# Zarema Balgabekova

| | |
|---|---|
| 12-5<br>Given<br>$a = (12t - 3t^{1/2})\ ft/s^2$<br>When $t=0$, $v=0$ $s=15ft$ | $v$ - ?<br>$s$ - ? |

## Solution

$$(\xrightarrow{+}) \quad a = \frac{dv}{dt} = 12t - 3t^{1/2}$$

$$\int_0^v dv = \int_0^t (12t - 3t^{1/2})\,dt$$

$$v\Big|_0^v = (6t^2 - 2t^{3/2})\Big|_0^t$$

$$v = (6t^2 - 2t^{3/2})\ ft/s$$

$$(\xrightarrow{+}) \quad v = \frac{ds}{dt} = 6t^2 - 2t^{3/2}$$

$$\int_{15ft}^{s} ds = \int_0^t (6t^2 - 2t^{3/2})\,dt$$

$$s\Big|_{15ft}^{s} = 2t^3 - \frac{4}{5}t^{5/2}\Big|_0^t$$

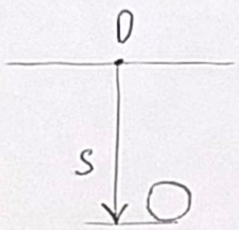
$$s = \left(2t^3 - \frac{4}{5}t^{5/2} + 15\right) ft$$

**Answer** $v = (6t^2 - 2t^{3/2})$ ft/s

$s = (2t^3 - \frac{4}{5}t^{5/2} + 15)$ ft

| | |
|---|---|
| 12-12 <br> Given: <br> $v_0 = 27$ m/s <br> $a = (-6t)$ m/s$^2$ | Distance traveled before the sphere stops - ? <br> ($s_T$) |

Solution




$(+\downarrow)$ $a = \frac{dv}{dt} = -6t$

$$\int_{27\,m/s}^{v} dv = \int_0^t (-6t)\,dt$$

$$v\Big|_{27\,m/s}^{v} = -3t^2\Big|_0^t$$

$$v = (-3t^2 + 27) \text{ m/s}$$

The sphere stops when $v = 0$

$$-3t^2 + 27 = 0$$

$$t^2 = 9$$

$$t = 3 \text{ s}$$

$(+\downarrow)$ $v = \frac{ds}{dt} = -3t^2 + 27$

$$\int_0^s ds = \int_0^t (-3t^2 + 27)\, dt$$

$$s \Big|_0^s = -t^3 + 27t \Big|_0^t$$

$$s = (-t^3 + 27t) \text{ m}$$

The sphere travels only downward, therefore, distance traveled equals to displacement

$$s_T = s\Big|_{t=3s} - s\Big|_{t=0} = 54 - 0 = 54 \text{ m}$$

<u>Answer</u> the distance traveled before the sphere stops:

$$s_T = 54 \text{ m}$$

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12-25

Given:

Particle is projected vertically upwards with $v_0$

$$a = -(g + kv^2)$$

$h_{max}$ - ?

Solution

$(+\uparrow)$ $\quad a\,ds = v\,dv$

$$ds = \frac{v\,dv}{a} = -\frac{v\,dv}{g + kv^2}$$

$$\int_0^s ds = \int_{v_0}^{v} -\frac{v\,dv}{g + kv^2}$$

$$s\Big|_0^s = -\frac{1}{2k} \ln|g + kv^2| \Big|_{v_0}^{v}$$

$$s = -\frac{1}{2k} \ln|g + kv^2| + \frac{1}{2k} \ln|g + kv_0^2|$$

$$s = \frac{1}{2k} \ln\left|\frac{g + kv_0^2}{g + kv^2}\right|$$

At the maximum height $s = h_{max}$, $v = 0$

$$h_{max} = \frac{1}{2k} \ln\left|\frac{g + kv_0^2}{g}\right| = \frac{1}{2k} \ln\left|1 + \frac{kv_0^2}{g}\right|$$

Answer $\quad h_{max} = \frac{1}{2k} \ln\left|1 + \frac{kv_0^2}{g}\right|$

12-30

Given

$v = v_0 - ks$

$s = 0$ when $t = 0$

$s(t)$ - ?

$a(t)$ - ?

$(\xrightarrow{+})$ $\quad v = \dfrac{ds}{dt} = v_0 - ks$

$$\int_0^s \frac{ds}{v_0 - ks} = \int_0^t dt$$

$$-\frac{1}{k} \ln|v_0 - ks| \Big|_0^s = t \Big|_0^t$$

$$\frac{1}{k} \ln\left|\frac{v_0}{v_0 - ks}\right| = t$$

$$\ln\left|\frac{v_0}{v_0 - ks}\right| = kt$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$kse^{kt} = v_0(e^{kt} - 1)$$

$$s = s(t) = \frac{v_0(e^{kt} - 1)}{ke^{kt}} = \frac{v_0}{k}(1 - e^{-kt})$$

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}\left(\frac{v_0}{k}(1-e^{-kt})\right) = v_0 e^{-kt}$$

$(\xrightarrow{+})$ $$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right) = -k v_0 e^{-kt}$$

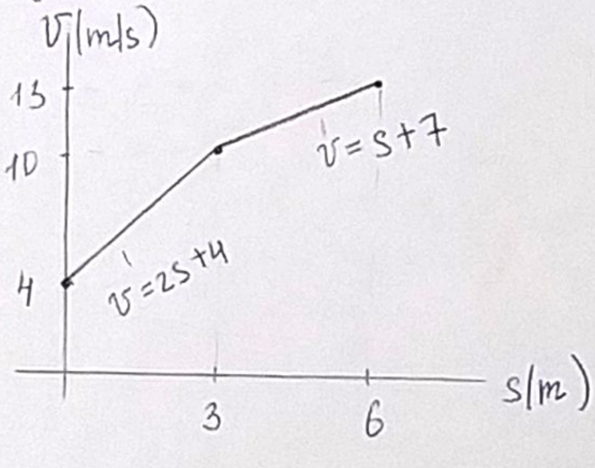
Answer

$$s(t) = \frac{v_0}{k}(1 - e^{-kt})$$

$$a(t) = -k v_0 e^{-kt}$$

12-47

Given




a-s graph - ?

