Zarema Balgabekova

15-9

Given:

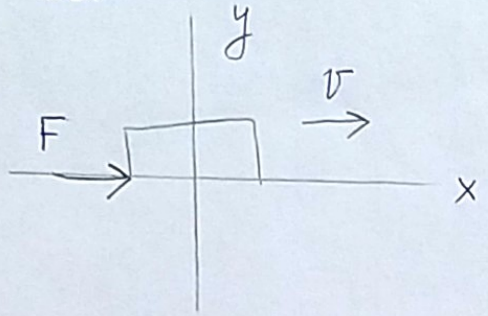
$m = 130\ Gg$

$v_0 = 0$

$F = 30(1 - e^{-0.1t})\ MN$

v when $t = 10s$ - ?

Solution:



$$\xrightarrow{+}\quad m v_{x1} + \sum \int_{t_1}^{t_2} F_x\, dt = m v_{x2}$$

$$0 + \int_0^{10s} (30 \cdot 10^6 - 30 \cdot 10^6 e^{-0.1t})\, dt = 130 \cdot 10^6 v$$

$$30 \cdot 10^6 t + 300 \cdot 10^6 e^{-0.1t} \Big|_0^{10s} = 130 \cdot 10^6 v$$

$$(30 \cdot 10^7 + 300 \cdot 10^6 e^{-1}) - 300 \cdot 10^6 = 130 \cdot 10^6 v$$

$$v = 0.849\ m/s$$

Answer: $v = 0.849\ m/s$

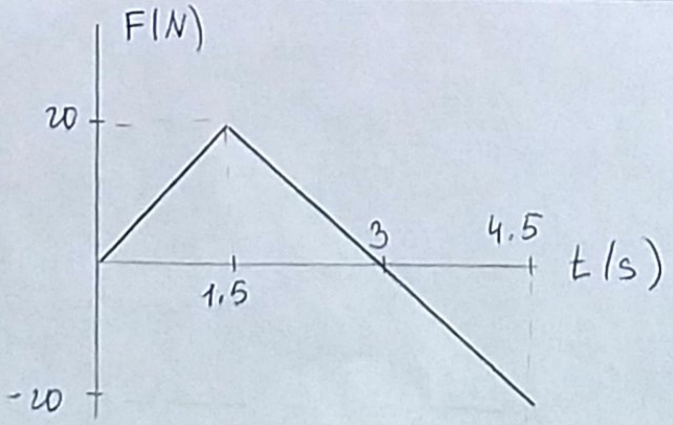
15-14

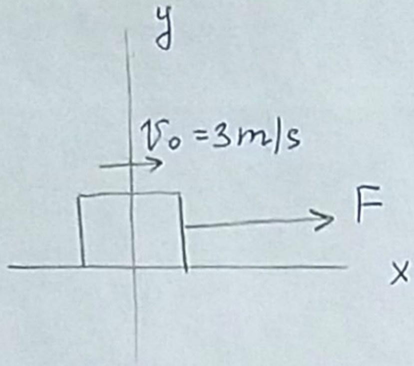
Given:

$m = 10\ kg$

$v_0 = 3\ m/s$

v when $t = 4.5s$ - ?





Solution:

$$F = \begin{cases} \frac{40}{3}t \quad N & 0 < t \leq 1.5 s \\ -\frac{40}{3}t + 40 \quad N & 1.5 s \leq t \leq 4.5 s \end{cases}$$

$$\xrightarrow{+} m v_{x1} + \sum \int_{t_1}^{t_2} F_x \, dt = m v_{x2}$$

$$10 \cdot 3 + \int_0^{1.5} \frac{40}{3} t \, dt + \int_{1.5}^{4.5} \left(-\frac{40}{3} t + 40\right) dt = 10 v$$

$$30 + \frac{20}{3} t^2 \Big|_0^{1.5} + \left(-\frac{20}{3} t^2 + 40t\right)\Big|_{1.5}^{4.5} = 10 v$$

$$30 + 15 + (45 - 45) = 10 v$$

$$v = 4.5 \ m/s$$

Answer: $v = 4.5 \ m/s$

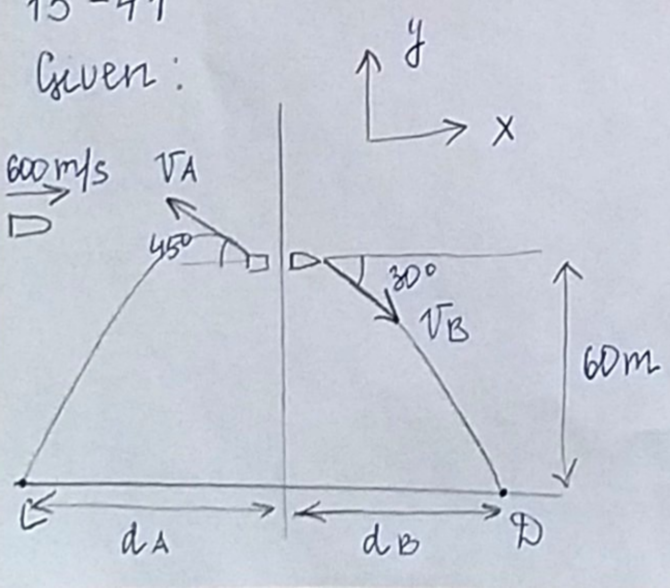
15-41

Given:

$m = 4 kg \quad m_A = 1.5 kg \quad m_B = 2.5 kg$

v_A, v_B - ?

d_B - ?



