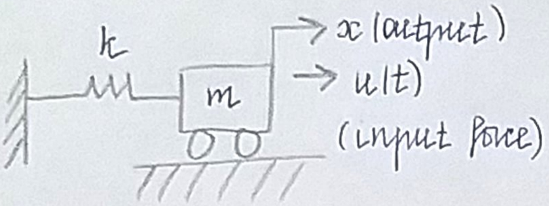
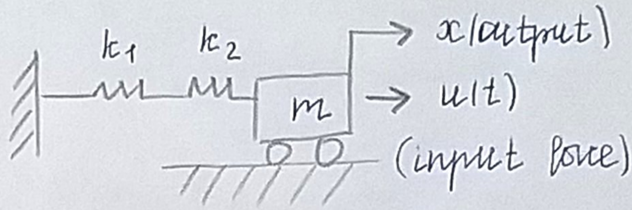
Zarema Balgabekova HW4

1. B-5-1

a) 

b) 

a) The equation of motion:

$$m\ddot{x}(t) = -kx(t) + u(t)$$

$$\ddot{x}(t) = -\frac{k}{m}x(t) + \frac{1}{m}u(t)$$

State variables:

$$x_1(t) = x(t)$$

$$x_2(t) = \dot{x}(t)$$

Output: $y(t) = x(t)$

=> State-space representation:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\frac{k}{m}x_1(t) + \frac{1}{m}u(t)$$

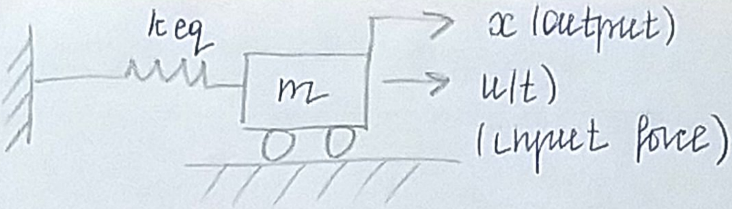
$$y(t) = x_1(t)$$

State-space representation (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

b) $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$



The equation of motion:

$$m\ddot{x}(t) = -\frac{k_1 k_2}{k_1 + k_2} x(t) + u(t)$$

$$\ddot{x}(t) = -\frac{k_1 k_2}{m(k_1 + k_2)} x(t) + \frac{1}{m} u(t)$$

State variables:

$x_1(t) = x(t)$

$x_2(t) = \dot{x}(t)$

Output $y(t) = x(t)$

$\Rightarrow$ State-space representation:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\frac{k_1 k_2}{m(k_1 + k_2)} x_1(t) + \frac{1}{m} u(t)$$

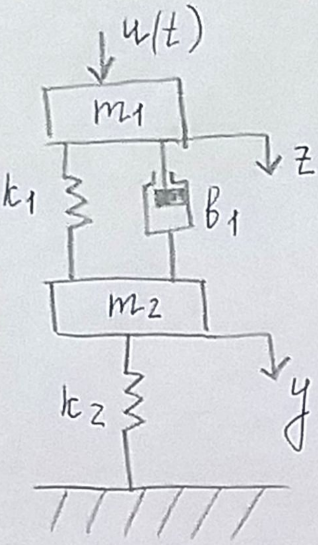
$$y(t) = x_1(t)$$

<u>State-space representation</u> (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1 k_2}{m(k_1 + k_2)} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

2. B-5-3



The equations of motion:

$$m_1 \ddot{z} = -k_1(z-y) - b_1(\dot{z}-\dot{y}) + u$$

$$\ddot{z} = -\frac{k_1}{m_1} z + \frac{k_1}{m} y - \frac{b_1}{m_1} \dot{z} + \frac{b_1}{m_1} \dot{y} + \frac{u}{m_1}$$

$$m_2 \ddot{y} = -k_1(y-z) - b_1(\dot{y}-\dot{z}) - k_2 y$$

$$\ddot{y} = -\frac{k_1+k_2}{m_2} y + \frac{k_1}{m_2} z - \frac{b_1}{m_2} \dot{y} + \frac{b_1}{m_2} \dot{z}$$

