Robt 303 Zarema Balgabekova HW 3

1. $G_{p1}(s) = \dfrac{s^2+2s+2}{s^2(s+3)}$

$$L(s) = G_c(s) \cdot G_{p1}(s) = k \cdot \frac{s^2+2s+2}{s^2(s+3)} = \rho \frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s^2+2s+2$

$D(s) = s^3+3s^2$

$\Rightarrow$ $n = 3$

$m = 2$

$r = n - m = 1 \Rightarrow 1$ asymptote

Zeros: $s_1 = -1+j$ $\quad s_2 = -1-j$

Poles: $s_{1,2} = 0$ $\quad s_3 = -3$

$$x_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{(1+j+1-j)-(0+0+3)}{1} = -1$$

$$r\psi_a = \begin{cases} (2h+1)\pi & h=0 \quad PL \\ 2h\pi & h=0 \quad NL \end{cases}$$

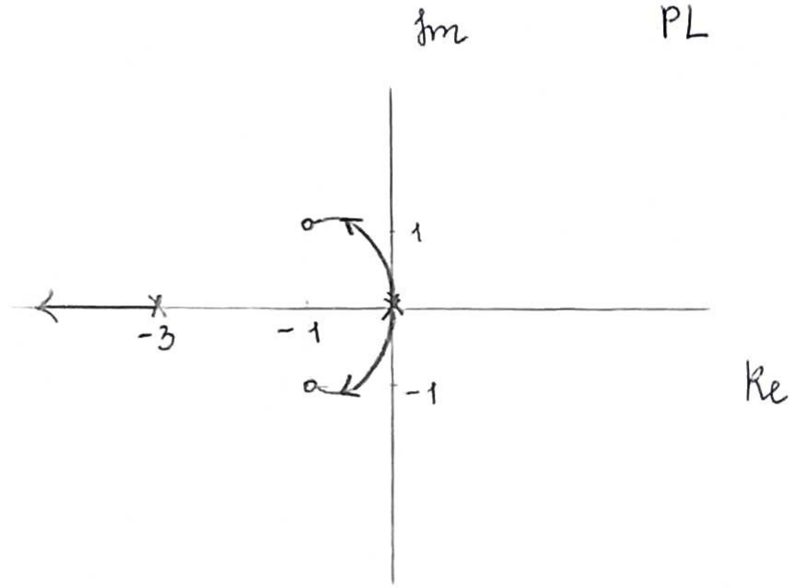
$$\psi_a = \begin{cases} \pi & PL \\ 0 & NL \end{cases}$$

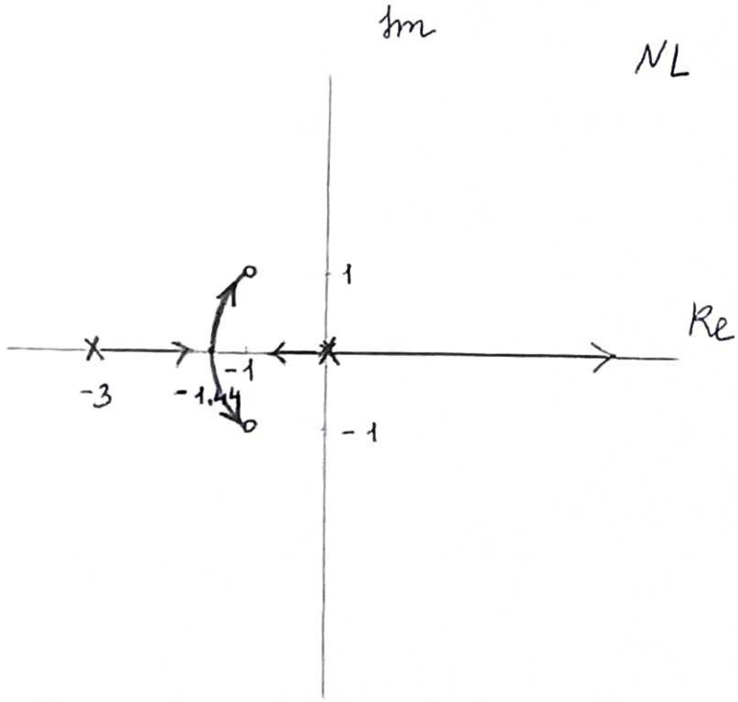
Real axis: $(-\infty, -3)$ $\quad 5 \quad \in PL$

