Zarema Balgabekova HW3

1.

$$G_1(s) = \frac{s-1}{s+2}$$

$s+2=0 \Rightarrow$ Pole at $s=-2$

$\Rightarrow$ the system is BIBO stable

$$G_2(s) = \frac{1}{s^2+2s}$$

$s^2+2s=0$

$s(s+2)=0 \Rightarrow$ Poles at $s=0$ and $s=-2$

$\Rightarrow$ the system is not BIBO stable.

$$G_3(s) = \frac{1}{s^2+3s+2}$$

$s^2+3s+2=0$

$(s+1)(s+2)=0 \Rightarrow$ Poles at $s=-1$ and $s=-2$

$\Rightarrow$ the system is BIBO stable.

$G_4(s) = \frac{1}{s}$

$s = 0 \quad \Rightarrow$ Pole at $s = 0$

$\Rightarrow$ the system is not BIBO stable.

$G_5(s) = \frac{s+2}{s^2-1}$

$s^2 - 1 = 0$

$(s-1)(s+1) = 0 \quad \Rightarrow$ Poles at $s = 1$ and $s = -1$

$\Rightarrow$ the system is not BIBO stable.

$G_6(s) = \frac{s}{(s^2+1)(s^2+4)}$

$(s^2+1)(s^2+4) = 0 \quad \Rightarrow$ Poles at $s = \pm j$ and $s = \pm 2j$

$\Rightarrow$ the system is not BIBO stable.

$G_7(s) = \frac{s+3}{s^2+2s+17}$

$s^2 + 2s + 17 = 0$

$\Delta = 1 - 17 = -16$

$\Rightarrow$ Poles at $s = -1 \pm 4j$

$\Rightarrow$ the system is BIBO stable

$$G_8(s) = \frac{7}{s^3 + 5s^2 + 4s}$$

$$s^3 + 5s^2 + 4s = 0$$

$$s(s^2 + 5s + 4) = 0$$

$s(s+1)\cdot(s+4) = 0$ => Poles at $s=0$, $s=-1$ and $s=-4$

=> the system is not BIBO stable

2.

$$G_1(s) = \frac{s-1}{s+2}$$

$$u(t) = 2\,\text{step}(t)$$

$$\lim_{t \to \infty} u(t) = 2$$

$$y_{ss}(t) = \lim_{t \to +\infty} y(t) = G_1(0) \cdot 2 = \frac{0-1}{0+2} \cdot 2 = -1$$

=> $y_{ss}(t) = -1$ when $u(t) = 2\,\text{step}(t)$

$$u(t) = 3\sin(t)$$

$$G_1(j\omega) = \frac{j\omega - 1}{j\omega + 2}$$

For $\omega = 1$ rad/s $\quad G_1(j) = \frac{-1+j}{2+j}$

$$|G_1(j)| = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$$

$$\angle G_1(j) = \arctan(-1) + 180^\circ - \arctan\left(\frac{1}{2}\right) = 108.4^\circ$$

$$\Rightarrow y_{ss}(t) = 3 \cdot \sqrt{\frac{2}{5}} \sin(t + 108.4^\circ) \quad \text{when} \quad u(t) = 3\sin(t)$$

$$G_3(s) = \frac{1}{s^2 + 3s + 2}$$

$$y_{ss}(t) = G_3(0) \cdot 2 = \frac{1}{0^2 + 3 \cdot 0 + 2} \cdot 2 = 1$$

$$\Rightarrow y_{ss}(t) = 1 \quad \text{when} \quad u(t) = 2\,\text{step}(t)$$

$$G_3(jw) = \frac{1}{(jw)^2 + 3jw + 2} = \frac{1}{-w^2 + 3jw + 2}$$

For $w = 1$ rad/s: $G_3(j) = \frac{1}{-1 + 3j + 2} = \frac{1}{1 + 3j}$

$$|G_3(j)| = \frac{1}{\sqrt{10}}$$

$$\angle G_3(j) = 0 - \arctan(3) = -71.6^\circ$$

$$\Rightarrow y_{ss}(t) = \frac{3}{\sqrt{10}} \sin(t - 71.6^\circ) \quad \text{when} \quad u(t) = 3\sin(t)$$

$$G_7(s) = \frac{s+3}{s^2+2s+17}$$

$$y_{ss}(t) = G_7(0)\cdot 2 = \frac{0+3}{0^2+2 0+17}\cdot 2 = \frac{6}{17}$$

$\Rightarrow y_{ss}(t) = \frac{6}{17}$ when $u(t) = 2\,step(t)$

$$G_7(j\omega) = \frac{j\omega+3}{(j\omega)^2+2j\omega+17} = \frac{j\omega+3}{-\omega^2+2j\omega+17}$$