Solution

$$a\,ds = v\,dv \Rightarrow a = v\frac{dv}{ds}$$

$0 \leq s < 3\,m$ $\qquad v = (2s+4)\ m/s$

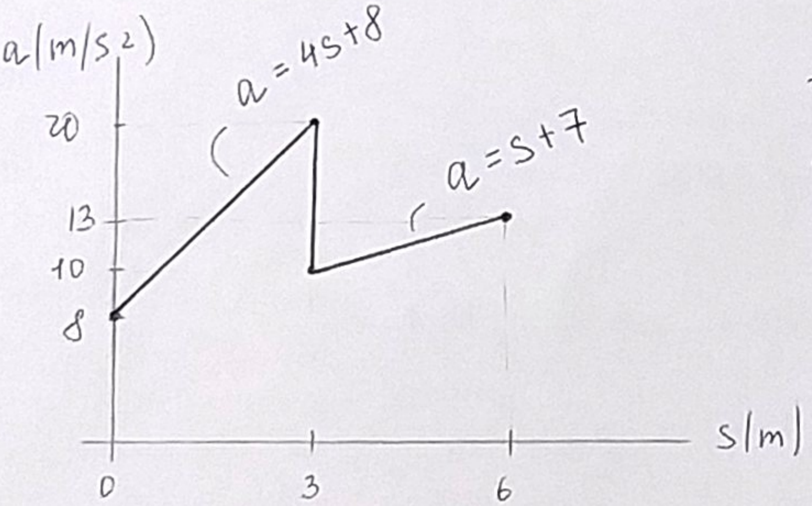
$$a = (2s+4)\frac{d}{ds}(2s+4) = (2s+4)\cdot 2 = (4s+8)\ m/s^2$$

$a|_{s=0} = 8\ m/s^2$ $\qquad a|_{s=3m} = 20\ m/s^2$

$3\,m < s \leq 6\,m$ $\qquad v = (s+7)\ m/s$

$$a = (s+7)\frac{d}{ds}(s+7) = (s+7)\cdot 1 = (s+7)\ m/s^2$$

$a|_{s=3m} = 10\ m/s^2$ $\qquad a|_{s=6m} = 13\ m/s^2$

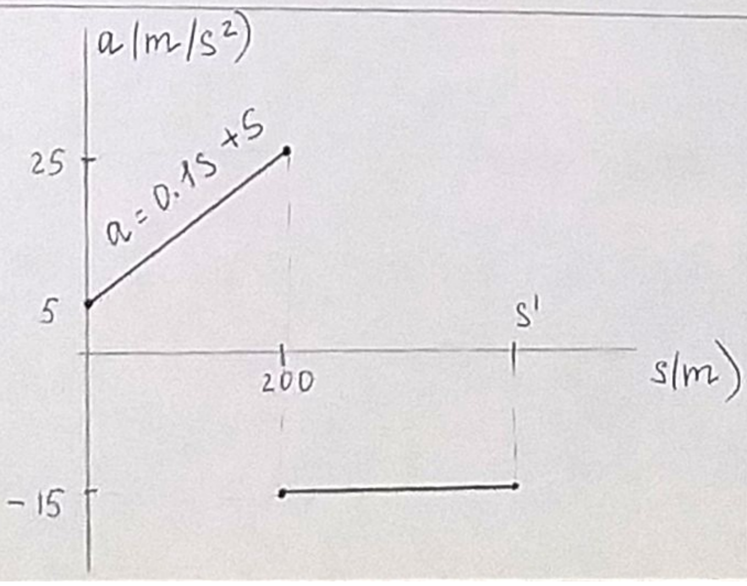



The a-s graph

12-57

Given

$v_0 = 0$




v-s graph for $0 \leq s \leq s'$ - ?

$s'$ - ?

Solution

$$a\,ds = v\,dv$$

$0 \leq s < 200\,m$ $\qquad a = (0.1s + 5)\ m/s^2$

initial conditions $v = 0$ when $s = 0$

$$\int_0^v v\,dv = \int_0^s (0.1s + 5)\,ds$$

$$\left.\frac{v^2}{2}\right|_0^v = 0.05s^2 + 5s \Big|_0^s$$

$$\frac{v^2}{2} = 0.05s^2 + 5s$$

$$v = \sqrt{0.1s^2 + 10s} \quad m/s$$

(the positive root is taken, since the dragster travels along a straight track)

$200m < s \leq s'$ $\qquad a = -15 \quad m/s^2$

initial conditions when $s = 200\,m$

$$v = \sqrt{0.1 \cdot 200^2 + 10\,200} = 20\sqrt{15} = 77.5\,m/s$$

$$\int_{20\sqrt{15}\,\text{m/s}}^{v} v\,dv = \int_{200\,\text{m}}^{s} -15\,ds$$

$$\frac{v^2}{2}\Bigg|_{20\sqrt{15}\,\text{m/s}}^{v} = -15s\Bigg|_{200\,\text{m}}^{s}$$

$$\frac{v^2}{2} - 3000 = -15s + 3000$$

$$\frac{v^2}{2} = -15s + 6000$$

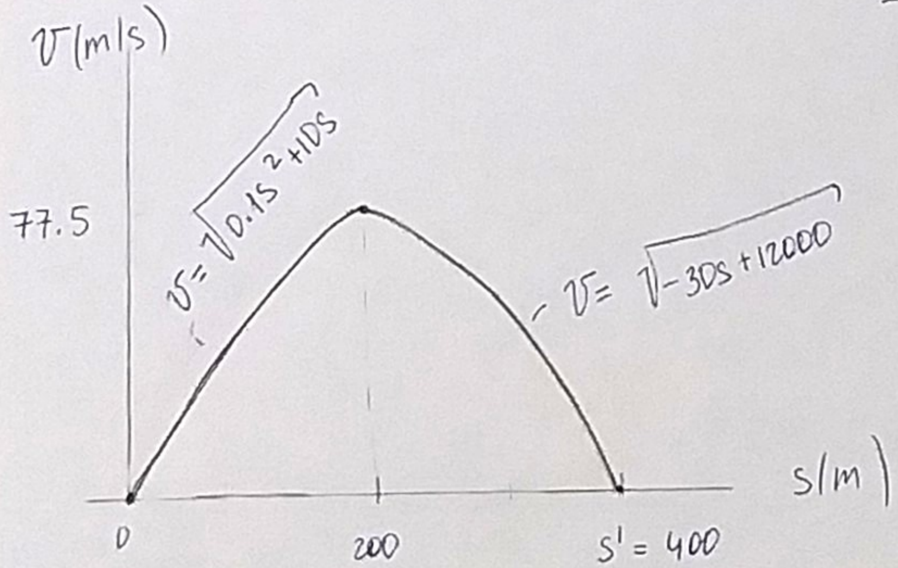
$$v = \sqrt{-30s + 12000} \quad \text{m/s}$$

When $s = s'$, $v = 0$

$$0 = \sqrt{-30s' + 12000}$$

$$-30s' + 12000 = 0$$

$$s' = 400\,\text{m}$$



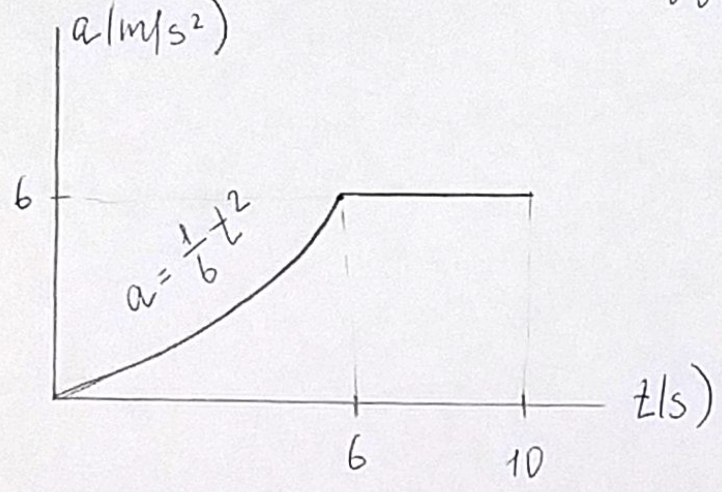

The v-s graph

Answer $s' = 400\,\text{m}$

12 - 60

Given

$v_0 = 0$




$v$-$t$ graph - ?

