4-12

Given

$F = 6\,kN$

θ for $M_{A\,max}$ - ?

θ for $M_{A\,min}$ - ?

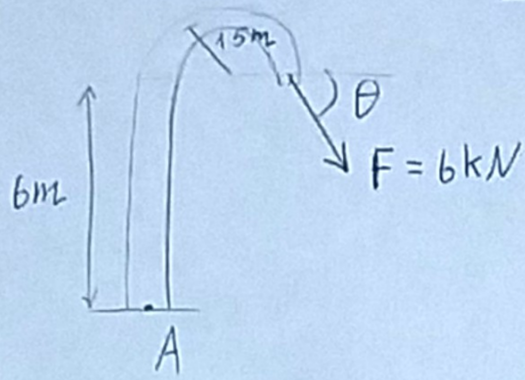
$(0^\circ \leq \theta \leq 180^\circ)$

$M_{A\,max}$ - ?

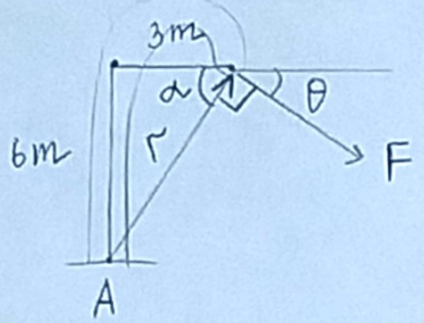
$M_{A\,min}$ - ?

Solution

Zarema Balgabekova



For $M_{A\,max}$, $F \perp r$



$\alpha = \tan^{-1}\left(\frac{6}{3}\right) = 63.4^\circ$

$\theta = 180 - (90 + 63.4) = 26.6^\circ$

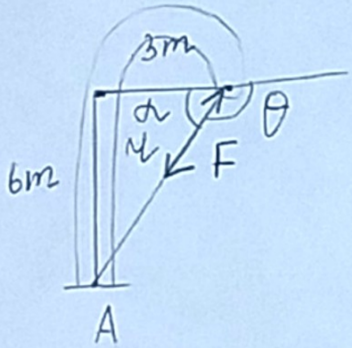
$r = \sqrt{6^2 + 3^2} = 6.71\,m$

$M_{A\,max} = rF\sin 90^\circ =$

$6.71 \cdot 6 = 40.3\ kN \cdot m$

For $M_{A\,min}$, F and r are collinear

$\theta = 180 - 63.4 = 117^\circ$



$M_{A\,min} = 6.71 \cdot 6 \cdot \sin 180^\circ = 0$

Answer

$\theta = 26.6^\circ$ for $M_{A\,max} = 40.3\ kN \cdot m$

$\theta = 117^\circ$ for $M_{A\,min} = 0$

4-42

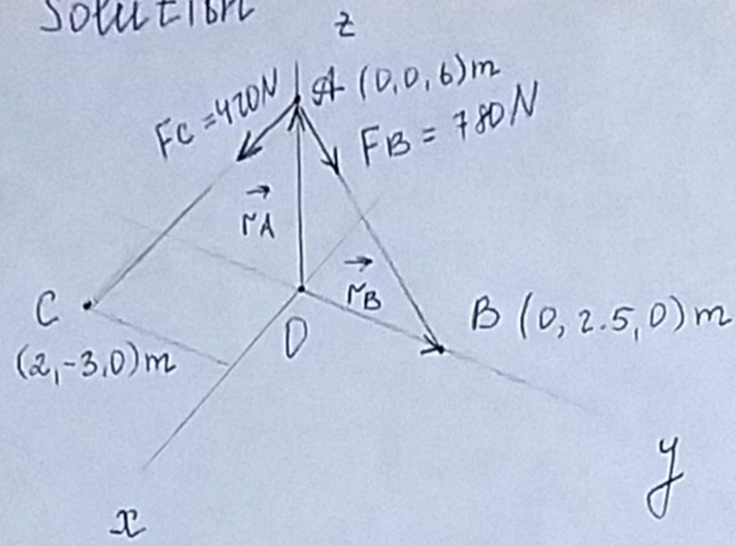
Given

$F_B = 780 N$

$F_C = 420 N$

$\vec{M}_{RO}$ - ?

Solution



Position vectors $\vec{r}_A = \{6 k\}$ m

$\vec{r}_B = \{2.5 j\}$ m

$$\vec{F}_B = F_B \hat{u}_{AB} = F_B \frac{\vec{r}_{AB}}{r_{AB}} = 780 \left(\frac{2.5\hat{j} - 6\hat{k}}{\sqrt{2.5^2 + (-6)^2}} \right) =$$

$$780 \left(\frac{5}{13}\hat{j} - \frac{12}{13}\hat{k} \right) = \{300\hat{j} - 720\hat{k}\} N$$

$$\vec{F}_C = F_C \hat{u}_{AC} = 420 \left(\frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{\sqrt{2^2 + (-3)^2 + (-6)^2}} \right) = \{120\hat{i} - 180\hat{j} - 360\hat{k}\} N$$

$$\vec{M}_{RO} = \Sigma(\vec{r} \times \vec{F}) = \vec{r}_B \times \vec{F}_B + \vec{r}_A \times \vec{F}_C =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2.5 & 0 \\ 0 & 300 & -720 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix} = -1800\hat{i} + 1080\hat{i} + 720\hat{j} =$$

$$\{-720\hat{i} + 720\hat{j}\} N\cdot m$$

Answer: $\vec{M}_{RO} = \{-720\hat{i} + 720\hat{j}\}$ N m

4-47

Given:

$\vec{F} = \{6\hat{i} + 8\hat{j} + 10\hat{k}\}\ N$

$\vec{M}_O = \{-14\hat{i} + 8\hat{j} + 2\hat{k}\}\ N \cdot m$

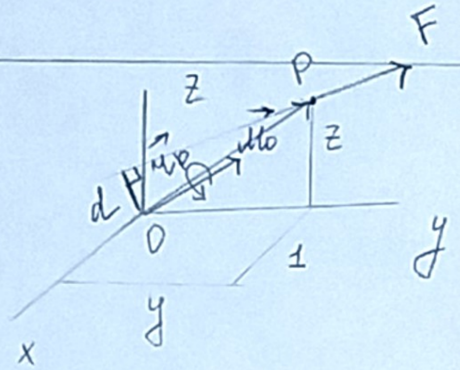
$P\ (1,\ y,\ z)\ m$

y, z - ?

d - ?

