



Performance Metrics of Collective Coordinated Motion in Flocks



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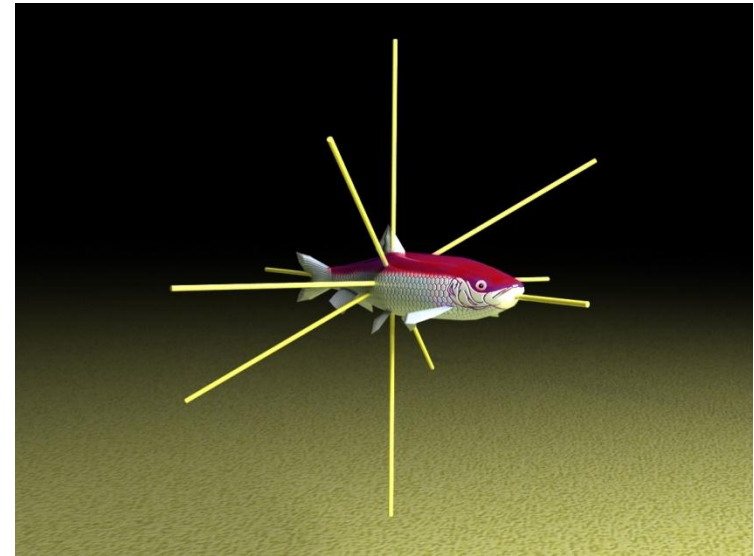
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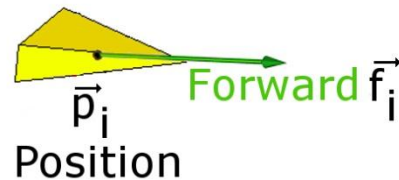
Overview

- **Boids model:** a brief review of *boids* model where our metrics are applied.
- **Previous metrics:** a review of the metric that have been used in order to evaluate fish school: extension, polarization and frequency of collision.
- **Proposed metrics:** a set of three new quantitative metrics are propose in order to evaluate and design flocks: consistency in extension, consistency in polarization and quality.

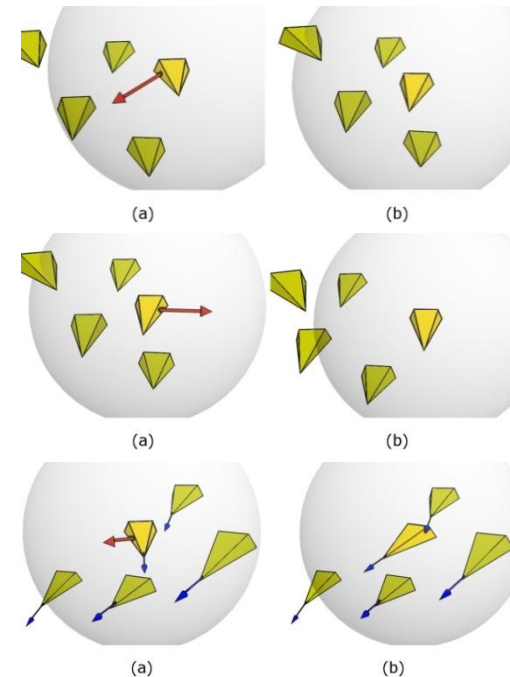


Boids model

Boids is an artificial life model which simulates the flocking behavior of birds (or fishes). In the *Boids* model each agent has a position vector \vec{p}_i and a forward vector \vec{f}_i that describe its motion in space.

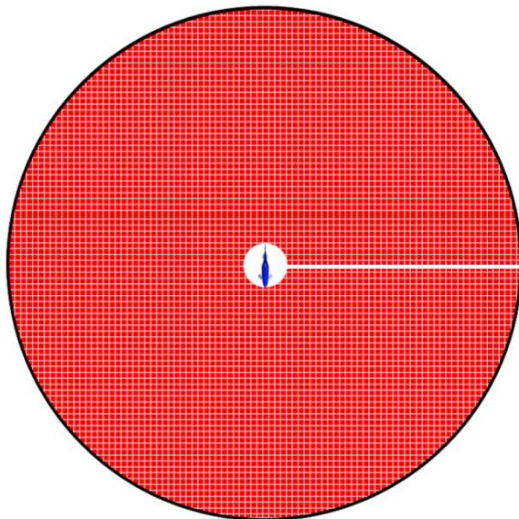


- **Cohesion:** the purpose of cohesion behavior is to move an agent towards the center of a group perceived within its neighborhood.
- **Separation:** the purpose of separation behavior is to move an agent to avoid a collision with their neighbors.
- **Alignment:** The purpose of alignment behavior is to move an agent in the same direction as their neighbors.

