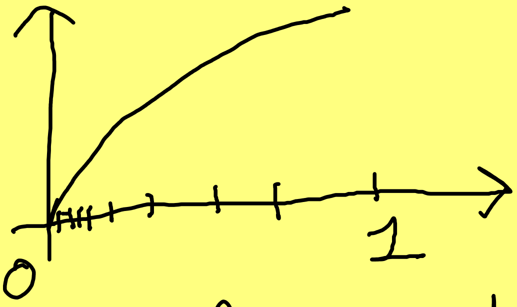
L-∞ error

$f(r) = r^{2/3}$ for L-shaped

$[0,\varepsilon]$

$x_J = \left(J/N\right)^{\beta}$ $\quad J = 0,\dots,[N\varepsilon]$



$p_{1,J}$ linear interpolant

$$\|f-p_{1,J}\|_{\infty,[0,1]} = \max\Big\{\underbrace{\|f-p_{1,J}\|_{[0,x_1)}}_{A},\ \underbrace{\|f-p_{1,J}\|_{[x_1,1]}}_{B}\Big\}$$

Split the error as $f \notin C^2[0,\varepsilon]$

$(|f'(x)| \to \infty \quad x \to 0)$

A Triangle ineq

$$\|f-p_{1,J}\|_{[0,x_1]} \le x_1^{2/3} + p_{1,J}(x_1) = 2\left(\frac{1}{N}\right)^{\beta\frac{2}{3}}$$

B $\|f-p_1\|_{[x_{J-1},x_J]} \le \frac{1}{8} h_J^2 \|f''\|_{[y_{J-1},y_J]}$

$$g(t) = t^{\beta} = \left(\frac{t}{N}\right)^{\beta}$$

M.V.T if $\beta > 1$

$$h_J = x_J - x_{J-1} = \frac{1}{N} g'(\xi) \le \frac{1}{N} g'(J/N) = \left(\frac{J}{N}\right)^{\beta-1}\frac{\beta}{N}$$

$$\|f''\|_{[x_{j-1},x_j]} \le \left|\frac{2}{3}\left(\frac{2}{3}-1\right)\right| \left(x_{j-1}\right)^{2/3-2}$$

$$\|f-p_1\|_{[x_{j-1},x_j]} \le \frac{1}{8}\left[\left(\frac{j}{N}\right)^{\beta-1}\frac{\beta}{N}\right]^2 \left(\frac{j-1}{N}\right)^{-\frac{4}{3}} \cdot \frac{2}{9}$$

$$= \frac{1}{36} N^{-2} \beta^2 \left(\frac{j}{N}\right)^{2(\beta-1)} \left(\frac{j-1}{N}\right)^{-4/3}$$

$$= \frac{1}{36} N^{-2+\frac{4}{3}+2(1-\beta)} \beta^2 \frac{j^{2(\beta-1)}}{(j-1)^{4/3}}$$

$$= \frac{1}{36} N^{\frac{4}{3}-2\beta} \beta^2 \frac{j^{2(\beta-1)}}{(j-1)^{4/3}}$$

$$\frac{4}{3}-6 = N^{-\frac{12}{3}} = N^{-4}$$

$$2(\beta-1) = \frac{4}{3} \Rightarrow$$

$$2\beta = \frac{4}{3}+2 = \frac{10}{3} = \boxed{5/3} > 1 \quad \checkmark$$

$$= \frac{1}{36} N^{\frac{4}{3}-\frac{10}{3}} \left(\frac{j}{j-1}\right)^{+4/3}$$

$$= \frac{1}{36}\left(1+\frac{1}{j-1}\right)^{4/3} N^{-2}$$

$$\|f - p_{1,N}\|_{\infty,[0,\varepsilon]} = \max\left\{ 2N^{-\frac{10}{9}}, \frac{2^{4/3}}{36} N^{-2} \right\}$$

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otherwise $\beta \cdot \frac{2}{3} = \frac{4}{3} - 2\beta$

$$\frac{4}{3} = \frac{2}{3}\beta + \frac{6}{3}\beta = + \frac{8}{3}\beta$$

$\boxed{\beta = 1/2}$

NO $\beta > 1$

$\frac{2}{3}\beta \nearrow \qquad 2\beta - \frac{4}{3} \nearrow$

$\frac{J^{2(\beta-1)}}{(J-1)^{4/3}} \longrightarrow$ must be decreasing

$$\frac{2(\beta-1) \cdot J^{2(\beta-1)-1} \cdot (J-1)^{4/3} - \frac{4}{3}(J-1)^{1/3} J^{2(\beta-1)}}{//} < 0$$

$$(J-1)^{\frac{1}{3}} J^{2(\beta-1)-1}\left[2(\beta-1)(J-1) - \frac{4}{3}J\right] < 0$$

$(\beta-1) < \frac{2}{3}\left(J/J-1\right) \qquad \beta < 1 + \frac{2}{3}\left(\frac{J}{J-1}\right)$

$$\beta < 1 + \frac{2}{3} = \boxed{5/3}$$

$$r^{-2\gamma+1} = r^{-5/3} \Rightarrow -2\gamma = -\frac{8}{3}$$

$$\boxed{\gamma = 4/3}$$

L2 norm

Piecewise lin. interpolation for $u \in H^2$

optimal mesh density function:

$$\rho = \left(1 + \frac{1}{\alpha}|u''|^2\right)^{1/5} \qquad h \sim |u''|^{-2/5}$$

$u(r) = r^{2/3}$ $\quad u'' = c\, r^{2/3 - 2 = -4/3}$

equidistribute e.r $\quad r^{-\frac{1}{3}} \quad h \sim r^{\frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}}$?

From $\|u - \Pi_h u\|^2_{L^2(I_J)} \leq C h_J^2 |u|^2_{H^1(I_J)}$

$$\Rightarrow \|u - \Pi_h u\|^2_{L^2(\Omega)} \leq \sum_{J=2}^{N} c\left(h_J \langle u \rangle^{2/3}_{H^1}\right)^3$$

$u = r^{2/3}$ $\quad \longrightarrow \quad \rho = \left(1 + \frac{1}{\alpha}|u'|^2\right)^{\frac{1}{3}}$

$u' = c\, r^{-1/3}$ $\quad$ $h \sim |u'|^{-2/3}$

$u' r = r^{2/3}$ $\quad$ $h \sim r^{-4/9}$