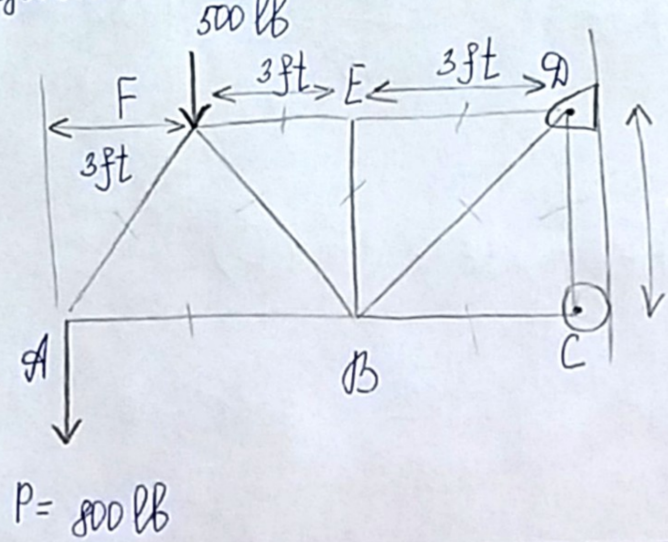
Zarema Balgabekova

6-8

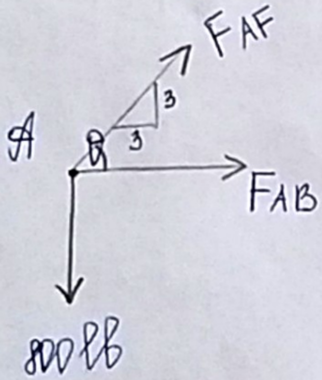
Given:

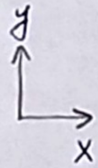


Determine the force in each member of the truss, and state if the members are in tension or compression.

Solution:

There are 1 known and 2 unknown forces at joint A:





$$\theta = \arctan\left(\frac{3}{3}\right) = 45^\circ$$

$\Sigma F_y = 0$ $\quad F_{AF} \sin 45^\circ - 800 = 0$

$$F_{AF} = 1131 \text{ lb} \quad (T)$$

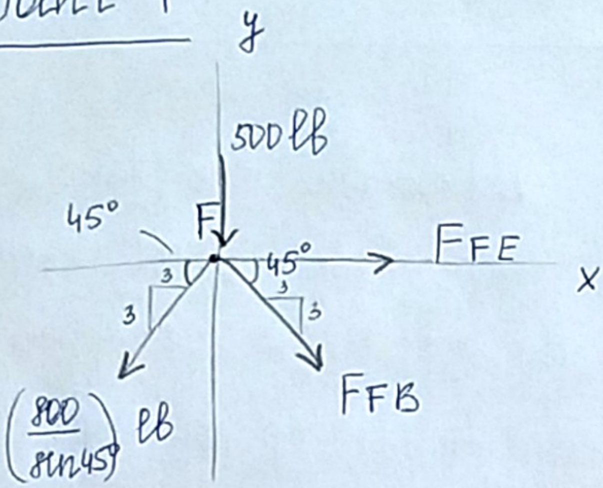
$\Sigma F_x = 0$ $\quad F_{AF} \cos 45^\circ + F_{AB} = 0$

$$F_{AB} = -\left(\frac{800}{\sin 45^\circ}\right) \cdot \cos 45^\circ =$$

-800 lb (opposite dir)

$$F_{AB} = 800 \text{ lb} \quad (C)$$

Joint F



$\Sigma F_y = 0$

$$-500 - \left(\frac{800}{\sin 45^\circ}\right) \cdot \sin 45^\circ - F_{FB} \sin 45^\circ = 0$$

$$F_{FB} = -\frac{1300}{\sin 45^\circ} = -1838 \text{ lb}$$

(oppos dir)

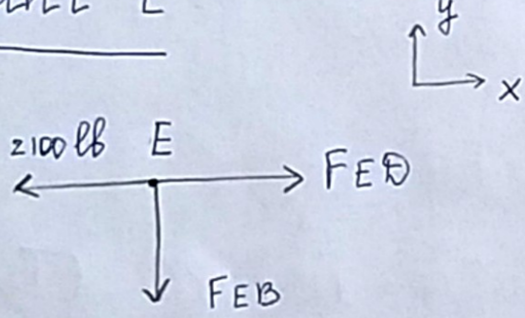
$$F_{FB} = 1838 \text{ lb } (C)$$

$\Sigma F_x = 0$

$$F_{FE} - \left(\frac{800}{\sin 45^\circ}\right) \cos 45^\circ + \left(-\frac{1300}{\sin 45^\circ}\right) \cos 45^\circ = 0$$

$$F_{FE} = 2100 \text{ lb } (T)$$

Joint E



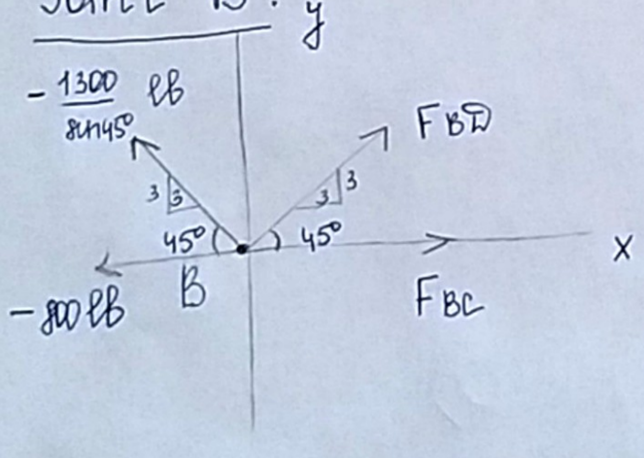
$\Sigma F_x = 0 \qquad F_{ED} - 2100 = 0$

$$F_{ED} = 2100 \text{ lb } (T)$$

$\Sigma F_y = 0 \qquad -F_{EB} = 0$

$$F_{EB} = 0$$

Joint B:



$\Sigma F_y = 0$

$$F_{BD} \sin 45^\circ + \left(-\frac{1300}{\sin 45^\circ}\right) \cdot \sin 45^\circ = 0$$

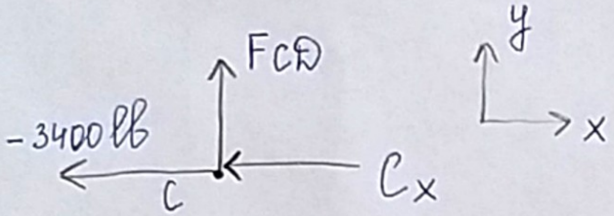
$$F_{BD} = \frac{1300}{\sin 45^\circ} = 1838 \text{ lb } (T)$$

$\Sigma F_x = 0$ $\quad F_{BC} - (-800) - \left(-\frac{1300}{\sin 45^\circ}\right)\cdot \cos 45^\circ + \left(\frac{1300}{\sin 45^\circ}\right)\cos 45^\circ = 0$

$F_{BC} = -3400$ lb (oppos dir)

$F_{BC} = 3400$ lb (C)

<u>Joint C</u>



$\Sigma F_y = 0$

$F_{CD} = 0$

<u>Answer</u>

$F_{AF} = 1131$ lb (T)

$F_{AB} = 800$ lb (C)

$F_{FB} = 1838$ lb (C)

$F_{FE} = 2100$ lb (T)

$F_{ED} = 2100$ lb (T)

$F_{EB} = 0$

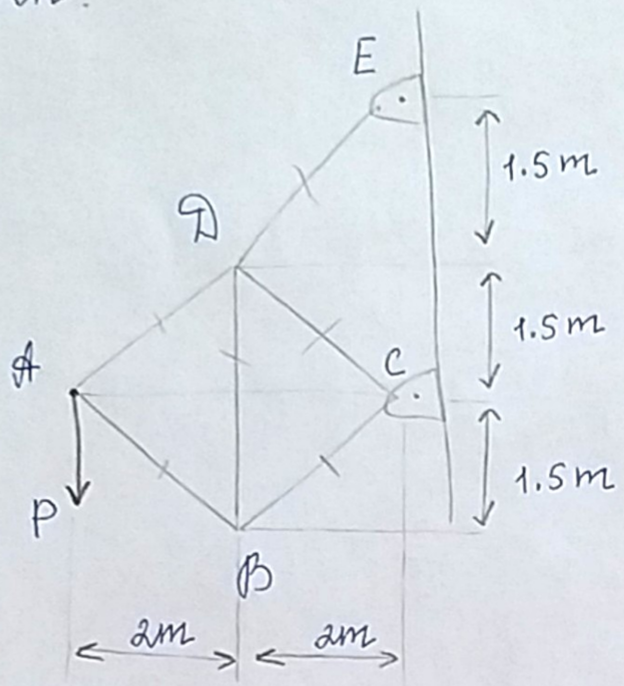
$F_{BD} = 1838$ lb (T)

$F_{BC} = 3400$ lb (C)

$F_{CD} = 0$

6-17

Given:

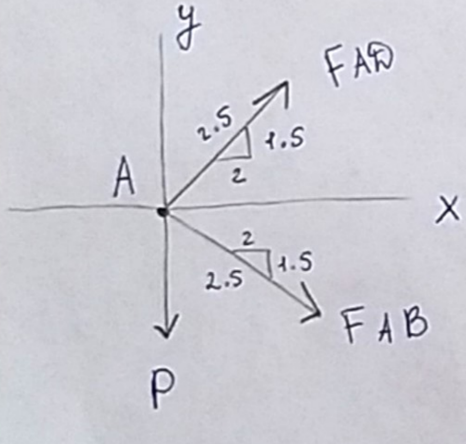


P_{max} - ?

, so that none of the members are subjected to a force exceeding 2.5 kN in T or 2 kN in C.

Solution

There are 1 known (P) and 2 unknown forces at joint A



$$\sqrt{2^2 + 1.5^2} = 2.5$$

$$\Sigma F_x = 0 \qquad F_{AD} \cdot \frac{2}{2.5} + F_{AB} \cdot \frac{2}{2.5} = 0$$

$$F_{AD} = -F_{AB} \qquad [1]$$

$$\Sigma F_y = 0 \qquad F_{AD} \frac{1.5}{2.5} - F_{AB} \cdot \frac{1.5}{2.5} - P = 0 \qquad [2]$$

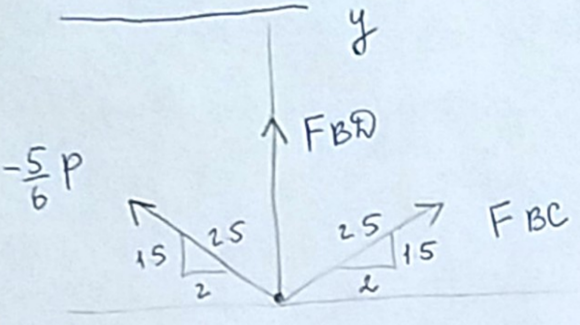
Substituting [1] into [2]: $-\frac{6}{5} F_{AB} = P$

$$F_{AB} = -\frac{5}{6} P \text{ (oppos dir)}$$

$$F_{AB} = \frac{5}{6} P \text{ (C)}$$

From [1] , $F_{AD} = -\left(-\frac{5}{6}P\right) = \frac{5}{6}P \quad (T)$

Joint B



$\Sigma F_x = 0$

$$F_{BC}\frac{2}{2.5} - \left(-\frac{5}{6}P\right)\cdot\frac{2}{2.5} = 0$$

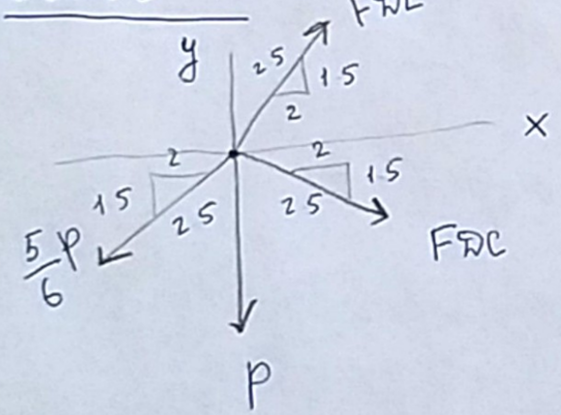
$$F_{BC} = -\frac{5}{6}P \quad \text{(oppos dir)}$$