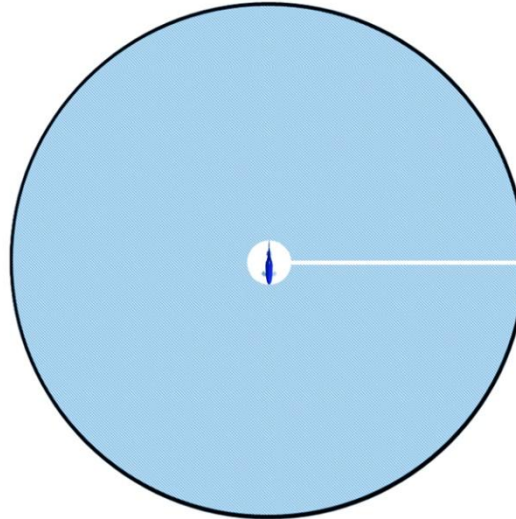


Boids model

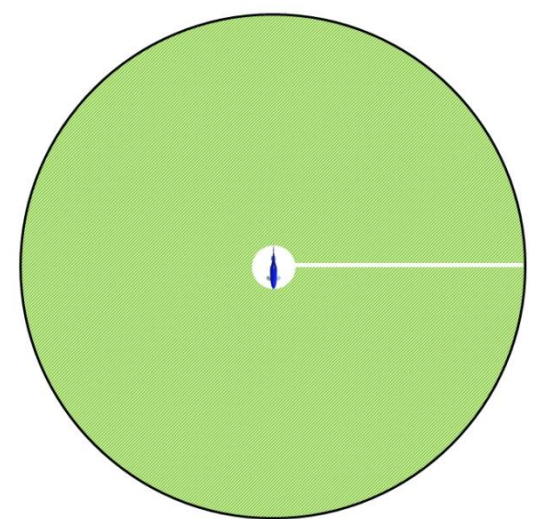
- Each *bird* has a local view of its environment called “area of perception” related to a steering behavior.
- The area of perception is determined by a radius where only neighbors who are in the area of perception are selected for calculating certain steering behavior.



 r_s Separation radius



 r_a Alignment radius



 r_c Cohesion radius



- **Previous Metrics: extension, polarization and frequency of collisions.**



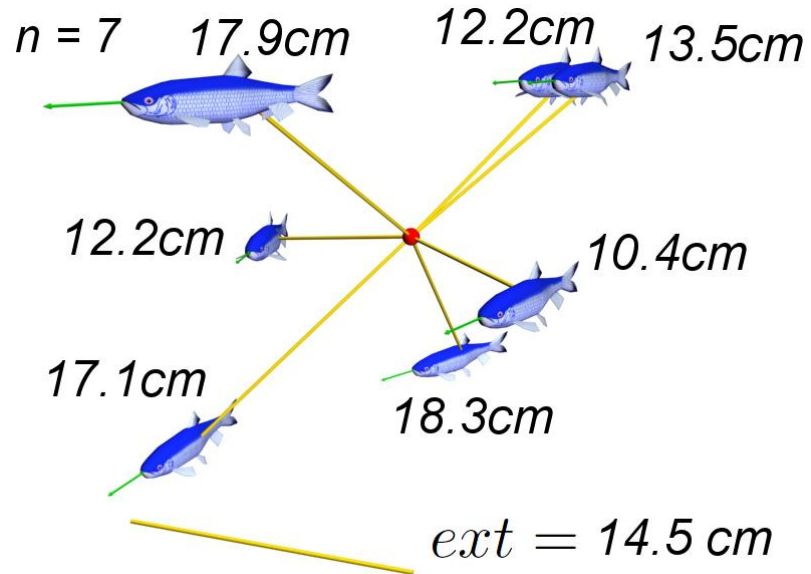
Extension

- The **extension** reflects the amount of cohesion of the flock and is determined by the average distance between one agent and the center of the flock.
- The center of the flock is expressed:

$$c\vec{e}n = \frac{1}{n} \sum_{i=1}^n \vec{p}_i$$

- The **extension** of the flock is expressed:

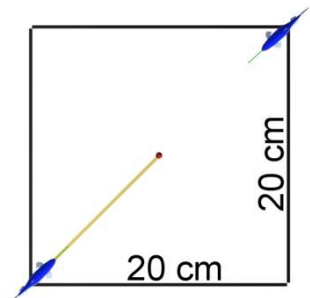
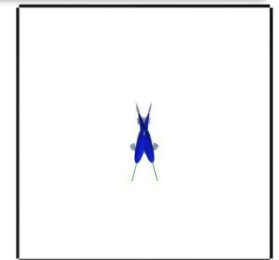
$$ext = \frac{1}{n} \sum_{i=1}^n \|c\vec{e}n - \vec{p}_i\|$$



Size of flock $n = 2$.

