Zarema Balgabekova HW1

1.

a) 7562.45 to octal

7562 | 8
-7560 | 945 | 8
2 | -944 | 118 | 8
1 | -112 | 14 | 8
6 | -8 | 1
6

$(7562)_{10} = (16612)_8$

$0.45 \times 8 = 3.6$

$0.6 \times 8 = 4.8$

$0.8 \times 8 = 6.4$

$0.4 \times 8 = 3.2$

$0.2 \times 8 = 1.6$

$(0.45)_{10} = (0.34631)_8$

$\Rightarrow (7562.45)_{10} = (16612.34631)_8$

b) 1938.257 to hexadecimal

1938 | 16
-1936 | 121 | 16
2 | -112 | 7
9

$(1938)_{10} = (792)_{16}$

$0.257 \times 16 = 4.112$

$0.112 \times 16 = 1.792$

$0.792 \times 16 = C.672$

$0.672 \times 16 = A.752$

$0.752 \times 16 = C.032$

$(0.257)_{10} = (0.41CAC)_{16}$

$\Rightarrow (1938.257)_{10} = (792.41CAC)_{16}$

c) 175.175 to Binary

$175 - 174 = (1)$, $175 : 2 = 87$
$87 - 86 = (1)$, $87 : 2 = 43$
$43 - 42 = (1)$, $43 : 2 = 21$
$21 - 20 = (1)$, $21 : 2 = 10$
$10 - 10 = (0)$, $10 : 2 = 5$
$5 - 4 = (1)$, $5 : 2 = 2$
$2 - 2 = (0)$, $2 : 2 = (1)$

$(175)_{10} = (10101111)_2$

$0.175 \times 2 = 0.35$
$0.35 \times 2 = 0.7$
$0.7 \times 2 = 1.4$
$0.4 \times 2 = 0.8$
$0.8 \times 2 = 1.6$
$0.6 \times 2 = 1.2$

$(0.175)_{10} = (0.001011)_2$

$\Rightarrow (175.175)_{10} = (10101111.001011)_2$

2.

a) $(BEE)_r = (2699)_{10}$

$11 \cdot r^2 + 14r + 14r^0 = 2699$

$11r^2 + 14r - 2685 = 0$

$\Delta' = 49 + 11 \cdot 2685 = 29584 \ (172^2)$

$r_1 = \frac{-7+172}{11} = 15 \qquad r_2 = \frac{-7-172}{11} = -\frac{179}{11}$

Choose positive root : $r = 15$

$\Rightarrow (BEE)_{15} = (2699)_{10}$

b) $(365)_r = (194)_{10}$

$3r^2 + 6r + 5r^0 = 194$

$3r^2 + 6r - 189 = 0$

$D = 9 + 3 \; 189 = 576 \; (24^2)$

$r_1 = \dfrac{-3+24}{3} = 7 \qquad r_2 = \dfrac{-3-24}{3} = -9$

Choose positive root $r = 7$

$\Rightarrow (365)_7 = (194)_{10}$

3.

a) 0100 1000 0110 0111

$(0100 \; 1000 \; 0110 \; 0111)_{BCD} = (4867)_{10}$

$\begin{array}{r|l} 4867 & 2 \\ -4866 & 2433 \\ \hline 1 & \end{array}$
$\begin{array}{r|l} 2433 & 2 \\ -2432 & 1216 \\ \hline 1 & \end{array}$
$\begin{array}{r|l} 1216 & 2 \\ -1216 & 608 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 608 & 2 \\ -608 & 304 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 304 & 2 \\ -304 & 152 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 152 & 2 \\ -152 & 76 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 76 & 2 \\ -76 & 38 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 38 & 2 \\ -38 & 19 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 19 & 2 \\ -18 & 9 \\ \hline 1 & \end{array}$
$\begin{array}{r|l} 9 & 2 \\ -8 & 4 \\ \hline 1 & \end{array}$
$\begin{array}{r|l} 4 & 2 \\ -4 & 2 \\ \hline 0 & \end{array}$
$\begin{array}{r|l} 2 & 2 \\ -2 & 1 \\ \hline 0 & \end{array}$