For $\omega = 1$ rad/s : $G_7(j) = \frac{3+j}{-1+2j+17} = \frac{3+j}{16+2j}$

$$|G_7(j)| = \frac{\sqrt{10}}{\sqrt{260}} = \frac{1}{\sqrt{26}}$$

$$\angle G_7(j) = \arctan\left(\frac{1}{3}\right) - \arctan\left(\frac{2}{16}\right) = 11.3^\circ$$

$\Rightarrow y_{ss}(t) = \frac{3}{\sqrt{26}}\sin(t + 11.3^\circ)$ when $u(t) = 3\sin(t)$

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3. $G_2(s) = \frac{1}{s^2+2s}$

Step input of any amplitude makes the output unbounded, for example, $u(t) = step(t)$

$$\left(U(s) = \frac{1}{s}\right)$$

$G_4(s) = \frac{1}{s}$

Step input of any amplitude makes the output unbounded, for example, $u(t) = \text{step}(t)$

$\left(U(s) = \frac{1}{s} \right)$

$G_5(s) = \frac{s+2}{s^2-1}$

Any input signal whose Laplace transform has no zeros at $s = 1$ makes the output unbounded, for example, $u(t) = e^{-t}$ $\left(U(s) = \frac{1}{s+1} \right)$

$G_6(s) = \frac{s}{(s^2+1)(s^2+4)}$

Sinusoidal input with frequency $\omega = 1$ rad/s or $\omega = 2$ rad/s and any amplitude makes the output unbounded, for example, $u(t) = \sin(2t)$

$\left(U(s) = \frac{2}{s^2+4} \right)$

$$G_8(s) = \frac{7}{s^3+5s^2+4s}$$

Step input of any amplitude makes the output unbounded, for example, $u(t) = \text{step}(t)$

$$\left(U(s) = \frac{1}{s}\right)$$

4.

$$G_1(s) = \frac{200}{s^4+6s^3+11s^2+6s+200}$$

$$\begin{array}{c|ccc}
s^4 & 1 & 11 & 200 \\
s^3 & \cancel{6}\,1 & \cancel{6}\,1 & 0 \\
s^2 & 1\cancel{0} & 20\cancel{0} & \\
s^1 & -1\cancel{9}\ -1 & & \\
s^0 & 20 & &
\end{array}$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 11 \\ 1 & 1 \end{vmatrix}}{1} = 10$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 200 \\ 1 & 0 \end{vmatrix}}{1} = 200$$

$$c_1 = \frac{-\begin{vmatrix} 1 & 1 \\ 1 & 20 \end{vmatrix}}{1} = -19$$

$$d_1 = \frac{-\begin{vmatrix} 1 & 20 \\ -1 & 0 \end{vmatrix}}{-1} = 20$$

The order of the characteristic polynomial is 4, and there are 2 sign changes

=> $G_1(s)$ has 2 poles in the RHP and 2 poles in the LHP

=> the system is not BIBO stable

$$G_2(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}\,\varepsilon$	$\frac{7}{2}$	
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	3	
s^1	1		
s^0	3		

$$b_1 = \frac{-\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}}{2} = 0$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix}}{2} = \frac{7}{2}$$

$$c_1 = \frac{-\begin{vmatrix} 2 & 6 \\ \varepsilon & 7/2 \end{vmatrix}}{\varepsilon} = \frac{6\varepsilon - 7}{\varepsilon} \quad \text{(negative)}$$

$$c_2 = \frac{-\begin{vmatrix} 2 & 3 \\ \varepsilon & 0 \end{vmatrix}}{\varepsilon} = \frac{3\varepsilon}{\varepsilon} = 3$$

$$d_1 = \frac{-\begin{vmatrix} \varepsilon & 7/2 \\ \frac{6\varepsilon - 7}{\varepsilon} & 3 \end{vmatrix}}{\frac{6\varepsilon - 7}{\varepsilon}} = -\frac{3\varepsilon^2 - 21\varepsilon + 24.5}{6\varepsilon - 7} \quad \text{(positive)}$$

$$e_1 = \frac{-\begin{vmatrix} \frac{6\varepsilon - 7}{\varepsilon} & 3 \\ 1 & 0 \end{vmatrix}}{1} = 3$$

The order of the characteristic polynomial is 5, and there are 2 sign changes.