$(M_O = Fd)$

Solution



Position vector $\vec{r}_P = \{\hat{i} + y\hat{j} + z\hat{k}\}\ m$

$$\vec{M}_O = \vec{r}_P \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix} = \{(10y - 8z)\hat{i} - (10 - 6z)\hat{j} + (8 - 6y)\hat{k}\}\ N \cdot m$$

$$(10y - 8z)\hat{i} - (10 - 6z)\hat{j} + (8 - 6y)\hat{k} = -14\hat{i} + 8\hat{j} + 2\hat{k}$$

Therefore,

$$10y - 8z = -14$$
$$-10 + 6z = 8 \quad \Rightarrow \quad z = 3\ m$$
$$8 - 6y = 2 \quad \Rightarrow \quad y = 1\ m$$

$$M_O = Fd \quad \Rightarrow \quad d = \frac{M_O}{F} = \frac{\sqrt{(-14)^2 + 8^2 + 2^2}}{\sqrt{6^2 + 8^2 + 10^2}} = 1.15\ m$$

Answer: $y = 1\ m$, $z = 3\ m$

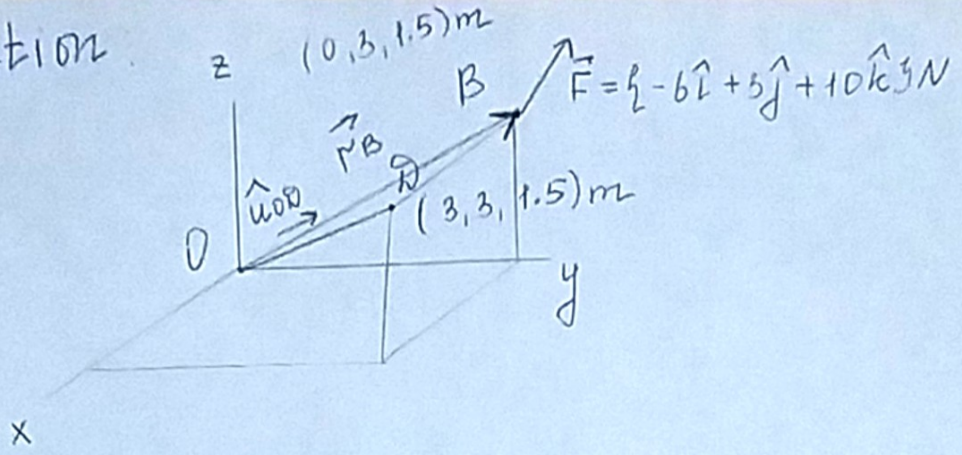
$d = 1.15\ m$

4-52

Given

$\vec{F} = \{-6\hat{i} + 3\hat{j} + 10\hat{k}\}$ N

Solution



$\vec{M}_{OD}$ - ?

Unit vector $\hat{u}_{OD}$ defines the direction of the diagonal OD

$$\hat{u}_{OD} = \frac{\vec{r}_{OD}}{r_{OD}} = \frac{3\hat{i} + 3\hat{j} + 1.5\hat{k}}{\sqrt{3^2 + 3^2 + 1.5^2}} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$$

Position vector $\vec{r}_B = \{3\hat{j} + 1.5\hat{k}\}$ m

$$M_{OD} = \hat{u}_{OD} \cdot (\vec{r}_B \times \vec{F}) = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 3 & \frac{3}{2} \\ -6 & 3 & 10 \end{vmatrix} = \frac{2}{3}\, 25.5 - \frac{2}{3}(-9) + \frac{1}{3}(-18)$$

$$= 17 \ \text{N}\cdot\text{m}$$

$$\vec{M}_{OD} = M_{OD}\,\hat{u}_{OD} = 17\left(\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}\right) = \{11.3\hat{i} + 11.3\hat{j} + 5.67\hat{k}\} \ \text{N}\cdot\text{m}$$

Answer $\vec{M}_{OD} = \{11.3\hat{i} + 11.3\hat{j} + 5.67\hat{k}\}$ N·m

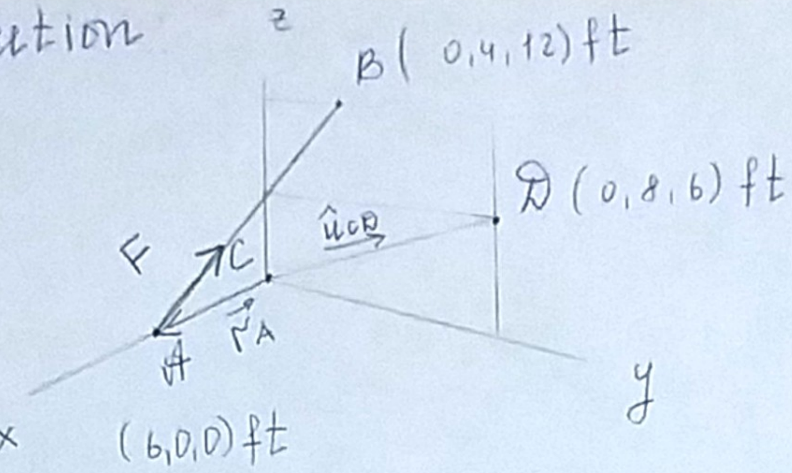
4-62

Given

$M_{CD} = 500$ lb·ft

F - ?

Solution



Unit vector $\hat{u}_{CD}$ defines the direction of the axis CD

$$\hat{u}_{CD} = \frac{\vec{r}_{CD}}{r_{CD}} = \frac{8\hat{j} + 6\hat{k}}{\sqrt{8^2 + 6^2}} = \frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$$

Position vector $\vec{r}_A = \{6\hat{i}\}$ ft

$$\vec{F} = F\hat{u}_{AB} = F\frac{\vec{r}_{AB}}{r_{AB}} = F\left(\frac{-6\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{(-6)^2 + 4^2 + 12^2}}\right) = \left\{-\frac{3}{7}F\hat{i} + \frac{2}{7}F\hat{j} + \frac{6}{7}F\hat{k}\right\} \text{lb}$$

$$M_{CD} = \hat{u}_{CD}\cdot(\vec{r}_A \times \vec{F}) = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix} = -\frac{4}{5}\,\frac{36}{7}F + \frac{3}{5}\,\frac{12}{7}F = -\frac{108}{35}F \text{ lb ft}$$

Therefore, $-\frac{108}{35}F = 500 \Rightarrow F = -162$ lb

Magnitude $F = |-162| = 162$ lb

Answer: $F = 162$ lb

4 - 80

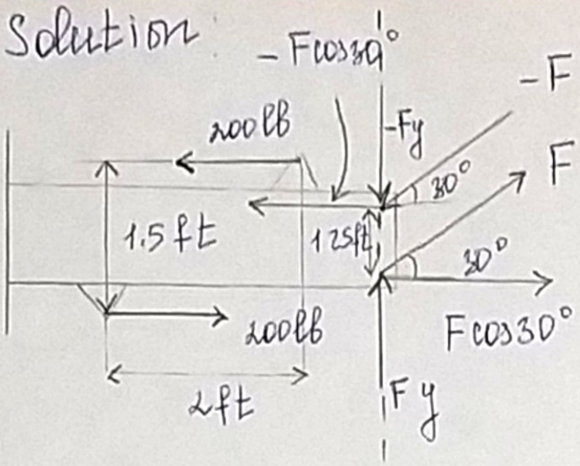
Given

$M_R = 450$ lb ft (counterclockwise)

F - ?

where on the beam does M_R act?

Solution



Fy and - Fy have the same line of action => they cancel each other

$\circlearrowleft + M_R = \Sigma M$

$M_R = 200 \cdot 1.5 + F\cos 30^\circ \cdot 1.25 =$

$(300 + 1.0825F)$ lb ft

Therefore $300 + 1.0825F = 450$

$F = 139$ lb

M_R is a free vector, therefore, it can act at any point on the beam

Answer $F = 139$ lb

The resultant couple moment can act at any point on the beam.