$(4867)_{10} = (1001100000011)_2$

$\Rightarrow (0100\ 1000\ 0110\ 0111)_{BCD} = (1001100000011)_2$

в) 0011 0111 1000.0111 0101

$(0011\ 0111\ 1000.0111\ 0101)_{BCD} = (378.75)_{10}$

```
378 |2
-378|189 |2
  0 -188 |94 |2
       1  -94|47 |2
            0 -46|23 |2
                1 -22|11 |2
                    1 -10|5 |2
                        1 -4|2 |2
                           1 -2|1
                              0
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$0.75 \times 2 = 1.5$

$0.5 \times 2 = 1.0$

$(378.75)_{10} = (101111010.11)_2$

$\Rightarrow (0011\ 0111\ 1000.0111\ 0101)_{BCD} = (101111010.11)_2$

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4. Zarema M. Balgabekova

a) 01011010 01100001 01110010 01100101 01101101 01100001 00100000
01001101 00101110 00100000 01000010 01100001 01101100 01100111
01100001 01100010 01100101 01101011 01101111 01110110 01100001

в) 01011010 11100001 01110010 01100101 11101101 11100001 10100000
01001101 00101110 10100000 01000010 11100001 01101100 11100111
11100001 11100010 01100101 11101011 01101111 11110110 11100001

5.

1000111 1101111 0100000 1000010 1100001 1100100
1100111 1100101 1110010 1110011 0100001

Go Badgers!

6.

a) De Morgan's theorem for three variables:

$$\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$$

x	y	z	XYZ	$\overline{XYZ}$	$\bar{X}+\bar{Y}+\bar{Z}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

b) The second distributive law:

$$X + YZ = (X+Y)(X+Z)$$

X	Y	Z	YZ	$X+YZ$	$X+Y$	$X+Z$	$(X+Y)(X+Z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

c) $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

x	y	z	$\bar{X}Y$	$\bar{Y}Z$	$X\bar{Z}$	$\bar{X}Y + \bar{Y}Z + X\bar{Z}$	$X\bar{Y}$	$Y\bar{Z}$	$\bar{X}Z$	$X\bar{Y} + Y\bar{Z} + \bar{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

7.

a) $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$

Left-hand side:

$$AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = BC + \bar{C}(AB + B\bar{D} + D) = BC +$$