Solution:

$$\xrightarrow{+} \quad m v_x = m_A v_{Ax} + m_B v_{Bx}$$

$$4 \cdot 600 = -1.5 \cdot v_A \cdot \cos 45° + 2.5 \, v_B \cos 30° \quad [1]$$

$$\uparrow + \quad m v_y = m_A v_{Ay} + m_B v_{By}$$

$$0 = 1.5 \, v_A \sin 45° - 2.5 \, v_B \sin 30°$$

$$v_A = \frac{2.5 \sin 30°}{1.5 \sin 45°} v_B \quad [2]$$

Substituting [2] into [1].

$$2400 = -\cancel{1.5 \cos 45°}\left(\frac{2.5 \sin 30°}{\cancel{1.5 \sin 45°}} v_B\right) + 2.5 \, v_B \cos 30°$$

$$v_B = \frac{2400}{2.5(\cos 30° - \sin 30°)} = 2622.8 \text{ m/s} = 2.62 \text{ km/s}$$

$$\text{From [2]}, \; v_A = \frac{2.5 \sin 30°}{1.5 \sin 45°} 2622.8 = 3091 \text{ m/s} = 3.09 \text{ km/s}$$

$$+\uparrow \quad y = y_0 + (v_{B})_y t - \frac{1}{2} g t^2$$

$$0 = 60 - 2622.8 \cdot \sin 30° t - \frac{9.81}{2} t^2$$

$$4.905t^2 + 1311.4t - 60 = 0$$

$$t = 0.0457\,s$$

$$\xrightarrow{+} \quad x = x_0 + (v_{B0})_x t$$

$$d_B = 0 + 2622.8\cos 30° \cdot 0.0457 = 104\,m$$

Answer: $v_A = 3.09$ km/s $\quad v_B = 2.62$ km/s

$d_B = 104\,m$

15-51

Given

$m_b = 8$ kg

$v_{m_0} = 0$

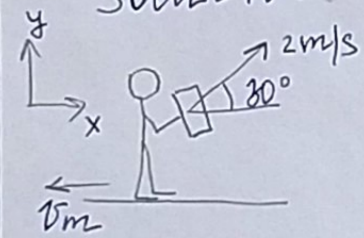
$t = 1.5s$

$m_m = 70$ kg

v_m (just after releasing the block) - ?

N (during the throw) - ?

Solution:



$$\xrightarrow{+} \quad m_b v_{bx_1} + m_m v_{mx_1} = m_b v_{bx_2} - m_m v_{mx_2}$$

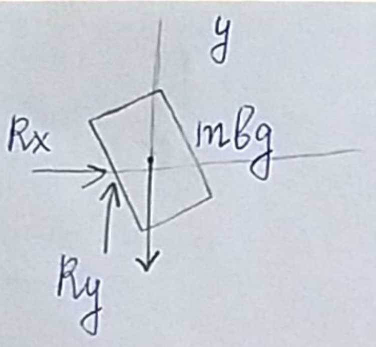
$$\vec{v}_{b/m} = \vec{v}_b - \vec{v}_m \qquad \vec{v}_b = \vec{v}_m + \vec{v}_{b/m}$$

$$v_{bx} = -v_m + 2\cos 30° \qquad v_{by} = 0 + 2\sin 30° = 1\,m/s$$

$$\Rightarrow \quad 0 = 8(-v_m + 2\cos 30°) - 70 v_m$$

$$78 v_m = 16\cos 30°$$

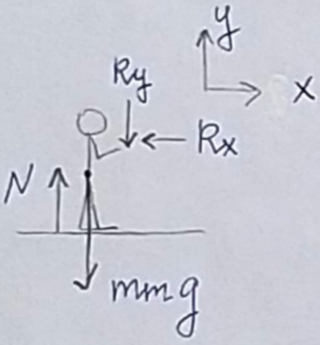
$$v_m = 0.178\,m/s \leftarrow$$



$$+\uparrow \; m_B v_{By1} + \sum \int_{t_1}^{t_2} F_y \, dt = m_B v_{By2}$$

$$0 + R_y \cdot 1.5 - 8 \cdot 9.81 \cdot 1.5 = 8 \cdot 1$$

$$R_y = 83.813 \ N$$



$$+\uparrow \sum F_y = 0$$

$$N - R_y - m_m g = 0$$

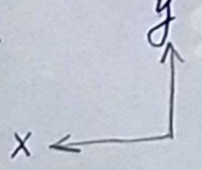
$$N = 83.813 + 70 \cdot 9.81 = 771 \ N$$