$$F_{BC} = \frac{5}{6}P \; (C)$$

$\Sigma F_y = 0$ $\quad F_{BD} + \left(-\frac{5}{6}P\right)\frac{1.5}{2.5} + \left(-\frac{5}{6}P\right)\frac{1.5}{2.5} = 0$

$$F_{BD} = P \quad (T)$$

Joint D



$\Sigma F_x = 0$

$$F_{DE}\frac{2}{2.5} + F_{DC}\frac{2}{2.5} - \left(\frac{5}{6}P\right)\frac{2}{2.5} = 0$$

$$\frac{4}{5}F_{DE} + \frac{4}{5}F_{DC} = \frac{2}{3}P \quad [3]$$

$\Sigma F_y = 0$ $\quad F_{DE}\frac{1.5}{2.5} - F_{DC}\frac{1.5}{2.5} - P - \left(\frac{5}{6}P\right)\frac{1.5}{2.5} = 0$

$$\frac{3}{5}F_{DE} - \frac{3}{5}F_{DC} = \frac{3}{2}P \quad [4]$$

Solving [3] and [4] simultaneously:

$$\begin{array}{l} \cdot\frac{1}{4} \\ \frac{1}{3} \end{array}\left\{\begin{array}{l} \frac{4}{5} F_{DE} + \frac{4}{5} F_{DC} = \frac{2}{3} P \\ \frac{3}{5} F_{DE} - \frac{3}{5} F_{DC} = \frac{3}{2} P \end{array}\right. \qquad + \left\{\begin{array}{l} \frac{1}{5} F_{DE} + \frac{1}{5} F_{DC} = \frac{P}{6} \\ \frac{1}{5} F_{DE} - \frac{1}{5} F_{DC} = \frac{P}{2} \end{array}\right.$$

$$\frac{2}{5} F_{DE} = \frac{2}{3} P$$

$$F_{DE} = \frac{5}{3} P \ (T)$$

From [3], $F_{DC} = \left[\frac{2}{3} P - \frac{4}{5}\left(\frac{5}{3} P\right)\right] \cdot \frac{5}{4} = -\frac{5}{6} P$

(oppos dir)

$$F_{DC} = \frac{5}{6} P \ (C)$$

The first tensile force that will exceed 2.5 kN is

$F_{DE} = \frac{5}{3} P \quad \Rightarrow \quad \frac{5}{3} P = 2.5$

$$P = 1.5 \text{ kN}$$

All compressive forces are equal to $\frac{5}{6} P$ $\Rightarrow$ $\frac{5}{6} P = 2$

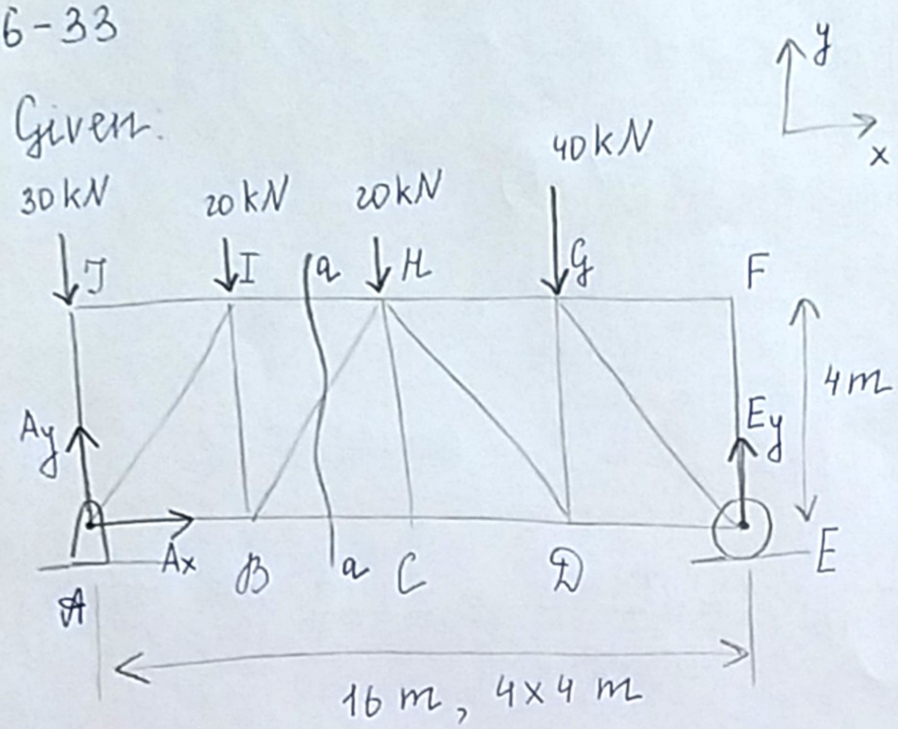
$$P = 2.4 \text{ kN}$$

$1.5 \text{ kN} < 2.4 \text{ kN} \Rightarrow P_{max} = 1.5 \text{ kN}$, so that none of the members are subjected to a force exceeding 2.5 kN in T or 2 kN in C

Answer $P_{max} = 1.5$ kN

6-33

Given:



Determine the force in members HI, HB, BC, and state if the members are in T or C

Solution:

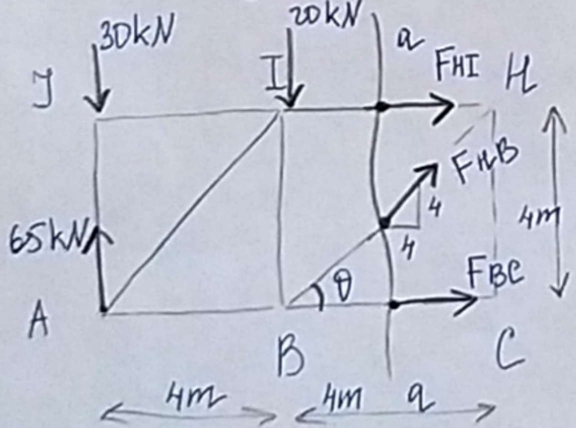
External support reactions:

$\Sigma F_x = 0 \qquad A_x = 0$

$\circlearrowleft + \Sigma M_E = 0 \qquad 40 \cdot 4 + 20 \cdot 8 + 20 \cdot 12 + 30 \cdot 16 - A_y \cdot 16 = 0$

$A_y = 65 \text{ kN}$

It is not needed to find E_y, because the left portion of the sectioned truss will be analyzed



$\theta = \arctan\left(\frac{4}{4}\right) = 45°$

$\circlearrowleft + \Sigma M_B = 0$

$-4F_{HI} + 30 \cdot 4 - 65 \cdot 4 = 0$

$F_{HI} = -35 \text{ kN}$ (oppos dir)

$F_{HI} = 35 \text{ kN}$ (C)

$\circlearrowleft + \sum M_H = 0 \qquad 4F_{BC} + 20.4 + 30.8 - 65.8 = 0$

$$F_{BC} = 50\ kN\ (T)$$

$\sum F_x = 0 \qquad 50 + (-35) + F_{HB}\cos 45^\circ = 0$

$$F_{HB} = -\frac{15}{\cos 45^\circ} = -21.2\ kN\ (\text{oppos dir})$$

$$F_{HB} = 21.2\ kN\ (C)$$

Answer

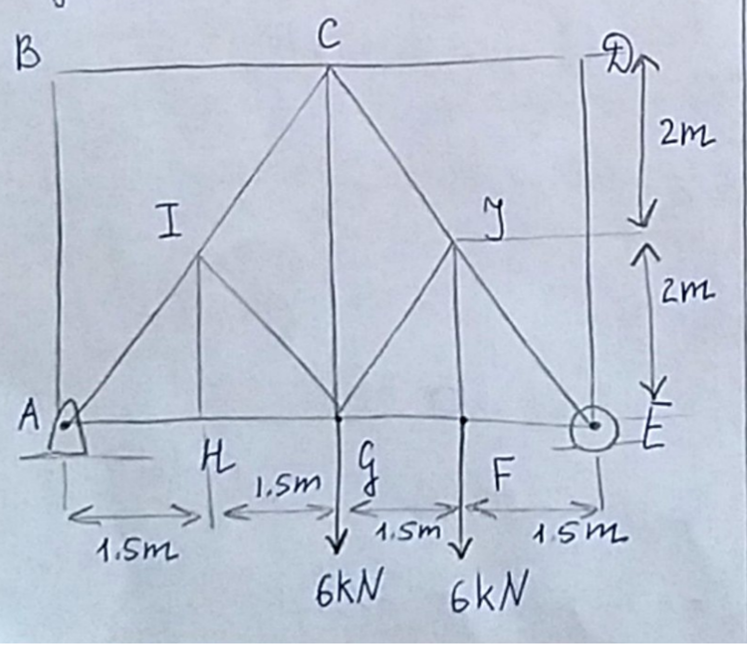
$F_{HI} = 35\ kN\ (C)$

$F_{HB} = 21.2\ kN\ (C)$

$F_{BC} = 50\ kN\ (T)$

6-42

Given:

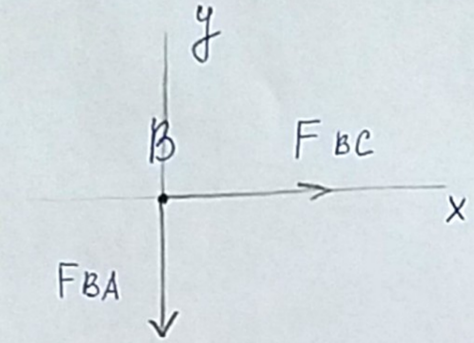


Determine the force in members IC and CG of the truss and state if these members are in T or C.

Indicate all zero-force members

Zero - force members: AB, BC, CD, DE, HI, GI

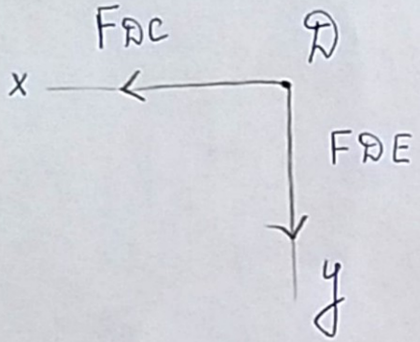
Joint B:



$\Sigma F_x = 0 \quad F_{BC} = 0$

$\Sigma F_y = 0 \quad -F_{BA} = 0 \quad F_{BA} = 0$

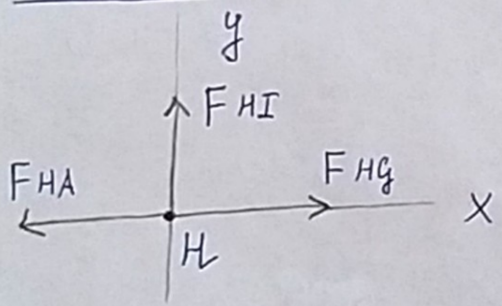
Joint D:



$\Sigma F_x = 0 \quad F_{DC} = 0$

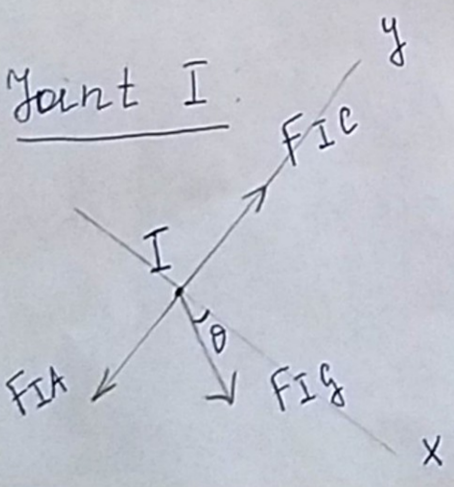
$\Sigma F_y = 0 \quad F_{DE} = 0$

Joint H:



