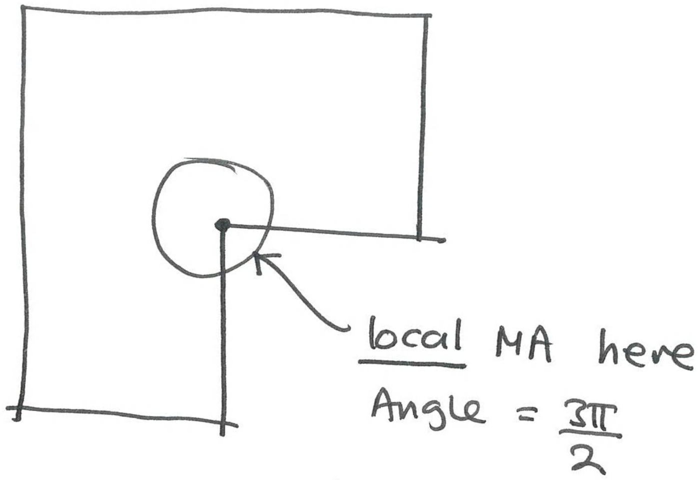
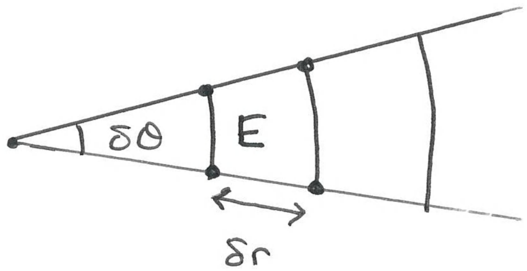


Local implementation of OT. (L_∞ case)

$$\alpha = \pi/\omega = 2/3$$

$$u \sim r^{2/3} \, Ⓗ(\theta) \qquad Ⓗ \text{ regular function of } \theta .$$

- Dominant interpolation error due to the r variation.
- Consider a mesh with constant angle $\delta\theta$



- L_∞ interpolation error E in an element is

$$E \sim \delta r^2 \, r^{\alpha - 2}$$

Seek to equidistribute E over an element

$$\delta r^2 \, r^{\alpha-2} \sim C$$

So if we have a computational variable s then in the limit:

$$\left(\frac{dr}{ds}\right)^2 r^{\alpha-2} \sim 1$$ (1 is a perfectly dle constan)

$$dr\, r^{\alpha/2-1} \sim ds$$

$$r^{\alpha/2} \sim s$$

$$\boxed{r \sim s^{2/\alpha}} \quad \text{eg} \quad \alpha = 2/3 \; ; \quad \boxed{r \sim s^3}$$

<u>THM 1</u> $r \sim s^{2/\alpha}$ equidistributes the L_∞ interpolation error.

⌈h-adapt is $\equiv$'t to equidistribution⌋

Now OT deals with area maps.

$$M(r)\,\text{Area}(r) = \text{Area}(s)$$

The area of a typical element is given by

$$E(r) \sim r\,dr\,d\theta \longleftarrow \text{same } \theta$$

r only map $\Rightarrow$ $E(s) \sim s\,ds\,d\theta$

$$\therefore \quad m(r)\, r\, dr = s\, ds$$

$$\therefore \quad m(r)\, r\, \frac{dr}{ds} = s \quad \Rightarrow \quad m(r) = \frac{1}{r}\, s \cdot \frac{ds}{dr}$$

$$s = r^{\alpha/2}$$

$$\frac{ds}{dr} = r^{\alpha/2 - 1}$$

$$\therefore \; M(r) = \frac{1}{r} \cdot r^{\alpha/2} \cdot r^{\alpha/2 - 1}$$

$$M(r) = r^{\alpha - 2}$$

THM 2 The monitor function

$$M(r) = r^{\alpha - 2}$$

in MA leads to a mesh which equidistributes the L_∞ interpolation error.