$(-3, -1)$ $\quad 4 \quad \in NL$

$(-1, 0)$ $\quad 2 \quad \in NL$

$(0, +\infty)$ $\quad 0 \quad \in NL$





Break-in/away:

$$N(s)D'(s) - N'(s)D(s) = 0$$

$$N'(s) = 2s + 2$$

$$D'(s) = 3s^2 + 6s$$

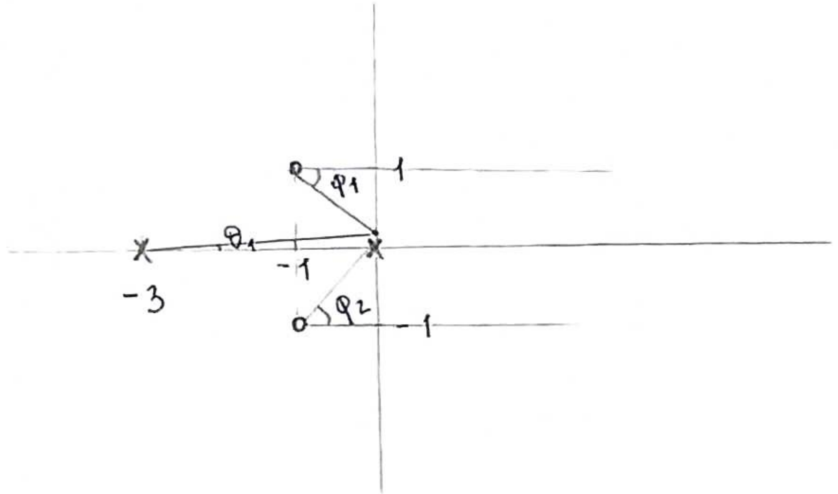
$$(s^2+2s+2)(3s^2+6s) - (2s+2)(s^3+3s^2) = 0$$

$$3s^4 + 12s^3 + 18s^2 + 12s - (2s^4 + 8s^3 + 6s^2) = 0$$

$$s^4 + 4s^3 + 12s^2 + 12s = 0$$

$$s(s^3 + 4s^2 + 12s + 12) = 0$$

$s = 0 \qquad s = -1.44424 \approx -1.44$



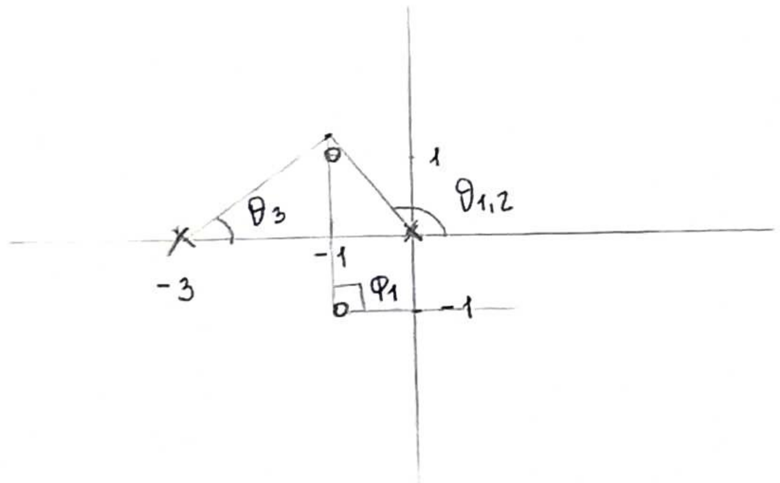
$$\varphi_1 = -\frac{\pi}{4}$$

$$\varphi_2 = \frac{\pi}{4}$$

$$\theta_1 = 0$$

$$q\,\alpha_{dep} = \begin{cases} \varphi_1 + \varphi_2 - \theta_1 + (2k+1)\pi & k = 0,1 \quad PL \\ \varphi_1 + \varphi_2 - \theta_1 + 2k\pi & k = 0,1 \quad NL \end{cases}$$

$$\alpha_{dep} = \begin{cases} \frac{\pi}{2},\ \frac{3\pi}{2} & PL \\ 0,\ \pi \end{cases}$$



$$\varphi_1 = 90°$$

$$\theta_1 = \theta_2 = 135°$$

$$\theta_3 = \tan^{-1}\left(\frac{1}{2}\right) = 26,565°$$

$$\beta_{arr} = \begin{cases} -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + (2k+1)\pi & k = 0 \quad PL \\ -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + 2k\pi & k = 0 \quad NL \end{cases}$$

$$\beta_{arr} = \begin{cases} 26,565° & PL \\ 206,565° & NL \end{cases}$$

$$D(s) + \rho N(s) = 0$$

$$s^2(s+3) + k(s^2+2s+2) = 0$$

$$s^3 + 3s^2 + ks^2 + 2ks + 2k = 0$$

$$s^3 + (3+k)s^2 + 2ks + 2k = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 2k \\ s^2 & 3+k & 2k \\ s^1 & \dfrac{2k(2+k)}{3+k} & \\ s^0 & 2k & \end{array}$$