State variables:

$x_1 = z$

$x_2 = \dot{z}$

$x_3 = y$

$x_4 = \dot{y}$

Output variables:

$y_1 = z$

$y_2 = y$

State-space representation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m_1} x_1 - \frac{b_1}{m_1} x_2 + \frac{k_1}{m_1} x_3 + \frac{b_1}{m_1} x_4 + \frac{u}{m_1}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_1}{m_2} x_1 + \frac{b_1}{m_2} x_2 - \frac{k_1+k_2}{m_2} x_3 - \frac{b_1}{m_2} x_4$$

$$y_1 = x_1$$

$$y_2 = x_3$$

State-space representation (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{b_1}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

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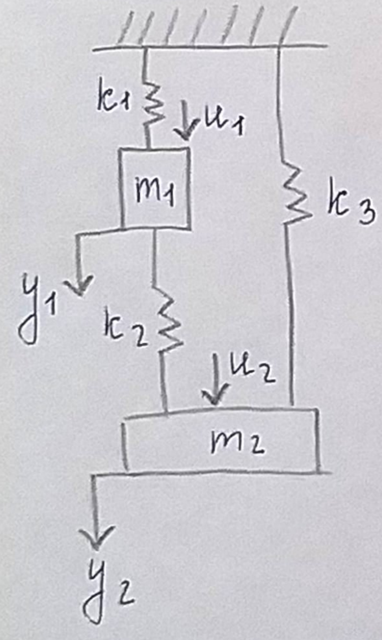
3. B-5-4



The equations of motion:

$$m_1\ddot{y}_1 = -k_1y_1 - k_2(y_1 - y_2) + u_1$$

$$\ddot{y}_1 = -\frac{k_1+k_2}{m_1}y_1 + \frac{k_2}{m_1}y_2 + \frac{1}{m_1}u_1$$

$$m_2\ddot{y}_2 = -k_2(y_2 - y_1) - k_3y_2 + u_2$$

$$\ddot{y}_2 = -\frac{k_2+k_3}{m_2}y_2 + \frac{k_2}{m_2}y_1 + \frac{1}{m_2}u_2$$

State variables:

$x_1 = y_1$

$x_2 = \dot{y}_1$

$x_3 = y_2$

$x_4 = \dot{y}_2$

Output variables:

$y_1 = x_1$

$y_2 = x_3$

State-space representation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1 + k_2}{m_1} x_1 + \frac{k_2}{m_1} x_3 + \frac{1}{m_1} u_1$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_2}{m_2} x_1 - \frac{k_2 + k_3}{m_2} x_3 + \frac{1}{m_2} u_2$$

$$y_1 = x_1$$

$$y_2 = x_3$$

State-space representation (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2+k_3}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

4. B-5-5

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B + D$$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ -1 & 3 & s-3 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{\det(sI-A)} \operatorname{Adj}(sI-A)$$

$$\det(sI-A) = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ -1 & 3 & s-3 \end{vmatrix} = s\left(s(s-3)+3\right) + (-1) =$$

$$s^3 - 3s^2 + 3s - 1$$

$$\operatorname{Adj}(sI-A) = \begin{bmatrix} s(s-3)+3 & s-3 & 1 \\ 1 & s(s-3) & s \\ s & -(3s-1) & s^2 \end{bmatrix} = \begin{bmatrix} s^2-3s+3 & s-3 & 1 \\ 1 & s^2-3s & s \\ s & -3s+1 & s^2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^3-3s^2+3s-1}\begin{bmatrix} s^2-3s+3 & s-3 & 1 \\ 1 & s^2-3s & s \\ s & -3s+1 & s^2 \end{bmatrix}$$

$$G(s) = \frac{1}{s^3-3s^2+3s-1}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} s^2-3s+3 & s-3 & 1 \\ 1 & s^2-3s & s \\ s & -3s+1 & s^2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$

$$\frac{1}{s^3-3s^2+3s-1}\begin{bmatrix} s^2-3s+3 & s-3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$

$$\frac{s^2-3s+3+s-3+1}{s^3-3s^2+3s-1} = \frac{s^2-2s+1}{s^3-3s^2+3s-1}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s^2-2s+1}{s^3-3s^2+3s-1} \Rightarrow (s^3-3s^2+3s-1)\,Y(s) = (s^2-2s+1)\,U(s)$$

