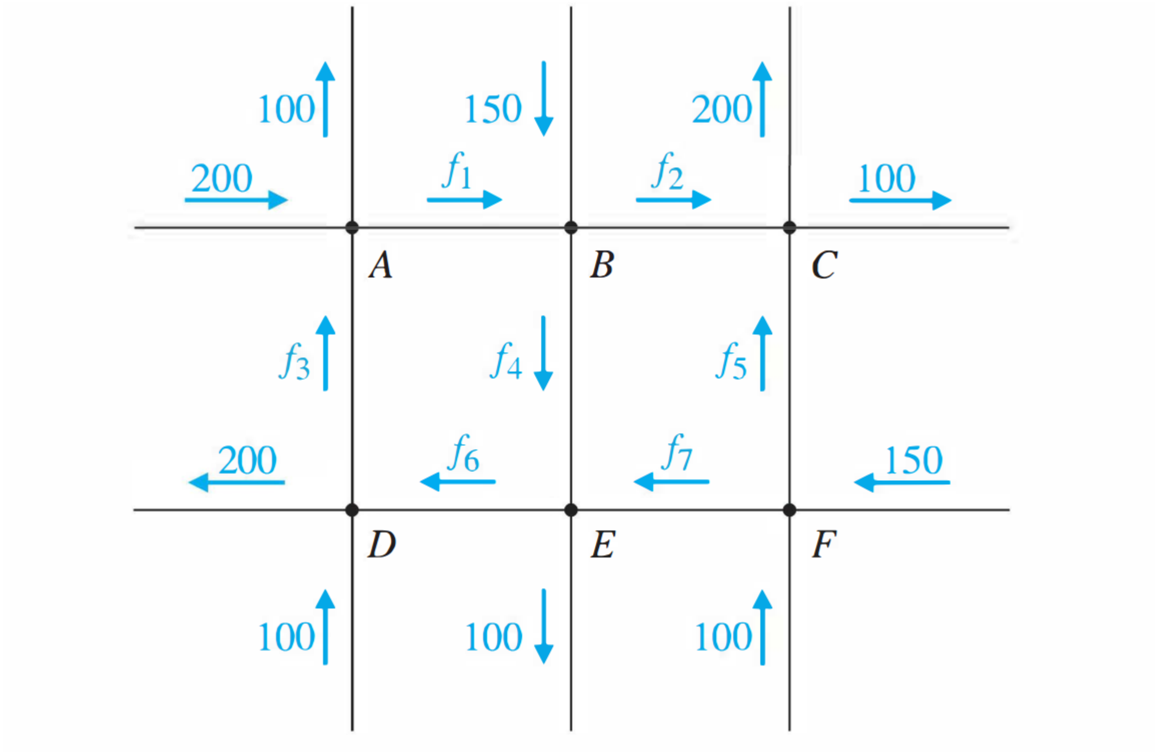
- Write a code to find the number of individuals in each group of class age in T, where T is some year after 1990 (for instance, $T = 2022$).
- In a long run, can you give some conclusions about the number of members in each group?

4) A mouse trap is placed in room 1 of the house with the pictured floor plan. Each time the mouse comes into room 1, he is trapped with probability p = 0.1. If he is not trapped, he leaves each room by one of its exits, chosen at random.



- Find the matrix transition describing the path of the mouse through the house.
- A vector q is called **steady** vector of Markov model if $Pq = q$ where P is matrix transition, find q.
- Suppose that the mouse starts in Room 4, what is the probability that the mouse in Room 1 after 3 steps?

5) The next figure indicates a traffic network, the number of vehicles being in and out at each node. Suppose that all streets are one way.



– Set up a system to find the unknown flows.
– Solve the system when $f_1 = 100,\ f_6 = 150$.
– Solve the system when $f_4 = 0$, then what will the range of flow be on each of the other branches?

2 Requirements

- Students write a report of project with the following contents:
  – Mathematical background of LU, Leslei model, Markov model.
  – The detailed solution of each problem
  – Code and examples.

  Some must have INFORMATION to be included in the report
  – Full, EXACT information of each member and they must coincide with those in the EXCEL file on BKEL, if not, your grade will not be valuable.
  – Contribution of each member. The total is 100%.
  – List all the references you use.

References

[1] D. Poole, *Linear Algebra: A Modern Introduction*, Boston, MA, USA:Cengage Learning, 2014.