<u>Answer</u>: $v_m = 0.178 \ m/s \leftarrow$

$N = 771 \ N$

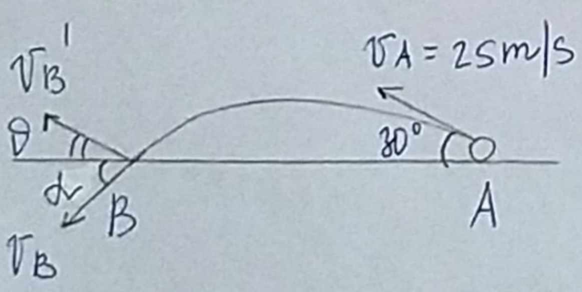
15-77

Given:



$m = 300 \ g$

$e = 0.4$



$v_B' - ?$

$\theta - ?$

Solution:

$v_{Bx} = v_{Ax}$ $\quad +\uparrow \; v_{By}^2 = v_{Ay}^2 - 2g(0-0)$

$$v_{By}^2 = v_{Ay}^2$$

$\Rightarrow v_B = v_A = 25 m/s$ $\quad \alpha = 30°$

$\overset{+}{\leftarrow} \quad \cancel{m} v_{Bx} = \cancel{m} v'_{Bx}$

$$v'_{Bx} = 25 \cos 30° = 21.65 \, m/s$$

The field remains at rest both before and after impact

$\Rightarrow \; +\uparrow \; e = \dfrac{0 - v'_{By}}{v_{By} - 0}$ $\quad v'_{By} = -0.4(-25 \sin 30°) = 5 m/s$

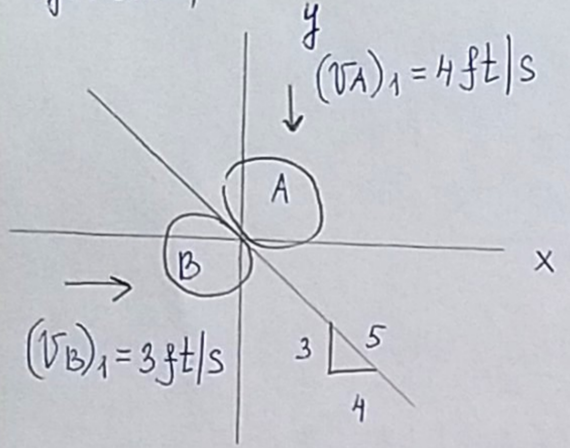
$$v'_B = \sqrt{v'^2_{Bx} + v'^2_{By}} = \sqrt{21.65^2 + 5^2} = 22.2 \, m/s$$

$$\theta = \tan^{-1}\left(\frac{v'_{By}}{v'_{Bx}}\right) = \tan^{-1}\left(\frac{5}{21.65}\right) = 13°$$

<u>Answer</u>: $v'_B = 22.2 \, m/s$ $\quad \theta = 13°$

15 - 89

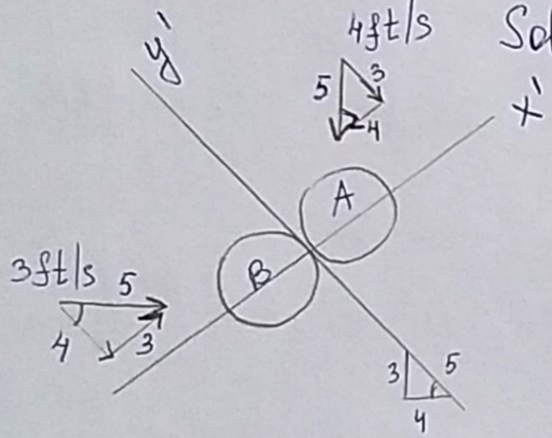
Given:



$W_A = W_B = 2\ lb$

$e = 0.5$

$(v_A)_2,\ (v_B)_2$ - ?



Solution:

$$+\nearrow \quad \cancel{m}_A V_{Ax'1} + \cancel{m}_B V_{Bx'1} = \cancel{m}_A V_{Ax'2} + \cancel{m}_B V_{x'2}$$

$$-4 \cdot \frac{4}{5} + 3\,\frac{3}{5} = V_{Ax'2} + V_{Bx'2} \quad [1]$$

$$+\nearrow \quad e = \frac{v_{Bx'2} - V_{Ax'2}}{V_{Ax'1} - V_{Bx'1}} = \frac{V_{Bx'2} - V_{Ax'2}}{-4\,\frac{4}{5} - 3 \cdot \frac{3}{5}} = 0.5 \quad [2]$$

Solving [1] and [2] simultaneously

$$\begin{cases} V_{Ax'2} + V_{Bx'2} = -\frac{7}{5} \\ -V_{Ax'2} + V_{Bx'2} = -\frac{5}{2} \end{cases} \qquad \begin{cases} v_{Ax'2} = 0.55\ ft/s \\ v_{Bx'2} = -1.95\ ft/s \end{cases} \qquad \begin{cases} V_{Ax'2} = 0.55\ ft/s \ \nearrow \\ v_{Bx'2} = 1.95\ ft/s \ \swarrow \end{cases}$$