$\Sigma F_y = 0 \quad F_{HI} = 0$

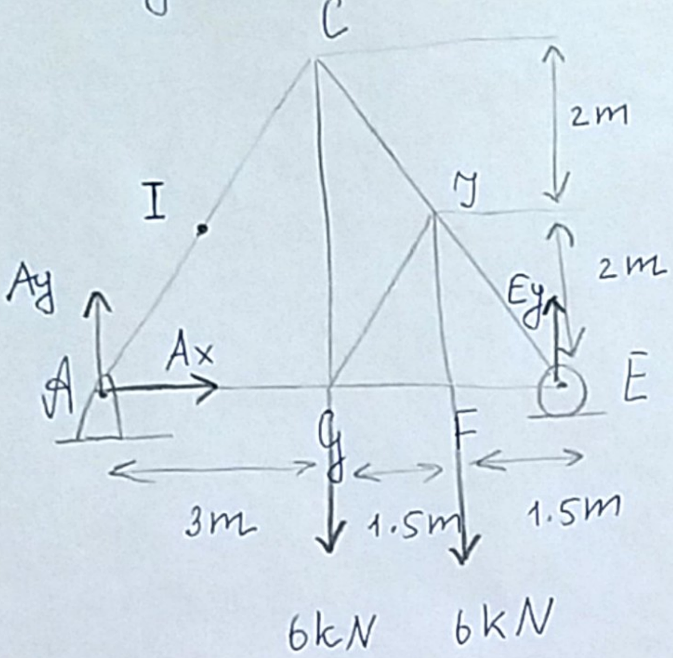
Joint I:



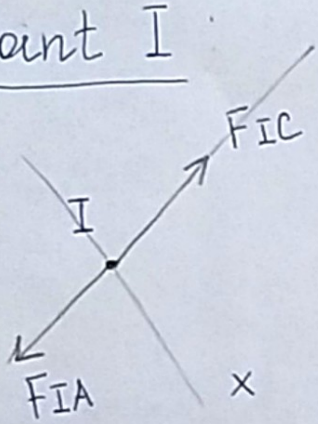
$\Sigma F_x = 0 \quad F_{IG} \cos\theta = 0$

$F_{IG} = 0 \quad (\theta \neq 90^\circ)$

Removing zero-force members



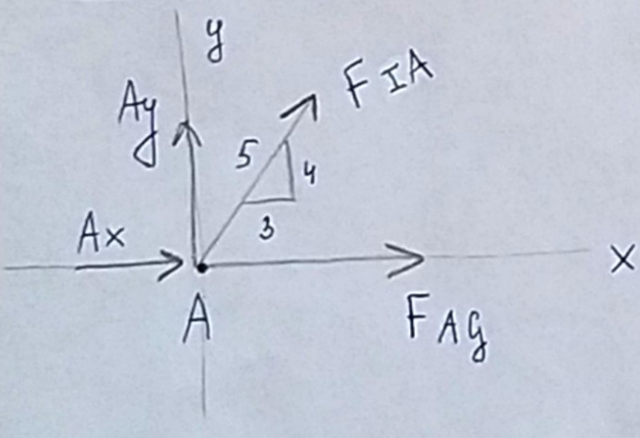
Joint I



$\sum F_y = 0 \qquad F_{IC} - F_{IA} = 0$

$F_{IC} = F_{IA} \qquad [1]$

Joint A



External support reactions (FBD of the entire truss)

$\sum F_x = 0 \qquad A_x = 0$

$\circlearrowleft + \sum M_E = 0$

$6 \cdot 1.5 + 6 \cdot 3 - A_y \cdot 6 = 0$

$A_y = 4.5 \text{ kN}$

FBD of joint A:

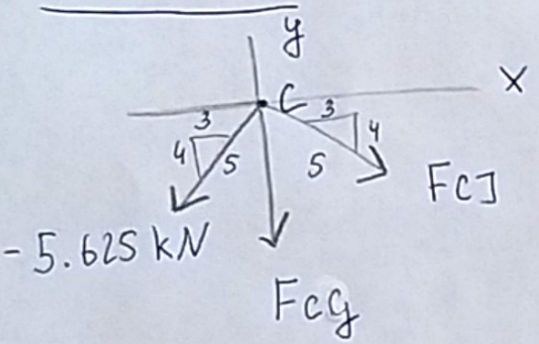
$\Sigma F_y = 0 \qquad 4.5 + F_{IA} \frac{4}{5} = 0$

$F_{IA} = -5.625 \text{ kN}$

From [1], $\quad F_{IC} = -5.625 \text{ kN}$ (oppos dir)

$F_{IC} = 5.625 \text{ kN} \ (C)$

Joint C



$\Sigma F_x = 0$

$F_{CJ} \cdot \frac{3}{5} - (-5.625) \cdot \frac{3}{5} = 0$

$F_{CJ} = -5.625 \text{ kN}$

$\Sigma F_y = 0 \qquad -F_{CG} - (-5.625) \cdot \frac{4}{5} - (-5.625) \cdot \frac{4}{5} = 0$

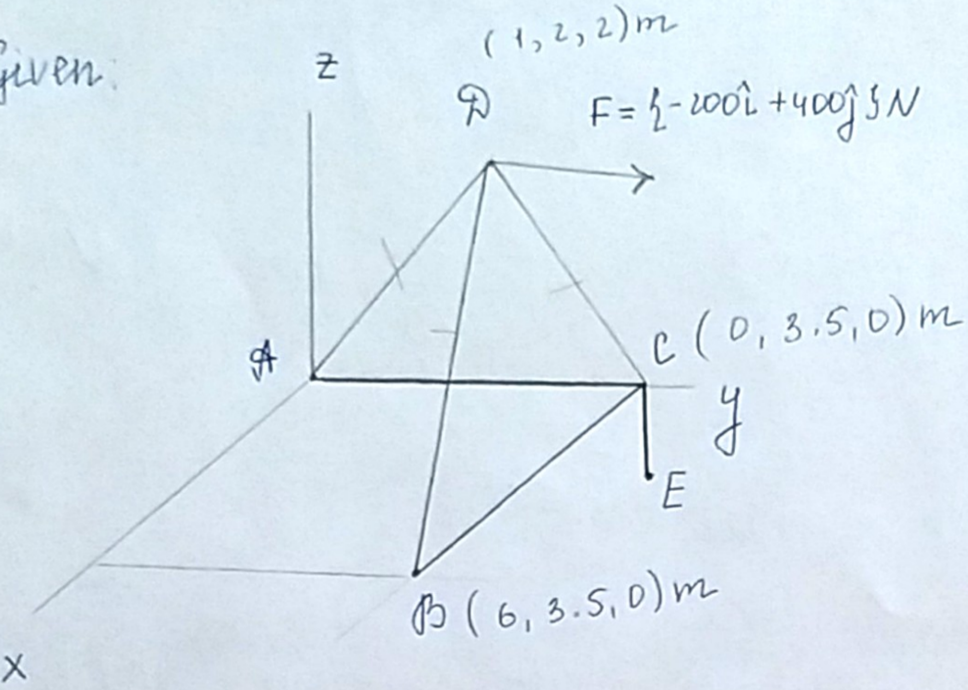
$F_{CG} = 9 \text{ kN} \ (T)$

Answer: $\quad F_{IC} = 5.625 \text{ kN} \ (C), \quad F_{CG} = 9 \text{ kN} \ (T)$

Zero-force members: AB, BC, CD, DE, HI, GI

6-57

Given:

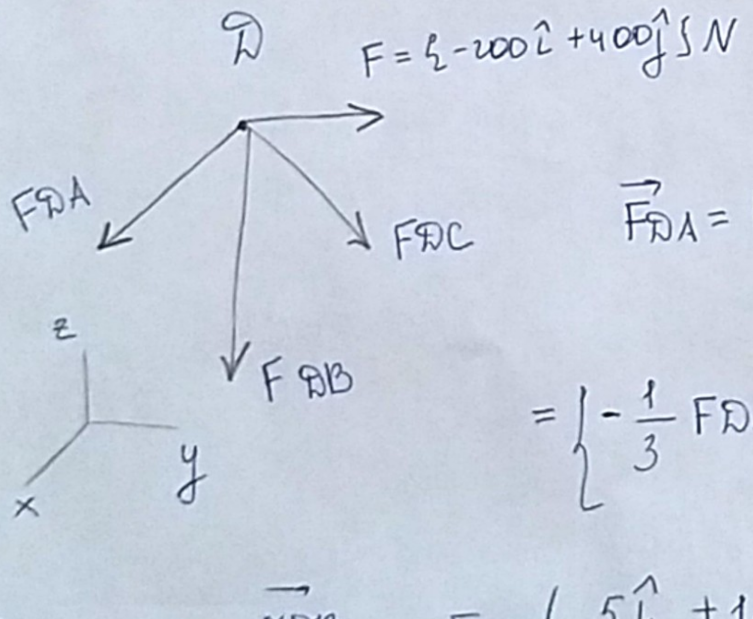


ball-and-socket joints at A, B, E

Determine the force in each member of the space truss and state if the members are in T or C

Solution:

There are 1 known and 3 unknown forces at joint D



$$\vec{F}_{DA} = F_{DA}\frac{\vec{r}_{DA}}{r_{DA}} = F_{DA}\left(\frac{-\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{(-1)^2+(-2)^2+(-2)^2}}\right)$$

$$= \left\{-\frac{1}{3}F_{DA}\hat{i} - \frac{2}{3}F_{DA}\hat{j} - \frac{2}{3}F_{DA}\hat{k}\right\} N$$

$$\vec{F}_{DB} = F_{DB}\frac{\vec{r}_{DB}}{r_{DB}} = F_{DB}\left(\frac{5\hat{i}+1.5\hat{j}-2\hat{k}}{\sqrt{5^2+1.5^2+(-2)^2}}\right) =$$

$$\left\{\frac{2\sqrt{5}}{5}F_{DB}\hat{i} + \frac{3\sqrt{5}}{25}F_{DB}\hat{j} - \frac{4\sqrt{5}}{25}F_{DB}\hat{k}\right\} N$$

$$\vec{F}_{DC} = F_{DC}\frac{\vec{r}_{DC}}{r_{DC}} = F_{DC}\left(\frac{-\hat{i}+1.5\hat{j}-2\hat{k}}{\sqrt{(-1)^2+1.5^2+(-2)^2}}\right) =$$

$$\left\{-\frac{2\sqrt{29}}{29}F_{DC}\hat{i} + \frac{3\sqrt{29}}{29}F_{DC}\hat{j} - \frac{4\sqrt{29}}{29}F_{DC}\hat{k}\right\} N$$

$$\sum \vec{F} = 0 \qquad \vec{F}_{DA} + \vec{F}_{DB} + \vec{F}_{DC} + \vec{F} = 0$$

$$\sum F_x = 0 \qquad -\frac{1}{3}F_{DA} + \frac{2\sqrt{5}}{5}F_{DB} - \frac{2\sqrt{29}}{29}F_{DC} - 200 = 0 \qquad [1]$$

$$\sum F_y = 0 \qquad -\frac{2}{3}F_{DA} + \frac{3\sqrt{5}}{25}F_{DB} + \frac{3\sqrt{29}}{29}F_{DC} + 400 = 0 \qquad [2]$$

$$\sum F_z = 0 \qquad -\frac{2}{3}F_{DA} - \frac{4\sqrt{5}}{25}F_{DB} - \frac{4\sqrt{29}}{29}F_{DC} = 0 \qquad [3]$$

From [3], $F_{DA} = \left(-\frac{4\sqrt{5}}{25}F_{DB} - \frac{4\sqrt{29}}{29}F_{DC}\right)\cdot\frac{3}{2} =$

$$-\frac{6\sqrt{5}}{25}F_{DB} - \frac{6\sqrt{29}}{29}F_{DC}$$

Substituting F_{DA} into [1].

