Zarema Balgabekova HW6

1. $\dot{x}_1 = -2x_1 + x_2(\sin x_1 + 1) + 2u$

$\dot{x}_2 = x_2$

$y = x_1$

Equilibrium state:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2x_{1r} + x_{2r}(\sin x_{1r} + 1) + 2u_r \\ x_{2r} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_{2r} = 0$

if $u_r = 0 \ \forall t \geq 0$, $x_{1r} = 0$

$\Rightarrow x_{1r} = 0, x_{2r} = 0$ is equilibrium state when $u_r = 0 \ \forall t \geq 0$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\substack{x = x_r \\ u = u_r}} = \begin{bmatrix} -2 + x_2 \cos x_1 & \sin x_1 + 1 \\ 0 & 1 \end{bmatrix}_{\substack{x = x_r \\ u = u_r}} =$$

$$\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad C = [1 \quad 0] \qquad D = 0$$

A - triangular matrix $\Rightarrow \lambda_1 = -2, \ \lambda_2 = 1$

$Re(\lambda_1) < 0, \ Re(\lambda_2) > 0 \Rightarrow$ equilibrium point is unstable

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2.
$$\dot{x}_1 = -x_1^3 + x_1 + x_2 + u + 1$$
$$\dot{x}_2 = x_1 + x_2 + u$$
$$y = x_1 + x_2$$

Equilibrium state:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_{1r}^3 + x_{1r} + x_{2r} + u_r + 1 \\ x_{1r} + x_{2r} + u_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $u_r = -1 \ \forall t \geq 0, \ x_{2r} = 1 - x_{1r}$

$$\Rightarrow -x_{1r}^3 + x_{1r} + (1 - x_{1r}) - 1 + 1 = 0$$
$$-x_{1r}^3 + 1 = 0$$

$\Rightarrow x_{1r} = 1, \ x_{2r} = 0$ is equilibrium state when $u_r = -1 \ \forall t \geq 0$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\substack{x=x_r \\ u=u_r}} = \begin{bmatrix} -3x_1^2+1 & 1 \\ 1 & 1 \end{bmatrix}_{\substack{x=x_r \\ u=u_r}} =$$

$$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$$

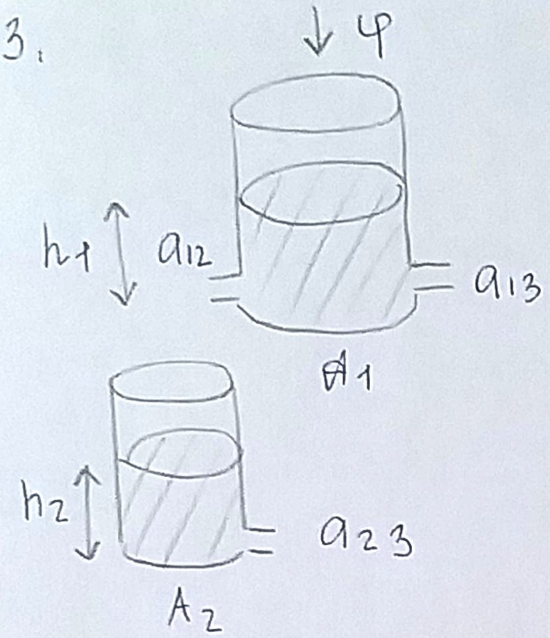
$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad D = 0$$

$$|\lambda I - A| = \begin{vmatrix} \lambda+2 & -1 \\ -1 & \lambda-1 \end{vmatrix} = (\lambda+2)(\lambda-1) - 1 = \lambda^2 + \lambda - 3 = 0$$

$$\lambda_1 = \frac{-1+\sqrt{13}}{2} \approx 1.3 \qquad \lambda_2 = \frac{-1-\sqrt{13}}{2} \approx -2.3$$

$Re(\lambda_1) > 0$, $Re(\lambda_2) < 0$ $\Rightarrow$ equilibrium point is <u>unstable</u>

3.