$+\nwarrow \quad \cancel{m_A} v_{Ay'1} = \cancel{m_A} v_{Ay'2}$

$$v_{Ay'2} = -4\frac{3}{5} = -2.4 \text{ ft/s} = 2.4 \text{ ft/s} \searrow$$

$+\nwarrow \quad \cancel{m_B} v_{By'1} = \cancel{m_B} v_{By'2}$

$$v_{By'2} = -3\frac{4}{5} = -2.4 \text{ ft/s} = 2.4 \text{ ft/s} \searrow$$

$$(v_A)_2 = \sqrt{v_{Ax'2}^2 + v_{Ay'2}^2} = \sqrt{0.55^2 + (-2.4)^2} = 2.46 \text{ ft/s}$$

$$(v_B)_2 = \sqrt{v_{Bx'2}^2 + v_{By'2}^2} = \sqrt{(-1.95)^2 + (-2.4)^2} = 3.09 \text{ ft/s}$$

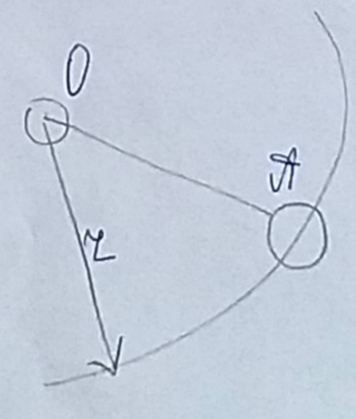
Answer: $(v_A)_2 = 2.46$ ft/s $\quad (v_B)_2 = 3.09$ ft/s

15-99

Given:

$v_1 = 4$ ft/s when $r = 12$ ft

$\frac{dr}{dt} = 0.5$ ft/s



v in 3s - ?

Solution:

$\Delta r = 0.5 \cdot 3 = 1.5\ ft \quad \Rightarrow \quad r_2 = 12 - 1.5 = 10.5\ ft$

$$(H_O)_1 = (H_O)_2$$

$$r_1 \cancel{m} v_1 = r_2 \cancel{m} v_2'$$

$$v_2' = \frac{12 \cdot 4}{10.5} = \frac{32}{7} = 4.57\ ft/s$$

$$v = \sqrt{\left(\frac{32}{7}\right)^2 + 0.5^2} = 4.6\ ft/s$$

Answer: $v = 4.6\ ft/s$

15-104

Given:

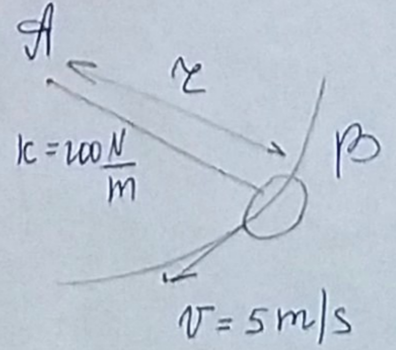
$m = 5 kg$

$v = 5 m/s$ at $r = 1.5 m$

(perpendicular to the cord)

v and v_r when $r = 1.2 m$ - ?

$l_{unstretched} = 0.5 m$



Solution:

$$(H_A)_1 = (H)_2$$

$$r_1 \cancel{m} v_1 = r_2 \cancel{m} v_2'$$

$$v_2' = \frac{1.5 \cdot 5}{1.2} = 6.25 \text{ m/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{5 \cdot 5^2}{\cancel{2}} + \frac{200\,(1.5-0.5)^2}{\cancel{2}} = \frac{5 \cdot v^2}{\cancel{2}} + \frac{200\,(1.2-0.5)^2}{\cancel{2}}$$

$$v = \sqrt{\frac{5^3 + 200(1.5-0.5)^2 - 200\,(1.2-0.5)^2}{5}} = 6.74 \text{ m/s}$$

$$v_r = \sqrt{6.74^2 - 6.25^2} = 2.52 \text{ m/s}$$

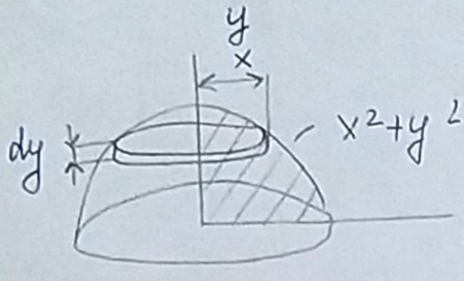
Answer:

$v = 6.74$ m/s

$v_r = 2.52$ m/s

17 - 6

Given:



constant ρ

I_y (in terms of m) - ?