$3+k > 0 \qquad k > -3$

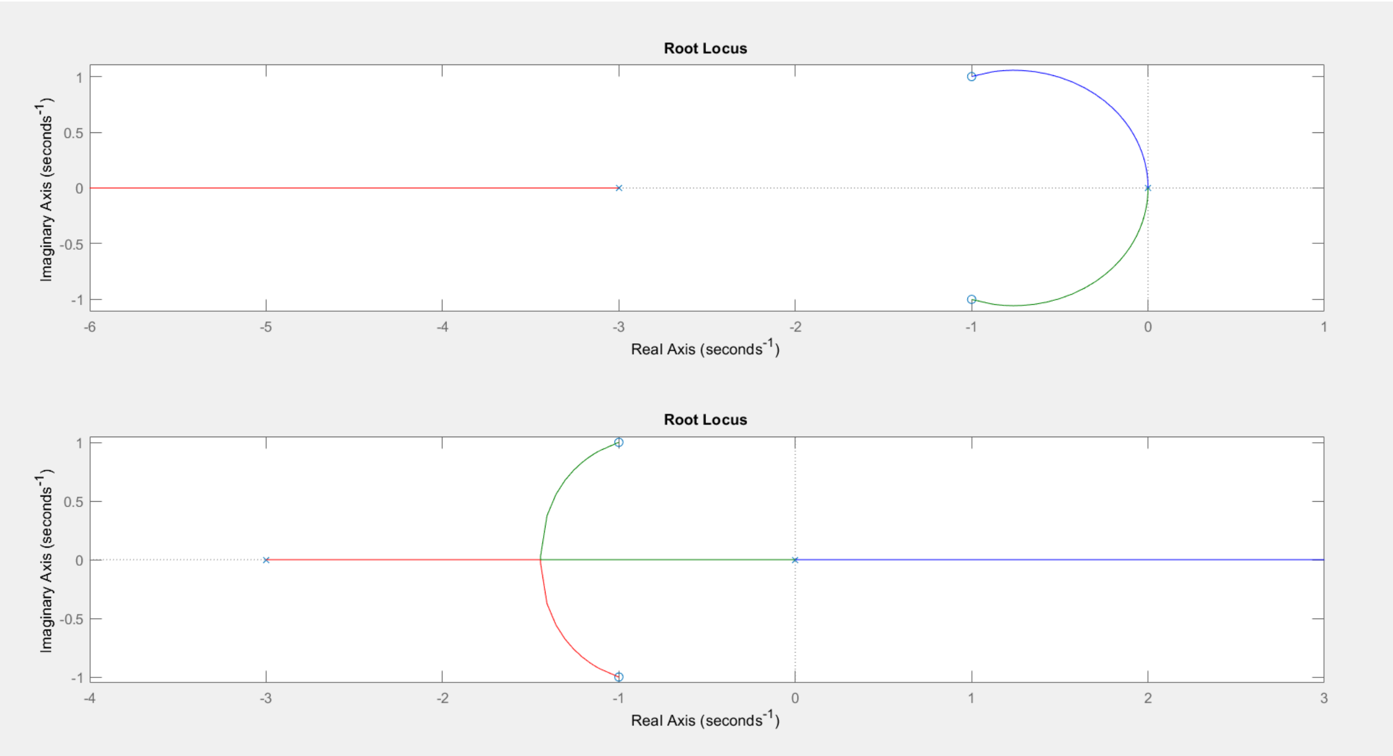
$2k > 0 \qquad k > 0$

$\dfrac{2k(2+k)}{3+k} > 0 \qquad k > -2$

$\Rightarrow$ The closed loop system is stable for $k > 0$.

MATLAB code:

```
sys1 = tf([1 2 2], [1 3 0 0]);
figure
subplot(2, 1, 1)
rlocus(sys1)
subplot(2, 1, 2)
rlocus(-sys1)
```



The obtained sketch is pretty similar to the MATLAB plot.