State variables:

$x_1 = h_1$

$x_2 = h_2$

Output: $y = h_2 = x_2$

$$A_1 h_1'(t) = -a_{12}\sqrt{2g\,h_1(t)} - a_{13}\sqrt{2g\,h_1(t)} + \varphi(t)$$

$$A_2 h_2'(t) = a_{12}\sqrt{2g h_1(t)} - a_{23}\sqrt{2g\,h_2(t)}$$

State-space representation:

$$\dot{x}_1(t) = -\frac{a_{12}+a_{13}}{A_1}\sqrt{2g x_1(t)} + \frac{u(t)}{A_1}$$

$$\dot{x}_2(t) = \frac{a_{12}\sqrt{2g x_1(t)}}{A_2} - \frac{a_{23}\sqrt{2g\,x_2(t)}}{A_2}$$

$$y(t) = x_2(t)$$

if $a_{12} = a_{23} = a_{13} = 1\ m^2$, $A_1 = 200\ m^2$, $A_2 = 100\ m^2$, $g \approx 10 m/s^2$,

$$\dot{x}_1 = -\frac{\sqrt{20}}{100}\sqrt{x_1} + \frac{u}{200}$$

$$\dot{x}_2 = \frac{\sqrt{20}}{100}\sqrt{x_1} - \frac{\sqrt{20}}{100}\sqrt{x_2}$$

$$y = x_2$$

Equilibrium state:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{20}}{100}\sqrt{\bar{x}_1} + \frac{\bar{u}}{200} \\ \frac{\sqrt{20}}{100}\sqrt{\bar{x}_1} - \frac{\sqrt{20}}{100}\sqrt{\bar{x}_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $\bar{u} = 10\ \frac{m^3}{s}$, $\sqrt{\bar{x}_1} = \frac{5}{\sqrt{20}}$ $\qquad \sqrt{\bar{x}_2} = \frac{5}{\sqrt{20}}$

$$\bar{x}_1 = \frac{5}{4} \qquad \bar{x}_2 = \frac{5}{4}$$

$\Rightarrow \bar{x}_1 = \frac{5}{4}$, $\bar{x}_2 = \frac{5}{4}$ is equilibrium state when $\bar{u} = 10\ \frac{m^3}{s}$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{bmatrix} -\frac{\sqrt{5}}{100\sqrt{x_1}} & 0 \\ \frac{\sqrt{5}}{100\sqrt{x_1}} & -\frac{\sqrt{5}}{100\sqrt{x_2}} \end{bmatrix}_{\substack{x=\bar{x} \\ u=\bar{u}}} =$$

$$\begin{bmatrix} -\frac{1}{50} & 0 \\ \frac{1}{50} & -\frac{1}{50} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{200} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = 0$$

A - triangular matrix $\Rightarrow \lambda_1 = -\frac{1}{50} \quad \lambda_2 = -\frac{1}{50}$

$Re(\lambda_1) < 0, \; Re(\lambda_2) < 0 \Rightarrow$ equilibrium point is asymptotically stable

4.

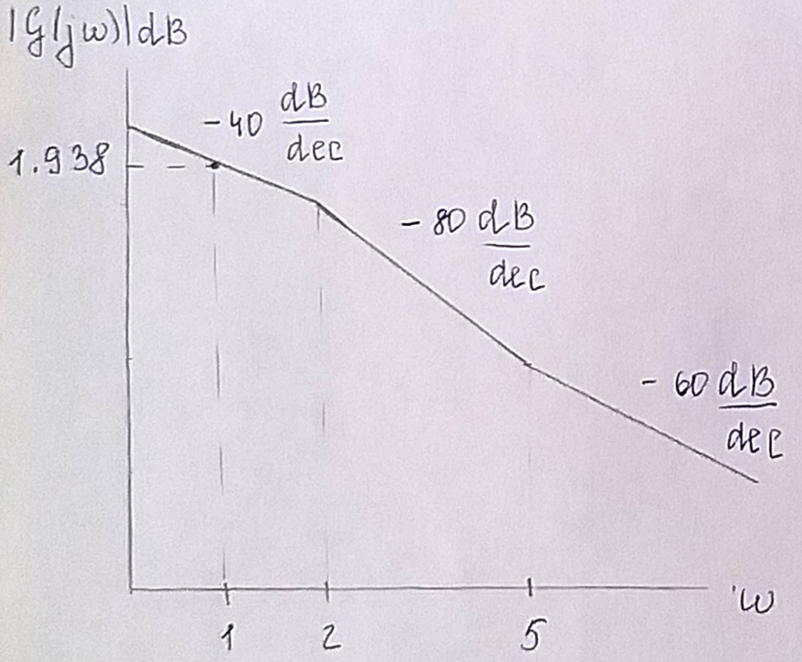
1) $G_1(s) = \dfrac{s+5}{s^2(s^2+3s+4)} = \dfrac{5(0.2s+1)}{4s^2\left(\frac{s^2}{4}+\frac{3}{4}s+1\right)} = \dfrac{1.25\,(0.2s+1)}{s^2\left(\frac{s^2}{4}+\frac{3}{4}s+1\right)}$