Solution:

$$dm = \rho dV = \rho \pi x^2 dy \qquad x^2 = r^2 - y^2$$

$$m = \rho \pi \int_0^r (r^2 - y^2) dy = \rho \pi \left(r^2 y - \frac{y^3}{3}\right)\Big|_0^r = \frac{2}{3} \rho \pi r^3$$

$$dI_y = \frac{1}{2}(dm) x^2 = \frac{1}{2}(\rho \pi x^2 dy) x^2 = \frac{1}{2} \rho \pi x^4 dy$$

$$I_y = \frac{\rho \pi}{2} \int_0^r (r^2 - y^2)^2 dy = \frac{\rho \pi}{2} \int_0^r (r^4 - 2r^2 y^2 + y^4) dy =$$

$$\frac{\rho \pi}{2}\left(r^4 y - 2r^2 \frac{y^3}{3} + \frac{y^5}{5}\right)\Big|_0^r = \frac{4}{15} \rho \pi r^5$$

$$\Rightarrow I_y = \frac{2}{5} m r^2$$

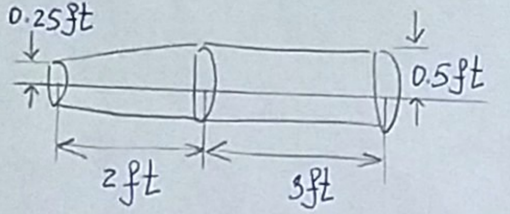
Answer: $I_y = \frac{2}{5} m r^2$

17-17

Given:

$\gamma_{st} = 490\ lb/ft^3$

$I_x - ?$



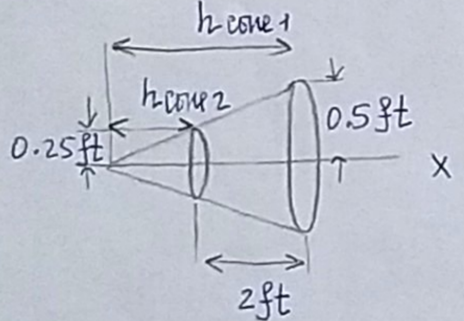
Solution:

$$(I_{cyl})_x = \frac{1}{2} m_{cyl} r_{cyl}^2 \qquad m_{cyl} = \rho V_{cyl} = \left(\frac{\gamma_{st}}{g}\right) \pi r_{cyl}^2 h_{cyl}$$

$$(I_{cyl})_x = \frac{1}{2}\left(\frac{\gamma_{st}}{g}\right)\pi h_{cyl} r_{cyl}^4 = \frac{1}{2}\,\frac{490}{32.2}\,\pi \cdot 3 \cdot 0.5^4 = 4.482\ slug \cdot ft^2$$

For the truncated cone: $(I_{tr\,cone})_x = (I_{cone\,1})_x - (I_{cone\,2})_x =$

$$\frac{3}{10} m_{cone\,1} r_{cone\,1}^2 - \frac{3}{10} m_{cone\,2} r_{cone\,2}^2$$



$$m_{cone\,1} = \rho V_{cone\,1} = \left(\frac{\gamma_{st}}{g}\right)\frac{1}{3}\pi r_{cone\,1}^2 h_{cone\,1}$$

$$m_{cone\,2} = \rho V_{cone\,2} = \left(\frac{\gamma_{st}}{g}\right)\frac{1}{3}\pi r_{cone\,2}^2 h_{cone\,2}$$

$h_{cone\,1} = h_{cone\,2} + 2$

$$\frac{h_{cone\,2}}{h_{cone\,2}+2} = \frac{0.25}{0.5}$$

$h_{cone\,2} = 2\ ft$

$h_{cone\,1} = 4\ ft$

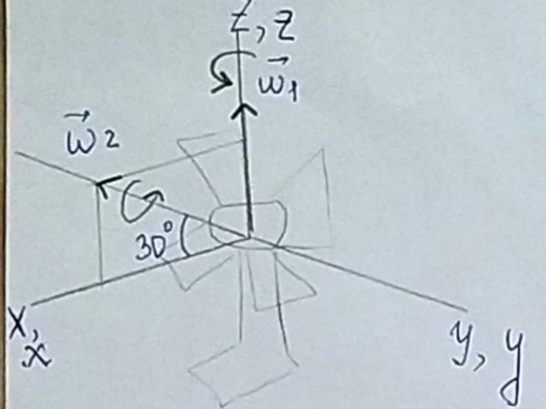
$$(I_{tr\,cone})_x = \frac{1}{10}\frac{\gamma_{st}}{g}\pi h_{cone\,1} r_{cone\,1}^4 - \frac{1}{10}\frac{\gamma_{st}}{g}\pi h_{cone\,2} r_{cone\,2}^4 =$$

$$\frac{1}{10}\cdot\frac{490}{32.2}\pi\left(4 \cdot 0.5^4 - 2 \cdot 0.25^4\right) = 1.158\ slug \cdot ft^2$$

$$I_x = (I_{cyl})_x + (I_{tr\,cone})_x = 4.482 + 1.158 = 5.64\ slug\,ft^2$$

Answer: $I_x = 5.64\ slug\ ft^2$

20-4 Given:



$\omega_1 = 0.8$ rad/s

$\dot{\omega}_1 = 12$ rad/s^2 (increasing)

$\omega_2 = 16$ rad/s

$\dot{\omega}_2 = -2$ rad/s^2 (decreasing)

$\vec{\omega}, \vec{\alpha}$ - ?