$$\bar{C}(AB + (B+D)\underbrace{(D+\bar{D})}_{1}) = BC + \bar{C}(AB + B\cdot 1 + D) = BC +$$

$$\bar{C}(B\underbrace{(A+1)}_{1} + D) = BC + B\bar{C} + \bar{C}D = B\underbrace{(C+\bar{C})}_{1} + \bar{C}D = B + \bar{C}D$$

(the same as right-hand side)

$$\Rightarrow AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$$

b) $wy + \bar{w}y\bar{z} + wxz + \bar{w}x\bar{y} = wy + \bar{w}x\bar{z} + \bar{x}y\bar{z} + x\bar{y}z$

Right-hand side

$$wy + \bar{w}x\bar{z} + \bar{x}y\bar{z} + x\bar{y}z = wy + \bar{w}x\bar{z} + \bar{x}y\bar{z}\underbrace{(w+\bar{w})}_{1} +$$

$$x\bar{y}z\underbrace{(w+\bar{w})}_{1} = wy + \bar{w}x\bar{z} + w\bar{x}y\bar{z} + \bar{w}\bar{x}y\bar{z} + wx\bar{y}z +$$

$$\bar{w}x\bar{y}z = w(y + \bar{x}y\bar{z} + x\bar{y}z) + \bar{w}(x\bar{z} + \bar{x}y\bar{z} + x\bar{y}z) =$$

$$w(y\underbrace{(1+\bar{x}\bar{z})}_{1} + x\bar{y}z) + \bar{w}(x\bar{z}\underbrace{(y+\bar{y})}_{1} + \bar{x}y\bar{z} + x\bar{y}z) =$$

$$w(\underbrace{(y+\bar{y})}_{1}(y+xz)) + \bar{w}(x\bar{z}y + x\bar{z}\bar{y} + \bar{x}y\bar{z} + x\bar{y}z) =$$

$$wy + wxz + \bar{w}((x\bar{z}y + \bar{x}y\bar{z}) + (x\bar{z}\bar{y} + x\bar{y}z)) =$$

$$wy + wxz + \bar{w}\,(y\bar{z}(\underbrace{x+\bar{x}}_{1}) + x\bar{y}\,(z+\bar{z})) =$$

$$wy + wxz + \bar{w}y\bar{z} + \bar{w}x\bar{y} = wy + \bar{w}y\bar{z} + wxz + \bar{w}x\bar{y}$$

(the same as left-hand side)

$$\Rightarrow wy + \bar{w}y\bar{z} + wxz + \bar{w}x\bar{y} = wy + \bar{w}x\bar{z} + \bar{x}y\bar{z} + x\bar{y}z$$

c) $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)$

Left-hand side

$$A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = \overline{\overline{A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C}} = \overline{\overline{A\bar{D}}\cdot\overline{\bar{A}B}\;\overline{\bar{C}D}\;\overline{\bar{B}C}} =$$

$$\overline{(\bar{A}+D)\,(A+\bar{B})\,(C+\bar{D})\,(B+\bar{C})} = \overline{(\underbrace{A\bar{A}}_{0} + \bar{A}\bar{B} + AD + \bar{B}D)\,(BC + \underbrace{C\bar{C}}_{0} + B\bar{D} + \bar{C}\bar{D})}$$