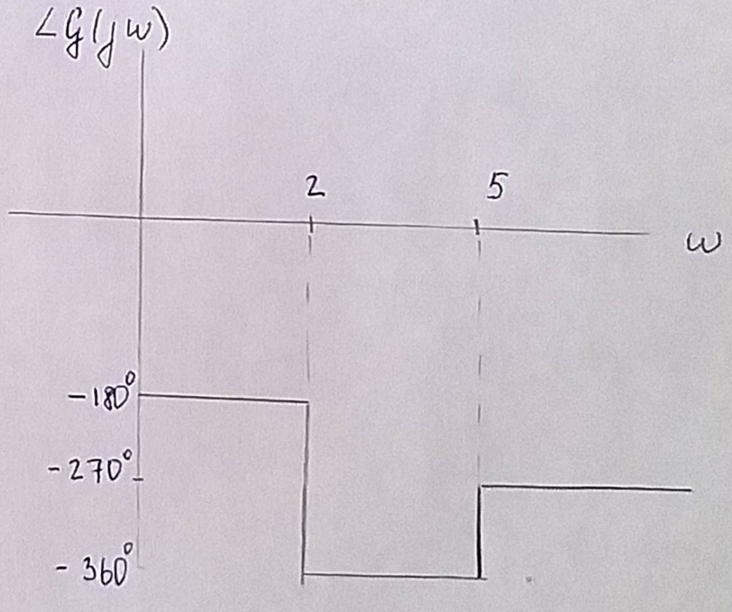
$\Rightarrow$ 2 integrators $(\omega=0)$: $-40\,\dfrac{dB}{dec}$, -180°

general gain $K=1.25$: $+20\log_{10}|1.25| = +1.938\,dB$, 0°

zero at $s=-5$ $(\omega=5)$: $+20\,\dfrac{dB}{dec}$, $+90^\circ$

2 complex poles: $\omega_n=2$, $\zeta=\frac{3}{4}$ $(\omega=2)$: $-40\,\dfrac{dB}{dec}$, -180°