- The minimum extension is 0 cm .
 $ext = 0 \text{ cm}$
- The maximum extension max_{ext} depends on the shape and size of the pond, the number of *boids* and their distribution.

$$max_{ext} \approx ext \approx 28.2 \text{ cm}$$



Polarization

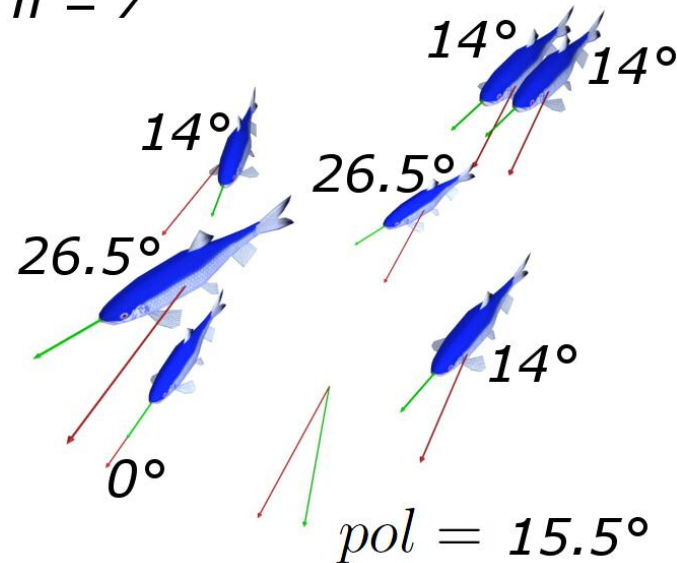
- The **polarization** expresses the degree of alignment of the agents headings and is defined as the average of the angular deviation of each agent with respect to the average orientation.
- The average orientation of the flock is expressed:

$$\vec{\mu}_p = \frac{1}{n} \sum_{i=1}^n \vec{f}_i$$

- The **polarization** of the flock is expressed:

$$pol = \frac{1}{n} \sum_{i=1}^n \angle(\vec{f}_i, \vec{\mu}_p)$$

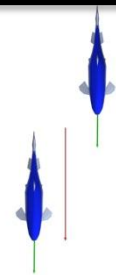
$n = 7$



Size of flock $n = 2$.

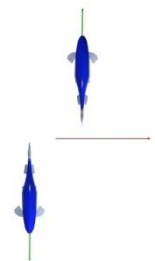
- The minimum polarization is 0° .

$$pol = 0^\circ$$



- The maximum polarization is 90° .

$$pol = 90^\circ$$



Frequency of collision

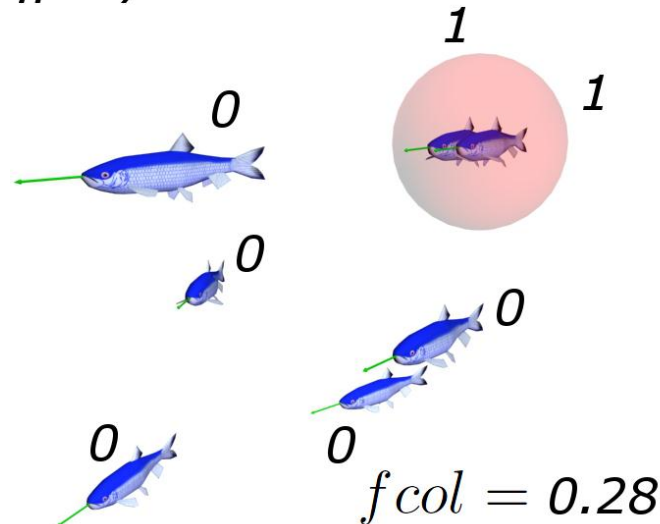
- The **frequency of collision** represents the degree of conflict among agents and is defined as the average of the number of agents in the collision state.
- The number of agents in collision state is expressed:

$$col = \sum_{i=1}^n c_i$$

- The **frequency of collision** of the flock is expressed:

$$fcol = \frac{1}{n} col$$

$n = 7$



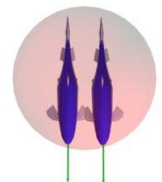
Size of flock $n = 2$.

- The minimum frequency of collision is 0.

$$fcol = 0$$

- The maximum frequency of collision is 1.

$$fcol = 1$$





- **Proposed metrics: consistency in extension, consistency in polarization and quality**