Distance traveled in 10 s - ?
(s)

Solution

$$a = \frac{dv}{dt}$$

$0 \leqslant t < 6\,s$ $\qquad a = \frac{1}{6}t^2 \quad m/s^2$

initial conditions $\quad v = 0 \quad$ when $t = 0$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2\,dt$$

$$v\Big|_0^v = \frac{t^3}{18}\Big|_0^t$$

$$v = \frac{t^3}{18} \quad m/s$$

$6s < t \leqslant 10\,s$ $\qquad a = 6 \quad m/s^2$

initial conditions when $t = 6\,s$

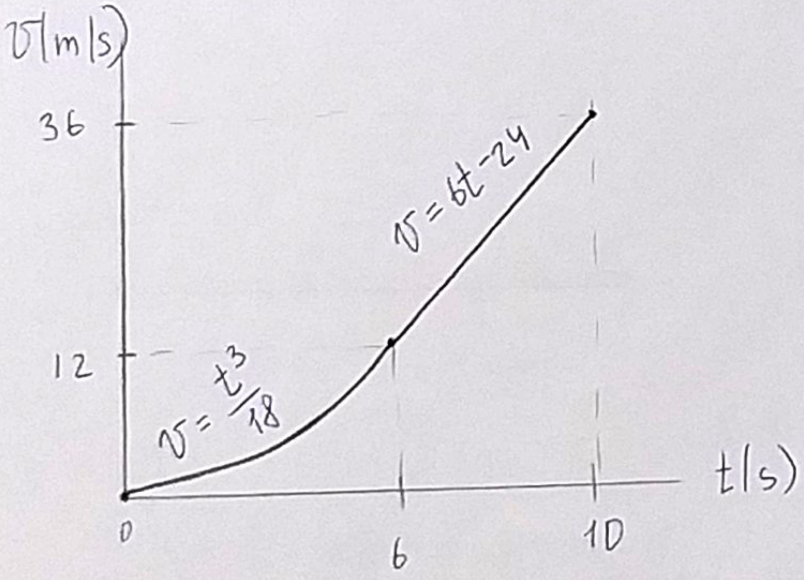
$$v = \frac{6^3}{18} = 12\ m/s$$

$$\int_{12\,m/s}^{v} dv = \int_{6}^{t} 6\,dt$$

$$v \Big|_{12\,m/s}^{v} = 6t \Big|_{6}^{t}$$

$$v = 6t - 36 + 12 = (6t - 24) \quad m/s$$

$$v\big|_{t=10s} = 6 \cdot 10 - 24 = 36\,m/s$$




The v-t graph

$$v = \frac{ds}{dt}$$

$0 \leq t < 6\,s$ $\quad v = \frac{t^3}{18}$ $\quad m/s$

Initial conditions. $s=0$ when $t=0$

$$\int_0^s ds = \int_0^t \frac{t^3}{18}\,dt$$

$$s\Big|_0^s = \frac{t^4}{72}\Big|_0^t$$

$$s = \frac{t^4}{72} \quad m$$

$6s < t \leq 10\,s$ $\qquad v = (6t-24)\ m/s$

Initial conditions $\qquad$ when $t = 6s$,

$$s = \frac{6^4}{72} = 18m$$

$$\int_{18m}^{s} ds = \int_6^t (6t-24)\,dt$$

$$s\Big|_{18m}^{s} = 3t^2 - 24t\Big|_6^t$$

$$s = 3t^2 - 24t + 36 + 18 = (3t^2 - 24t + 54)\ m$$

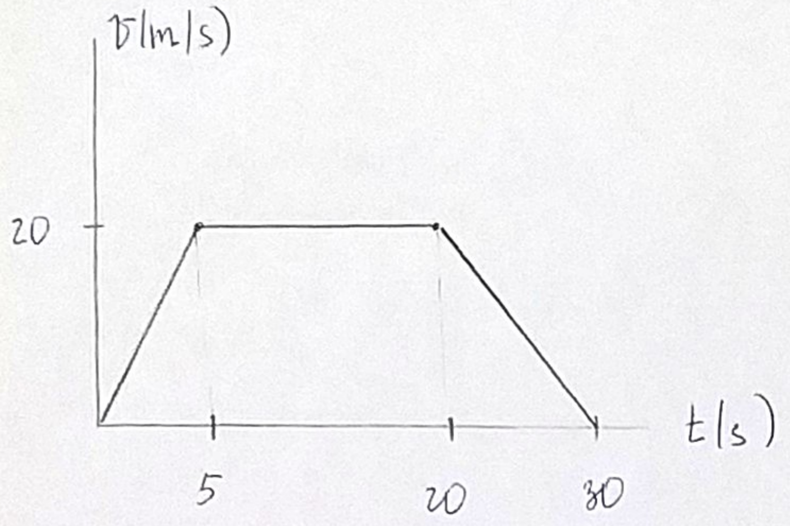
The motorcyclist travels along a straigh road, therefore, distance traveled equals to displacement

$$s = s\big|_{t=10s} - s\big|_{t=0} = (3\,10^2 - 24\,10 + 54) - \frac{0^4}{72} = 114\,m$$

Answer: $s = 114\ m$

12-61

Given




s-t graph - ?

a-t graph - ?

Solution

From the v-t graph,

$$\underset{(m/s)}{v} = \begin{cases} 4t & 0 \le t < 5\,s \\ 20 & 5s < t < 20\,s \\ -2t + 60 & 20s < t \le 30\,s \end{cases}$$