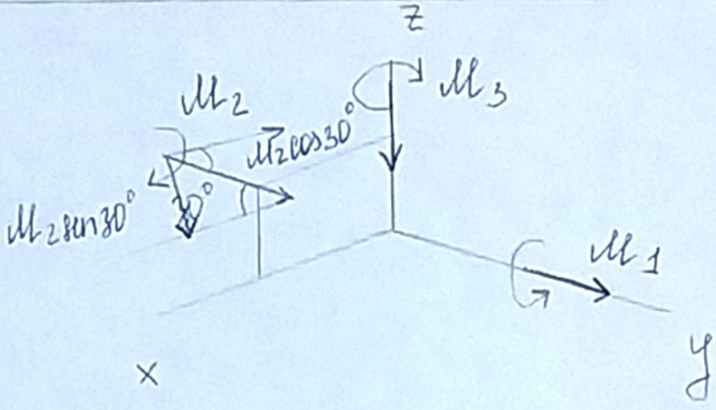
4.92

Given

$\vec{M}_R = \{-300\hat{i} + 450\hat{j} - 600\hat{k}\}$ N m

M_1, M_2, M_3 - ?

Solution



$\vec{M}_1 = \{M_1\hat{j}\}$ N m

$\vec{M}_3 = \{-M_3\hat{k}\}$ N m

$\vec{M}_2 = \{-M_2\cos 30°\hat{i} - M_2\sin 30°\hat{k}\}$ N m

$\vec{M}_R = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = \{-M_2\cos 30°\hat{i} + M_1\hat{j} - (M_2\sin 30° + M_3)\hat{k}\}$ N m

Therefore, $-M_2\cos 30° = -300 \Rightarrow M_2 = 346$ N·m

$M_1 = 450$ N·m

$346\sin 30° + M_3 = 600 \Rightarrow M_3 = 427$ N·m

Answer: $M_1 = 450$ N m

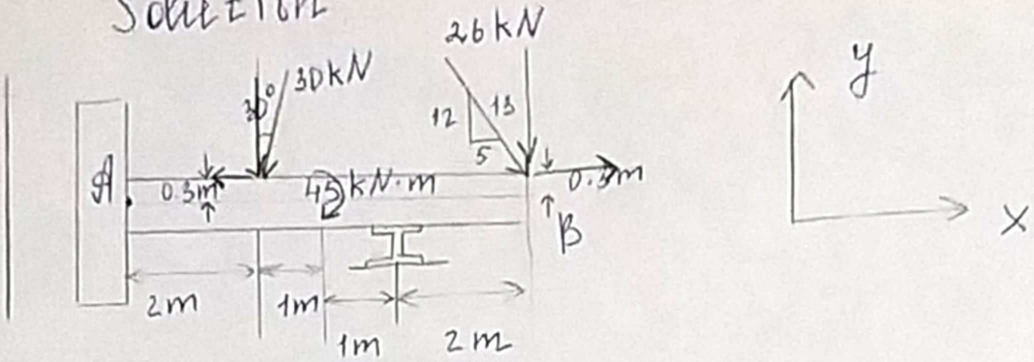
$M_2 = 346$ N m

$M_3 = 427$ N m

4-110 Solution

F_R - ?

M_{RA} - ?



$$F_{Rx} = \Sigma F_x = 26 \cdot \frac{5}{13} - 30 \sin 30^\circ = -5 \text{ kN}$$

$$F_{Ry} = \Sigma F_y = -26 \frac{12}{13} - 30 \cos 30^\circ = -50 \text{ kN}$$

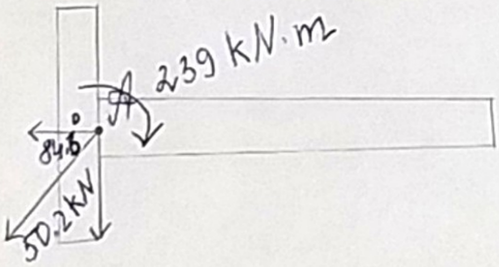
$$F_R = \sqrt{(-5)^2 + (-50)^2} = 50.2 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{-50}{-5}\right) = 84.3^\circ$$

$$\curvearrowright M_{RA} = \Sigma M_A + \Sigma M_C = -30\cos 30^\circ \cdot 2 + 30 \sin 30^\circ \, 0.3 -$$

$$26 \frac{12}{13} \cdot 6 - 26 \frac{5}{13} \cdot 0.3 - 45 = -239 \text{ kN}\cdot\text{m} =$$

$$239 \text{ kN m} \curvearrowright$$



Answer $F_R = 50.2$ kN $\quad \theta = 84.3^\circ$

$M_{RA} = 239$ kN·m $\curvearrowright$

4-117

Given

$F_1 = 6\,kN$

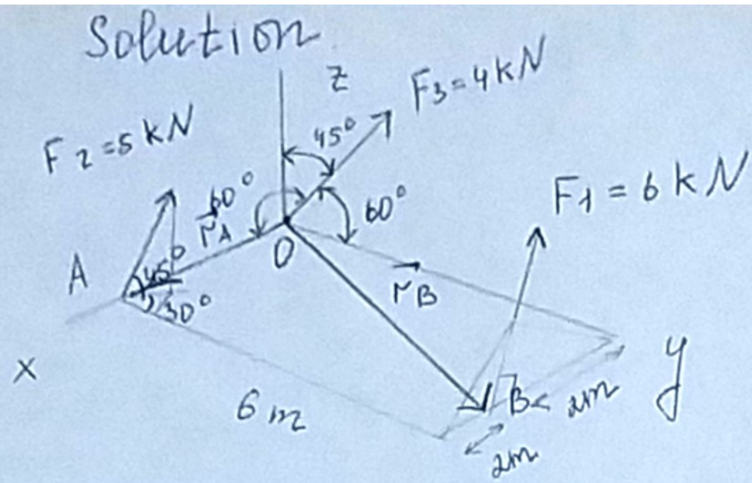
$F_2 = 5\,kN$

$F_3 = 4\,kN$

F_R - ?

M_{RO} - ?

Solution



$$\vec{F}_1 = \{6\hat{k}\}\ kN$$

$$\vec{F}_2 = -5\cos45^\circ\sin30^\circ\hat{i} + 5\cos45^\circ\cos30^\circ\hat{j} + 5\sin45^\circ\hat{k} =$$

$$\{-1.77\hat{i} + 3.06\hat{j} + 3.54\hat{k}\}\ kN$$

$$\vec{F}_3 = 4\cos60^\circ\hat{i} + 4\cos60^\circ\hat{j} + 4\cos45^\circ\hat{k} =$$

$$\{2\hat{i} + 2\hat{j} + 2.83\hat{k}\}\ kN$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \{0.230\hat{i} + 5.06\hat{j} + 12.4\hat{k}\}\ kN$$

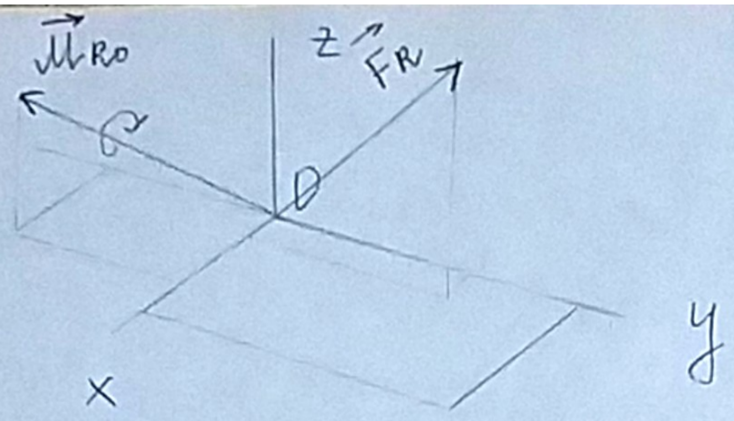
Position vectors $\vec{r}_A = \{4\hat{i}\}\ m$