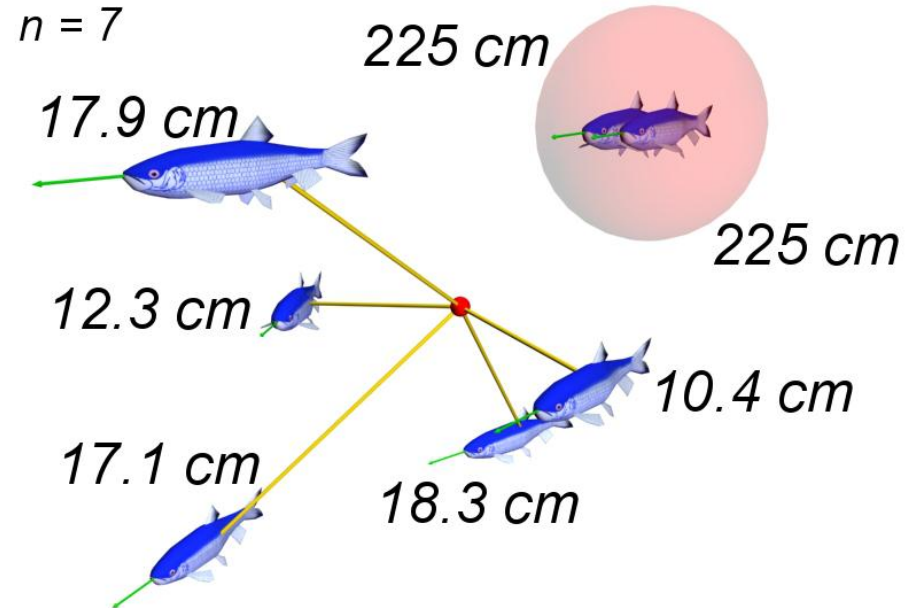


Consistency in extension

- The consistency in extension aims at balancing the values of extension and frequency of collision.
- The **consistency in extension** of the flock is expressed :

$$cns_{ext} = 1 - \frac{\sum_{i=1}^m \|c\vec{e}n - \vec{p}_i\| + k \cdot col}{max_e \cdot n}$$

- Where k is a distance penalty that holds a value in the range $[0, max_e]$ and m to denote the number of boids that do not collide with each other.
- cns_{ext} holds a value in the range $[0, 1]$, where 0 indicates the worst consistency and 1 the optimum consistency.



$$cns_{ext} = 1 - \frac{76 + 225 \cdot 2}{225 \cdot 7} = 0.67$$

- In our experiments the value of maximum extension max_e and constant penalty k is 225cm.

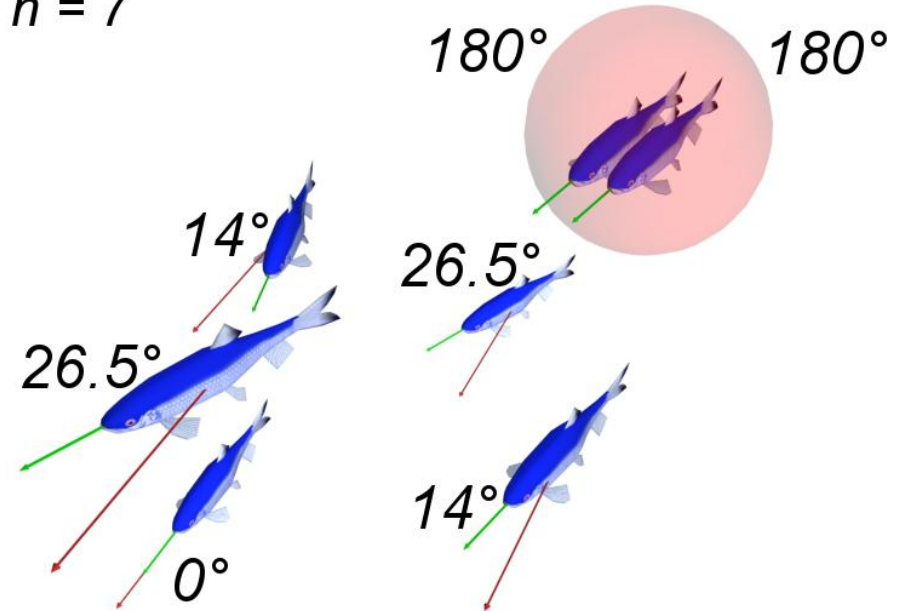
Consistency in polarization

- The consistency in polarization aims at balancing the values of polarization and frequency of collision.
- The consistency in polarization is denoted as cns_{pol} and it is calculated by applying:

$$cns_{pol} = 1 - \frac{\sum_{i=1}^m \angle(\vec{f}_i, \vec{\mu}_p) + \rho \cdot col}{180 \cdot n}$$

- Where ρ is an angle penalty that holds a value in the range $[0, 180^\circ]$ and m to denote the number of agents that do not collide with each other.
- The cns_{pol} holds a value in the range $[0, 1]$, where 0 indicates the worst consistency and 1 the optimum consistency.

$n = 7$



$$cns_{pol} = 1 - \frac{81 + 180 \cdot 2}{180 \cdot 7} = 0.65$$

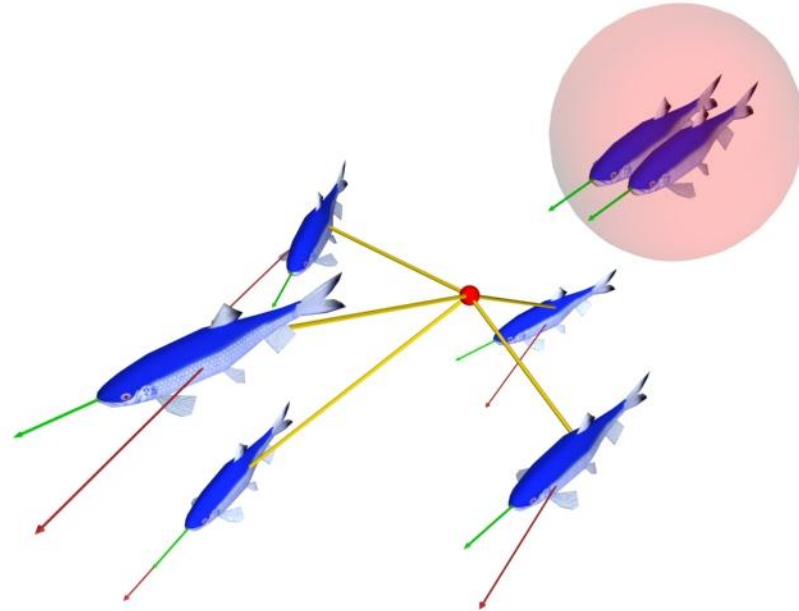
- In our experiments the value of constant penalty ρ is 180° .