$$v = \frac{ds}{dt}$$

$0 \le t < 5\,s$ $\quad v = 4t \ \ m/s$

initial conditions $\quad s = 0$ when $t = 0$

$$\int_0^s ds = \int_0^t 4t\,dt \qquad s = 2t^2$$

$$s\Big|_0^s = 2t^2\Big|_0^t$$

$5s < t < 20s \qquad v = 20 \quad m/s$

inital conditions when $t = 5s$, $s = 2\,5^2 = 50m$

$$\int_{50m}^{s} ds = \int_{5}^{t} 20dt$$

$$s = 20t - 100 + 50 = (20t - 50)\ m$$

$$s\Big|_{50m}^{s} = 20t\Big|_{5}^{t}$$

$20s < t \leq 30s \qquad v = (-2t + 60) \quad m/s$

inital conditions. when $t = 20s$, $s = 20\,20 - 50 = 350m$

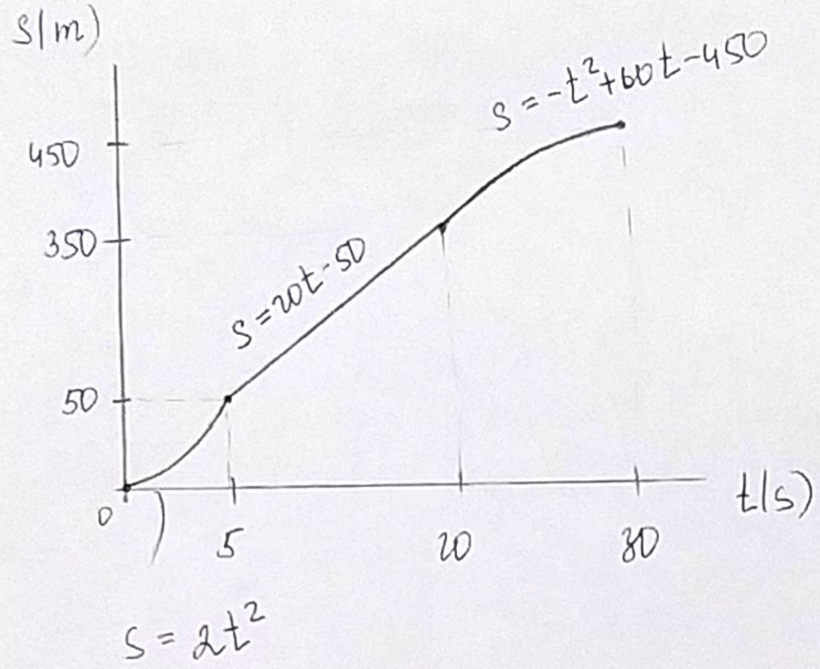
$$\int_{350m}^{s} ds = \int_{20}^{t} (-2t + 60)\,dt$$

$$s\Big|_{350m}^{s} = -t^2 + 60t\Big|_{20}^{t}$$

$$s = -t^2 + 60t - 800 + 350 = (-t^2 + 60t - 450)\ m$$

$$s|_{t=30s} = -30^2 + 60 \cdot 30 - 450 = 450\ m$$

## the s-t graph




$$a = \frac{dv}{dt}$$

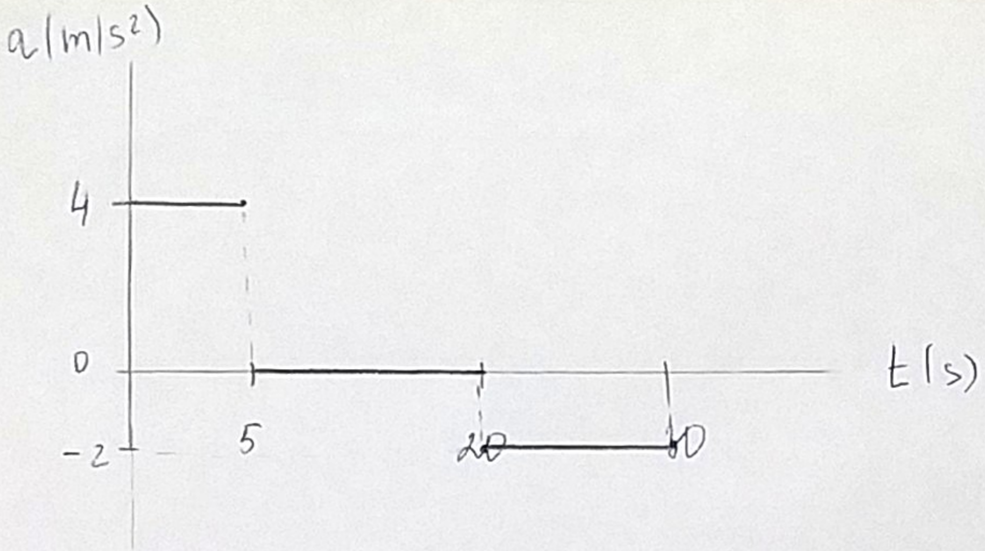
$0 \leq t < 5\,s$ $\qquad v = 4t$ m/s

$$a = \frac{d}{dt}(4t) = 4 \quad m/s^2$$

$5s < t < 20\,s$ $\qquad v = 20$ m/s $\qquad a = \frac{d}{dt}(20) = 0$

$20s < t \leq 30\,s$ $\qquad v = (-2t + 60)$ m/s

$$a = \frac{d}{dt}(-2t + 60) = -2 \; m/s^2$$




the a-t graph

12-80

Given.

$y = \left(-1.5(10^{-3})x^2 + 15\right)$ ft

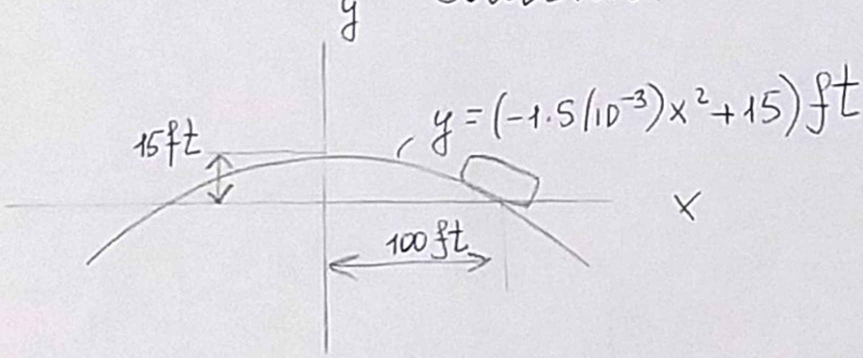
$v = 75$ ft/s

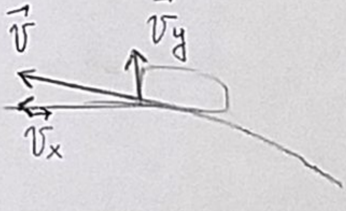
$x = 50$ ft

$v_x, v_y$ -?

$a_x, a_y$ -?

Solution.




$$v_y = \dot{y} = \frac{d}{dt}\left(-1.5\,10^{-3}x^2 + 15\right) = -1.5 \cdot 10^{-3} \cdot 2x\,\dot{x} =$$

$$-3 \cdot 10^{-3} x \cdot \dot{x} = -3\,10^{-3} x\, v_x$$

When $x = 50\ ft$, $v_y = -3\,10^{-3}\,50 \cdot v_x = -0.15\,v_x$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + (-0.15 v_x)^2} = 75$$

$$v_x^2 + (-0.15 v_x)^2 = 75^2$$

$$v_x^2 = \frac{75^2}{1 + (-0.15)^2}$$

$$v_x = 74.2\ ft/s \leftarrow$$

$$\Rightarrow \quad v_y = -0.15(-74.2) = 11.1\ ft/s \uparrow$$

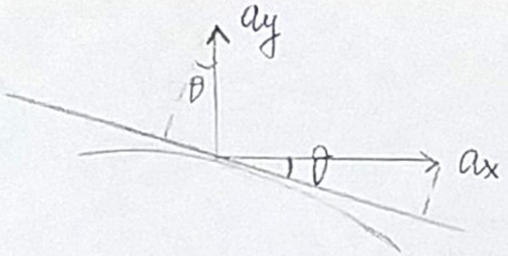
$$a_y = \dot{v}_y = \frac{d}{dt}(-3\,10^{-3} x\, v_x) = -3\,10^{-3}\,\dot{x}\, v_x - 3\,10^{-3}\, x\, \dot{v}_x =$$

$$-3\,10^{-3}(v_x^2 + x\, a_x)$$

When $x = 50\ ft$, $v_x = 74.2\ ft/s \leftarrow$

$$a_y = -3\,10^{-3}\left((-74.2)^2 + 50\, a_x\right) = -16.5 - 0.15\, a_x$$

$v = 75\ ft/s \Rightarrow a = \dfrac{dv}{dt} = 0$ (along the tangent of the path)