Assuming zero-initial conditions, anti-transform:

$$\dddot{y} - 3\ddot{y} + 3\dot{y} - y = \ddot{u} - 2\dot{u} + u$$

$\Rightarrow$ <u>Differential equation:</u>

$$\dddot{y} - 3\ddot{y} + 3\dot{y} - y = \ddot{u} - 2\dot{u} + u$$

5. B-5-6

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

State variables: Output: $y = x_1$

$x_1 = y$

$x_2 = \dot{y}$

$x_3 = \ddot{y}$

=> State-space representation:

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + 6u$

$y = x_1$

State-space representation (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

6, B-5-7

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} B + D$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+4)(s+1)+3} \begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix} =$$

$$\frac{1}{s^2+5s+7} \begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+5s+7} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

$$\frac{1}{s^2+5s+7} \begin{bmatrix} s+1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{s}{s^2+5s+7}$$

=> Transfer function: $G(s) = \dfrac{s}{s^2+5s+7}$

7. B-5-8

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $D = 1$

$$G(s) = C(sI - A)^{-1} B + D$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} = \frac{1}{s^2+2s+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 =$$

$$\frac{1}{s^2+2s+1} \begin{bmatrix} s+2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 = \frac{s+3}{s^2+2s+1} + 1 =$$

$$\frac{s^2+3s+4}{s^2+2s+1}$$

=> Transfer function: $G(s) = \dfrac{s^2+3s+4}{s^2+2s+1}$

8. B-5-9

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -600 & -100 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -600 & -100 & -10 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 600 & 100 & s+10 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{\det(sI-A)} \cdot Adj\,(sI-A)$$

$$\det(sI-A) = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 600 & 100 & s+10 \end{vmatrix} = s \cdot \left(s(s+10)+100\right) + 600 =$$

$$s^3 + 10s^2 + 100s + 600$$

$$Adj\,(sI-A) = \begin{bmatrix} s(s+10)+100 & s+10 & 1 \\ -600 & s(s+10) & s \\ -600s & -(100s+600) & s^2 \end{bmatrix} =$$

$$\begin{bmatrix} s^2+10s+100 & s+10 & 1 \\ -600 & s^2+10s & s \\ -600s & -100s-600 & s^2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^3+10s^2+100s+600}\begin{bmatrix} s^2+10s+100 & s+10 & 1 \\ -600 & s^2+10s & s \\ -600s & -100s-600 & s^2 \end{bmatrix}$$

$$G(s) = \frac{1}{s^3+10s^2+100s+600}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} s^2+10s+100 & s+10 & 1 \\ -600 & s^2+10s & s \\ -600s & -100s-600 & s^2 \end{bmatrix}\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} =$$

$$\frac{1}{s^3+10s^2+100s+600}\begin{bmatrix} s^2+10s+100 & s+10 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} =$$