$\vec{r}_B = \{2\hat{i} + 6\hat{j}\}\ m$

$$\vec{M}_{RO} = \vec{r}_A \times \vec{F}_2 + \vec{r}_B \times \vec{F}_1 =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ -1.77 & 3.06 & 3.54 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 0 \\ 0 & 0 & 6 \end{vmatrix} =$$

$$-14.16\hat{j} + 12.24\hat{k} - 12\hat{j} + 36\hat{i} = \{36.0\hat{i} - 26.2\hat{j} + 12.2\hat{k}\}\ kN\cdot m$$



<u>Answer</u> $\vec{F}_R = \{0.230\hat{i} + 5.06\hat{j} + 12.4\hat{k}\}$ kN

$\vec{M}_{RO} = \{36.0\hat{i} - 26.2\hat{j} + 12.2\hat{k}\}$ kN m

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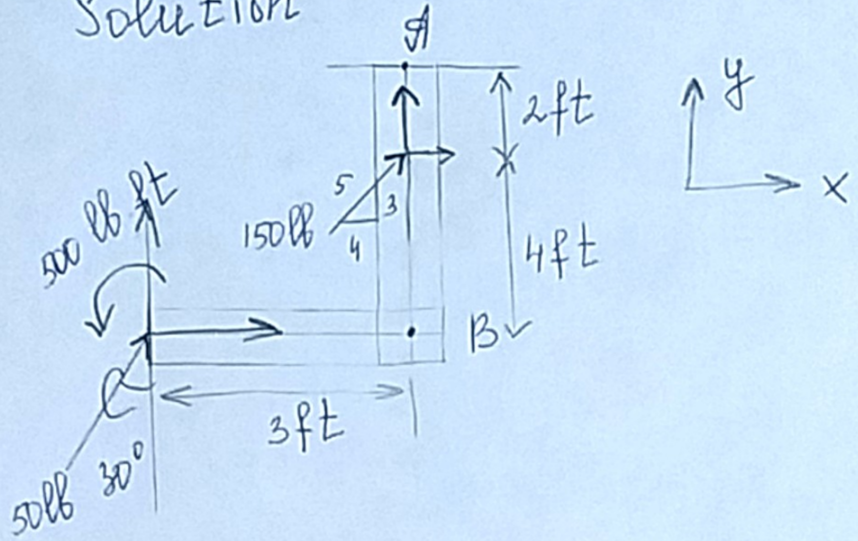
Solution

4-123

F_R - ?

d - ?

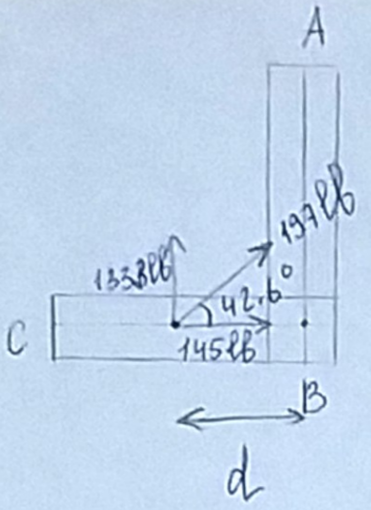


$$F_{Rx} = \Sigma F_x = 150\,\frac{4}{5} + 50\sin 30^\circ = 145 \text{ lb}$$

$$F_{Ry} = \Sigma F_y = 150\,\frac{3}{5} + 50\cos 30^\circ = 133.3 \text{ lb}$$

$$F_R = \sqrt{145^2 + 133.3^2} = 197 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6^\circ$$



$\curvearrowleft + (M_R)_B = \sum M_B$

$-133.3d = -4 \; 150 \frac{4}{5} - 50\cos 30^\circ \, 3 + 500$

$d = 0.824 \text{ ft}$

Answer $F_R = 197 \text{ lb}$ $\theta = 42.6^\circ$

$d = 0.824 \text{ ft}$

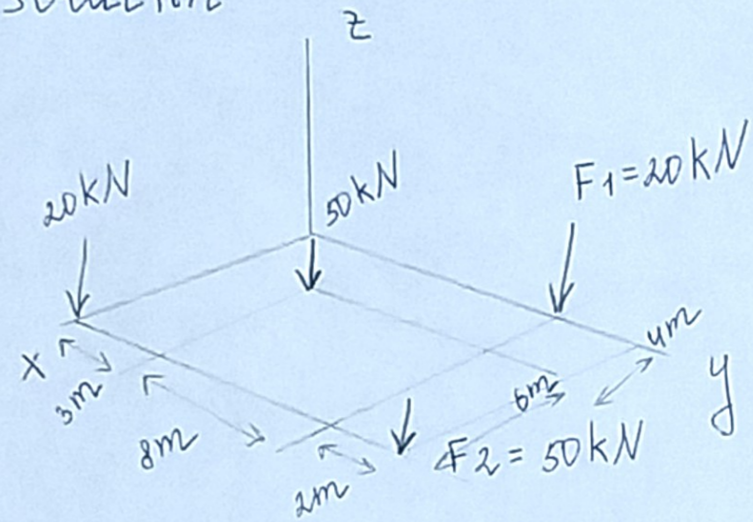
4-130

Given
$F_2 = 50 \text{ kN}$
$F_1 = 20 \text{ kN}$

F_R - ?

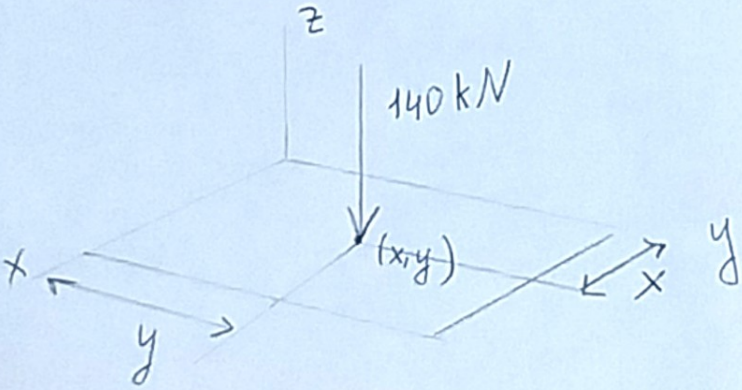
(x, y) - ?

Solution



$F_R = \sum F_z = -20 - 50 - 20 - 50 = -140 \text{ kN}$

$\vec{F}_R = \{-140 \hat{k}\} \text{ kN}$



$(M_R)_x = \Sigma M_x$

$-140\,y = -50\;3 \;-50\;13 \;-20\;11$

$y = 7.29\,m$

$(M_R)_y = \Sigma M_y$

$140x = 50\;4 \;+50\;10 \;+20\;10$

$x = 6.43\,m$

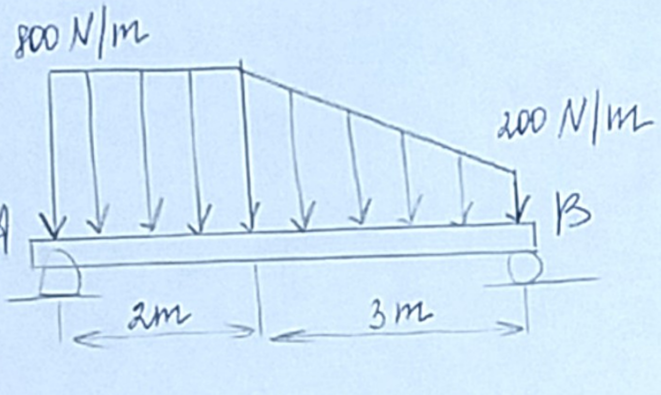
Answer $\vec{F}_R = \{-140\,\hat{k}\}\,kN$

$(x, y) = (6.43,\ 7.29)\,m$

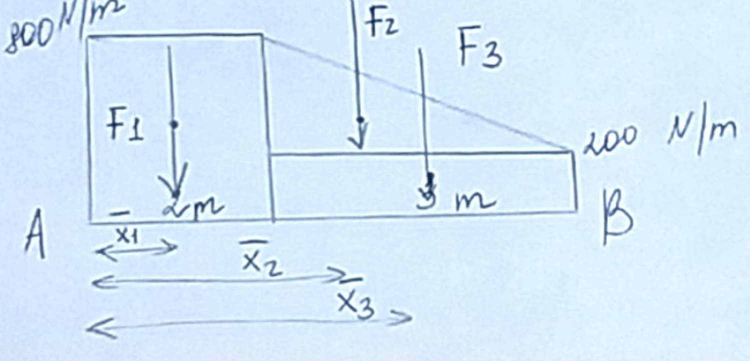
4-144

F_R - ?