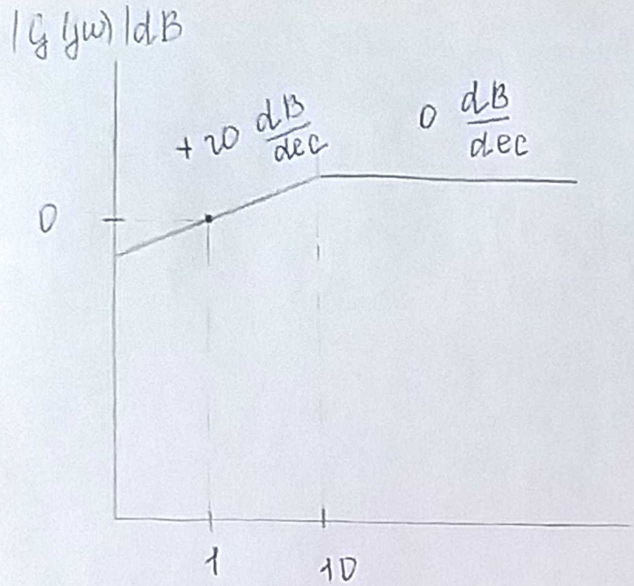


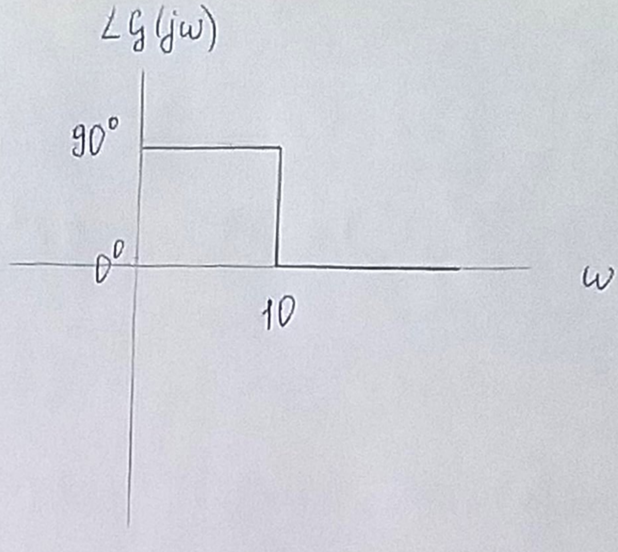


2) $G_2(s) = \dfrac{10s}{s+10} = \dfrac{10s}{10(0.1s+1)} = \dfrac{s}{0.1s+1}$

$\Rightarrow$ 1 differentiator $(\omega=0)$: $+20\,\dfrac{dB}{dec}$, $+90^\circ$

pole at $s=-10$ $(\omega=10)$: $-20\,\dfrac{dB}{dec}$, -90°



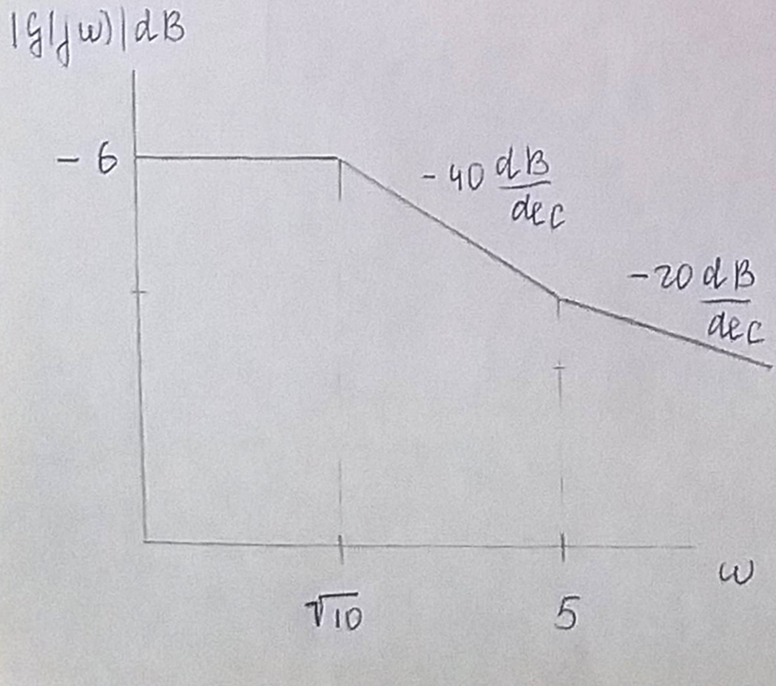


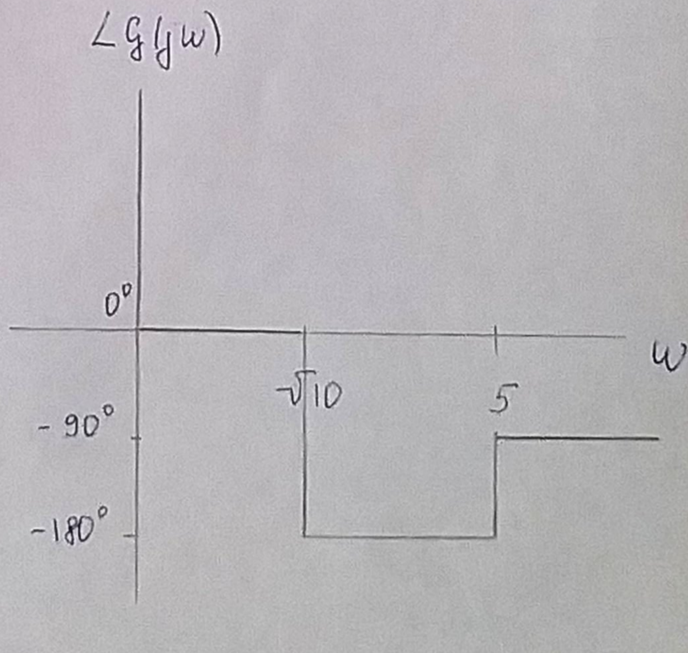
3) $$g_3(s) = \frac{s+5}{s^2+2s+10} = \frac{5\,(0.2s+1)}{10\left(\frac{s^2}{10}+\frac{1}{5}s+1\right)} = \frac{0.5\,(0.2s+1)}{\frac{s^2}{10}+\frac{1}{5}s+1}$$