$$\frac{10(s+10)}{s^3+10s^2+100s+600} = \frac{10s+100}{s^3+10s^2+100s+600}$$

$\Rightarrow$ <u>Transfer function:</u>

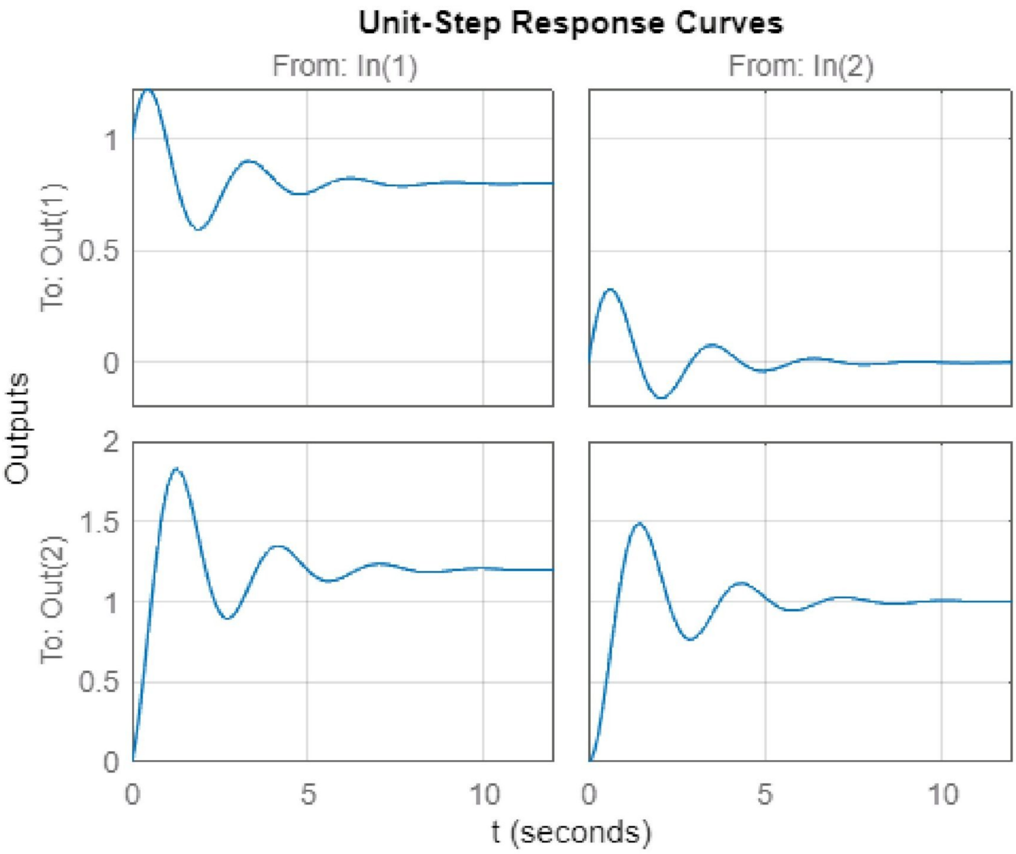
$$G(s) = \frac{10s+100}{s^3+10s^2+100s+600}$$

9) **B-5-10**

MATLAB code:

```
A = [-1 -1; 5 0];
B = [1 1; 1 0];
C = [1 0; 0 1];
D = [1 0; 0 0];

sys = ss(A, B, C, D);
step(sys)
grid
title('Unit-Step Response Curves')
xlabel('t');
ylabel('Outputs');
```

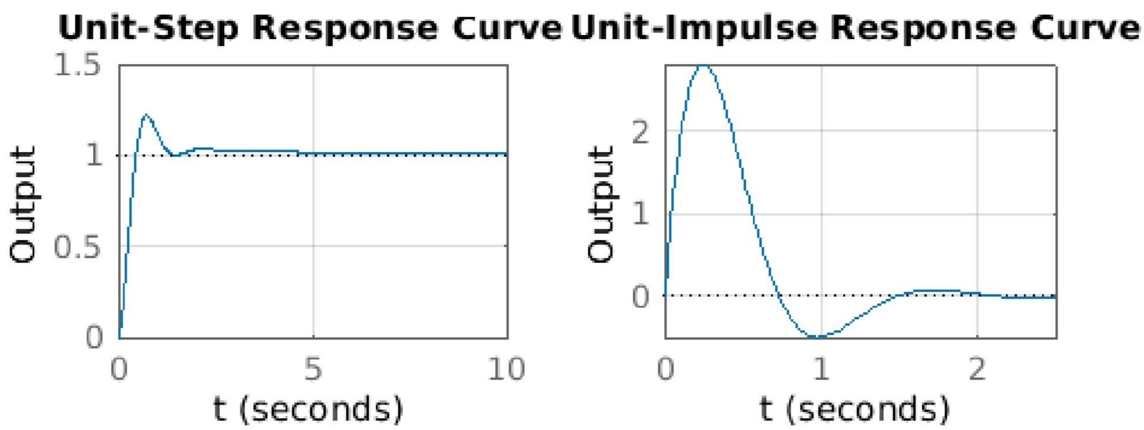


10) **B-5-11**

MATLAB code:

```
A = [-5 -25 -5; 1 0 0; 0 1 0];
B = [1; 0; 0];
C = [0 25 5];
D = 0;
sys = ss(A, B, C, D);
subplot(221);
step(sys)
grid
title('Unit-Step Response Curve')
xlabel('t');
ylabel('Output');

subplot(222);
impulse(sys);
grid
title('Unit-Impulse Response Curve')
xlabel('t');
ylabel('Output');
```