$\bar{x}$ - ?



The loading can be divided



$F_1 = 800 \cdot 2 = 1600\,N$

$F_2 = \frac{1}{2}(800-200) \cdot 3 = 900\,N$

$F_3 = 200 \cdot 3 = 600\,N$

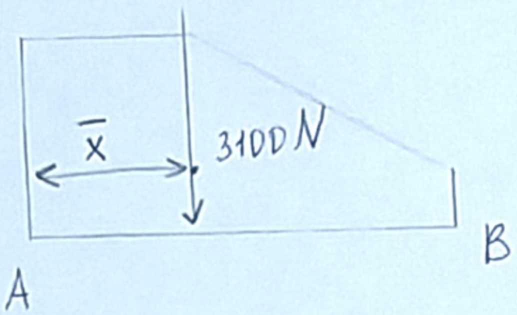
$\bar{x}_1 = \frac{1}{2} \cdot 2 = 1\,m$

$\bar{x}_2 = 2 + \frac{1}{3} \cdot 3 = 3\,m$

$\bar{x}_3 = 2 + \frac{1}{2} \cdot 3 = 3.5\,m$

$F_R = \Sigma F_z = -1600 - 900 - 600 = -3100\,N$

$\vec{F}_R = \{-3100\,\hat{k}\}\,N$



$\curvearrowleft + \; M_{RA} = \Sigma M_A$

$-\bar{x} \cdot 3100 = -1600 \cdot 1 - 900 \cdot 3 - 600 \cdot 3.5$

$\bar{x} = 2.06\,m$

Answer $\vec{F}_R = \{-3100\,\hat{k}\}\,N$

$\bar{x} = 2.06\,m$

4-154

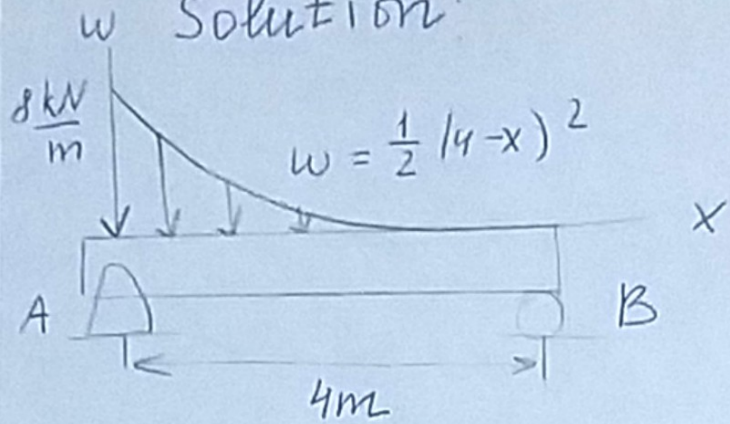
Given

$w = \frac{1}{2}(4-x)^2 \frac{kN}{m}$

F_R - ?

$\bar{x}$ - ?

Solution:

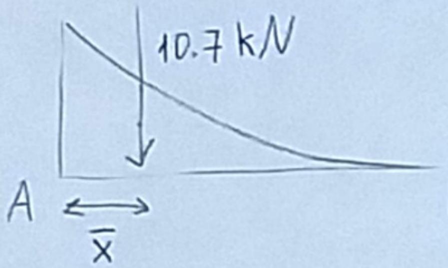


$$F_R = \int_A dA = \int_0^4 w\,dx = \frac{1}{2}\int_0^4 (16-8x+x^2)\,dx =$$

$$\frac{1}{2}\left(16x-4x^2+\frac{x^3}{3}\right)\Big|_0^4 = \frac{1}{2}\left(16\cdot 4-4\cdot 16+\frac{64}{3}\right) =$$

10.7 kN

$$\vec{F}_R = \{-10.7\,\hat{k}\}\ kN$$



$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^4 (16x-8x^2+x^3)\,dx}{10.7} =$$

$$\frac{\frac{1}{2}\left(8x^2-\frac{8x^3}{3}+\frac{x^4}{4}\right)\Big|_0^4}{10.7} = \frac{\frac{1}{2}\left(8\cdot 16-\frac{8\cdot 64}{3}+\frac{256}{4}\right)}{10.7} = 1\ m$$

Answer

$$\vec{F}_R = \{-10.7\,\hat{k}\}\ kN$$

$$\bar{x} = 1\ m$$

5-24 Given

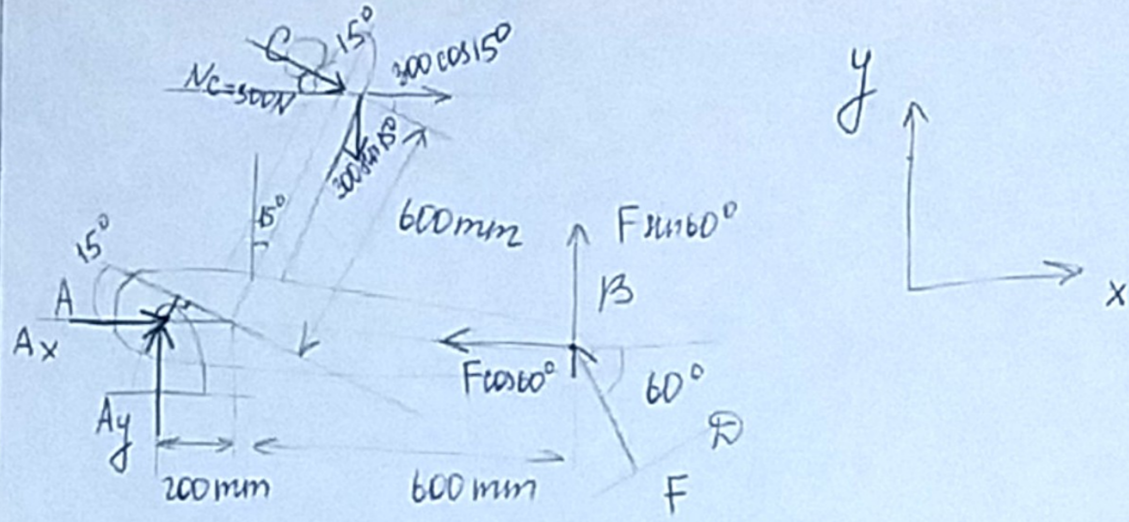
$N_C = 300\,N$

F - ?

A_x - ?

A_y - ?