Solution:

$\vec{\omega}_1 = \{0.8\hat{k}\}$ rad/s $\quad \vec{\omega}_2 = 16\cos 30°\hat{i} + 16\sin 30°\hat{k} =$

$\{8\sqrt{3}\hat{i} + 8\hat{k}\}$ rad/s

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = 0.8\hat{k} + (8\sqrt{3}\hat{i} + 8\hat{k}) = \{13.9\hat{i} + 8.8\hat{k}\}$ rad/s

$\dot{\vec{\omega}}_2 = (\dot{\vec{\omega}}_2)_{xyz} + \vec{\omega}_1 \times \vec{\omega}_2 = (-2\cos 30°\hat{i} - 2\sin 30°\hat{k}) +$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.8 \\ 8\sqrt{3} & 0 & 8 \end{vmatrix} = -2\cos 30°\hat{i} - 2\sin 30°\hat{k} + \hat{j}\, 8\sqrt{3}\cdot 0.8 =$

$\{-\sqrt{3}\hat{i} + 6.4\sqrt{3}\hat{j} - \hat{k}\}$ rad/s^2

$\dot{\vec{\omega}}_1 = (\dot{\vec{\omega}}_1)_{xyz} + \vec{\omega}_1 \times \omega_1 = 12\hat{k} + 0 = \{12\hat{k}\}$ rad/s^2

$\vec{\alpha} = \dot{\vec{\omega}} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2 = 12\hat{k} + (-\sqrt{3}\hat{i} + 6.4\sqrt{3}\hat{j} - \hat{k}) =$

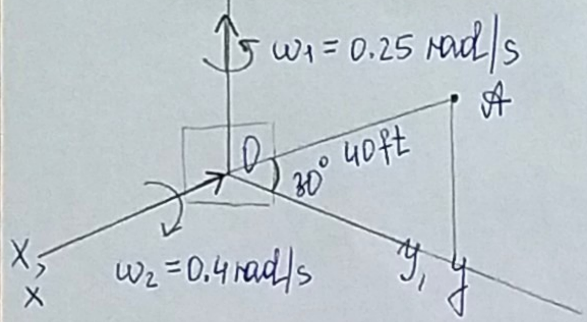
$\{-1.73\hat{i} + 11.1\hat{j} + 11\hat{k}\}$ rad/s^2

<u>Answer:</u> $\vec{\omega} = \{13.9\hat{i} + 8.8\hat{k}\}$ rad/s

$\vec{\alpha} = \{-1.73\hat{i} + 11.1\hat{j} + 11\hat{k}\}$ rad/s^2

20-13

Given: Z, z



$\dot{\omega}_1 = 0.6\ rad/s^2$ (increasing)

$\dot{\omega}_2 = 0.8\ rad/s^2$ (increasing)

$\vec{v}_A,\ \vec{a}_A$ - ?

Solution:

$\vec{\omega}_1 = \{0.25\hat{k}\}$ rad/s $\quad \vec{\omega}_2 = \{-0.4\hat{i}\}$ rad/s

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = \{-0.4\hat{i} + 0.25\hat{k}\}$ rad/s

$$\dot{\vec{\omega}}_2 = (\dot{\vec{\omega}}_2)_{xyz} + \vec{\omega}_1 \times \vec{\omega}_2 = -0.8\hat{i} + 0.25\hat{k} \times (-0.4\hat{i}) =$$

$$\{-0.8\hat{i} - 0.1\hat{j}\}\ rad/s^2$$

$$\dot{\vec{\omega}}_1 = (\dot{\vec{\omega}}_1)_{xyz} + \vec{\omega}_1 \times \vec{\omega}_1 = 0.6\hat{k} + 0 = \{0.6\hat{k}\}\ rad/s^2$$

$$\alpha = \dot{\vec{\omega}} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2 = \{-0.8\hat{i} - 0.1\hat{j} + 0.6\hat{k}\}\ rad/s^2$$

$$\vec{r}_A = 40\cos 30°\hat{j} + 40\sin 30°\hat{k} = \{20\sqrt{3}\hat{j} + 20\hat{k}\}\ m$$

$$\vec{v}_A = \vec{\omega} \times \vec{r}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.4 & 0 & 0.25 \\ 0 & 20\sqrt{3} & 20 \end{vmatrix} = -0.25\,20\sqrt{3}\hat{i} + 0.4\cdot 20\hat{j} - 0.4\ 20\sqrt{3}\hat{k}$$

$$= \{-8.66\hat{i} + 8\hat{j} - 13.9\hat{k}\}\ ft/s$$

$$\vec{a}_A = \vec{\alpha} \times \vec{r}_A + \vec{\omega} \times \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.8 & -0.1 & 0.6 \\ 0 & 20\sqrt{3} & 20 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.4 & 0 & 0.25 \\ -5\sqrt{3} & 8 & -8\sqrt{3} \end{vmatrix} =$$

$$-22.785\hat{i} + 16\hat{j} - 16\sqrt{3}\hat{k} - 2\hat{i} - 7.708\hat{j} - 3.2\hat{k} =$$