$\Rightarrow$ DC gain $K = 0.5$; $+20\log_{10}|0.5| = -6$ dB , $0°$

zero at $s=-5$ ($\omega = 5$) ; $+20$ dB/dec , $+90°$

2 complex poles : $\omega_n = \sqrt{10}$, $\zeta = \frac{\sqrt{10}}{10}$; $-40\frac{dB}{dec}$, $-180°$ ($\omega = \sqrt{10}$)



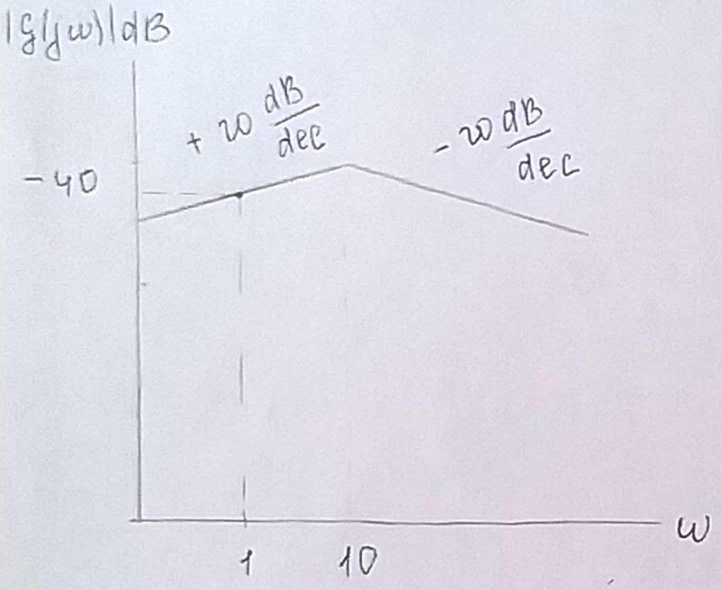


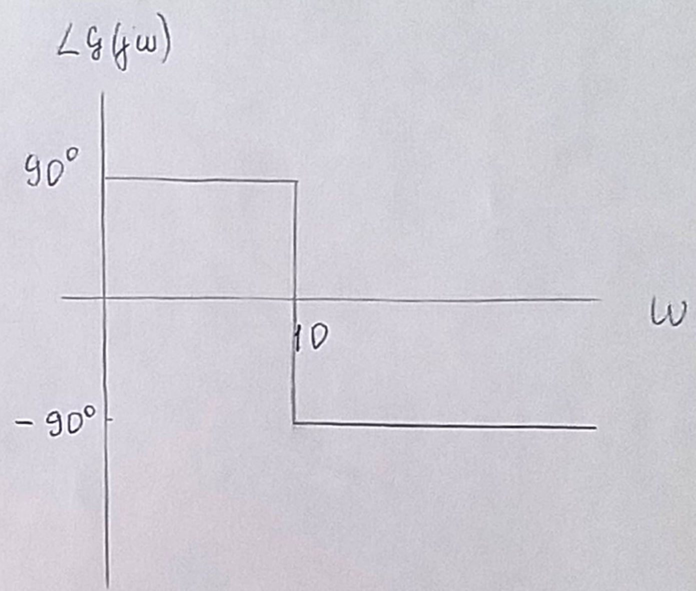
4) $G_4(s) = \dfrac{s}{s^2+2s+100} = \dfrac{s}{100\left(\frac{s^2}{100}+\frac{s}{50}+1\right)} = \dfrac{0.01s}{\frac{s^2}{100}+\frac{s}{50}+1}$

=> 1 differentiator $(\omega=0)$: $+20\,\frac{dB}{dec}$, $+90°$

general gain $K = 0.01$: $+20\log_{10}|0.01| = -40\,dB$, $0°$

2 complex poles : $\omega_n = 10$, $\zeta = \frac{1}{10}$ $(\omega=10)$: $-40\,\frac{dB}{dec}$, $-180°$