Solution



$\overset{+}{\circlearrowright} \Sigma M_A = 0 \qquad F\sin 60°\, 0.8 - 300(0.6 + 0.2\sin 15°) = 0$

$F = 282\,N$

$\Sigma F_x = 0 \qquad A_x - 282\cos 60° + 300\cos 15° = 0$

$A_x = -149\,N$

(the direction of A_x is opposite to that shown on FBD $\xleftarrow{A_x}$)

$\Sigma F_y = 0 \qquad A_y - 300\sin 15° + 282\sin 60° = 0$

$A_y = -167\,N$

(the direction of A_y is $\downarrow A_y$)

Answer: $F = 282\,N$

$A_x = 149\,N$, $A_y = 167\,N$

5-26

Given

$m_1 = 2kg$

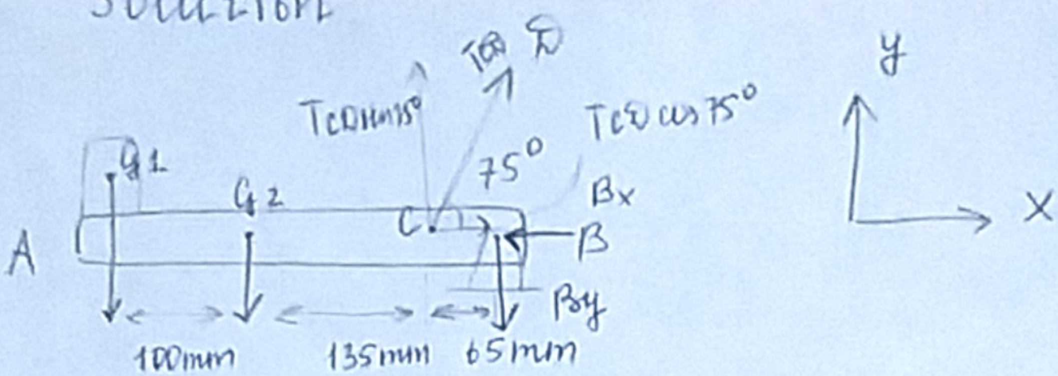
$m_2 = 1.2kg$

T_{CD} - ?

B_x - ?

B_y - ?

Solution



$2 \cdot 9.81 N$ $\quad 1.2 \cdot 9.81 N$

$$\circlearrowleft + \Sigma M_B = 0 \quad -T_{CD}\sin 75 \cdot 0.065 + 1.2 \, 9.81 \cdot 0.2 + 2 \, 9.81 \, 0.3 = 0$$

$$T_{CD} = 131 N$$

$\Sigma F_x = 0$

$$-B_x + 131\cos 75° = 0$$

$$B_x = 34 N$$

$\Sigma F_y = 0$

$$-B_y + 131\sin 75° - 1.2 \, 9.81 - 2 \cdot 9.81 = 0$$

$$B_y = 95 N$$

Answer $T_{CD} = 131 N$

$B_x = 34 N$, $B_y = 95 N$

5-37

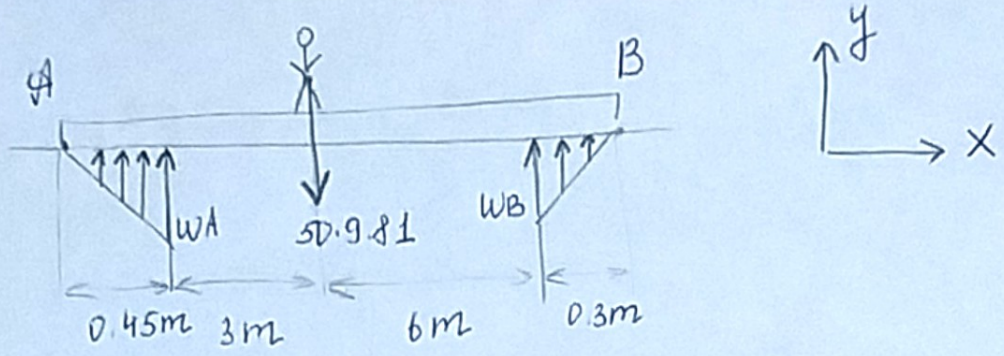
Given

$m = 50\ kg$

w_A - ?

w_B - ?

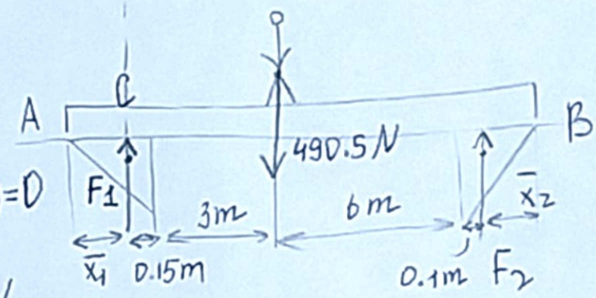
Solution.



Triangular loadings can be replaced by their equivalent resultant forces

$$F_1 = \left\{\frac{1}{2} w_A \cdot 0.45\right\} N \qquad \bar{x}_1 = 0.45 - \frac{0.45}{3} = 0.3m \text{ (from point A)}$$

$$F_2 = \left\{\frac{1}{2} w_B \cdot 0.3\right\} N \qquad \bar{x}_2 = 0.3 - \frac{0.3}{3} = 0.2m \text{ (from point B)}$$



$$\circlearrowleft + \sum M_C = 0: \quad -490.5(3+0.15) + \left(\frac{1}{2} w_B 0.3\right)(9+0.25) = 0$$

$$w_B = 1114 \frac{N}{m} = 1.11 \frac{kN}{m}$$

$$\sum F_y = 0 \qquad \left(\frac{1}{2} w_A \cdot 0.45\right) - 490.5 + \left(\frac{1}{2} \cdot 1114 \cdot 0.3\right) = 0$$