$$\frac{2\sqrt{5}}{25}F_{DB} + \cancel{\frac{2\sqrt{29}}{29}F_{DC}} + \frac{2\sqrt{5}}{5}F_{DB} - \cancel{\frac{2\sqrt{29}}{29}F_{DC}} - 200 = 0$$

$$\frac{12\sqrt{5}}{25} F_{DB} = 200$$

$$F_{DB} = \frac{200 \cdot 25}{12\sqrt{5}} = 186 \text{ N (T)}$$

Substituting F_{DA} into [2],

$$\frac{4\sqrt{5}}{25} F_{DB} + \frac{4\sqrt{29}}{29} F_{DC} + \frac{3\sqrt{5}}{25} F_{DB} + \frac{3\sqrt{29}}{29} F_{DC} + 400 = 0$$

$$\frac{7\sqrt{5}}{25} F_{DB} + \frac{7\sqrt{29}}{29} F_{DC} + 400 = 0$$

$$F_{DC} = \frac{\sqrt{29}}{7}\left(-400 - \frac{7\cancel{\sqrt{5}}}{\cancel{25}}\left(\frac{200 \cdot \cancel{25}}{12\cancel{\sqrt{5}}}\right)\right) =$$

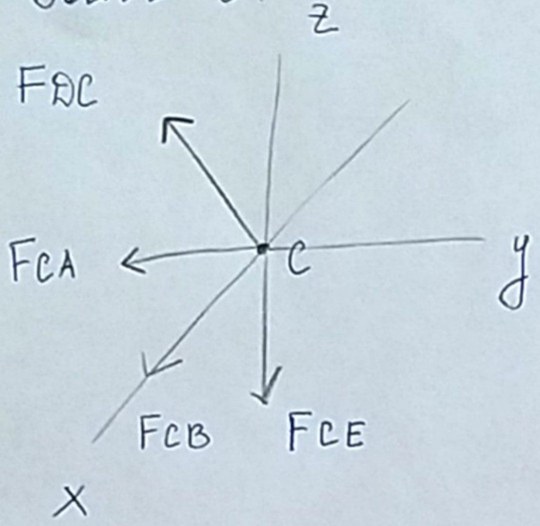
$$- \frac{\sqrt{29} \cdot 1550}{21} = -397 \text{ N (oppos dir)}$$

$$F_{DC} = 397 \text{ N (C)}$$

$$F_{DA} = -\frac{6\cancel{\sqrt{5}}}{\cancel{25}}\left(\frac{200 \cdot \cancel{25}}{12\cancel{\sqrt{5}}}\right) - \frac{6\cancel{\sqrt{29}}}{\cancel{29}}\left(-\frac{\cancel{\sqrt{29}} \cdot 1550}{21}\right) = \frac{2400}{7} =$$

343 N (T)

Joint C:



$$\vec{F}_{DC} = F_{DC}\frac{\vec{r}_{CD}}{r_{CD}} = \left(-\frac{\sqrt{29}\,1550}{21}\right)\cdot\left(\frac{\hat{i} - 1.5\hat{j} + 2\hat{k}}{\sqrt{1^2+(-1.5)^2+2^2}}\right) =$$

$$\left\{-\frac{3100}{21}\hat{i} + \frac{1550}{7}\hat{j} - \frac{6200}{21}\hat{k}\right\} N$$

$$\vec{F}_{CA} = \{-F_{CA}\hat{j}\} N$$

$$\vec{F}_{CB} = \{F_{CB}\hat{i}\} N$$

$$\vec{F}_{CE} = \{-F_{CE}\hat{k}\} N$$

$\Sigma \vec{F} = 0 \qquad \vec{F}_{DC} + \vec{F}_{CA} + \vec{F}_{CB} + \vec{F}_{CE} = 0$

$\Sigma F_x = 0 \qquad -\frac{3100}{21} + F_{CB} = 0 \qquad F_{CB} = 148\,N\,(T)$

$\Sigma F_y = 0 \qquad \frac{1550}{7} - F_{CA} = 0 \qquad F_{CA} = 221\,N\,(T)$

$\Sigma F_z = 0 \qquad -\frac{6200}{21} - F_{CE} = 0 \qquad F_{CE} = -295\,N$

(oppos dir)

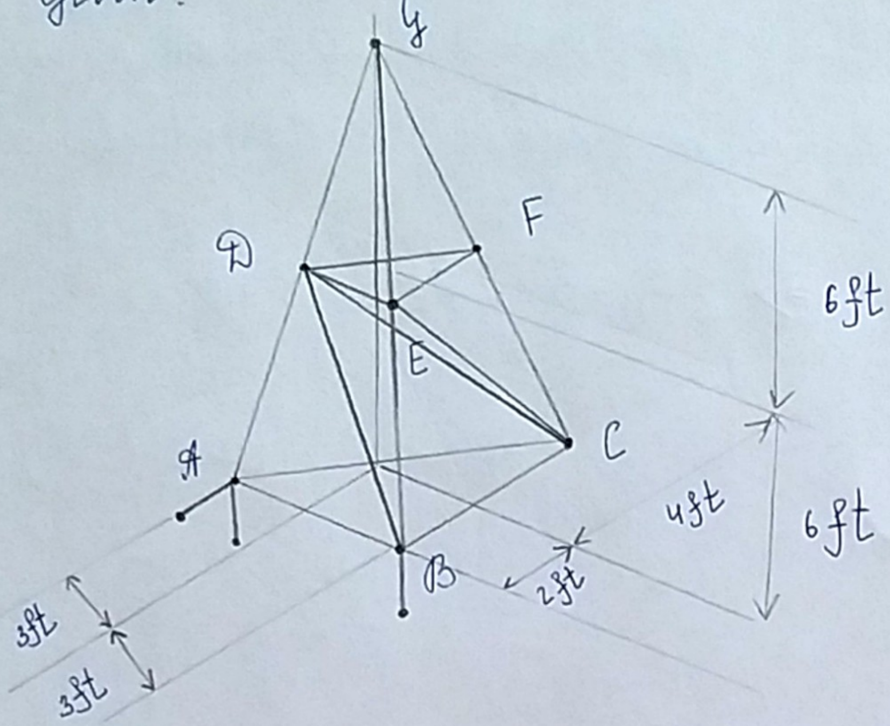
$F_{CE} = 295\,N\,(C)$

Answer: $F_{DB} = 186\,N\,(T)$, $F_{DC} = 397\,N\,(C)$

$F_{DA} = 343\,N\,(T)$, $F_{CB} = 148\,N\,(T)$, $F_{CA} = 221\,N\,(T)$, $F_{CE} = 295\,N\,(C)$

6-65

Given:

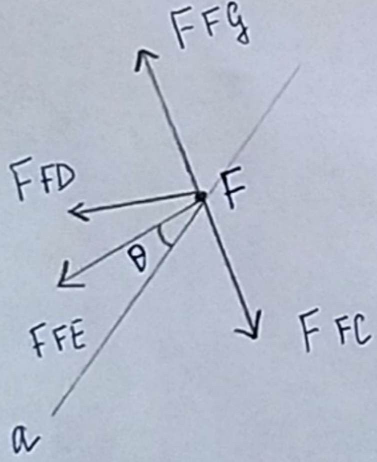


Determine the force in members FE and ED of the space truss and state if the members are in T or C

ball-and-socket at C
short links at A an B

Solution.

Joint F:

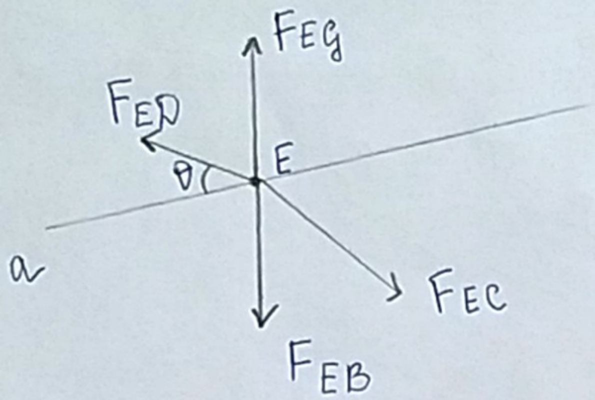


F_{FG}, F_{FC} and F_{FD} are lying in the same plane, and a-axis is perpendicular to this plane

$$\Rightarrow \Sigma F_a = 0 \quad F_{FE}\cos\theta = 0$$

$$F_{FE} = 0 \quad (\theta \neq 90^\circ)$$

Joint E:



F_{EG}, F_{EC} and F_{EB} are lying in the same plane, and a-axis is perpendicular to this plane

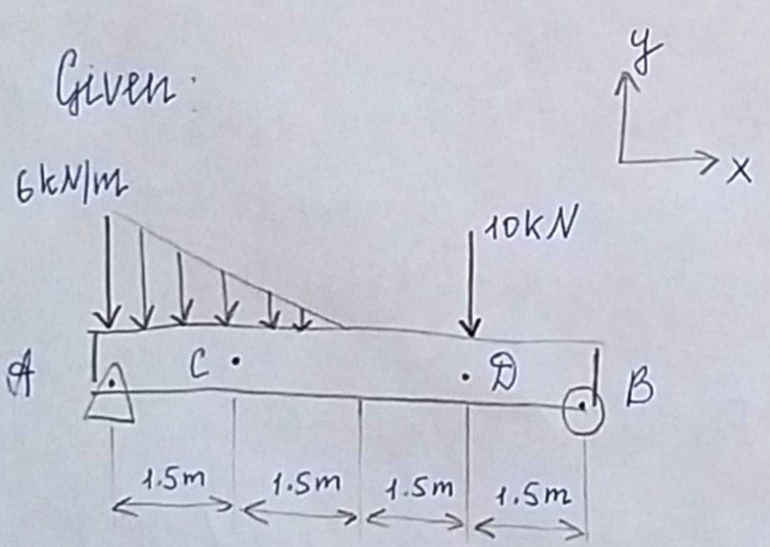
$\Rightarrow \sum F_a = 0 \qquad F_{ED}\cos\theta = 0$

$F_{ED} = 0 \quad (\cos\theta \neq 90°)$

Answer: $F_{FE} = 0$, $F_{ED} = 0$

7-11

Given:



Point D is located just to the left of the 10kN concentrated load

N_D, V_D, M_D - ?

N_C, V_C, M_C - ?

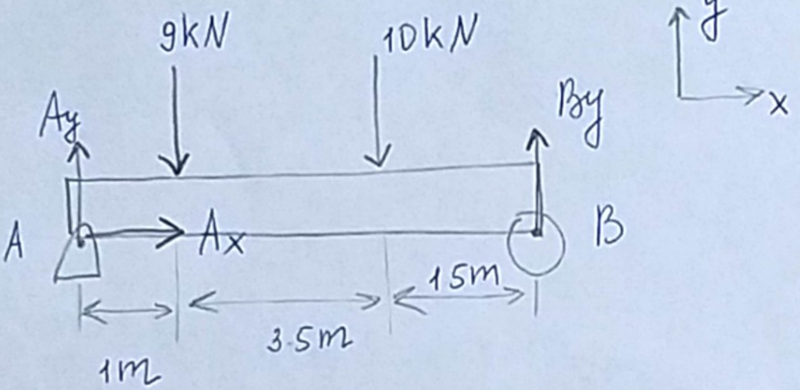
Solution:

External support reactions:

Distributed triangular loading can be replaced by its resultant force: $F_R = \frac{1}{\cancel{2}} \cdot \overset{3}{\cancel{6}} \cdot 3 = 9\ kN$

$$\vec{F_R} = \{-9\hat{j}\}\ kN$$

The location of F_R from point A: $\bar{x} = \frac{1}{3} \cdot 3 = 1\ m$



$\sum F_x = 0$

$A_x = 0$

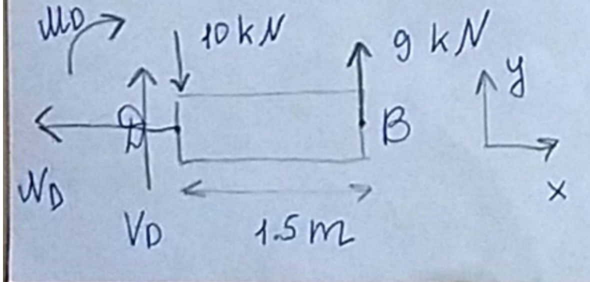
$\circlearrowleft + \sum M_A = 0 \qquad -9 \cdot 1 - 10 \cdot 4.5 + B_y \cdot 6 = 0$

$$B_y = 9\ kN$$

$\sum F_y = 0 \qquad A_y - 9 - 10 + 9 = 0$

$$A_y = 10\ kN$$

FBD of the right segment DB of the beam:



$\sum F_x = 0 \qquad -N_D = 0$

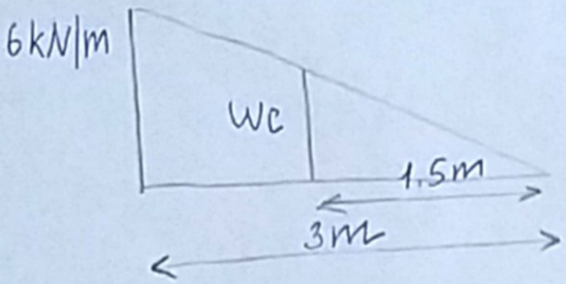
$$N_D = 0$$

$\Sigma F_y = 0 \qquad V_D - 10 + 9 = 0 \qquad V_D = 1\ kN$

$\circlearrowleft + \Sigma M_D = 0 \qquad -M_D + 9 \cdot 1.5 = 0$

$M_D = 13.5\ kN \cdot m$

Using similar triangles



$\frac{w_C}{6} = \frac{1.5}{3} \qquad w_C = 3\ \frac{kN}{m}$

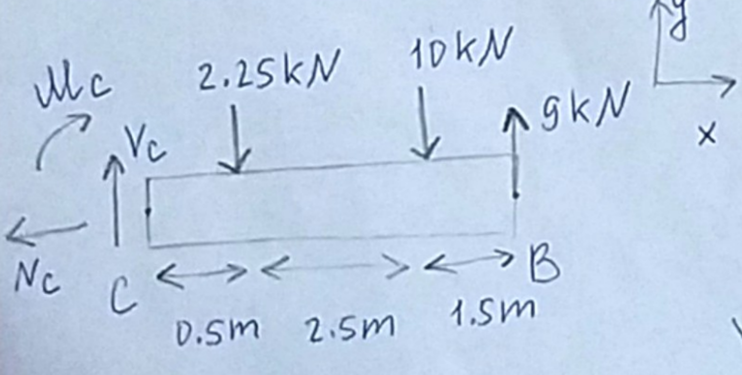
FBD of the right segment CB of the beam

Triangular loading can be replaced by its resultant force.

$F_R = \frac{1}{2} \cdot 1.5 \cdot 3 = 2.25\ kN$

$F_R = \{-2.25\ \hat{j}\}\ kN$

The location of F_R from point C: $\bar{x} = \frac{1}{3} \cdot 1.5 = 0.5\ m$



$\Sigma F_x = 0 \qquad -N_C = 0 \qquad N_C = 0$

$\Sigma F_y = 0$

$V_C - 2.25 - 10 + 9 = 0$

$V_C = 3.25\ kN$

$\circlearrowleft + \Sigma M_C = 0 \qquad -2.25 \cdot 0.5 - 10 \cdot 3 + 9 \cdot 4.5 - M_C = 0$

$$M_C = 9.375 \text{ kN}\cdot\text{m}$$

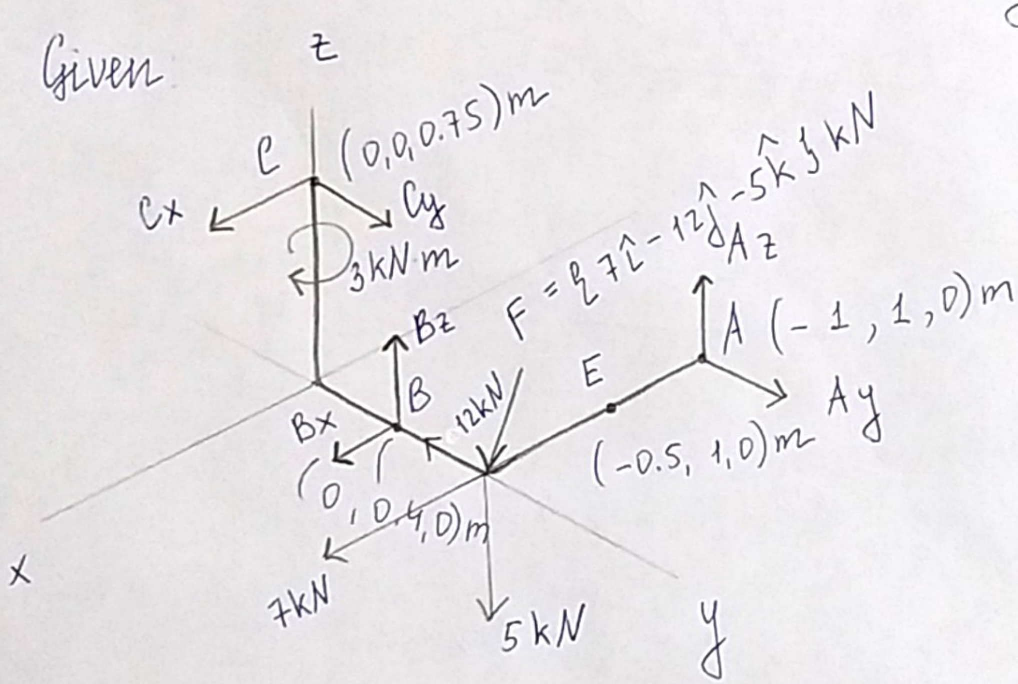
Answer: $N_C = 0$, $V_C = 3.25$ kN, $M_C = 9.375$ kN·m

$N_D = 0$, $V_D = 1$ kN, $M_D = 13.5$ kN·m

7-39

Given

journal bearings at A, B, and C



Determine x, y, z - components of internal loading in the rod at point E.

Solution.

External support reactions:

$\Sigma F_x = 0 \qquad C_x + B_x + 7 = 0 \qquad [1]$

$\Sigma F_y = 0 \qquad C_y + A_y - 12 = 0 \qquad [2]$

$\Sigma F_z = 0$ $\quad B_z + A_z - 5 = 0 \quad [3]$

$\Sigma M_x = 0$ $\quad -C_y \cdot 0.75 + B_z \cdot 0.4 + A_z \cdot 1 - 5 \cdot 1 = 0 \quad [4]$

$\Sigma M_y = 0$ $\quad C_x \cdot 0.75 + A_z \cdot 1 = 0 \quad [5]$

$\Sigma M_z = 0$ $\quad -B_x \cdot 0.4 - A_y \cdot 1 - 7 \cdot 1 - 3 = 0 \quad [6]$

From [5], $\underline{A_z = -0.75 C_x}$

From [3], $B_z = 5 - A_z$

$\underline{B_z = 5 + 0.75 C_x}$

From [1], $B_x = -7 - C_x$

From [6], $A_y = -10 - 0.4 B_x$

$A_y = -7.2 + 0.4 C_x$

From [2], $\underline{C_y = 12 - A_y = 19.2 - 0.4 C_x}$

Substituting A_z, B_z, C_y into [4]

$0.15 C_x = -17.4$

$C_x = -116 \text{ kN}$

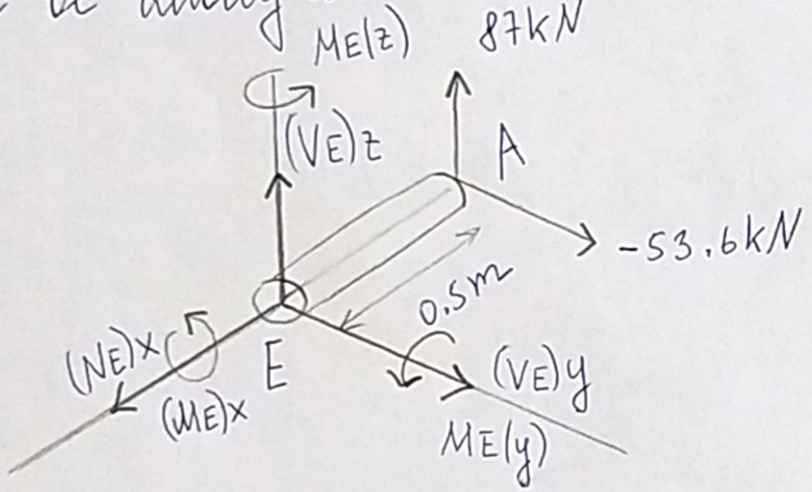
(The dir of C_x is opposite to that shown on FBD)

$$A_z = -0.75(-116) = 87 \text{ kN}$$

$$A_y = -7.2 + 0.4(-116) = -53.6 \text{ kN}$$

(The dir of A_y is opposite to that shown on FBD)

It is not needed to find other support reactions, as the right segment EA of the rod will be analyzed.



$\Sigma F_x = 0 \qquad (N_E)_x = 0$

$\Sigma F_y = 0 \qquad (V_E)_y + (-53.6) = 0$

$(V_E)_y = 53.6 \text{ kN}$

$\Sigma F_z = 0 \qquad (V_E)_z + 87 = 0$

$(V_E)_z = -87 \text{ kN}$

(The dir. of $(V_E)_z$ is opposite to that shown on FBD)

$\Sigma M_x = 0 \qquad (M_E)_x = 0$

$\Sigma M_y = 0 \qquad 87 \cdot 0.5 + (M_E)_y = 0$

$$(M_E)_y = -43.5 \text{ kN}\cdot\text{m}$$

(The dir of $(M_E)_y$ is opposite)

$\Sigma M_z = 0 \qquad -0.5(-53.6) + (M_E)_z = 0$

$$(M_E)_z = -26.8 \text{ kN}\cdot\text{m}$$

(The dir of $(M_E)_z$ is opposite)

Answer

$(N_E)_x = 0$

$(V_E)_y = 53.6 \text{ kN}$

$(V_E)_z = -87 \text{ kN}$

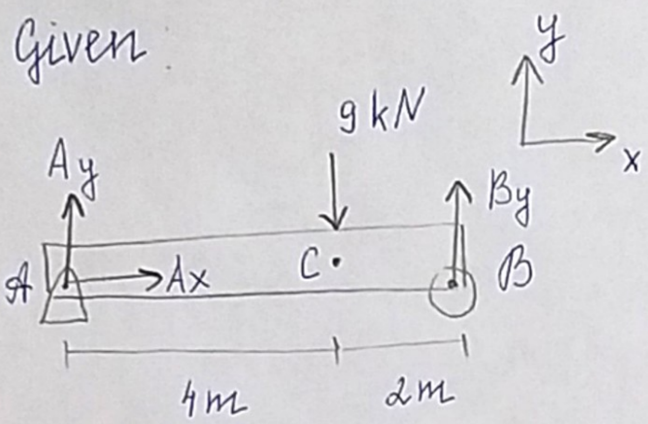
$(M_E)_x = 0$

$(M_E)_y = -43.5 \text{ kN m}$

$(M_E)_z = -26.8 \text{ kN m}$

7-41

Given



Draw the shear and moment diagrams for the simply supported beam

Solution

Support reactions $\sum F_x = 0 \quad A_x = 0$

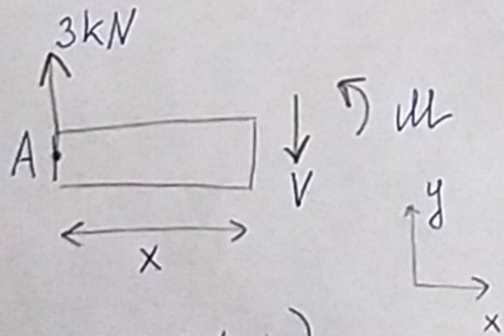
$\circlearrowright + \sum M_A = 0 \qquad -9 \cdot 4 + B_y \cdot 6 = 0$

$B_y = 6\ kN$

$\sum F_y = 0 \qquad A_y - 9 + 6 = 0 \qquad A_y = 3\ kN$

The beam is sectioned at an arbitrary distance x from point A, extending within the region AC

FBD of the left segment



$(0 \leq x < 4\ m)$

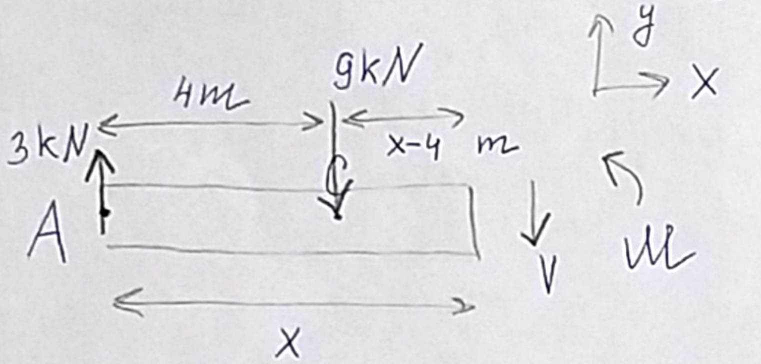
$\sum F_y = 0 \qquad 3 - V = 0$

$V = 3\ kN$

$\circlearrowright + \sum M = 0 \qquad -3x + M = 0$

$M = 3x \quad kN \cdot m$

FBD for a left segment of the beam extending a distance x within the region CB.