2. $G_{p_2}(s) = \dfrac{s^2+4}{(s^2+4s+8)(s-2)}$

$$L(s) = G_c(s) \cdot G_{p_2}(s) = k \cdot \frac{s^2+4}{(s^2+4s+8)(s-2)} = \rho \frac{N(s)}{D(s)}$$

$\rho = k$ $\qquad\qquad\qquad\qquad$ $n = 3$

$N(s) = s^2+4$ $\qquad \Rightarrow \qquad$ $m = 2$

$r = n - m = 1 \Rightarrow$ 1 asymptote

$D(s) = s^3 + 2s^2 - 16$

Zeros $\quad s_1 = 2j \qquad s_2 = -2j$

Poles: $\quad s_1 = -2+2j \qquad s_2 = -2-2j \qquad s_3 = 2$

$$\sigma_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{\overset{0}{(\cancel{2j} - \cancel{2j})} - (+2+\cancel{2j}+2-\cancel{2j}+(-2))}{1} = -2$$

$$r\varphi_a = \begin{cases} (2h+1)\pi & h=0 \quad PL \\ 2h\pi & h=0 \quad NL \end{cases}$$

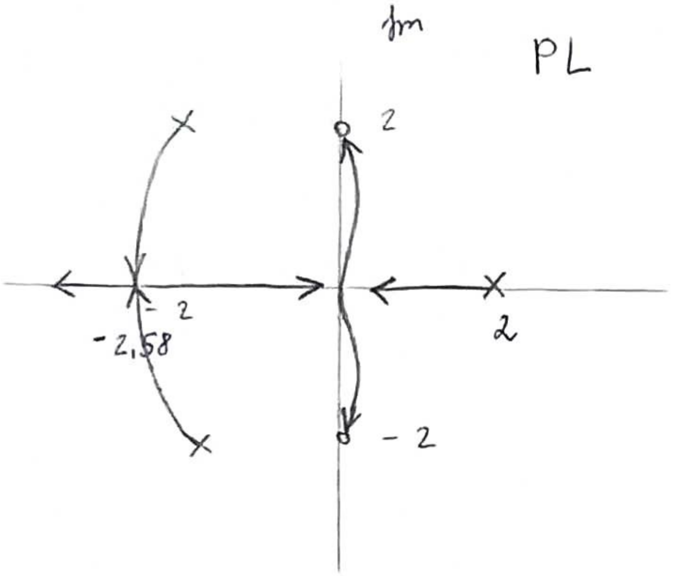
$$\varphi_a = \begin{cases} \pi & PL \\ 0 & NL \end{cases}$$