Quality

- The quality aims at establishing a criterion to combine the results in the consistency in both extension and polarization, in such a way that we can evaluate the global performance of a flock.
- The quality is weighted by the factors σ and γ which determine, respectively, the influence of consistency in the extension and polarization on the final result.
- The quality is expressed in:

$$qlty = \sigma \cdot cns_{ext} + \gamma \cdot cns_{pol}$$

where $0 \leq \sigma \leq 1$, $0 \leq \gamma \leq 1$ and $\sigma + \gamma = 1$

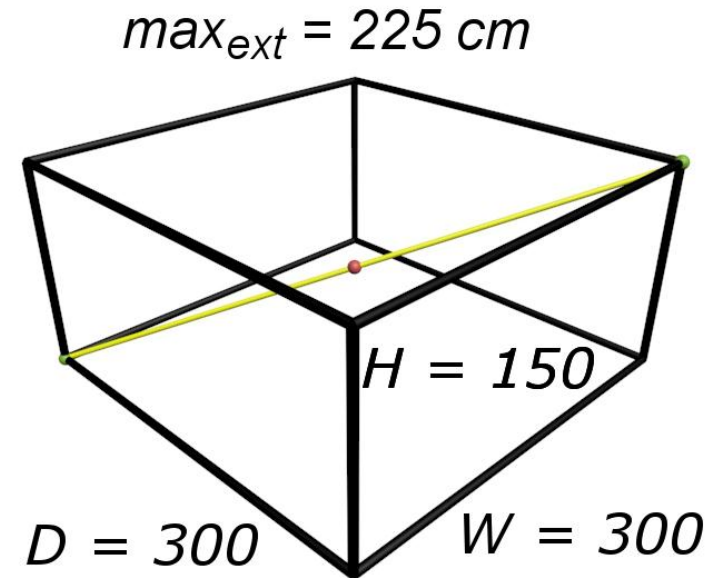
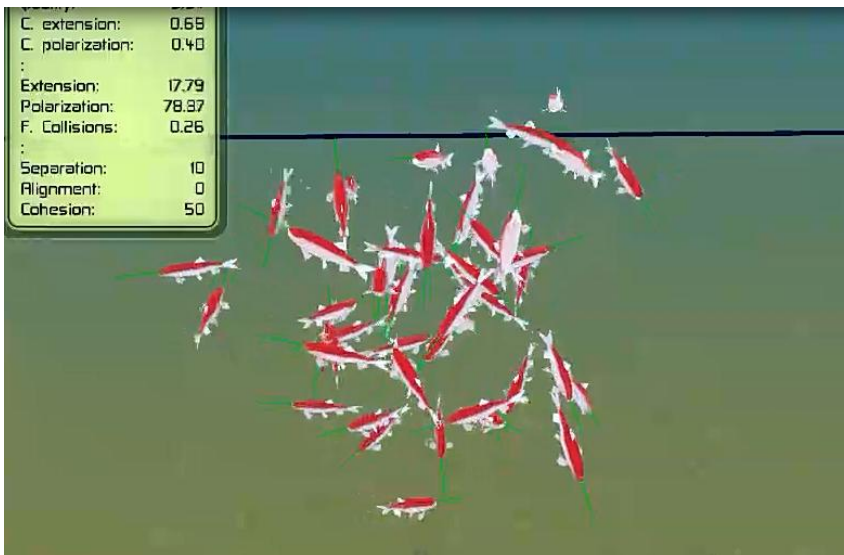


$$qlty = 0.5(0.67) + 0.5(0.65) = 0.66$$

- In our experiments the value of σ and γ is 0.5.

Experiments

- There are 50 agents initially distributed in random positions in a cuboid. Where the maximum distance that separates a agent from the central point of the pond is 50cm, in such a way that they can be perceived by each other.