$(4\text{ m} < x \leq 6\text{ m})$

$\Sigma F_y = 0$

$3 - 9 - V = 0$

$V = -6\text{ kN}$

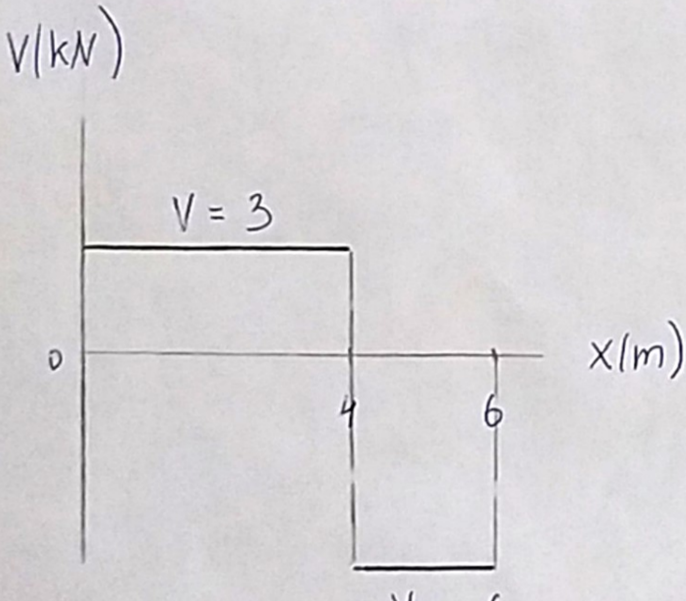
$\circlearrowleft + \Sigma M = 0$

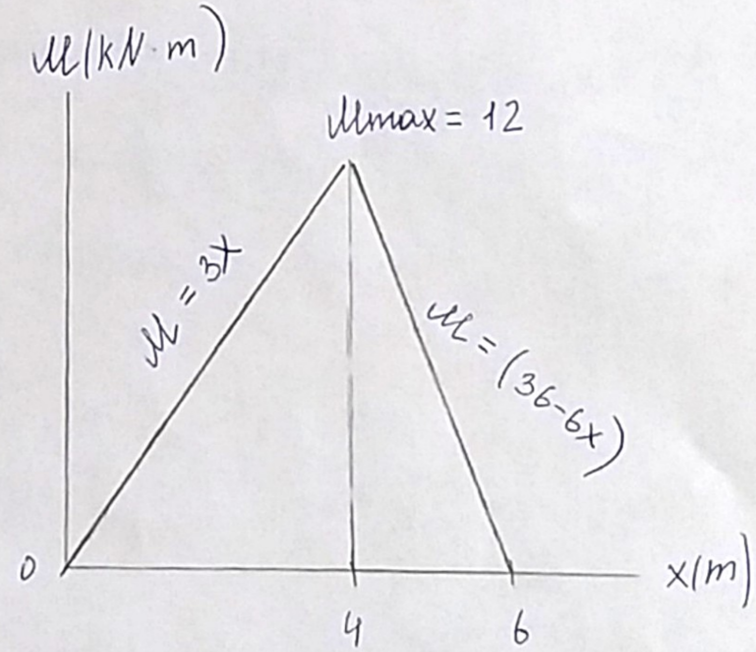
$-3x + 9(x-4) + M = 0$

$-3x + 9x - 36 + M = 0$

$M = (36 - 6x)\text{ kN}\cdot\text{m}$

$M_{max} = 3 \cdot 4 = 12\text{ kN}\cdot\text{m}$





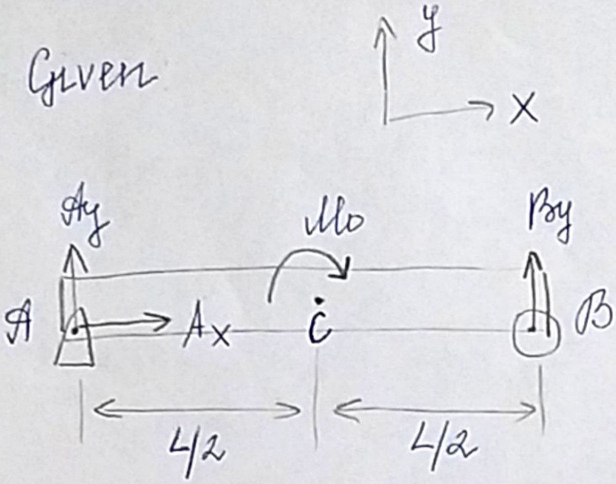
Shear Diagram

Moment Diagram

7-45

Given

$L = 9\,m$



The Beam will fail when

$V_{max} = 5\,kN$ or $M_{max} = 22\,kNm$

$M_{o\,max}$ - ?

Solution

Support reactions $\Sigma F_x = 0$ $A_x = 0$

$\circlearrowleft + \Sigma M_A = 0$ $B_y \cdot L - M_o = 0$

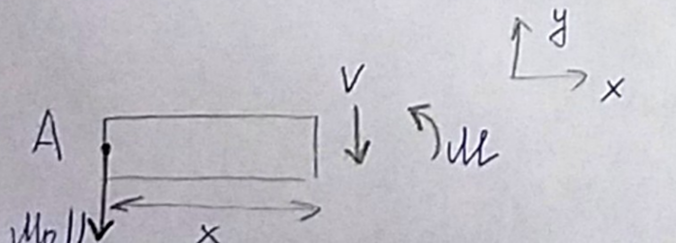
$$B_y = \frac{M_o}{L}$$

$\Sigma F_y = 0$ $A_y + \frac{M_o}{L} = 0$

$$A_y = -\frac{M_o}{L}$$

(The dir of A_y is opposite to that shown on FBD)

FBD for a left segment of the beam extending a distance x within the region AC

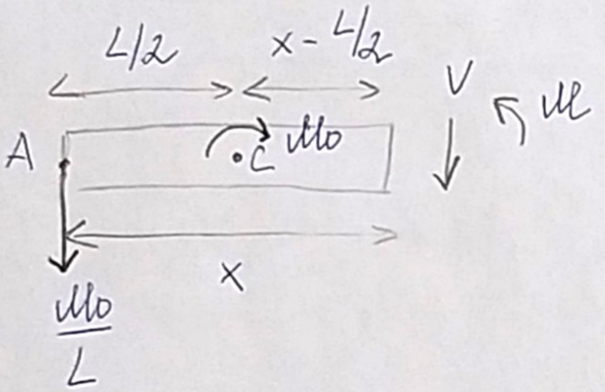


$\left(0 \leq x < \frac{L}{2} \right)$

$\Sigma F_y = 0 \qquad -\dfrac{M_o}{L} - V = 0 \qquad V = -\dfrac{M_o}{L} \qquad [1]$

$\circlearrowleft + \Sigma M = 0 \qquad \dfrac{M_o}{L}x + M = 0 \qquad M = -\dfrac{M_o}{L}x \qquad [2]$

FBD for a left segment of the beam extending a distance x within the region CB



$\left(\dfrac{L}{2} < x \leq L\right)$

$\Sigma F_y = 0 \qquad -\dfrac{M_o}{L} - V = 0 \qquad V = -\dfrac{M_o}{L} \qquad [3]$

$\circlearrowleft + \Sigma M = 0 \qquad \dfrac{M_o}{L}x - M_o + M = 0$

$$M = M_o - \frac{M_o}{L}x$$

$$M = M_o\left(1 - \frac{x}{L}\right) \qquad [4]$$

From [1] and [3], $V_{max} = \left|-\dfrac{M_o}{L}\right| = \dfrac{M_o}{L}$

From [2] and [4] $M_{max} = \left|-\dfrac{M_o}{L}\cdot\dfrac{L}{2}\right| = M_o\left(1 - \dfrac{L/2}{L}\right) = \dfrac{M_o}{2}$

If $L = 9\,m$, $V_{max} = \frac{M_o}{9} = 5$

$M_o = 45\ kN\cdot m$

$M_{max} = \frac{M_o}{2} = 22$ $\quad M_o = 44\ kN\cdot m$

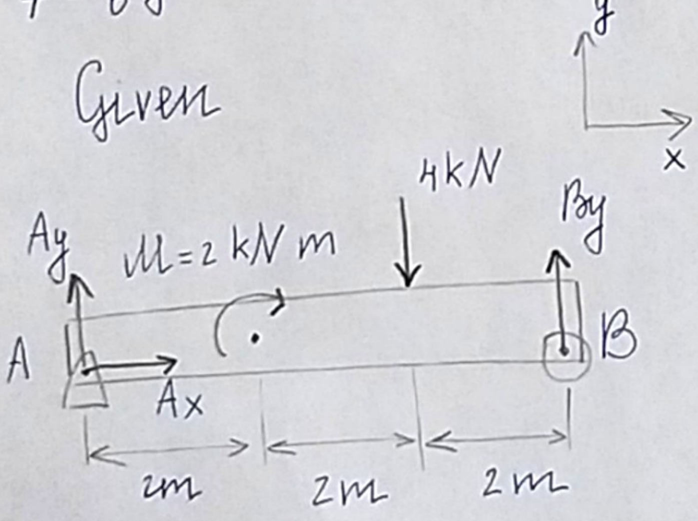
$44\ kN\,m < 45\ kN\,m \Rightarrow M_{o\,max} = 44\ kN\cdot m$

(If $M_{o\,max} = 45\ kN\cdot m$, $M_{max} > 22\ kN\cdot m$, that is not true)

Answer: $M_{o\,max} = 44\ kN\cdot m$

7-68

Given



Draw the shear and moment diagrams for the simply supported beam

Solution

Support reactions.