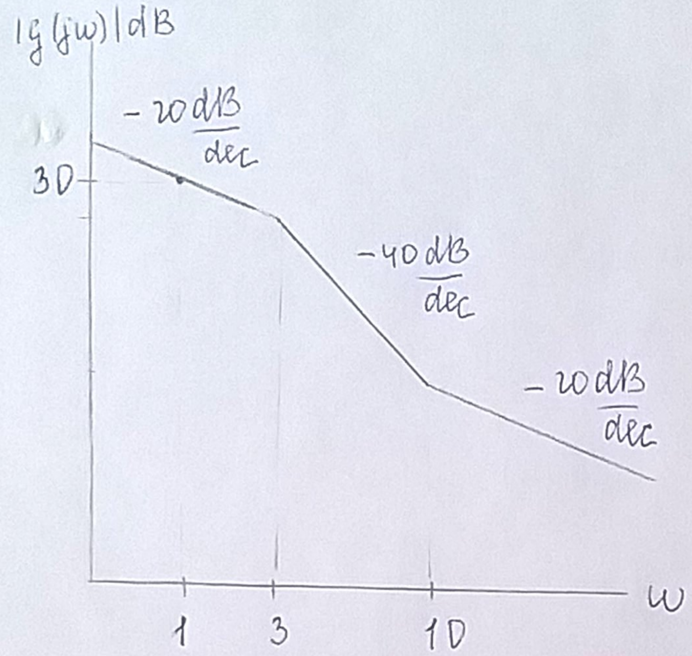
5) $G_5(s) = \dfrac{10(s+10)}{s(s+3)} = \dfrac{10\cdot 10(0.1s+1)}{3s\left(\frac{s}{3}+1\right)} = \dfrac{\frac{100}{3}(0.1s+1)}{s\left(\frac{s}{3}+1\right)}$

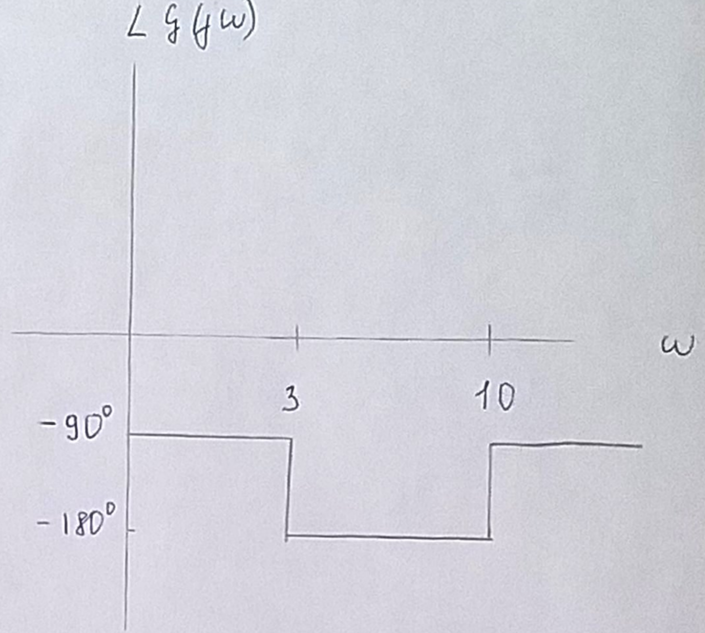
=> 1 integrator $(\omega=0)$: $-20\,\frac{dB}{dec}$, $-90°$

general gain $K = \frac{100}{3}$: $+20\log\left|\frac{100}{3}\right| = 30\,dB$, $0°$

zero at $s=-10$ $(\omega=10)$: $+20\,dB/dec$, $+90°$

pole at $s = -3$: $-20 \frac{dB}{dec}$, $-90°$
($\omega = 3$)