$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{11.1}{74.2}\right) =$$

$$8.51^\circ$$

$$\Rightarrow \quad a_y \sin(8.51^\circ) - a_x \cos(8.51^\circ) = 0$$

$$a_x = a_y \tan(8.51^\circ)$$

$$a_y = -16.5 - 0.15\,(a_y \tan(8.51^\circ))$$

$$a_y = \frac{-16.5}{1 + 0.15\tan(8.51^\circ)} = -16.1\ ft/s^2 = 16.1\ ft/s^2 \downarrow$$

$$a_x = -16.1 \tan(8.51^\circ) = -2.41\ ft/s^2 = 2.41\ ft/s^2 \leftarrow$$

Answer

$$v_x = 74.2\ ft/s \quad \leftarrow$$

$$v_y = 11.1\ ft/s \quad \uparrow$$

$$a_x = 2.41\ ft/s^2 \quad \leftarrow$$

$$a_y = 16.1\ ft/s^2 \quad \downarrow$$

12 - 91

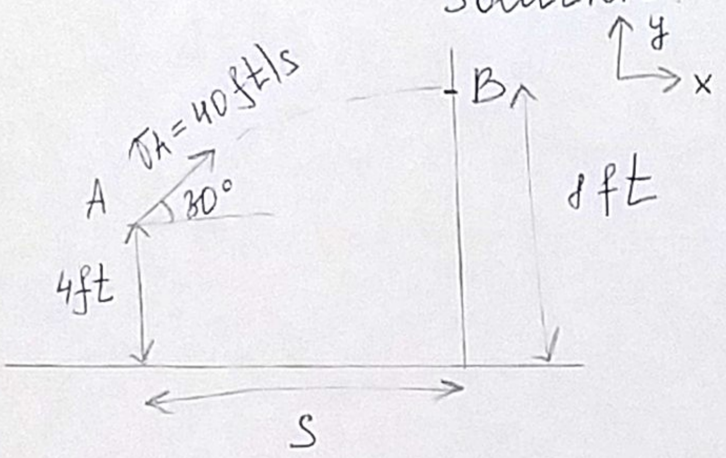
Given:

$\theta = 30°$

$v_A = 40\ ft/s$

two possible distances $s$ - ?

Solution




$(v_A)_x = 40 \cos 30° = 34.641\ m/s$

$(v_A)_y = 40 \sin 30° = 20\ m/s$

$(+\uparrow)\quad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \qquad a_y = -g = -32.2\ \frac{ft}{s^2}$

$8 = 4 + 20t - \frac{1}{2} \cdot 32.2 t^2$

$16.1 t^2 - 20t + 4 = 0$

$D = 100 - 16.1 \cdot 4 = 35.6$

$t_1 = \dfrac{10 + \sqrt{35.6}}{16.1} = 0.9917\ s$

$t_2 = \dfrac{10 - \sqrt{35.6}}{16.1} = 0.2505\ s$

$(\xrightarrow{+})\ x_B = x_A + (v_A)_x t$

$s_1 = 0 + 34.641\, t_1 = 34.641 \cdot 0.9917 = 34.4\ ft$

$s_2 = 0 + 34.641\, t_2 = 34.641 \cdot 0.2505 = 8.68\ ft$

Answer $s_1 = 34.4\ ft$ $\quad s_2 = 8.68\ ft$

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12-96

Given:

$v_A = 40\ ft/s$

$\theta_A = 60^\circ$

$v_B$ - ?

$d$ - ?

to make the catch at the same elevation at which the ball was hit

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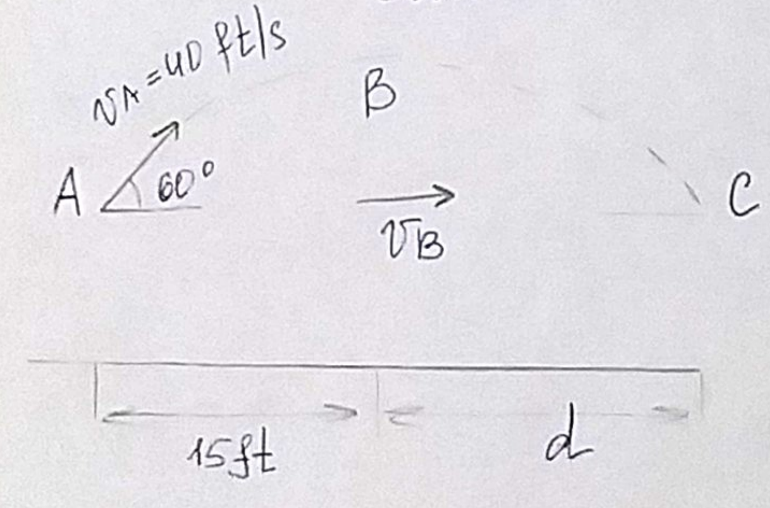
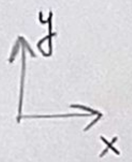
Solution




$(v_A)_x = 40 \cos 60^\circ = 20\ ft/s$

$(v_A)_y = 40 \sin 60^\circ = 34.641\ \frac{ft}{s}$

$y_A = y_C$

$(+\uparrow)$ $$y_C = y_A + (v_A)_y t - \frac{gt^2}{2}$$

$$y_A = y_A + (v_A)_y t - \frac{gt^2}{2}$$

$$(v_A)_y t - \frac{gt^2}{2} = 0 \qquad 34.641t - 16.1t^2 = 0$$

$$t = 0 \qquad \underline{t = 2.1516\,s}$$

$(\xrightarrow{+})$ $$x_C = x_A + (v_A)_x t$$

$$15 + d = 0 + 20t$$

$$d = 20 \cdot 2.1516 - 15 = 28\ ft$$

Since the horizontal component of velocity remains constant during the motion, player B must run to catch the ball at the same speed as $(v_A)_x$

$$v_B = (v_A)_x = 20\ ft/s$$

<u>Answer</u> $v_B = 20\ ft/s$

$d = 28\ ft$

12 - 121

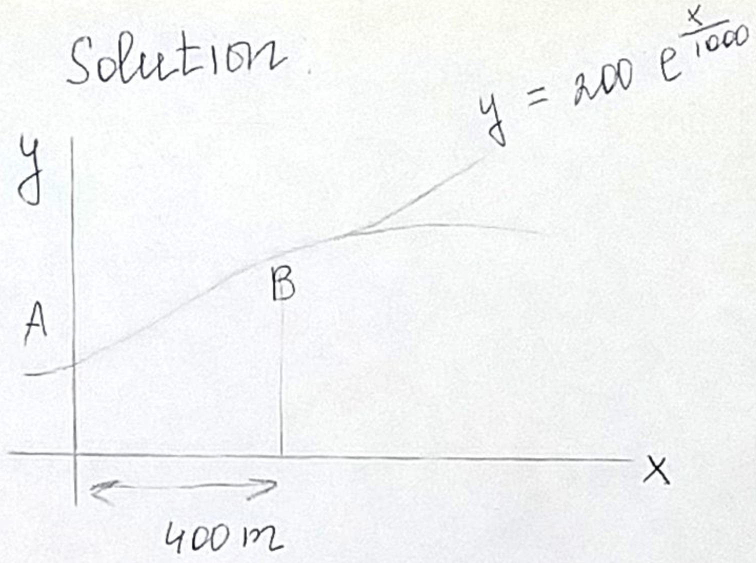
Given:

$V_B = 20\ m/s$

$a_t = -0.5\ m/s^2$

$a_B$ - ?