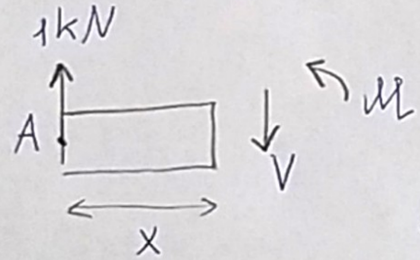
$\Sigma F_x = 0 \qquad A_x = 0$

$\circlearrowleft + \Sigma M_A = 0 \qquad -4 \cdot 4 + B_y \cdot 6 - 2 = 0$

$B_y = 3\ kN$

$\Sigma F_y = 0 \qquad A_y - 4 + 3 = 0 \qquad A_y = 1\ kN$

FBD for a left segment ($0 \leq x < 2m$)

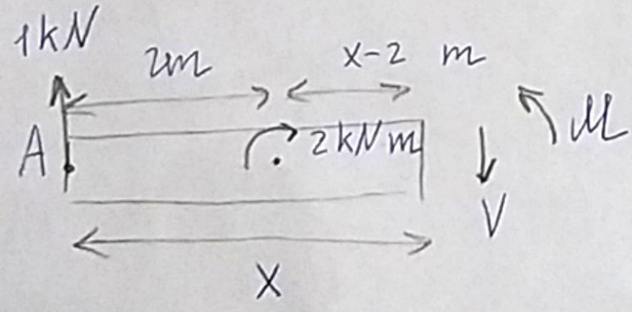


$\Sigma F_y = 0 \qquad 1 - V = 0 \qquad V = 1\ kN$

$\circlearrowleft + \Sigma M = 0 \qquad -1 \cdot x + M = 0 \qquad M = x\ kN \cdot m$

$M|_{x=0} = 0 \qquad M|_{x=2} = 2\ kN \cdot m$

FBD for a left segment ($2m < x < 4m$)



$\Sigma F_y = 0$

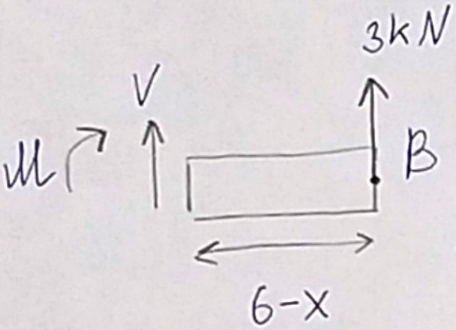
$1 - V = 0 \qquad V = 1\ kN$

$\circlearrowleft + \Sigma M = 0 \qquad -1 \cdot x - 2 + M = 0 \qquad M = x + 2$

$M|_{x=2} = 2+2 = 4 \text{ kN}\cdot\text{m}$

$M|_{x=4} = 4+2 = 6 \text{ kN}\cdot\text{m}$

FBD for a right segment $(4\text{m} < x \leq 6\text{m})$



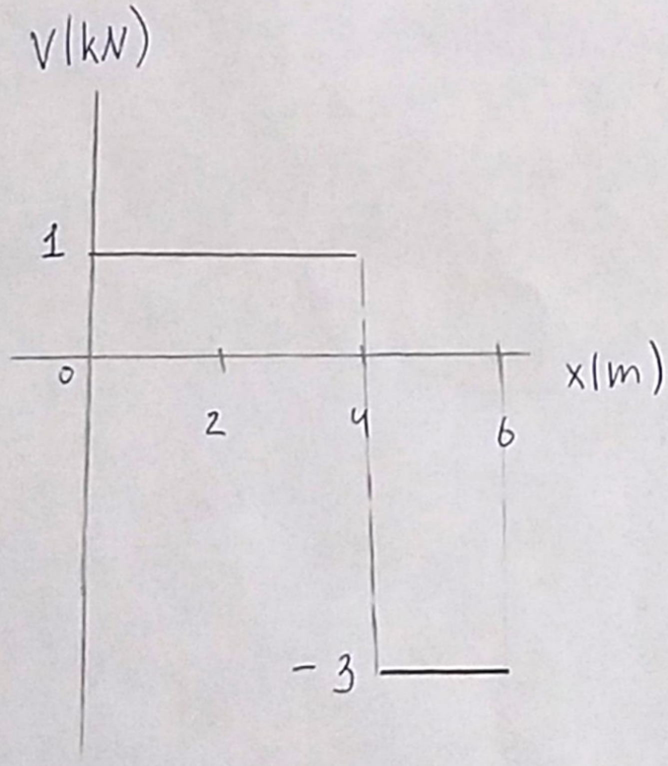
$\Sigma F_y = 0 \quad V + 3 = 0 \quad V = -3 \text{ kN}$

$\circlearrowleft + \Sigma M = 0 \quad 3(6-x) - M = 0$

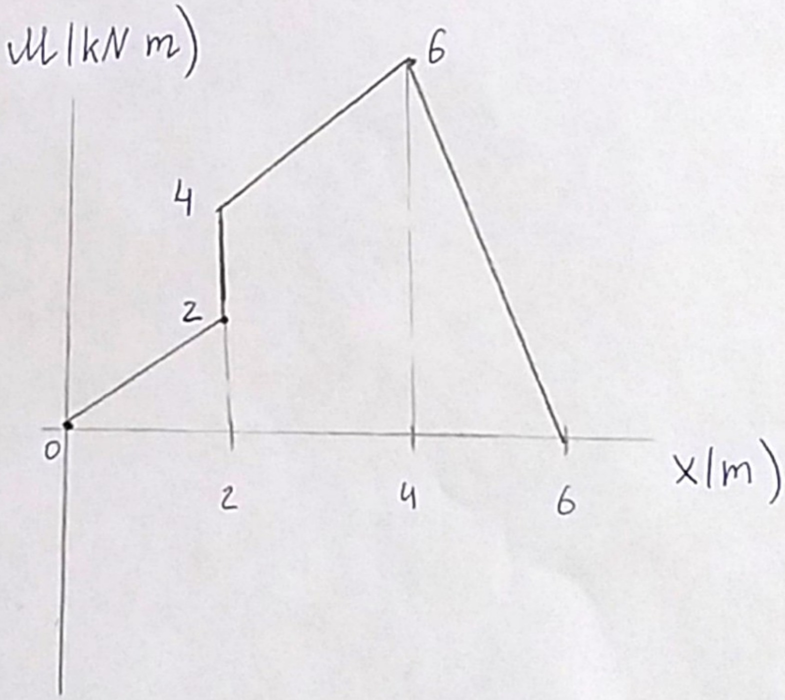
$M = 3(6-x) \text{ kN m}$

$M|_{x=4} = 3(6-4) = 6 \text{ kN m}$

$M|_{x=6} = 3(6-6) = 0$



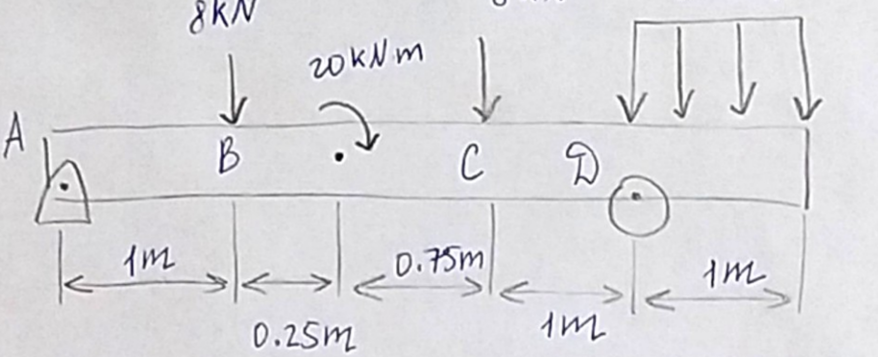
Shear Diagram



Moment Diagram

7-74

Given



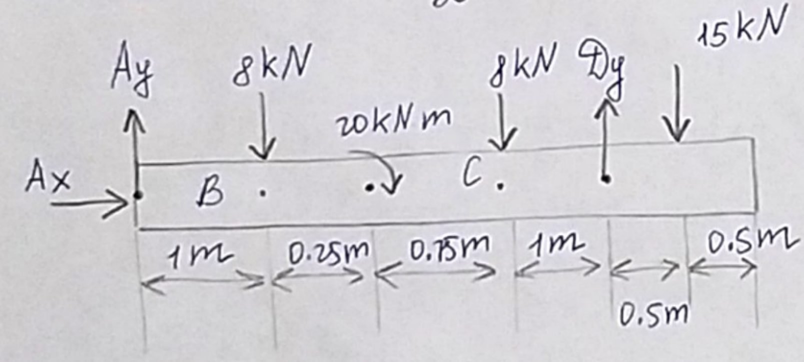
Draw the shear and moment diagrams for the Beam

Solution:

Support reactions

Rectangular loading can be replaced by its resultant force: $F_R = 15 \cdot 1 = 15$ kN $\quad \vec{F}_R = \{-15\hat{j}\}$ kN

The location of F_R from the right end of the beam. $\bar{x} = \frac{1}{2} \cdot 1 = 0.5$ m



$\Sigma F_x = 0 \quad A_x = 0$

$\circlearrowleft + \Sigma M_A = 0 \quad -8 \cdot 1 - 20 - 8 \cdot 2 + D_y\, 3 - 15 \cdot 3.5 = 0$

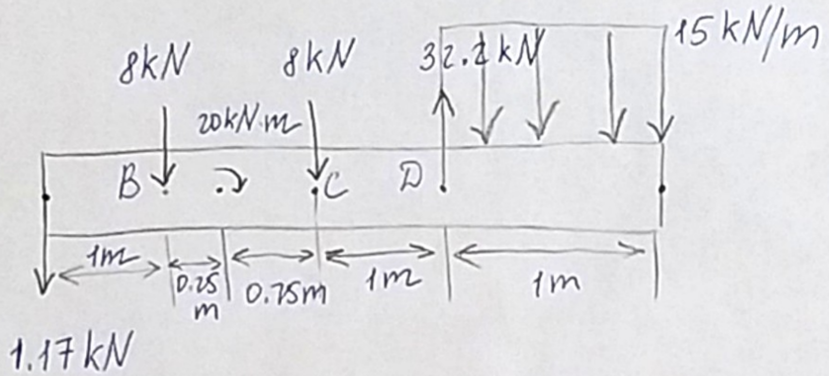
$$D_y = \frac{193}{6} = 32.2 \text{ kN}$$

$\Sigma F_y = 0 \quad A_y - 8 - 8 + \left(\frac{193}{6}\right) - 15 = 0$

$$A_y = -\frac{7}{6} = -1.17 \text{ kN}$$

(The dir of Ay is opposite to that shown on FBD)

It is simpler to use differential relations between distributed load, shear, and moment than to use the method of sections for this problem



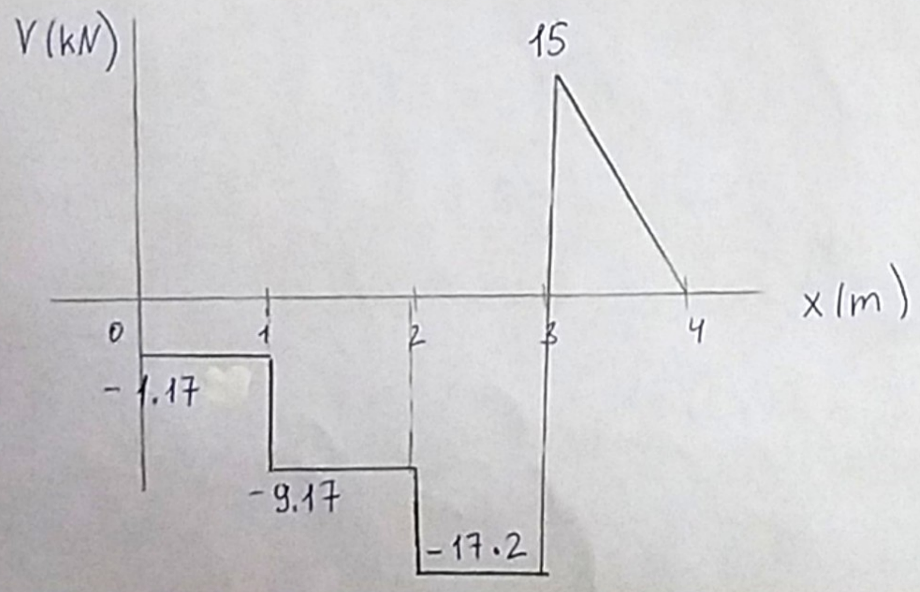
Knowing that $\frac{dV}{dx} = w(x)$ & $\Delta V =$ area under loading curve, or $\Delta V = F$,

$V|_{x=0} = -1.17\ kN$ $\qquad V|_{x=1m} = -\frac{7}{6} - 8 = -\frac{55}{6} = -9.17\ kN$

$$V|_{x=2m} = -\frac{55}{6} - 8 = -\frac{103}{6} = -17.2\ kN$$

$$V|_{x=3m} = -\frac{103}{6} + \frac{193}{6} = 15\ kN$$

$$V|_{x=4m} = 15 - 15 \cdot 1 = 0$$



Shear Diagram

Knowing that ΔM = area under shear diagram,

$\frac{dM}{dx} = V$ & or $\Delta M = M_0$,

$M|_{x=0} = 0$ $\quad$ $M|_{x=1m} = 0 - \frac{7}{6} \cdot 1 = -1.17$ kN·m

$M_1|_{x=1.25m} = -\frac{7}{6} - 0.25 \cdot \frac{55}{6} = -\frac{83}{24} = -3.46$ kN·m