NOTE Simone takes

$$M(r) = r^{-2\beta}$$

$$\therefore \; \beta = -\frac{\alpha}{2} + 1 \quad \text{is optimal}$$

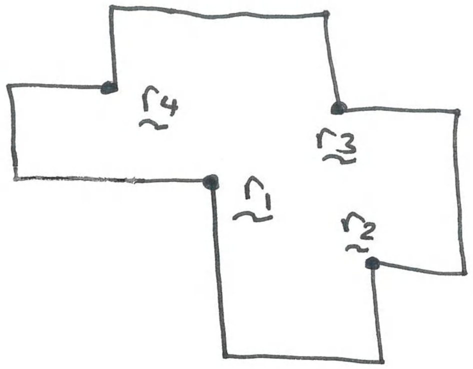
L-shape $\quad \beta = -\frac{1}{2} \cdot \frac{2}{3} + 1 = \frac{2}{3}$

This will lead to optimal L_∞- estimates.

* SIMONE'S *

CONFIRM THIS RESULT

Generalise to more corners &c



β_i can depend on the corner angle.

Take

$$M(\underset{\sim}{r}) = \sum_i \| \underset{\sim}{r} - \underset{\sim}{r}_i \|^{-2\beta_i} \qquad (*)$$

NOTE This has automatic regularisation.

*** Implementation 1** Solve full MA-eqn with monitor function (*)

!! This is costly !! And solving MA exactly is not needed.

*** Implementation 2**

- Let $M(\underset{\sim}{r}) = 1 + \| \underset{\sim}{r} - \underset{\sim}{r}_i \|^{-2\beta}$ locally
- solve (exactly) MA locally to give mesh at the corners
- Patch in the rest of the mesh using a standard mesh generator eg Delauney.

Skewness

From Budd et al. the skewness is given by

$$Q = \frac{1}{2}\left[\underbrace{\frac{1}{M(r)}\left(\frac{s}{r}\right)^2}_{r^{4/3}\, r^{-4/3}} + \underbrace{M(r)\left(\frac{r}{s}\right)^2}_{r^{-4/3}\, r^{4/3}} \right]$$

$= \text{const}$

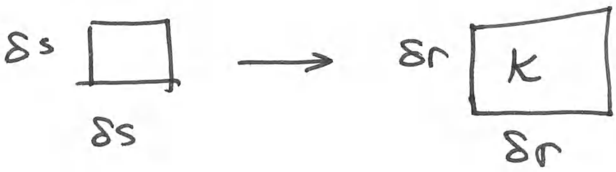
> THM 3 The OT generated meshes have CONSTANT skewness even as we approach the corners

(cf. Simone eqn (S2))

* Note this also implies dim V indept skewness *.

Local implementation of OT : L_2

Now consider a mesh element K



If $u \sim r^{2/3} f(\theta)$

The interpolation err is $\delta r^2\, r^{2/3-2} f(\theta)$

By THM 3 the mesh has constant skewness so we can approximate by a square.

The (L_2 estimate)^2 of the interpolation error is then proportional to:

$$\delta r^2 \cdot \underbrace{\left(\delta r^4 \, r^{4/3 - 4} \right)}_{\text{estimate}^2}$$

$$= \delta r^6 \, r^{-8/3} \equiv L_2^2 \, (\text{error})_k . \qquad (\#)$$

If we want to equidistribute this we get

$$\delta r^6 \, r^{-8/3} \sim \text{const}$$

ie $$\left(\frac{dr}{ds} \right)^6 r^{-8/3} \sim \text{const}$$

ie $$\left(\frac{dr}{ds} \right) r^{-4/9} \sim \text{const}$$

$$\Rightarrow \quad r^{5/9} \sim s$$

$$\boxed{r \sim s^{9/5}}$$

$$s \sim r^{5/9}$$

$$\frac{ds}{dr} \sim r^{-4/9}$$

$$m(r) = \frac{s}{r}\frac{ds}{dr} = \frac{r^{5/9}}{r} \, r^{-4/9} = r^{-8/9}$$

- Giving $4/9$ as the critical exponent.

- THIS is not quite consistent with Simone's result which gives 0.53 as the critical value

$\boxed{\text{?? THM 4 } \gamma = 4/9 \text{ will equidistribute the } \text{cell}_K \text{ estimate of the } L_2 \text{ interpolation err}}$

→ Simone shows in Fig 10 (a) that the total L_2 err is minimised at $\gamma \sim 0.53$ and in Fig 13 (a) that η_K is equidistributed (as $r \to 0$) at $\gamma \sim 0.53$.

$\boxed{\text{So}}$ how does η_K compare to the estimate (#) on p.6?

* NOTE FOR CRACK I get

L_∞ optimal $\gamma = 3/4$

L_2 optimal $\gamma = 1/2$