Solution.




$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|}$$

$$\frac{dy}{dx} = \frac{1}{5} e^{\frac{x}{1000}} \qquad \frac{d^2y}{dx^2} = \frac{1}{5000} e^{\frac{x}{1000}}$$

$$\rho = \left.\frac{\left[1 + \frac{1}{25} e^{\frac{x}{500}}\right]^{3/2}}{\left|\frac{1}{5000} e^{\frac{x}{1000}}\right|}\right|_{x=400\,m} = 3809\ m$$

$$a_n = \frac{V_B^2}{\rho} = \frac{20^2}{3809} = 0.105\ m/s^2$$

$$a_B = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.105^2} = 0.511\ m/s^2$$

Answer $a_B = 0.511\ m/s^2$

12-128

Given

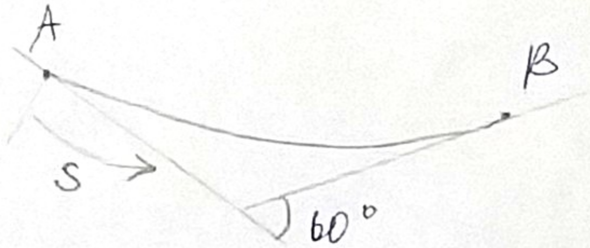
$t = 60\,s$

$v_A = 400\ ft/s$

$a_t = (-0.1t)\ ft/s^2$

$a_B - ?$

Solution




$$a_t = \frac{dv}{dt} = -0.1t$$

$$\int_{400ft/s}^{v} dv = \int_0^t -0.1t\,dt$$

$$v \Big|_{400\,ft/s}^{v} = -0.05t^2 \Big|_0^t$$

$$v = (-0.05t^2 + 400)\ ft/s$$

$$v_B = -0.05 \cdot 60^2 + 400 = 220\ ft/s$$

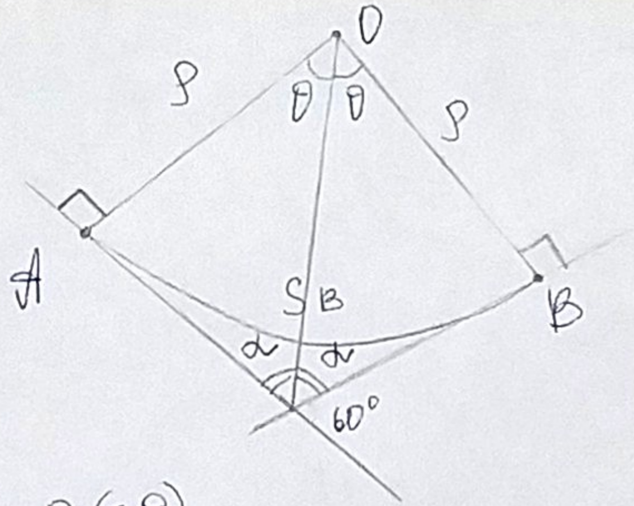
$$v = \frac{ds}{dt}$$

$$\int_0^s ds = \int_0^t (-0.05t^2 + 400)\,dt$$

$$s \Big|_0^s = -\frac{0.05t^3}{3} + 400t \Big|_0^t$$

$$s = \left(-\frac{0.05t^3}{3} + 400t\right)\ ft$$

$$s_B = -\frac{0.05\ 60^3}{3} + 400 \cdot 60 = 20\,400\ ft$$




$$2\alpha + 60 = 180$$
$$\alpha = 60^\circ$$

$$\theta + \alpha = 90$$
$$\theta = 30^\circ = \frac{\pi}{6} \text{ rad}$$

$$s_B = \rho \cdot (2\theta)$$

$$\rho = \frac{s_B}{2\theta} = \frac{20400}{2\left(\frac{\pi}{6}\right)} = 19481 \text{ ft}$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{220^2}{19481} = 2.484 \text{ ft/s}^2$$

$$(a_t)_B = -0.1 \cdot 60 = -6 \text{ ft/s}^2$$

$$a_B = \sqrt{2.484^2 + (-6)^2} = 6.49 \text{ ft/s}^2$$

Answer: $a_B = 6.49 \text{ ft/s}^2$

12 - 173

Given

$\theta = 30^\circ$

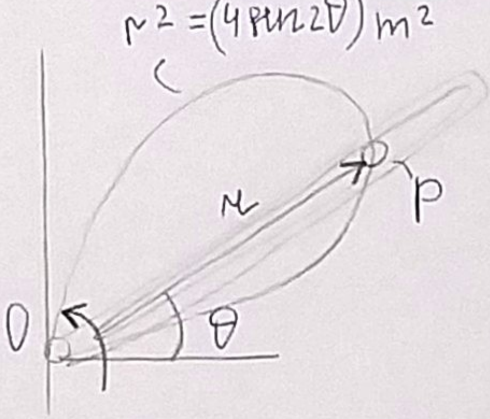
$\dot{\theta} = 2\ \text{rad/s}$

$\ddot{\theta} = 1.5\ \text{rad/s}^2$

$v$ - ?

$a$ - ?

Solution




$$r = \sqrt{4\sin 2\theta} = 2\sqrt{\sin 2\theta}$$

$$\theta = 30^\circ \Rightarrow r = 2\sqrt{\sin 60^\circ} = 1.861\ m$$

$$r' = \cancel{2}\,\frac{1}{\cancel{2}\sqrt{\sin 2\theta}} \cdot \cos 2\theta \cdot 2\,\dot{\theta} =$$

$$\frac{2\cos 2\theta\,\dot{\theta}}{\sqrt{\sin 2\theta}}$$

$$\theta = 30^\circ \Rightarrow r' = \frac{2\cos 60^\circ \cdot 2}{\sqrt{\sin 60^\circ}} = 2.149\ m/s$$

$$r'' = \frac{(-2\sin 2\theta \cdot 2\,\dot{\theta}\,\dot{\theta} + 2\cos 2\theta\,\ddot{\theta}) \cdot \sqrt{\sin 2\theta} - \cancel{2}\cos 2\theta \cdot \dot{\theta}\,\dfrac{\cos 2\theta \cdot 2\,\dot{\theta}}{\cancel{2}\sqrt{\sin 2\theta}}}{\sin 2\theta}$$

$$= \frac{(-4\sin 2\theta(\dot{\theta})^2 + 2\cos 2\theta\,\ddot{\theta})\sqrt{\sin 2\theta} - \dfrac{2\cos^2 2\theta\,(\dot{\theta})^2}{\sqrt{\sin 2\theta}}}{\sin 2\theta}$$