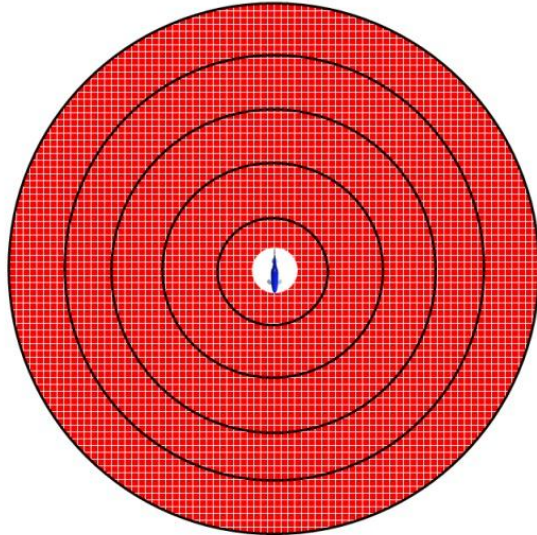


The agents avoid colliding with the walls of the pond so periodic boundary conditions are not applied.

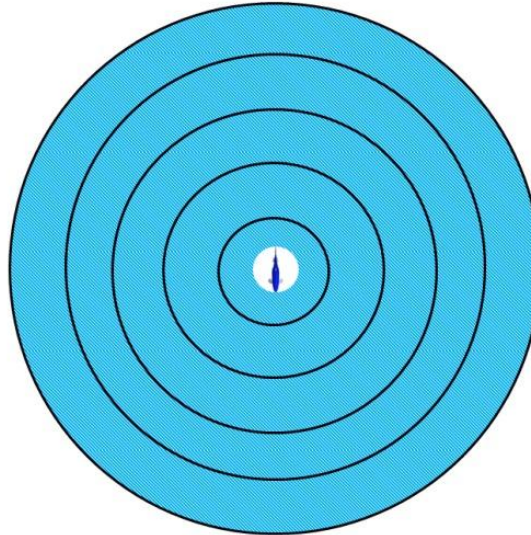
Experiments

- The experiments consisted in varying the radii r_s and r_a that determines the area of perception associated with the behaviors of separation and alignment in order to estimate the performance of the flocks under different configurations. Cohesion radius $r_c = 50\text{ cm}$ for all experiments.