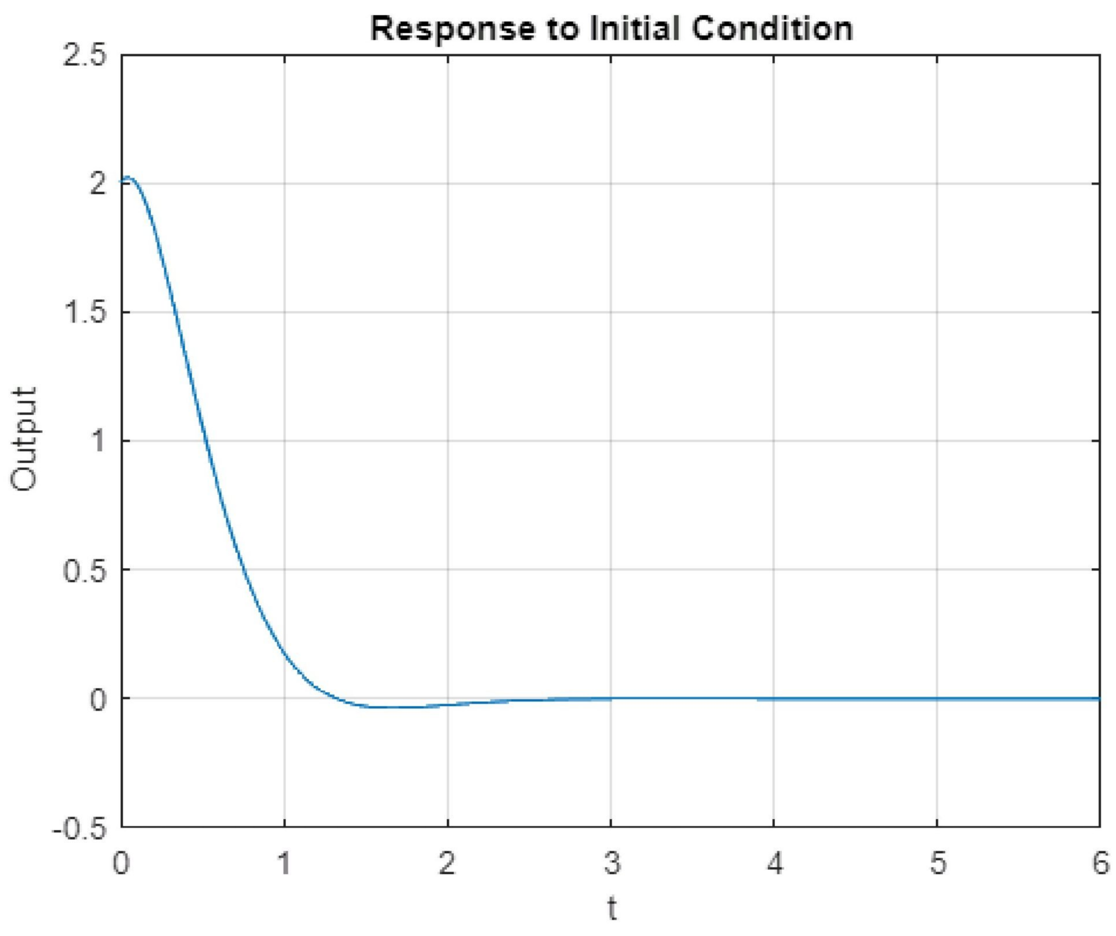
11) **B-5-12**

MATLAB code:

```
t = 0:0.01:6;

A= [0 1; -10 -5];
B = [0; 0];
C = [1 0];
D = 0;

[y] = initial(A, B, C, D, [2; 1], t);
plot (t, y)
grid
title('Response to Initial Condition')
xlabel('t');
ylabel('Output');
```