$\theta = 80°$ $\Rightarrow r'' = \dfrac{(-4\sin 60° \cdot 4 + 2\cos 60° \cdot 1.5)\sqrt{\sin 60°} - \dfrac{2\cos^2 60° \cdot 4}{\sqrt{\sin 60°}}}{\sin 60°} =$

$-15.759 \ m/s^2$

$\vec{v} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{2.149^2 + (1.861 \ 2)^2} = 4.3 \ m/s$

$\vec{a} = (\ddot{r} - r(\dot{\theta})^2)\,\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{u}_\theta$

$a = \sqrt{(\ddot{r} - r(\dot{\theta}^2))^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} =$

$\sqrt{(-15.759 - 1.861 \ 4)^2 + (1.861 \ 1.5 + 2 \ 2.149 \cdot 2)^2} = 25.8 \ m/s^2$

Answer

$v = 4.3 \ m/s$

$a = 25.8 \ m/s^2$

12 - 185

Given

$\dot{\theta} = 2t^2 \ \frac{rad}{s}$

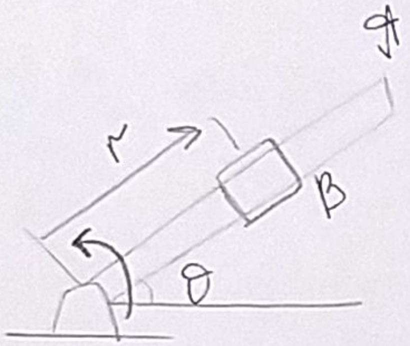
$\dot{r} = 4t^2 \ m/s$

$\theta = 0, r = 0$ when $t = 0$

$v$ at $\theta = 60° - ?$

$a$ at $\theta = 60° - ?$

Solution




$$\theta' = \frac{d\theta}{dt} = 2t^2$$

$$\int_0^\theta d\theta = \int_0^t 2t^2 dt$$

$$\theta \Big|_0^\theta = \frac{2t^3}{3} \Big|_0^t$$

$$\theta = \frac{2t^3}{3}$$

When $\theta = 60° = \frac{\pi}{3}$ rad

$$\frac{\pi}{3} = \frac{2t^3}{3}$$

$$t = \sqrt[3]{\frac{\pi}{2}} = 1.16 s$$

$\theta' = 2 \cdot 1.16^2 = 2.69$ rad/s

$\theta'' = \frac{d\theta'}{dt} = 4t = 4 \cdot 1.16 = 4.64$ rad/s$^2$

$$r' = \frac{dr}{dt} = 4t^2$$

$$\int_0^r dr = \int_0^t 4t^2\,dt$$

$$r \Big|_0^r = \frac{4t^3}{3} \Big|_0^t$$

$$r = \frac{4t^3}{3} = \frac{4 \cdot 1.16^3}{3} = 2.08 \text{ m}$$

$$r' = 4 \cdot 1.16^2 = 5.38 \text{ m/s} \qquad r'' = \frac{dr'}{dt} = 8t = 8 \cdot 1.16 = 9.28 \text{ m/s}^2$$

$$\vec{v} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$$

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{5.38^2 + (2.08 \cdot 2.69)^2} = 7.76 \text{ m/s}$$

$$\vec{a} = (\ddot{r} - r(\dot{\theta})^2)\,\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{u}_\theta$$

$$a = \sqrt{(\ddot{r} - r(\dot{\theta})^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} = \sqrt{(9.28 - 2.08\ 2.69^2)^2 + (2.08\ 4.64 + 2\ 5.38 . 2.69)^2}$$

$$= 39 \text{ m/s}^2$$

<u>Answer</u> $v = 7.76$ m/s, $a = 39$ m/s$^2$

12 - 203

Given

$v_B - ?$

$v_A = 4 m/s$

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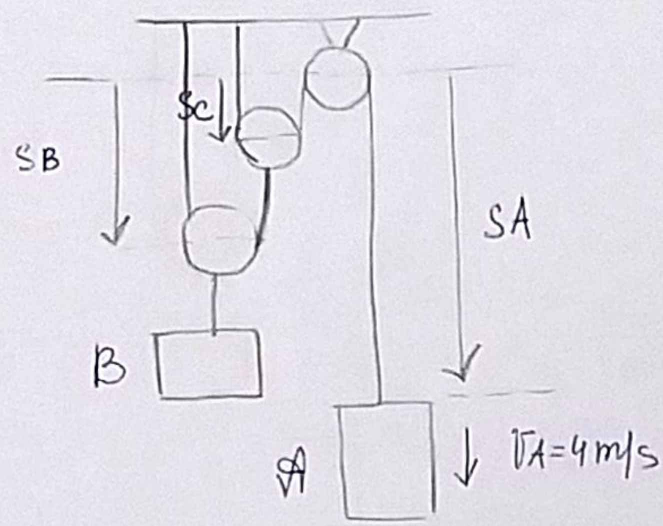
Solution




$$2S_C + S_A = l_1$$

$$2v_C + v_A = 0$$

$$v_C = -\frac{v_A}{2} = -\frac{4}{2} = -2\frac{m}{s}$$

$$S_B + (S_B - S_C) = l_2$$

$$2S_B - S_C = l_2$$

$$2v_B - v_C = 0$$

$$v_B = \frac{v_C}{2} = \frac{-2}{2} = -1 \ m/s = 1 m/s \uparrow$$

**Answer** $v_B = 1 m/s \uparrow$

12-209

Given

$a = (0.2t)\ m/s^2$

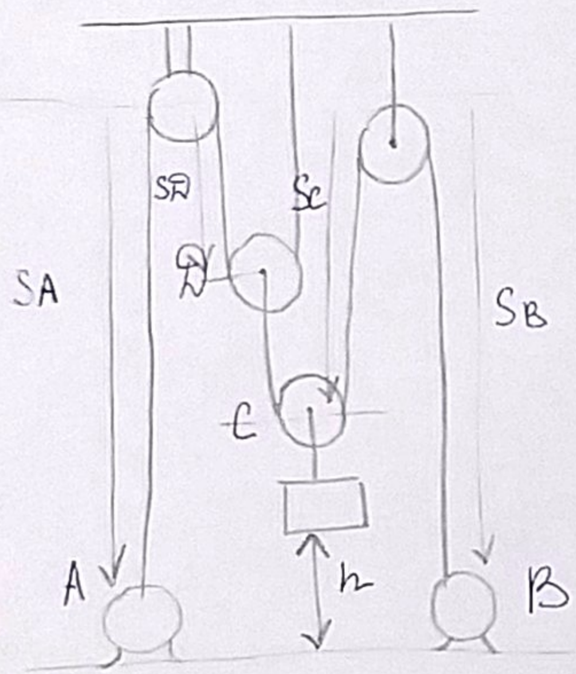
$h = 4m$

$v = 0$ at $h = 0$

$v_c$ - ?

$t$ - ?