$\Rightarrow$ $G_2(s)$ has $\underline{\text{2 poles in the RHP and 3 poles in the LHP}}$

$\Rightarrow$ the system is $\underline{\text{not BIBO stable}}$.

$$G_3(s) = \frac{1}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1}$$

$$\begin{array}{c|ccc}
s^5 & \cancel{2}^{1} & \cancel{2}^{1} & \cancel{2}^{1} \\
s^4 & 3 & 3 & 1 \\
s^3 & \cancel{0}\,\varepsilon & \frac{2}{3} & \\
s^2 & \frac{3\varepsilon - 2}{\varepsilon} & 1 & \\
s^1 & 1 & & \\
s^0 & 1 & &
\end{array}$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix}}{3} = 0$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}}{3} = \frac{2}{3}$$

$$c_1 = \frac{-\begin{vmatrix} 3 & 3 \\ \varepsilon & 2/3 \end{vmatrix}}{\varepsilon} = \frac{3\varepsilon - 2}{\varepsilon} \quad \text{(negative)}$$

$$c_2 = \frac{-\begin{vmatrix} 3 & 1 \\ \varepsilon & 0 \end{vmatrix}}{\varepsilon} = \frac{\varepsilon}{\varepsilon} = 1$$

$$d_1 = \frac{-\begin{vmatrix} \varepsilon & \frac{2}{3} \\ \frac{3\varepsilon-2}{\varepsilon} & 1 \end{vmatrix}}{\frac{3\varepsilon-2}{\varepsilon}} = -\frac{\varepsilon^2 - 2\varepsilon + \frac{4}{3}}{3\varepsilon - 2} \quad \text{(positive)}$$

$$e_1 = \frac{-\begin{vmatrix} \frac{3\varepsilon-2}{\varepsilon} & 1 \\ 1 & 0 \end{vmatrix}}{1} = 1$$

The order of the characteristic polynomial is 5, and there are 2 sign changes

$\Rightarrow G_3(s)$ has $\underline{\text{2 poles in the RHP and 3 poles in the LHP}}$

$\Rightarrow$ the system is $\underline{\text{not BIBO stable}}$

$$G_4(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

$$\begin{array}{c|ccc}
s^5 & 1 & 6 & 8 \\
s^4 & \cancel{7}\ 1 & \cancel{42}\ 6 & \cancel{56}\ 8 \\
s^3 & \cancel{0}\ \cancel{4}\ 1 & \cancel{0}\ \cancel{12}\ 3 & \\
s^2 & 3 & 8 & \\
s^1 & \cancel{\tfrac{1}{3}}\ 1 & & \\
s^0 & 8 & &
\end{array}$$

$$b_1 = -\frac{\begin{vmatrix} 1 & 6 \\ 1 & 6 \end{vmatrix}}{1} = 0 \qquad b_2 = -\frac{\begin{vmatrix} 1 & 8 \\ 1 & 8 \end{vmatrix}}{1} = 0$$

$$p(s) = s^4 + 6s^2 + 8$$

$$\frac{d\,p(s)}{ds} = 4s^3 + 12s$$

$$c_1 = -\frac{\begin{vmatrix} 1 & 6 \\ 1 & 3 \end{vmatrix}}{1} = 3$$

$$c_2 = -\frac{\begin{vmatrix} 1 & 8 \\ 1 & 0 \end{vmatrix}}{1} = 8$$

$$d_1 = -\frac{\begin{vmatrix} 1 & 3 \\ 3 & 8 \end{vmatrix}}{3} = \frac{1}{3}$$

$$e_1 = -\frac{\begin{vmatrix} 3 & 8 \\ 1 & 0 \end{vmatrix}}{1} = 8$$

$$p(s) = s^4 + 6s^2 + 8 = 0$$

$$(s^2+4)(s^2+2) = 0$$

$$s = \pm 2j \qquad s = \pm \sqrt{2}\,j$$

The order of the characteristic polynomial is 5, and there are no sign changes

$\Rightarrow$ $G_4(s)$ has <u>4 poles on the imaginary axis and</u>
<u>1 pole in the LHP</u>

$\Rightarrow$ the system is <u>not BIBO stable.</u>

$$G_5(s) = \frac{128}{s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 128}$$

s^8	1	10	48	128	128
s^7	$\cancel{3}\ 1$	$\cancel{24}\ 8$	$\cancel{96}\ 32$	$\cancel{192}\ 64$	0
s^6	$\cancel{2}\ 1$	$\cancel{16}\ 8$	$\cancel{64}\ 32$	$\cancel{128}\ 64$	
s^5	$\cancel{0}\ \cancel{6}\ 3$	$\cancel{0}\ \cancel{32}\ 16$	$\cancel{0}\ \cancel{64}\ 32$		
s^4	$\cancel{8/3}\ 1$	$\cancel{64/3}\ 8$	$\cancel{64}\ 24$		
s^3	$-\cancel{8}\ -1$	$-\cancel{40}\ -5$			
s^2	$\cancel{3}\ 1$	$\cancel{24}\ 8$			
s^1	$\cancel{3}\ 1$				
s^0	8				