12. B-5-20

$$\frac{Y(s)}{U(s)} = \frac{160(s+4)}{s^3+18s^2+192s+640} = \frac{160s+640}{s^3+18s^2+192s+640}$$

Let $\frac{Z(s)}{U(s)} = \frac{1}{s^3+18s^2+192s+640}$

$$\frac{Y(s)}{Z(s)} = 160s+640$$

Assume zero initial conditions:

$$\dddot{z} + 18\ddot{z} + 192\dot{z} + 640z = u$$

$$160\dot{z} + 640z = y$$

State variables: $x_1 = z$

$x_2 = \dot{z}$

$x_3 = \ddot{z}$

=> State-space representation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -640x_1 - 192x_2 - 18x_3 + u$$

$$y = 640x_1 + 160x_2$$

State-space representation (vector-matrix form):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -640 & -192 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 640 & 160 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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13. 1) $\dot{x}(t) = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 2 & 2 \end{bmatrix} x(t) + u(t)$$

$A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ - triangular matrix

$\Rightarrow$ Eigenvalues $\lambda_1 = -1$, $\lambda_2 = 2$ $\Rightarrow$ internally unstable

$R = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad \det R \neq 0 \Rightarrow \text{rank} = 2$

$\Rightarrow$ fully controllable

$O = \begin{bmatrix} 2 & 2 \\ -2 & 6 \end{bmatrix}$ $\quad \det O \neq 0 \Rightarrow \text{rank} = 2$

$\Rightarrow$ fully observable

$\Rightarrow$ Looking at internal stability, controllability, and observability, the system should be not BIBO stable

$$G(s) = C(sI-A)^{-1}B + D = [2 \;\; 2]\begin{bmatrix} s+1 & -1 \\ 0 & s-2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 =$$

$$\frac{1}{(s+1)(s-2)}[2 \;\; 2]\begin{bmatrix} s-2 & 1 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 =$$

$$\frac{2s+4}{(s+1)(s-2)} + 1 = \frac{2s+4+s^2-s-2}{(s+1)(s-2)} = \frac{s^2+s+2}{(s+1)(s-2)}$$

$\Rightarrow$ no zero/pole cancellation

Poles: $s=-1$ $\quad s=2 \Rightarrow$ <u>not BIBO stable</u>

2) $$\dot{x}(t) = \begin{bmatrix} -2 & 3 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0 \;\; 1]\, x(t) + u(t)$$

$A = \begin{bmatrix} -2 & 3 \\ 0 & -1 \end{bmatrix}$ - triangular matrix

$\Rightarrow$ Eigenvalues $\lambda_1 = -2,\ \lambda_2 = -1 \Rightarrow$ <u>asymptotically stable</u>

$R = \begin{bmatrix} 2 & -4 \\ 0 & 0 \end{bmatrix}$ $\quad \det R = 0 \Rightarrow$ rank $= 1$

$\Rightarrow$ <u>not fully controllable</u>

$$\mathcal{O} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \quad \det\mathcal{O} = 0 \Rightarrow \text{rank} = 1$$

$\Rightarrow$ not fully observable

As the system is asymptotically stable, it is also BIBO stable

$$G(s) = [0 \;\; 1] \begin{bmatrix} s+2 & -3 \\ 0 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 1 =$$

$$\frac{1}{(s+2)(s+1)} [0 \;\; 1] \begin{bmatrix} s+1 & 3 \\ 0 & s+2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 1 = 0 + 1 = 1$$

No poles in the RHS or imaginary axis $\Rightarrow$ BIBO stable

3) $$\dot{x}(t) = \begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [0 \;\; 1] \, x(t)$$

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & 1 \\ 2 & \lambda \end{bmatrix}$$

$$|\lambda I - A| = \lambda(\lambda-1) - 2 = \lambda^2 - \lambda - 2 = (\lambda+1)(\lambda-2) = 0$$