Solution




$$S_A + 2S_D = l_1$$

$$(S_C - S_D) + S_C + S_B = l_2$$

$$2S_C - S_D + S_B = l_2$$

$$\Rightarrow \quad v_A + 2v_D = 0$$

$$2v_C - v_D + v_B = 0$$

$$a = \frac{dv}{dt} = 0.2t$$

$$\int_0^v dv = \int_0^t 0.2t\,dt \qquad v = 0.1t^2 \quad m/s$$

$$v\Big|_0^v = 0.1t^2\Big|_0^t$$

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^s ds = \int_0^t 0.1t^2\,dt$$

$$s\Big|_0^s = \frac{0.1t^3}{3}\Big|_0^t$$

$$s = \frac{0.1t^3}{3}$$

$$\frac{16}{3} = \frac{0.1t^3}{3}$$

$$t = 5.43\,s \Rightarrow v = v_A = v_B = 0.1 \cdot 5.43^2 = 2.948\ m/s$$

$$\Delta S_A + 2\Delta S_D = \Delta l_1 = 0$$

$$2\Delta S_C - \Delta S_D + \Delta S_B = \Delta l_2 = 0$$

Knowing that $\Delta S_C = -4\ m$,

and $\Delta S_A = \Delta S_B$,

$$-8 = 3\Delta S_D \quad \Delta S_D = -\frac{8}{3}\ m$$

$$\Rightarrow \Delta S_A = \Delta S_B = S = \frac{16}{3}\ m$$

$$v_D = -\frac{v_A}{2} = -\frac{2.948}{2} = -1.474\ m/s$$

$$v_C = \frac{v_D - v_B}{2} = -\frac{1.474 - 2.948}{2} = -2.21\ m/s = 2.21\ m/s \uparrow$$

Answer: $v_C = 2.21\ m/s \uparrow$

$t = 5.43\ s$

12-220

Given:

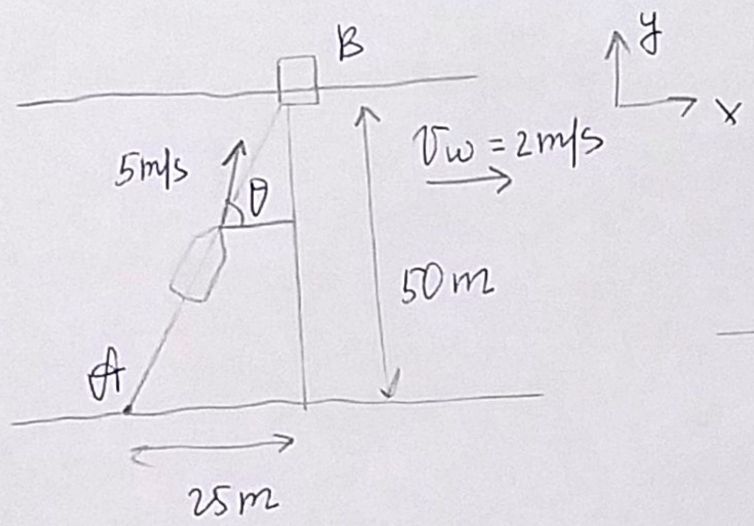
$v_{b/w} = 5\ m/s$

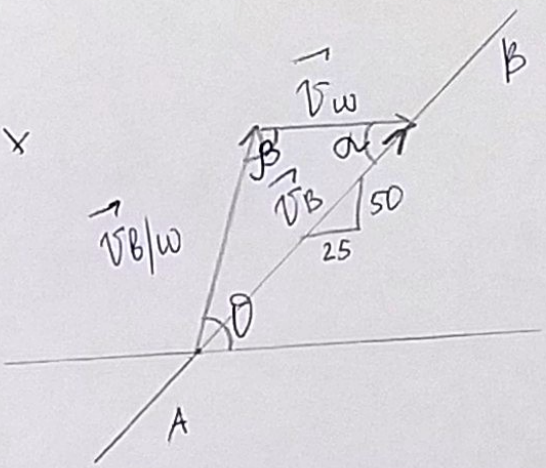
$v_w = 2\ m/s$

$v_b$ - ?

$\theta$ - ? (the boat travels from A to B)

Solution




$\alpha = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$

$\beta = 180° - \theta$

Using law of cosines

$v_{B/w}^2 = v_b^2 + v_w^2 - 2 v_b v_w \cos\alpha$

$25 = v_b^2 + 4 - 1.789 v_b$

$v_b^2 - 1.789 v_b - 21 = 0$

$D = 1.789^2 + 4 \cdot 21 = 87.2$

$v_b = \dfrac{1.789 + \sqrt{87.2}}{2} = 5.56\ m/s$

Using law of sines

$$\frac{V_{b/w}}{\sin\alpha} = \frac{V_b}{\sin\beta} \qquad \frac{5}{\sin 63.43^\circ} = \frac{5.56}{\sin(180-\theta)} = \frac{5.56}{\sin\theta}$$

$$\theta = \sin^{-1}\left(\frac{\sin 63.43^\circ \; 5.56}{5}\right) =$$

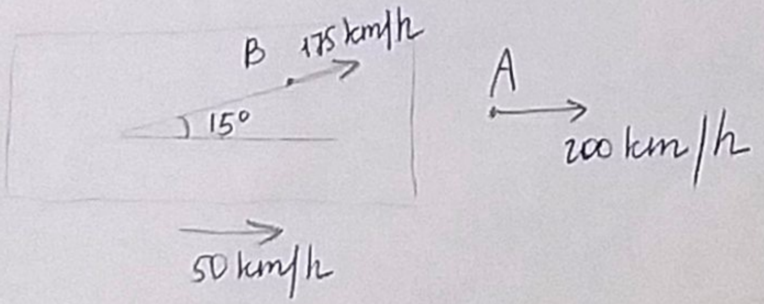
$$84^\circ$$

$$\theta = 84^\circ$$

<u>Answer</u> $V_b = 5.56$ m/s

| | |
|---|---|
| 12 - 226<br>Given:<br>$V_{carrier} = 50$ km/h<br>$V_A = 200$ km/h<br>$V_{B/carrier} = 175$ km/h | $\vec{V}_{A/B}$ - ? |

Solution:



$$\vec{v}_B = \vec{v}_{carrier} + \vec{v}_{B/carrier} =$$

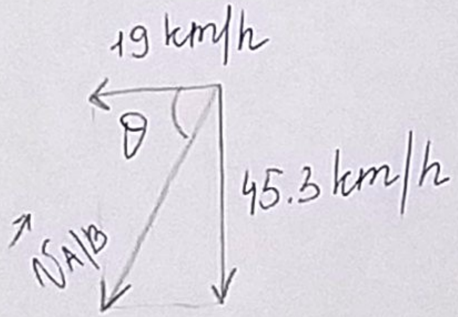
$$50\hat{i} + 175\cos 15^\circ\,\hat{i} + 175\sin 15^\circ\,\hat{j} = \{219\hat{i} + 45.3\hat{j}\}\ \frac{km}{h}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$200\hat{i} = 219\hat{i} + 45.3\hat{j} + \vec{v}_{A/B}$$

$$\vec{v}_{A/B} = \{-19\hat{i} - 45.3\hat{j}\}\ km/h$$

$$v_{A/B} = \sqrt{(-19)^2 + (-45.3)^2} = 49.1\ km/h$$




$$\theta = \tan^{-1}\left(\frac{45.3}{19}\right) = 67.2^\circ$$

<u>Answer</u> $v_{A/B} = 49.1\ km/h$

$$\theta = 67.2^\circ$$