$$b_1 = \frac{-\begin{vmatrix} 1 & 10 \\ 1 & 8 \end{vmatrix}}{1} = 2$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 48 \\ 1 & 32 \end{vmatrix}}{1} = 16$$

$$b_3 = \frac{-\begin{vmatrix} 1 & 128 \\ 1 & 64 \end{vmatrix}}{1} = 64$$

$$b_4 = \frac{-\begin{vmatrix} 1 & 128 \\ 1 & 0 \end{vmatrix}}{1} = 128$$

$$c_1 = c_2 = c_3 = 0$$

$$P(s) = s^6 + 8s^4 + 32s^2 + 64$$

$$\frac{dP(s)}{ds} = 6s^5 + 32s^3 + 64s$$

$$d_1 = \frac{-\begin{vmatrix} 1 & 8 \\ 3 & 16 \end{vmatrix}}{3} = \frac{8}{3}$$

$$d_2 = \frac{-\begin{vmatrix} 1 & 32 \\ 3 & 32 \end{vmatrix}}{3} = \frac{64}{3}$$

$$d_3 = \frac{-\begin{vmatrix} 1 & 64 \\ 3 & 0 \end{vmatrix}}{3} = \frac{192}{3} = 64$$

$$e_1 = \frac{-\begin{vmatrix} 3 & 16 \\ 1 & 8 \end{vmatrix}}{1} = -8$$

$$e_2 = \frac{-\begin{vmatrix} 3 & 32 \\ 1 & 24 \end{vmatrix}}{1} = -40$$

$$f_1 = \frac{-\begin{vmatrix} 1 & 8 \\ -1 & -5 \end{vmatrix}}{-1} = 3$$

$$f_2 = \frac{-\begin{vmatrix} 1 & 24 \\ -1 & 0 \end{vmatrix}}{-1} = 24$$

$$g_1 = \frac{-\begin{vmatrix} -1 & -5 \\ 1 & 8 \end{vmatrix}}{1} = 3$$

$$h_1 = \frac{-\begin{vmatrix} 1 & 8 \\ 1 & 0 \end{vmatrix}}{1} = 8$$

The order of the characteristic polynomial is 8

Before $P(s)$: no sign changes

Then : 2 sign changes

$\Rightarrow$ $P(s)$ has 2 roots in the RHP, 2 roots in the LHP, and 2 roots on the imaginary axis

$\Rightarrow$ $G_5(s)$ has <u>2 poles in the RHP, 4 poles in the LHP, and 2 poles on the imaginary axis</u>

$\Rightarrow$ the system is <u>not BIBO stable</u>.

5.

$$G_1(s) = \frac{1}{s^4 + 2s^3 + (4+K)s^2 + 9s + 25}$$

$$
\begin{array}{c|ccc}
s^4 & 1 & 4+K & 25 \\
s^3 & 2 & 9 & 0 \\
s^2 & K-\frac{1}{2} & 25 & \\
s^1 & \frac{9K-54.5}{K-0.5} & & \\
s^0 & 25 & &
\end{array}
$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 4+K \\ 2 & 9 \end{vmatrix}}{2} = \frac{2K-1}{2} = K - \frac{1}{2}$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 25 \\ 2 & 0 \end{vmatrix}}{2} = 25$$

$$c_1 = \frac{-\begin{vmatrix} 2 & 9 \\ K-\frac{1}{2} & 25 \end{vmatrix}}{K-\frac{1}{2}} = \frac{9K-54.5}{K-0.5}$$

$$d_1 = - \frac{\begin{vmatrix} K - \frac{1}{2} & 25 \\ \frac{9K - 54.5}{K - 0.5} & 0 \end{vmatrix}}{\frac{9K - 54.5}{K - 0.5}} = 25$$

For the system to be BIBO stable:

$K - \frac{1}{2} > 0$ and $\frac{9K - 54.5}{K - 0.5} > 0$

$K > \frac{1}{2}$

$K > \frac{109}{18}$, $K \neq \frac{1}{2}$

$\Rightarrow$ For BIBO stability: $K > \frac{109}{18}$

$$G_2(s) = \frac{K}{s^3 + 18 s^2 + 77 s + K}$$

$$
\begin{array}{c|cc}
s^3 & 1 & 77 \\
s^2 & 18 & K \\
s^1 & \frac{1386-K}{18} & \\
s^0 & K &
\end{array}
$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 77 \\ 18 & K \end{vmatrix}}{18} = \frac{1386-K}{18}$$

$$c_1 = \frac{-\begin{vmatrix} 18 & K \\ \frac{1386-K}{18} & 0 \end{vmatrix}}{\frac{1386-K}{18}} = K$$

For the system to be BIBO stable:

$$\frac{1386-K}{18} > 0 \qquad K > 0$$

$$K < 1386$$

$\Rightarrow$ For BIBO stability: $0 < K < 1386$