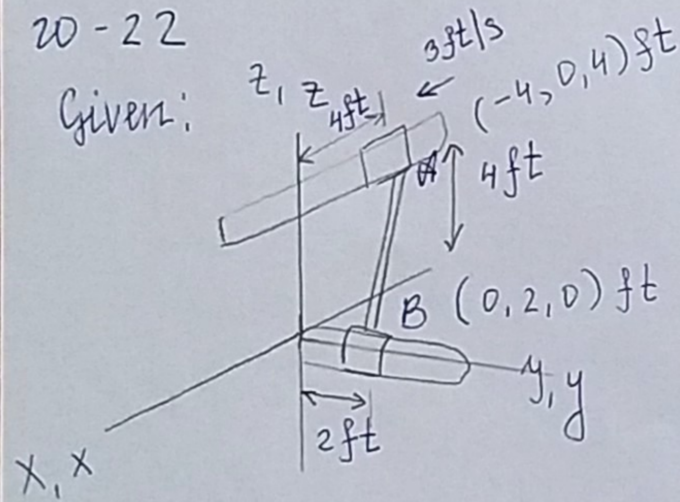
$$\{-24.8\hat{i} + 8.29\hat{j} - 30.9\hat{k}\} \text{ ft/s}^2$$

<u>Answer</u> $\vec{v}_A = \{-8.66\hat{i} + 8\hat{j} - 13.9\hat{k}\} \text{ ft/s}$

$\vec{a}_A = \{-24.8\hat{i} + 8.29\hat{j} - 30.9\hat{k}\} \text{ ft/s}^2$

20-22

Given:



$\vec{a}_A = \{8\hat{i}\} \text{ ft/s}^2$

$\vec{v}_A = \{3\hat{i}\} \text{ ft/s}$

$\vec{\alpha}$ - ? (directed perpendicular to the rod)

$\vec{a}_B$ - ?

Solution:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{v}_B = v_B\hat{j} \qquad v_A = \{3\hat{i}\} \text{ ft/s}$$

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = \{4\hat{i} + 2\hat{j} - 4\hat{k}\} \text{ ft}$$

$$\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$$

$$v_B\hat{j} = 3\hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 4 & 2 & -4 \end{vmatrix} = 3\hat{i} + (-4\omega_y - 2\omega_z)\hat{i} + (4\omega_x + 4\omega_z)\hat{j} + (2\omega_x - 4\omega_y)\hat{k}$$

$\Rightarrow \quad 3 - 4\omega_y - 2\omega_z = 0 \quad [1]$

$4\omega_x + 4\omega_z = v_B \quad [2]$

$2\omega_x - 4\omega_y = 0 \quad [3]$

If $\vec{\omega}$ is directed perpendicular to the rod,

$\vec{\omega} \cdot \vec{r}_{B|A} = 0 \qquad (\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k})\cdot(4\hat{i} + 2\hat{j} - 4\hat{k}) = 0$

$4\omega_x + 2\omega_y - 4\omega_z = 0 \quad [4]$

From [1], $\omega_z = 1.5 - 2\omega_y$

From [3], $\omega_x = 2\omega_y$

substituting ω_z and ω_x into [4]

$8\omega_y + 2\omega_y - 6 + 8\omega_y = 0$

$18\omega_y = 6 \qquad \omega_y = \frac{1}{3}$ rad/s

$\Rightarrow \quad \omega_x = \frac{2}{3}$ rad/s $\qquad \omega_z = \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$ rad/s

From [2], $v_B = \frac{8}{3} + \frac{20}{6} = 6$ ft/s

$\vec{\omega} = \left\{\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{5}{6}\hat{k}\right\}$ rad/s

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}\times\vec{r}_{B/A} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{B/A})$$

$$\vec{a}_B = a_B\hat{j} \qquad \vec{a}_A = \{8\hat{i}\}\ ft/s^2$$

$$\vec{\alpha} = \alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k}$$

$$\vec{\omega}\times\vec{r}_{B/A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{3} & \frac{4}{3} & \frac{5}{6} \\ 4 & 2 & -4 \end{vmatrix} = -3\hat{i} + 6\hat{j}$$

$$a_B\hat{j} = 8\hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 4 & 2 & -4 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{3} & \frac{1}{3} & \frac{5}{6} \\ -3 & 6 & 0 \end{vmatrix} =$$

$$8\hat{i} + (-4\alpha_y - 2\alpha_z)\hat{i} + j(4\alpha_x + 4\alpha_z) + k(2\alpha_x - 4\alpha_y)$$