$\Rightarrow$ Eigenvalues $\lambda_1 = -1,\ \lambda_2 = 2$ $\Rightarrow$ internally unstable

$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$ $\det R \neq 0 \Rightarrow \text{rank} = 2$

$\Rightarrow$ fully controllable

$$O = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$$ $\det O \neq 0 \Rightarrow \text{rank} = 2$

$\Rightarrow$ fully observable

$\Rightarrow$ system should be not BIBO stable

$$G(s) = [0\ \ 1] \begin{bmatrix} s-1 & 1 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$\frac{1}{s(s-1)-2} [0\ \ 1] \begin{bmatrix} s & -1 \\ -2 & s-1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{s-1}{s^2 - s - 2} =$$

$$\frac{s-1}{(s+1)(s-2)}$$

$\Rightarrow$ no pole-zero cancellation

Poles $s = -1$ $s = 2$ $\Rightarrow$ not BIBO stable

4) $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$y(t) = [\,2 \quad 0\,]\; x(t)$

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = \lambda^2 = 0$

$\Rightarrow$ Eigenvalues $\lambda_1 = \lambda_2 = 0$ (algebraic multiplicity $= 2$)

$A x = 0 x \qquad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 \,, \; x_1 \in \mathbb{R} \quad \Rightarrow$ geometric multiplicity $= 1$ (line)

As geometric multiplicity $\neq$ algebraic multiplicity, the system is internally unstable.

$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad \det R \neq 0 \Rightarrow$ rank $= 2$

$\Rightarrow$ fully controllable

$O = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad \det O \neq 0 \Rightarrow$ rank $= 2$

$\Rightarrow$ fully observable

$\Rightarrow$ the system should be not BIBO stable

$$g(s) = [2 \ 0] \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2} [2 \ 0] \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$\frac{2}{s^2}$$

$\Rightarrow$ no zero-pole cancellation

Poles: $s_1 = s_2 = 0$ $\Rightarrow$ not BIBO stable.

5. $$\dot{x}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \ 0] \ x(t)$$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow$ Eigenvalues: $\lambda_1 = \lambda_2 = 0$ (algebraic mult. $=2$)

$Ax = 0x$ $\quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_2$ $\Rightarrow$ geometric multiplicity $= 2$ (plane)

$\Rightarrow$ marginally stable

$R = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$ det $R = 0$ $\Rightarrow$ rank $= 1$

$\Rightarrow$ not fully controllable

$O = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ det $O = 0$ $\Rightarrow$ rank $= 1$

$\Rightarrow$ not fully observable

$\Rightarrow$ the system could be BIBO stable or not BIBO stable, depending on zero-pole cancellation

$$g(s) = [1 \quad 0] \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{s^2} [1 \quad 0] \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

$$\frac{2s}{s^2} = \frac{2}{s}$$ (only 1 pole is cancelled)

Pole at $s = 0$ $\Rightarrow$ not BIBO stable

6) $$\dot{x}(t) = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [0 \quad 1] \, x(t)$$

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & 1 \\ -2 & \lambda \end{vmatrix} = \lambda^2 + 2 = 0$$

$\Rightarrow$ Eigenvalues $\lambda_1 = \sqrt{2}j \quad \lambda_2 = -\sqrt{2}j$

$Re(\lambda_1) = Re(\lambda_2) = 0$

λ_1: geometric multiplicity = algebraicd multiplicity = 1

λ_2 geometric multiplicity = algebraic multiplicity = 1

$\Rightarrow$ marginally stable

$$R = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$$

$\det R \neq 0 \Rightarrow rank = 2$

$\Rightarrow$ fully controllable

$$O = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$\det O \neq 0 \Rightarrow rank = 2$

$\Rightarrow$ fully observable

$\Rightarrow$ the system should be not BIBO stable

$$g(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

$$\frac{1}{s^2+2} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{s+4}{s^2+2}$$

$\Rightarrow$ no zero-pole cancellation

Poles at $\pm\sqrt{2}j \Rightarrow$ not BIBO stable