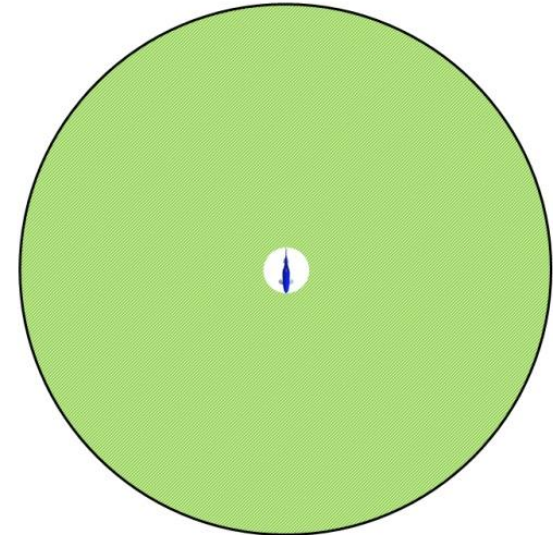
Separation radius



Alignment radius



Cohesion radius



$r_s = (0, 10, 20, 30, 40, 50)\text{ cm}$



$r_a = (0, 10, 20, 30, 40, 50)\text{ cm}$

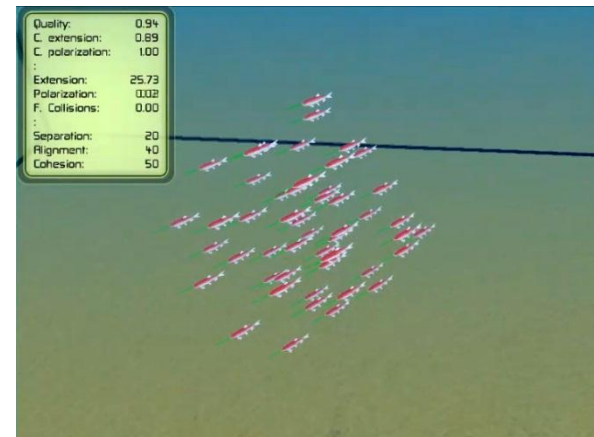
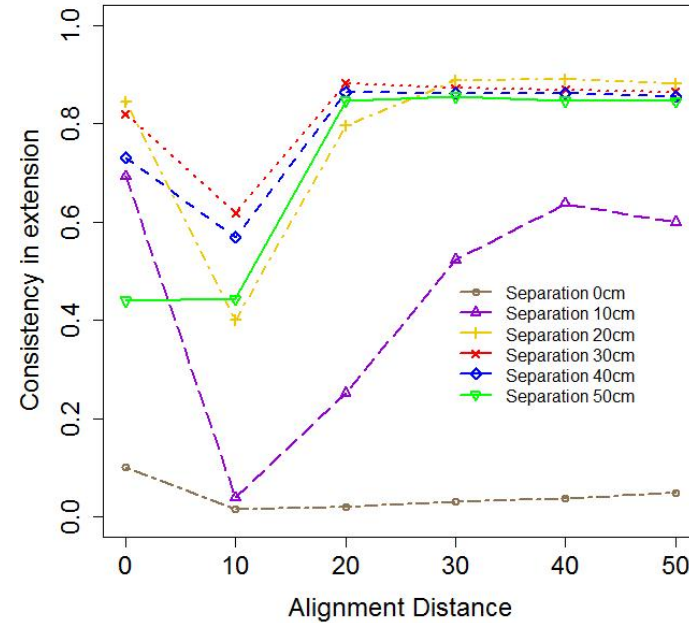
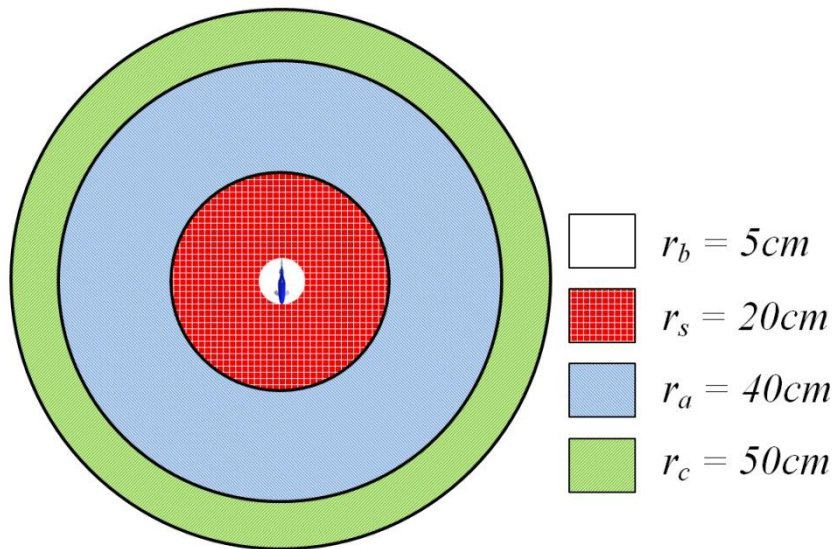


$r_c = 50\text{ cm}$

- 36 combinations are applied.

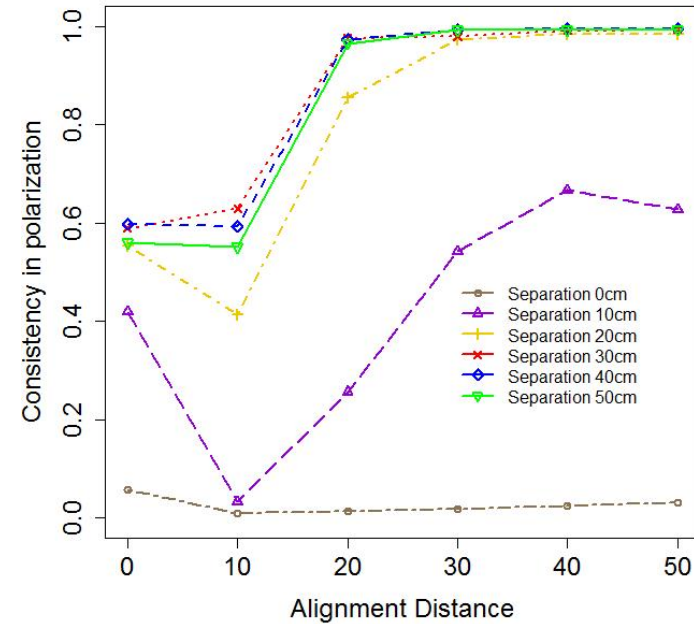
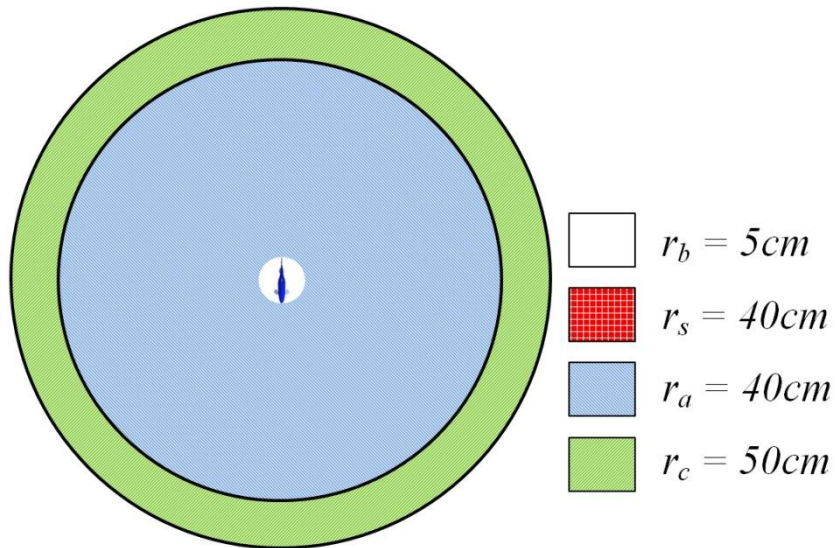
Results with consistency in extension