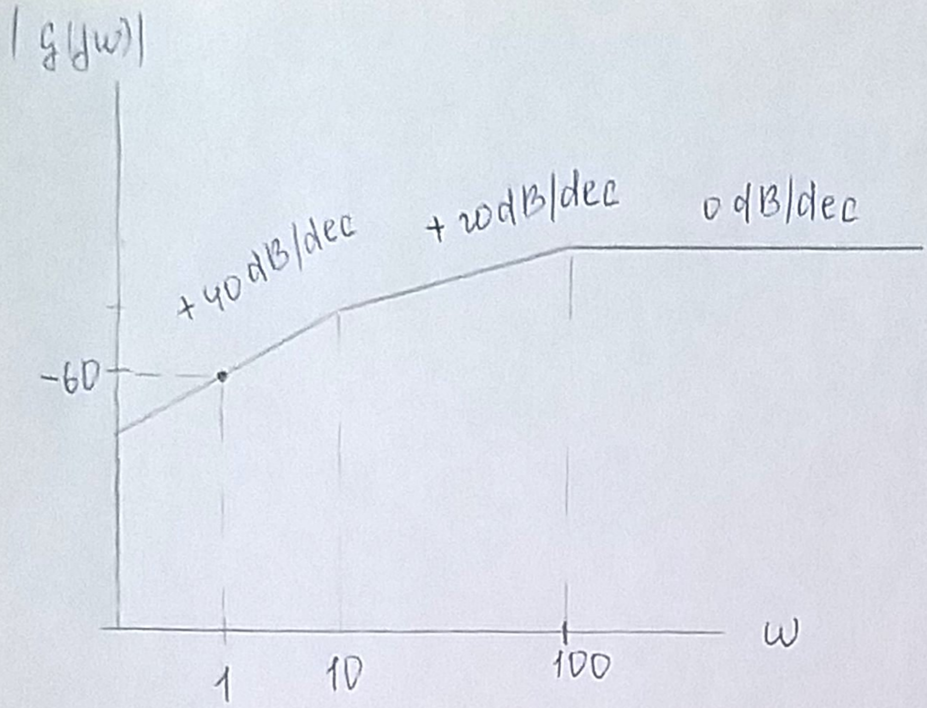
6) $g_6(s) = \frac{s^2}{(s+10)(s+100)} = \frac{s^2}{1000(0.1s+1)(0.01s+1)} = \frac{0.001 s^2}{(0.1s+1)(0.01s+1)}$

=> 2 differentiators : $+40 \frac{dB}{dec}$, $+180°$
($\omega = 0$)

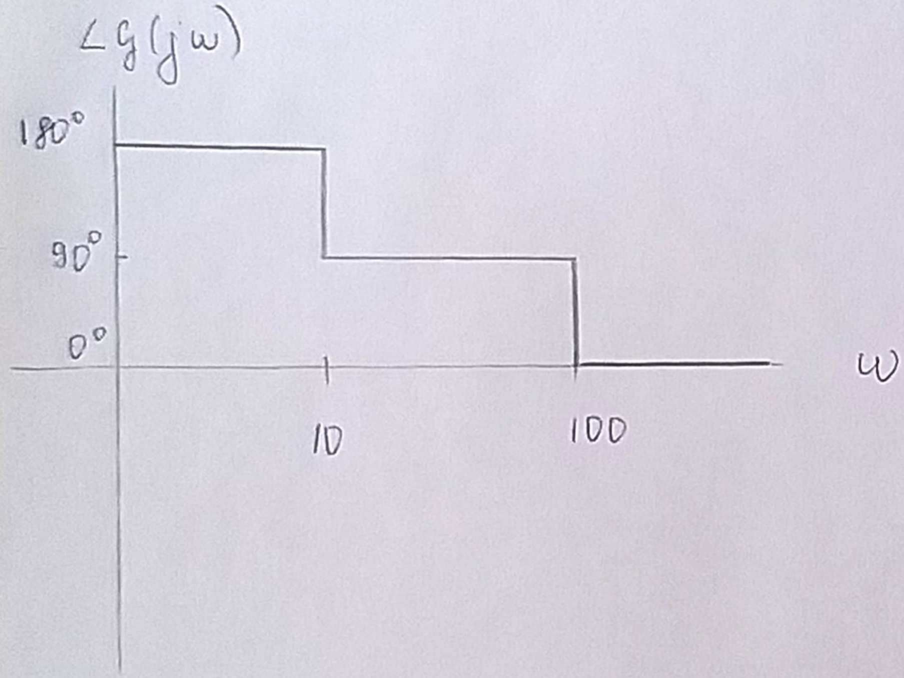
general gain $K = 0.001$: $+20\log_{10}|0.001| = -60 dB$, $0°$

pole at $s = -10$: $-20 \frac{dB}{dec}$, $-90°$
($\omega = 10$)

pole at $s = -100$ $-20 \frac{dB}{dec}$, $-90°$
($\omega = 100$)



|g(jw)|
+40dB/dec
+20dB/dec
0dB/dec
-60
1
10
100
ω



∠g(jw)
180°
90°
0°
10
100
ω