$$w_A = 1437 \frac{N}{m} = 1.44 \frac{kN}{m}$$

Answer: $w_A = 1.44 \frac{kN}{m}$

$w_B = 1.11 \frac{kN}{m}$

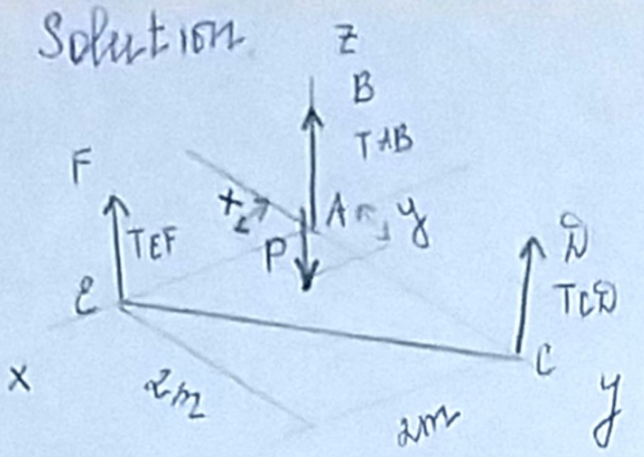
5-66

Given

$T_{AB} = T_{CD} = T_{EF} = T$

$x, y - ?$

Solution



$\Sigma F_z = 0 \qquad 3T - P = 0$

$T = \dfrac{P}{3}$

$\circlearrowleft + \Sigma M_x = 0 \qquad -Py + T_{CD}\,2 = 0$

$-Py + \dfrac{2P}{3} = 0$

$y = \dfrac{2}{3} = 0.667\,m$

$\circlearrowleft + \Sigma M_y = 0 \qquad Px - T_{EF} \cdot 2 = 0$

$Px - \dfrac{2}{3}P = 0$

$x = \dfrac{2}{3} = 0.667\,m$

Answer

$x = 0.667\,m$

$y = 0.667\,m$

5-68

Given

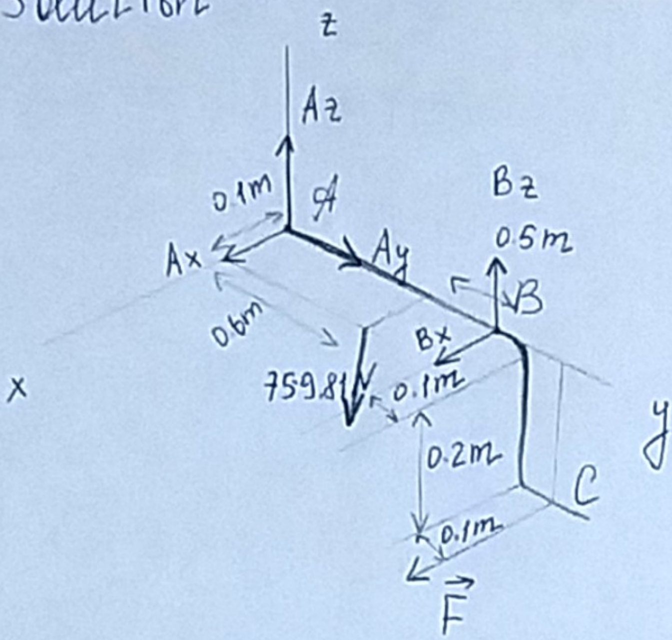
$m = 75\,kg$

F - ?

A_x, A_y, A_z - ?

B_x, B_z - ?

Solution



$\Sigma F_y = 0 \qquad A_y = 0$

$\circlearrowleft + \Sigma M_x = 0 \qquad -75 \cdot 9.81 \cdot 0.6 + B_z(0.6+0.5) = 0$

$B_z = 401\,N$

$\Sigma F_z = 0 \qquad A_z + B_z - 75 \cdot 9.81 = 0$

$A_z = 75 \cdot 9.81 - 401 = 335\,N$

$\circlearrowleft + \Sigma M_y = 0 \qquad 75 \cdot 9.81 \cdot 0.1 - F \cdot 0.2 = 0$

$F = 368\,N$

$\circlearrowleft + \Sigma M_z = 0 \qquad -B_x(0.5+0.6) - F(0.6+0.5+0.1+0.1) = 0$

$B_x = \dfrac{-368 \cdot 1.3}{1.1} = -435\,N$

(The direction of B_x is opposite to that shown on FBD: $\nearrow B_x$)

$\Sigma F_x = 0 \qquad A_x + B_x + F = 0$

$$A_x + (-435) + 368 = 0$$

$$A_x = 67\ N$$

Answer

$F = 368\ N$

$A_x = 67\ N,\ A_y = 0,\ A_z = 335\ N$

$B_x = 435\ N$ (-x-direction)

$B_z = 401\ N$

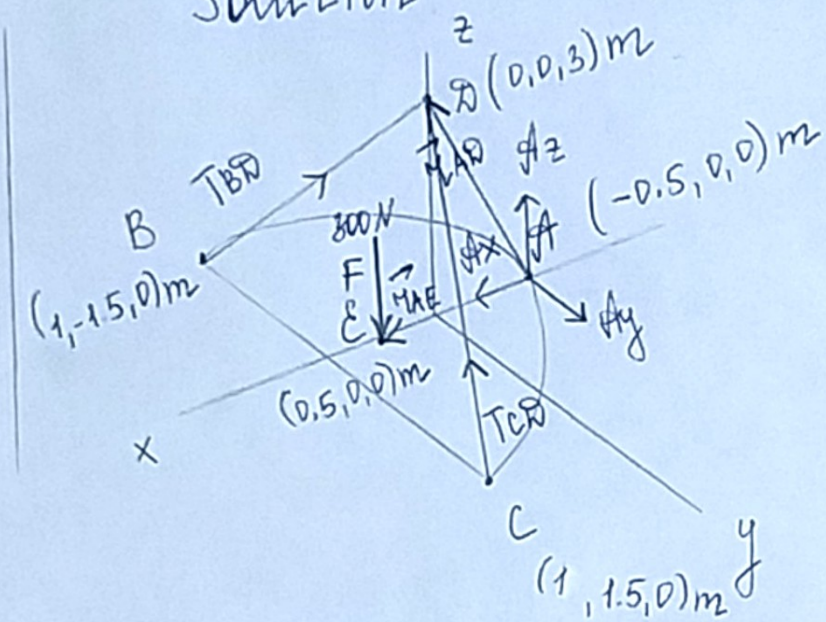
5-70

T_{BD} - ?

T_{CD} - ?

A_x, A_y, A_z - ?

Solution



$$\vec{T}_{BD} = T_{BD}\,\hat{u}_{BD} = T_{BD}\frac{\vec{r}_{BD}}{r_{BD}} = T_{BD}\left(\frac{-1\hat{i} + 1.5\hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + 1.5^2 + 3^2}}\right) =$$

$$T_{BD}\left(-\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \left\{-\frac{2}{7}T_{BD}\hat{i} + \frac{3}{7}T_{BD}\hat{j} + \frac{6}{7}T_{BD}\hat{k}\right\} N$$

$$\vec{T}_{CD} = T_{CD}\,\hat{u}_{CD} = T_{CD}\,\frac{\vec{r}_{CD}}{r_{CD}} = T_{CD}\left(\frac{-1\hat{i} - 1.5\hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1.5)^2 + 3^2}}\right) =$$

$$T_{CD}\left(-\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \left\{-\frac{2}{7}T_{CD}\,\hat{i} - \frac{3}{7}T_{CD}\,\hat{j} + \frac{6}{7}T_{CD}\,\hat{k}\right\} N$$

$$\vec{F}_A = \{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\}\ N$$

$$\vec{F} = \{-300\hat{k}\}\ N$$

$$\Sigma\vec{F} = 0 \qquad \vec{T}_{BD} + \vec{T}_{CD} + \vec{F}_A + \vec{F} = 0$$

$$\left(-\frac{2}{7}T_{BD} - \frac{2}{7}T_{CD} + A_x\right)\hat{i} + \left(\frac{3}{7}T_{BD} - \frac{3}{7}T_{CD} + A_y\right)\hat{j} + \left(\frac{6}{7}T_{BD} + \frac{6}{7}T_{CD} + A_z - 300\right)\hat{k} = 0$$

$$\Sigma F_x = 0 \qquad -\frac{2}{7}T_{BD} - \frac{2}{7}T_{CD} + A_x = 0 \qquad [1]$$

$$\Sigma F_y = 0 \qquad \frac{3}{7}T_{BD} - \frac{3}{7}T_{CD} + A_y = 0 \qquad [2]$$

$$\Sigma F_z = 0 \qquad \frac{6}{7}T_{BD} + \frac{6}{7}T_{CD} + A_z - 300 = 0 \qquad [3]$$