$$= \overline{\bar{A}\underbrace{\bar{B}B}_{0}C + \bar{A}\underbrace{\bar{B}B}_{0}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + ABCD + AB\underbrace{D\bar{D}}_{0} + A\bar{C}\underbrace{D\bar{D}}_{0} + \underbrace{\bar{B}B}_{0}CD + \underbrace{\bar{B}B}_{0}\underbrace{D\bar{D}}_{0} + \bar{B}\bar{C}\underbrace{D\bar{D}}_{0}}$$

$$= \overline{\bar{A}\bar{B}\bar{C}\bar{D} + ABCD} = \overline{\bar{A}\bar{B}\bar{C}\bar{D}}\cdot\overline{ABCD} =$$

$$(A+B+C+D)\,(\bar{A}+\bar{B}+\bar{C}+\bar{D}) = (\bar{A}+\bar{B}+\bar{C}+\bar{D})\,(A+B+C+D)$$

(the same as right-hand side)

$$\Rightarrow A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A}+\bar{B}+\bar{C}+\bar{D})\,(A+B+C+D)$$

8.

a) $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C = \bar{A}\bar{C} + C(\bar{A}B + \bar{B}) = \bar{A}\bar{C} + C((\bar{A}+\bar{B})\underbrace{(B+\bar{B})}_{1}) =$

$\bar{A}\bar{C} + \bar{A}C + \bar{B}C = \bar{A}\underbrace{(\bar{C}+C)}_{1} + \bar{B}C = \bar{A} + \bar{B}C$

b) $(\overline{A+B+C}) \cdot \overline{ABC} = \bar{A}\,\bar{B}\,\bar{C}\,(\bar{A}+\bar{B}+\bar{C}) = \bar{A}\cdot\bar{A}\,\bar{B}\,\bar{C} +$

$\bar{A}\cdot\bar{B}\,\bar{B}\,\bar{C} + \bar{A}\,\bar{B}\,\bar{C}\,\bar{C} = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\cdot\bar{B}\,\bar{C} + \bar{A}\cdot\bar{B}\,\bar{C} =$

$\bar{A}\cdot\bar{B}\,\bar{C}$

c) $AB\bar{C} + AC = A\,(B\bar{C}+C) = A((B+C)\,\underbrace{(\bar{C}+C)}_{1}) =$

$A\,(B+C)$

d) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD = D\,(\bar{A}\bar{B} + \bar{A}\bar{C} + B) =$

$D\,(\bar{A}\bar{C} + (\bar{A}+B)\underbrace{(\bar{B}+B)}_{1}) = D\,(B + \bar{A}\cdot\underbrace{(\bar{C}+1)}_{1}) =$

$D\,(\bar{A}+B)$

e) $(\overline{\bar{A}+B})\,(\overline{\bar{A}+\bar{C}})\,(\overline{A\bar{B}C}) = (A\cdot\bar{B})\,(A\cdot C)\,(\bar{A}+B+\bar{C}) =$

$A\bar{B}C\,(\bar{A}+B+\bar{C}) = \underbrace{A\bar{A}}_{0}\,\bar{B}C + A\underbrace{\bar{B}B}_{0}C + A\bar{B}\underbrace{C\bar{C}}_{0} = 0$

9. $F = A\bar{B}C + \bar{A}\,\bar{C} + AB$

a) $F = \overline{\bar{A} + B + \bar{C}} + \overline{A + C} + \overline{\bar{A} + \bar{B}}$