$$-5\hat{i} - 2.5\hat{j} + 5\hat{k}$$

$$\Rightarrow \quad 3 - 4\alpha_y - 2\alpha_z = 0 \quad [1]$$

$$-2.5 + 4\alpha_x + 4\alpha_z = a_B \quad [2]$$

$$5 + 2\alpha_x - 4\alpha_y = 0 \quad [3]$$

If $\vec{\alpha}$ is directed perpendicular to the rod,

$$\vec{\alpha}\cdot\vec{r}_{B/A} = 0$$

$$(\alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k})\cdot(4\hat{i} + 2\hat{j} - 4\hat{k}) = 0$$

$$4\alpha_x + 2\alpha_y - 4\alpha_z = 0 \quad [4]$$

From [1], $\alpha z = 1.5 - 2\alpha y$

From [3], $\alpha x = 2\alpha y - 2.5$

substituting αx and αz into [4],

$$8\alpha y - 10 + 2\alpha y - 6 + 8\alpha y = 0$$

$$18\alpha y = 16 \qquad \alpha y = \frac{8}{9} \text{ rad/s}^2$$

$$\Rightarrow \alpha x = \frac{16}{9} - 2.5 = -\frac{13}{18} \text{ rad/s}^2$$

$$\alpha z = 1.5 - \frac{16}{9} = -\frac{5}{18} \text{ rad/s}^2$$

From [2], $a_B = -2.5 + \left(-\frac{52}{18}\right) + \left(-\frac{20}{18}\right) = -6.5 \text{ ft/s}^2$

$$\Rightarrow \vec{\alpha} = \left\{-\frac{13}{18}\hat{i} + \frac{8}{9}\hat{j} - \frac{5}{18}\hat{k}\right\} \text{ rad/s}^2$$

$$\vec{a}_B = \{-6.5\hat{j}\} \text{ ft/s}^2$$

Answer

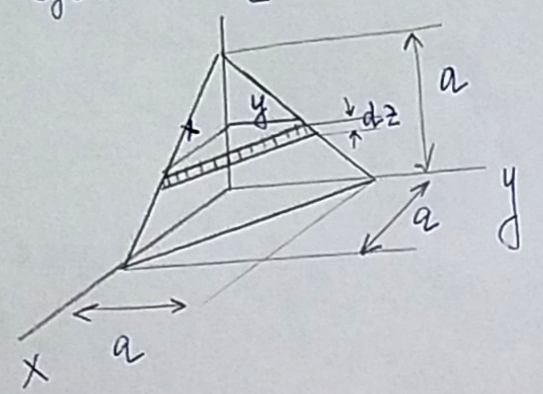
$$\vec{\alpha} = \left\{-\frac{13}{18}\hat{i} + \frac{8}{9}\hat{j} - \frac{5}{18}\hat{k}\right\} \text{ rad/s}^2$$

$$\vec{a}_B = \{-6.5\hat{j}\} \text{ ft/s}^2$$

21-6

density ρ

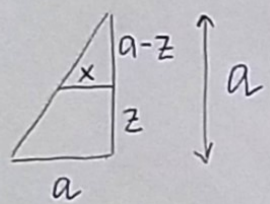
Given:



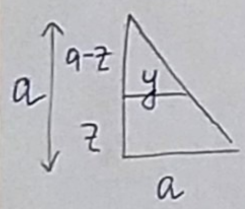
I_{xy} (in terms of m) - ?

Solution:

$$dm = \rho dV = \rho \frac{xy}{2} \cdot dz$$



$$\frac{a-z}{a} = \frac{x}{a} \Rightarrow x = a - z$$



$$\frac{a-z}{a} = \frac{y}{a} \Rightarrow y = a - z$$

$$dm = \rho \frac{(a-z)^2}{2} dz$$

$$m = \frac{\rho}{2} \int_0^a (a-z)^2 dz = \frac{\rho}{2} \int_0^a (a^2 - 2az + z^2) dz =$$

$$\frac{\rho}{2} \left(a^2 z - a z^2 + \frac{z^3}{3} \right) \Big|_0^a = \frac{\rho}{2} \frac{a^3}{3} = \frac{\rho a^3}{6}$$

For a triangular prism: $I_{xy} = \dfrac{\rho a^4 h}{24}$

$$\Rightarrow dI_{xy} = \frac{\rho (a-z)^4}{24} dz$$

$$I_{xy} = \frac{\rho}{24} \int_0^a (a-z)^4 dz = \frac{\rho}{24} \int_0^a (a^4 - 4a^3 z + 6a^2 z^2 - 4a z^3 + z^4) dz$$

$$= \frac{\rho}{24} \left(a^4 z - 2a^3 z^2 + 2a^2 z^3 - a z^4 + \frac{z^5}{5} \right) \Big|_0^a =$$

$$\frac{\rho}{24} \frac{a^5}{5} = \frac{\rho a^5}{120}$$

$$\Rightarrow I_{xy} = \left(\frac{\rho a^3}{6} \right) \frac{a^2}{20} = \frac{m a^2}{20}$$

<u>Answer</u>: $I_{xy} = \dfrac{m a^2}{20}$