Position vectors $\quad \vec{r}_{AD} = \{0.5\hat{i} + 3\hat{k}\}\ m$

$$\vec{r}_{AE} = \{\hat{i}\}\ m$$

$$\Sigma\vec{M}_A = 0 \qquad \vec{r}_{AD}\times\vec{T}_{BD} + \vec{r}_{AD}\times\vec{T}_{CD} + \vec{r}_{AE}\times\vec{F} = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & 0 & 3 \\ -\frac{2}{7}T_{BD} & \frac{3}{7}T_{BD} & \frac{6}{7}T_{BD} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & 0 & 3 \\ -\frac{2}{7}T_{CD} & -\frac{3}{7}T_{CD} & \frac{6}{7}T_{CD} \end{vmatrix} + \hat{i} \times (-300\hat{k}) = 0$$

$$-\frac{9}{7}T_{BD}\hat{i} - \frac{9}{7}T_{BD}\hat{j} + \frac{3}{14}T_{BD}\hat{k} + \frac{9}{7}T_{CD}\hat{i} - \frac{9}{7}T_{CD}\hat{j} - \frac{3}{14}T_{CD}\hat{k}$$

$$+300\hat{j} = 0$$

$\Sigma M_x = 0 \quad -\frac{9}{7}T_{BD} + \frac{9}{7}T_{CD} = 0 \qquad [4]$

$\Sigma M_y = 0 \quad -\frac{9}{7}T_{BD} - \frac{9}{7}T_{CD} + 300 = 0 \qquad [5]$

$\Sigma M_z = 0 \quad \frac{3}{14}T_{BD} - \frac{3}{14}T_{CD} = 0 \qquad [6]$

From [4] and [6], $T_{BD} = T_{CD}$

[5] $\quad \frac{18}{7}T_{BD} = 300 \Rightarrow T_{BD} = T_{CD} = 117\ N$

[1]: $A_x = 66.7\ N$

[2] $A_y = 0$

[3]: $A_z = 100\ N$

Answer. $T_{BD} = T_{CD} = 117\ N$

$A_x = 66.7\ N,\ A_y = 0,\ A_z = 100\ N$

5-81

Given:

$W = 55\ lb$

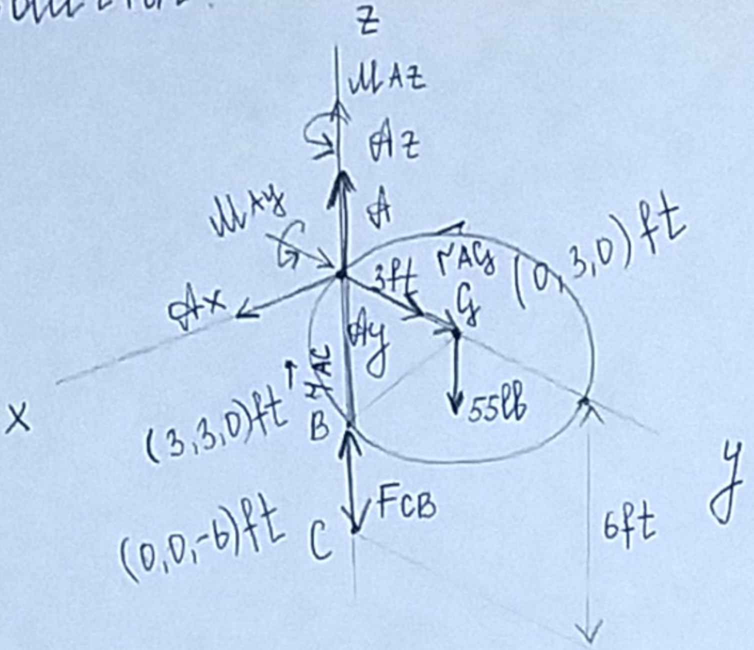
$\theta = 90^\circ$

A_x, A_y, A_z - ?

M_{Ay}, M_{Az} - ?

F_{CB} - ?

Solution.



$$\vec{F}_{CB} = F_{CB}\,\hat{u}_{CB} = F_{CB}\frac{\vec{r}_{CB}}{r_{CB}} = F_{CB}\left(\frac{3\hat{i}+3\hat{j}+6\hat{k}}{\sqrt{3^2+3^2+6^2}}\right) =$$

$$F_{CB}\left(\frac{\hat{i}}{\sqrt{6}}+\frac{\hat{j}}{\sqrt{6}}+\frac{2\hat{k}}{\sqrt{6}}\right) = \left\{\frac{F_{CB}}{\sqrt{6}}\hat{i}+\frac{F_{CB}}{\sqrt{6}}\hat{j}+\frac{2F_{CB}}{\sqrt{6}}\hat{k}\right\} lb$$

$$\vec{F}_A = \{A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\}\ lb$$

$$\vec{W} = \{-55\hat{k}\}\ lb$$

$\Sigma\vec{F}=0 \qquad \vec{F}_{CB}+\vec{F}_A+\vec{W}=0$

$$\left(\frac{F_{CB}}{\sqrt{6}}+A_x\right)\hat{i}+\left(\frac{F_{CB}}{\sqrt{6}}+A_y\right)\hat{j}+\left(\frac{2F_{CB}}{\sqrt{6}}+A_z-55\right)\hat{k}=0$$

$\Sigma F_x=0 \qquad \frac{F_{CB}}{\sqrt{6}}+A_x=0 \qquad [1]$

$\Sigma F_y=0 \qquad \frac{F_{CB}}{\sqrt{6}}+A_y=0 \qquad [2]$

$\Sigma F_z = 0 \qquad 2\dfrac{F_{CB}}{\sqrt{6}} + A_z - 55 = 0 \qquad [3]$

Position vectors $\quad \vec{r}_{AC} = \{-6\hat{k}\}\ m$

$\vec{r}_{AG} = \{3\hat{j}\}\ m$

$\Sigma \vec{M}_A = 0 \qquad \vec{r}_{AC} \times \vec{F}_{CB} + \vec{r}_{AG} \times \vec{W} + \vec{M}_{Ay} + \vec{M}_{Az} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -6 \\ \dfrac{F_{CB}}{\sqrt{6}} & \dfrac{F_{CB}}{\sqrt{6}} & \dfrac{2F_{CB}}{\sqrt{6}} \end{vmatrix} + 3\hat{j} \times (-55\hat{k}) + M_{Ay}\hat{j} + M_{Az}\hat{k} = 0$$

$$\sqrt{6}F_{CB}\hat{i} - \hat{j}\sqrt{6}F_{CB} - 165\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k} = 0$$

$$(\sqrt{6}F_{CB} - 165)\hat{i} + (M_{Ay} - \sqrt{6}F_{CB})\hat{j} + M_{Az}\hat{k} = 0$$

$\Sigma M_x = 0 \qquad \sqrt{6}F_{CB} - 165 = 0 \qquad [4]$

$F_{CB} = 67.4\ lb$

$\Sigma M_y = 0 \qquad M_{Ay} - \sqrt{6} \cdot 67.4 = 0 \qquad [5]$

$M_{Ay} = 165\ lb \cdot ft$

$\Sigma M_z = 0$ $\qquad M_{Az} = 0$ $\qquad [6]$

From [1] $\qquad A_x = -\dfrac{67.4}{\sqrt{6}} = -27.5$ lb

(The direction of A_x is along $-x$ axis)

From [2]. $\qquad A_y = -\dfrac{67.4}{\sqrt{6}} = -27.5$ lb

(The direction of A_y is along $-y$ axis)

From [3]. $\qquad A_z = 55 - \dfrac{2 \cdot 67.4}{\sqrt{6}} = 0$

Answer:

$A_x = 27.5$ lb ($-x$ direction)

$A_y = 27.5$ lb ($-y$ direction)

$A_z = 0$

$M_{Ay} = 165$ lb·ft, $M_{Az} = 0$

$F_{CB} = 67.4$ lb