b) $F = \overline{\overline{A\bar{B}C} \cdot \overline{\bar{A}\bar{C}} \cdot \overline{AB}}$

10.

a) $(XY + Z)(Y + XZ)$

X	Y	Z	XY	XY+Z	XZ	Y+XZ	(XY+Z)(Y+XZ)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	0	0	0	1	0
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

sum-of-minterms $\bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ = m_3 + m_5 + m_6 + m_7$

$= \sum m(3,5,6,7)$

product-of-maxterms $(X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)\cdot(\bar{X}+Y+Z) =$

$M_0 \, M_1 \, M_2 \, M_4 = \prod M(0,1,2,4)$

b) $(\bar{A}+B)(\bar{B}+C)$

A	B	C	$\bar{A}+B$	$\bar{B}+C$	$(\bar{A}+B)\cdot(\bar{B}+C)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

sum-of-minterms $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + ABC = m_0 + m_1 + m_3 + m_7$

$= \sum m(0,1,3,7)$

product-of-maxterms $(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C) =$

$M_2 + M_4 + M_5 + M_6 = \prod M(2,4,5,6)$

c) $wx\bar{y} + wx\bar{z} + wxz + y\bar{z}$

w	x	y	z	$wx\bar{y}$	$wx\bar{z}$	wxz	$y\bar{z}$	$wx\bar{y}+wx\bar{z}+wxz+y\bar{z}$
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	1	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	0	0	1	1
0	1	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	1	1	0	0	1
1	1	0	1	1	0	1	0	1
1	1	1	0	0	1	0	1	1
1	1	1	1	0	0	1	0	1

sum-of-minterms: $\bar{w}\bar{x}y\bar{z} + \bar{w}xy\bar{z} + w\bar{x}y\bar{z} + wx\bar{y}\bar{z} +$
$wx\bar{y}z + wxy\bar{z} + wxyz = m_2 + m_6 + m_{10} +$
$m_{12} + m_{13} + m_{14} + m_{15} = \sum m(2, 6, 10, 12, 13, 14, 15)$

product-of-maxterms:

$(w+x+y+z)(w+x+y+\bar{z})(w+x+\bar{y}+\bar{z})(w+\bar{x}+y+z)$

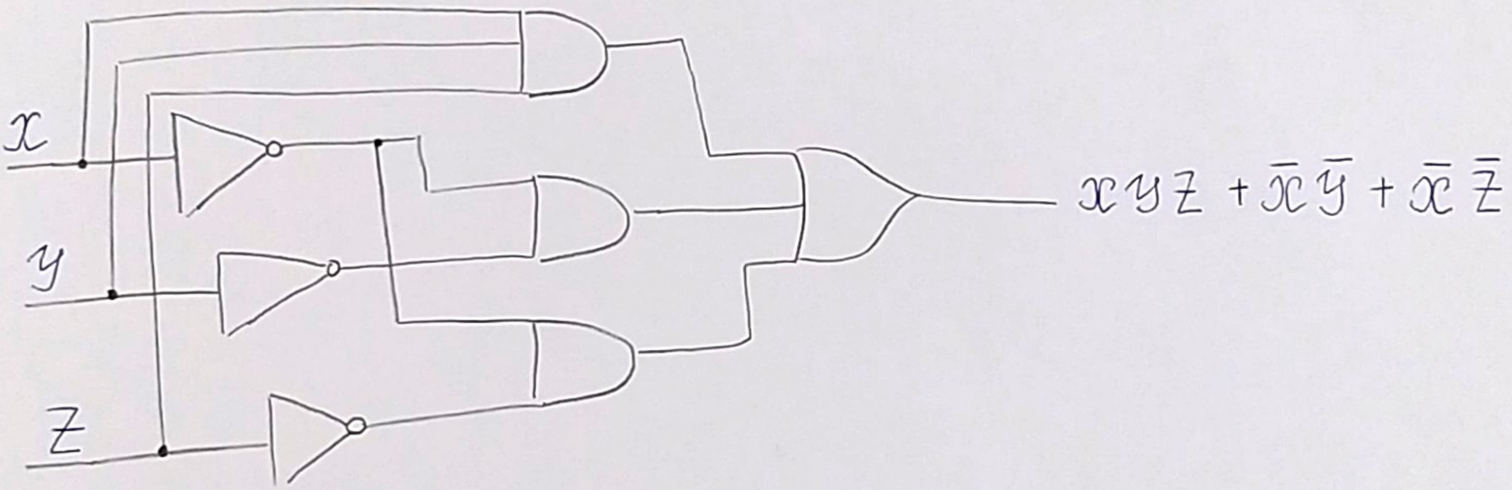
$(w+\bar{x}+y+\bar{z})(w+\bar{x}+\bar{y}+\bar{z})(\bar{w}+x+y+z)(\bar{w}+x+y+\bar{z})$

$(\bar{w}+x+\bar{y}+\bar{z}) = M_0 M_1 M_3 M_4 M_5 M_7 M_8 M_9 M_{11} =$

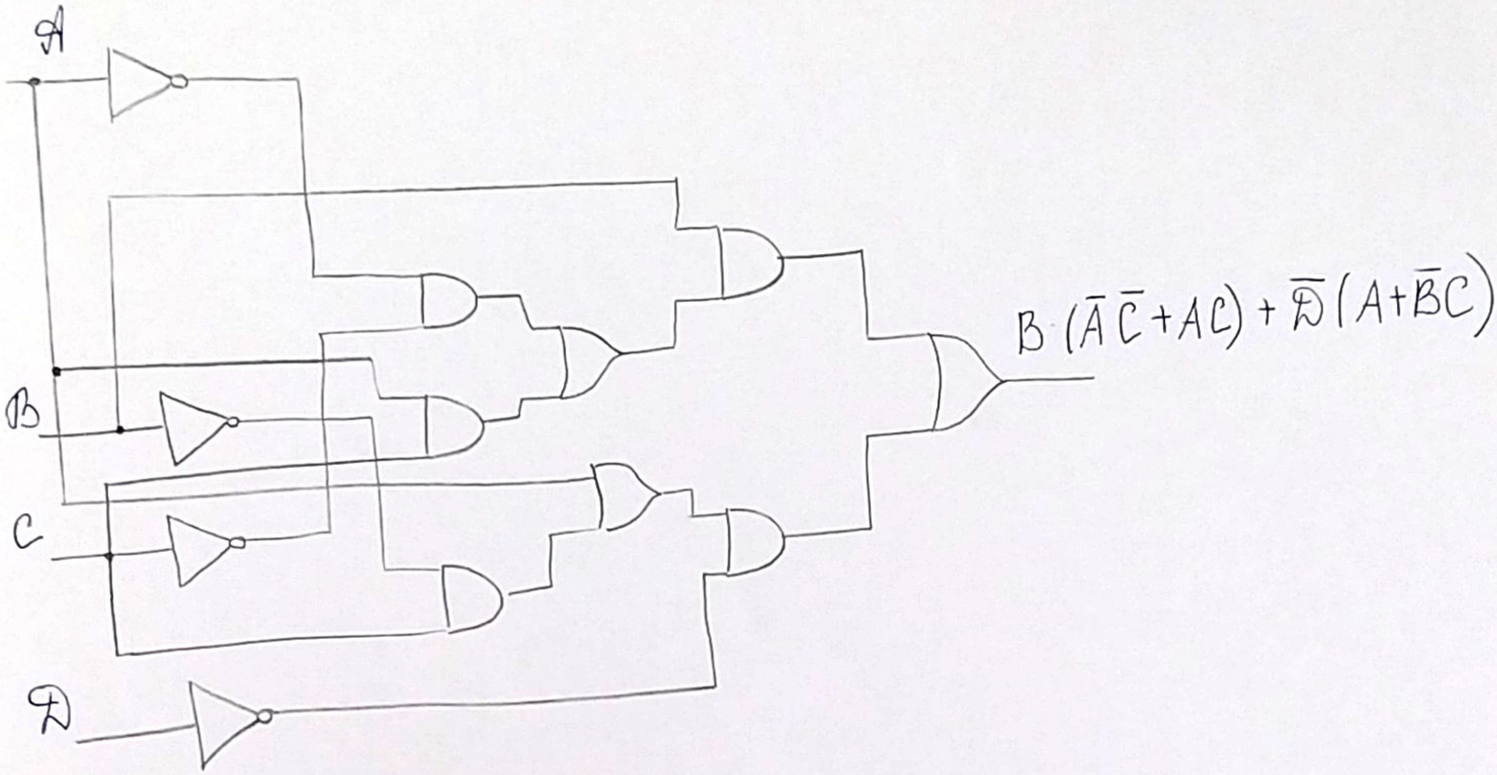
$\prod M(0,1,3,4,5,7,8,9,11)$

11.

a) $xyz + \bar{x}\bar{y} + \bar{x}\bar{z}$



b) $B(\bar{A}\bar{C} + AC) + \bar{D}(A + \bar{B}C)$



c) $x\bar{y}(\bar{w}+\bar{z})+\bar{w}y(\bar{x}+\bar{z})+wy(\bar{x}+z)$