- The maximum value of average consistency in the extension is in the order of 0.88 with the combination show below:



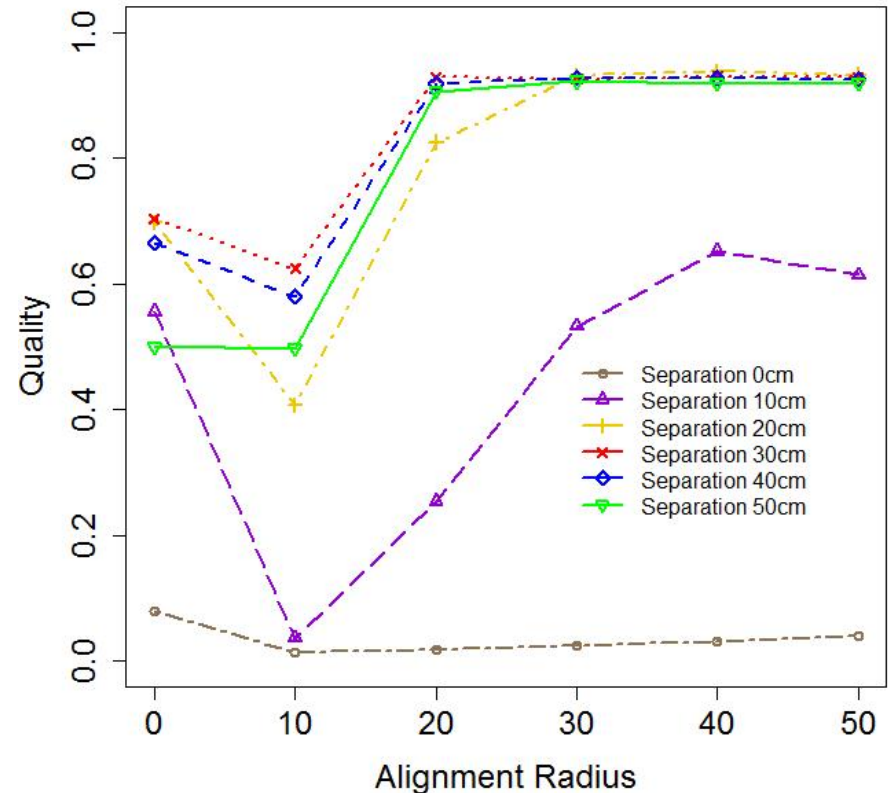
Results with consistency in polarization

- The maximum value of average consistency in the polarization is in the order of 0.99 with the combination show below:



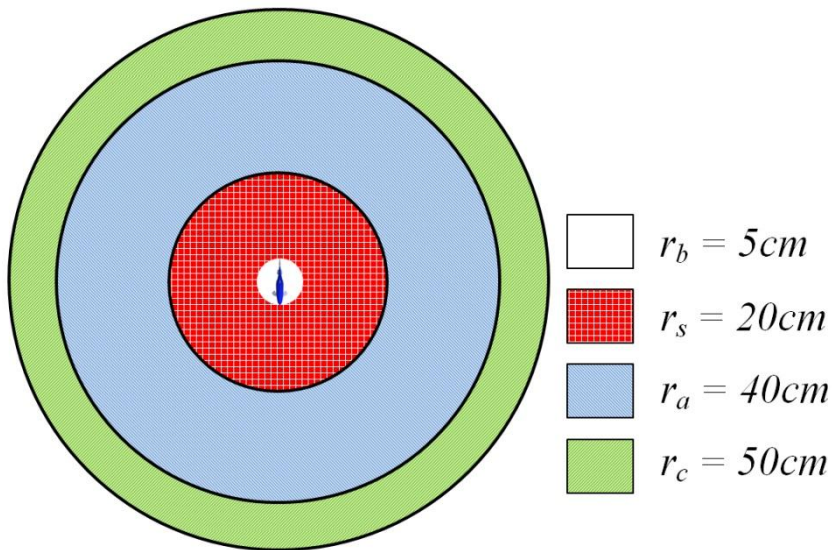
Results with quality

- Figure shows that the maximum value of total quality is in the order of ≈ 0.93 resulting from different configurations with which we can conclude that, If the integrity of the members of the flock is highly weighted, the flock is evaluated with a good quality under these scenarios:
- (1) The value of the separation radius is in a range that enables collision avoidance. This radius is in the range [20; 50].
- (2) The value of the alignment radius is greater than or equal to the radius of separation.
- (3) The value of the alignment radius is less than or equal to the radius of cohesion



Result with quality

- The maximum value of average quality is in the order of 0.93 with the combination show below:





Conclusions

- The metrics allow to establish a general benchmark for the evaluation of models of the type flock in terms of the spatial features.
- The parameters such as radius of perception for producing flocks able to reach the highest quality can be certainly estimated by our metrics. Therefore, the metrics proposed in this work can be used for the design flocks.





Thank you!