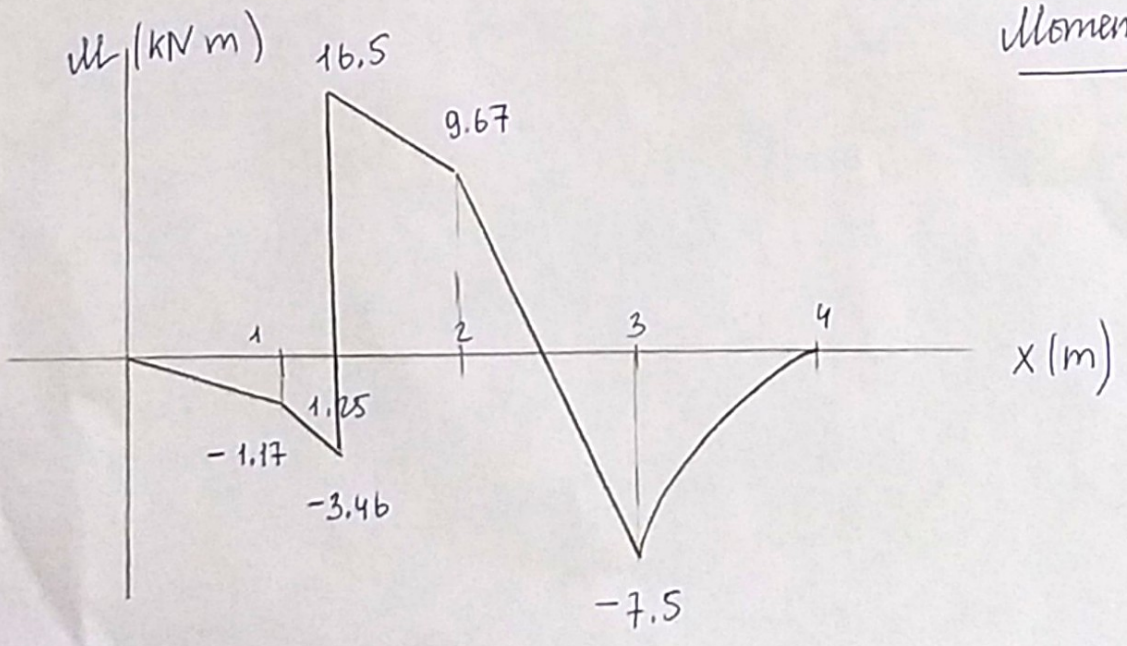
$M_2|_{x=1.25m} = -\frac{83}{24} + 20 = \frac{397}{24} = 16.5$ kN·m (positive jump)

$M|_{x=2m} = \frac{397}{24} - \frac{55}{6} \cdot 0.75 = \frac{29}{3} = 9.67$ kN·m

$M|_{x=3m} = \frac{29}{3} - \frac{103}{6} \cdot 1 = -\frac{15}{2} = -7.5$ kN·m

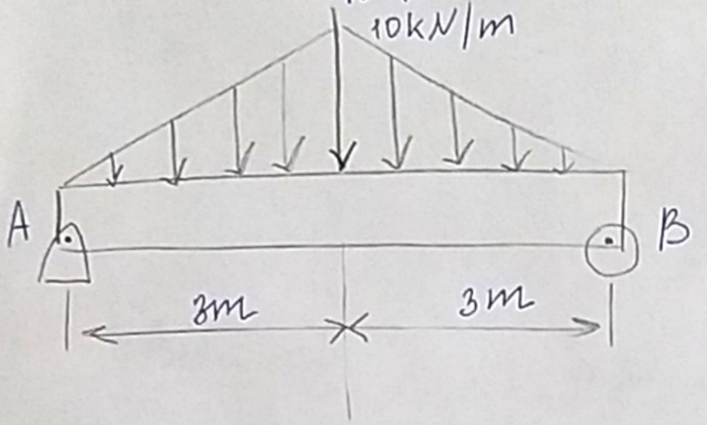
$M|_{x=4m} = -7.5 + \frac{15 \cdot 1}{2} = 0$

Moment Diagram



7-80

Given:



Draw the shear and moment diagrams for the simply supported beam

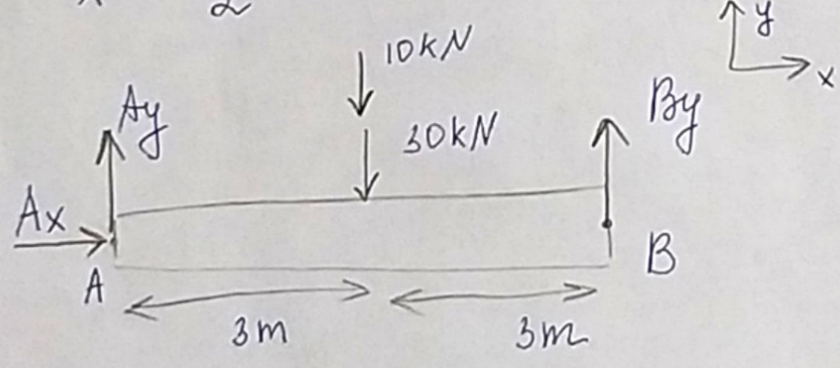
Solution

Support reactions:

Triangular loading can be replaced by its resultant force : $F_R = \frac{10 \cdot 6}{2} = 30 \text{ kN}$

The location of F_R from point A

$\bar{x} = \frac{1}{2} \cdot 6 = 3 \text{ m}$ (centroid of isosceles triangle)

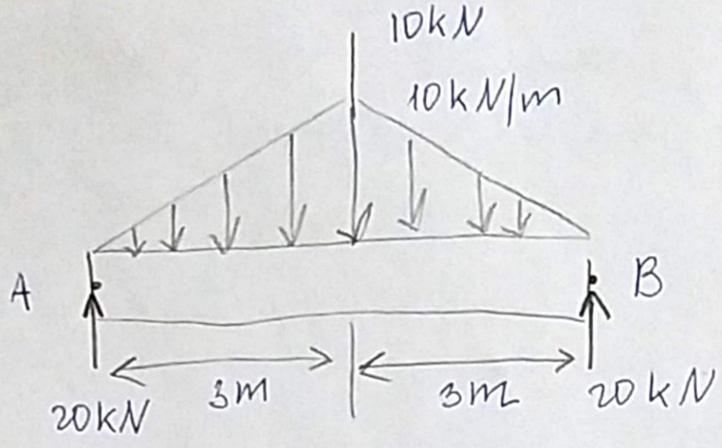


$\Sigma F_x = 0 \quad A_x = 0$

$\circlearrowleft + \Sigma M_A = 0 \qquad -10 \cdot 3 - 30 \cdot 3 + B_y \cdot 6 = 0$

$B_y = 20 \text{ kN}$

$\Sigma F_y = 0 \quad A_y - 10 - 30 + 20 = 0 \qquad A_y = 20 \text{ kN}$



Knowing that $\frac{dV}{dx} = w(x)$, and $\Delta V =$ area under loading curve,

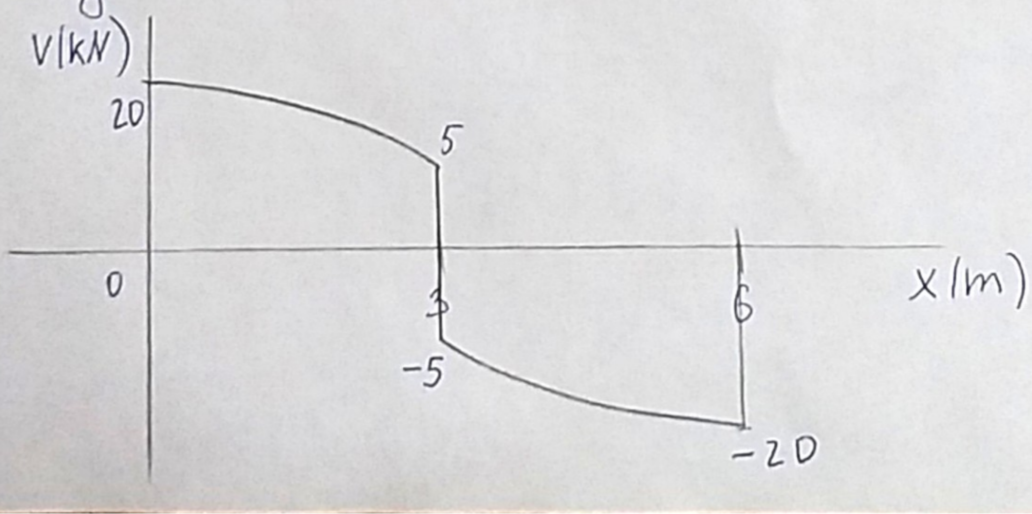
or $\Delta V = F$, it is possible to draw shear diagram

$V|_{x=0} = 20$ kN $\quad V_1|_{x=3} = 20 - \frac{10 \cdot 3}{2} = 5$ kN

$V_2|_{x=3} = 5 - 10 = -5$ kN (negative jump)

$V|_{x=6} = -5 - \frac{3 \cdot 10}{2} = -20$ kN

As distributed loadings are linear, shear diagram consists of parabolic curves

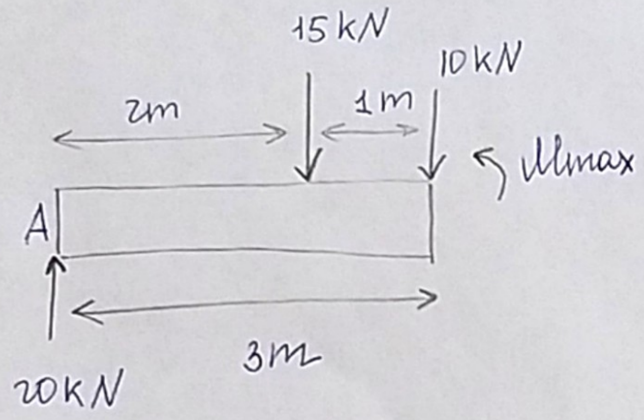


Shear Diagram

As shear changes sign at $x = 3m$, maximum moment occurs at this location $\left(\frac{dM}{dx} = V \right)$

It can be computed using the method of sections

FBD for a left segment of the beam

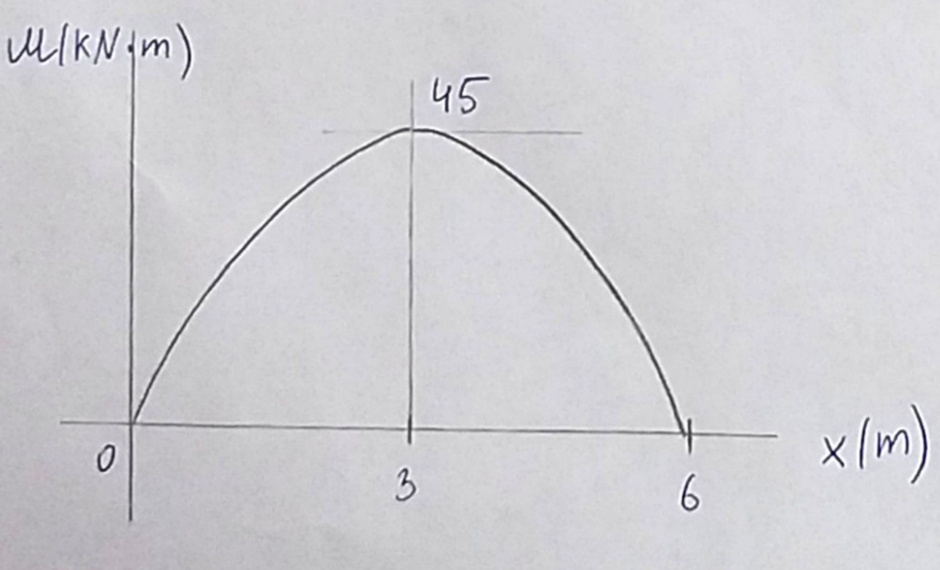


Distributed loading has been replaced by $F_R = \frac{10 \cdot 3}{2} = 15\ kN$ at $\bar{x} = 3 - \frac{3}{3} = 2m$ from point A

$\circlearrowright + \Sigma M = 0 \qquad 15 \cdot 1 - 20 \cdot 3 + M_{max} = 0$

$$M_{max} = 45\ kN \cdot m$$

Moment diagram is cubic, with $M|_{x=0} = M|_{x=6m} = 0$



Moment Diagram