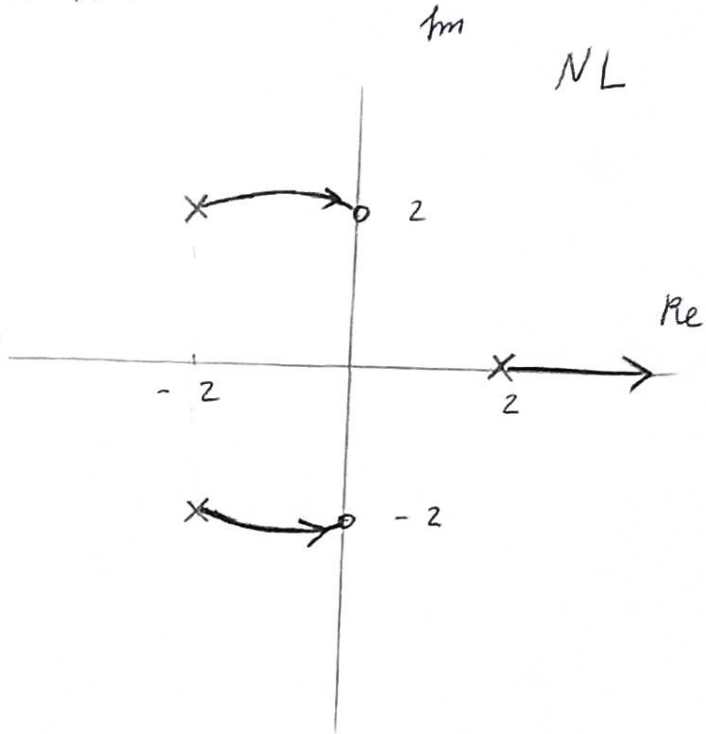
Real axis: $(-\infty, -2)$ 5 $\in PL$

$(-2, 0)$ 3 $\in PL$

$(0, 2)$ 1 $\in PL$

$(2, +\infty)$ 0 $\in NL$





Break-in/away

$N(s)D'(s) - N'(s)D(s) = 0$

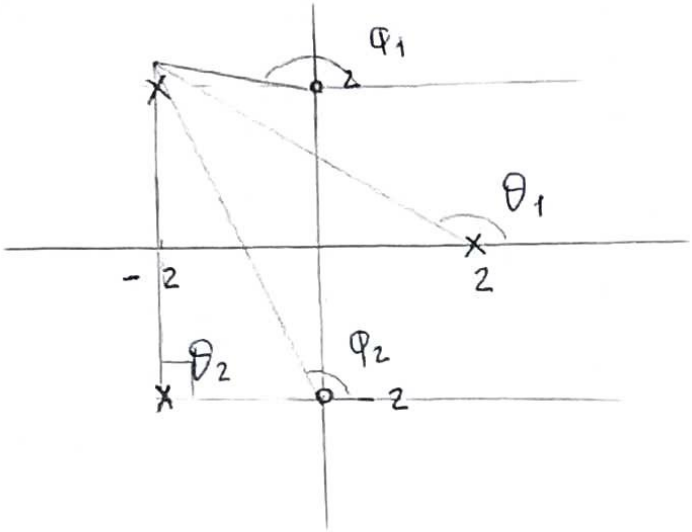
$(s^2+4)\cdot(3s^2+4s) - (2s)(s^3+2s^2-16) = 0$

$3s^4 + \cancel{4s^3} + 12s^2 + 16s - 2s^4 - \cancel{4s^3} + 32s = 0$

$s^4 + 12s^2 + 48s = 0$

$s(s^3 + 12s + 48) = 0$

$s = 0$ $s = -2.57582 \approx -2.58$



$\varphi_1 = 180^\circ$

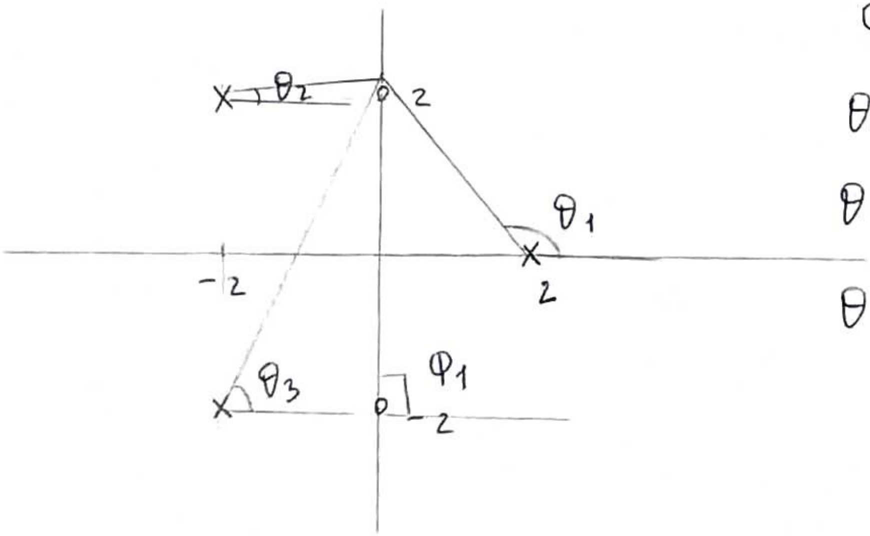
$\varphi_2 = 180^\circ - \tan^{-1}\left(\frac{4}{2}\right)$

$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{2}{4}\right)$

$\theta_2 = 90^\circ$

$$\alpha_{dep} = \begin{cases} \varphi_1 + \varphi_2 - \theta_1 - \theta_2 + (2k+1)\pi & k=0 \quad PL \\ \varphi_1 + \varphi_2 - \theta_1 - \theta_2 + 2k\pi & k=0 \quad NL \end{cases}$$

$$\alpha_{dep} = \begin{cases} 233^\circ & PL \\ 53^\circ & NL \end{cases}$$



$\varphi_1 = 90^\circ$

$\theta_1 = 180^\circ - 45^\circ = 135^\circ$

$\theta_2 = 0^\circ$

$\theta_3 = \tan^{-1}\left(\frac{4}{2}\right) = 63.435^\circ$

$$\beta_{arr} = \begin{cases} -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + (2k+\pi) & k=0 \quad PL \\ -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + 2k\pi & k=0 \quad NL \end{cases}$$

$$\beta_{arr} = \begin{cases} 288.435^\circ & PL \\ 108.435^\circ & NL \end{cases}$$

$D(s) + P\,N(s) = 0$

$s^3 + 2s^2 - 16 + k(s^2+4) = 0$

$s^3 + (2+k)s^2 + (4k-16) = 0$

s^3	1	0
s^2	$2+k$	$4k-16$
s^1	$\frac{4(4-k)}{2+k}$	
s^0	$4k-16$	

$2+k > 0 \qquad k > -2$

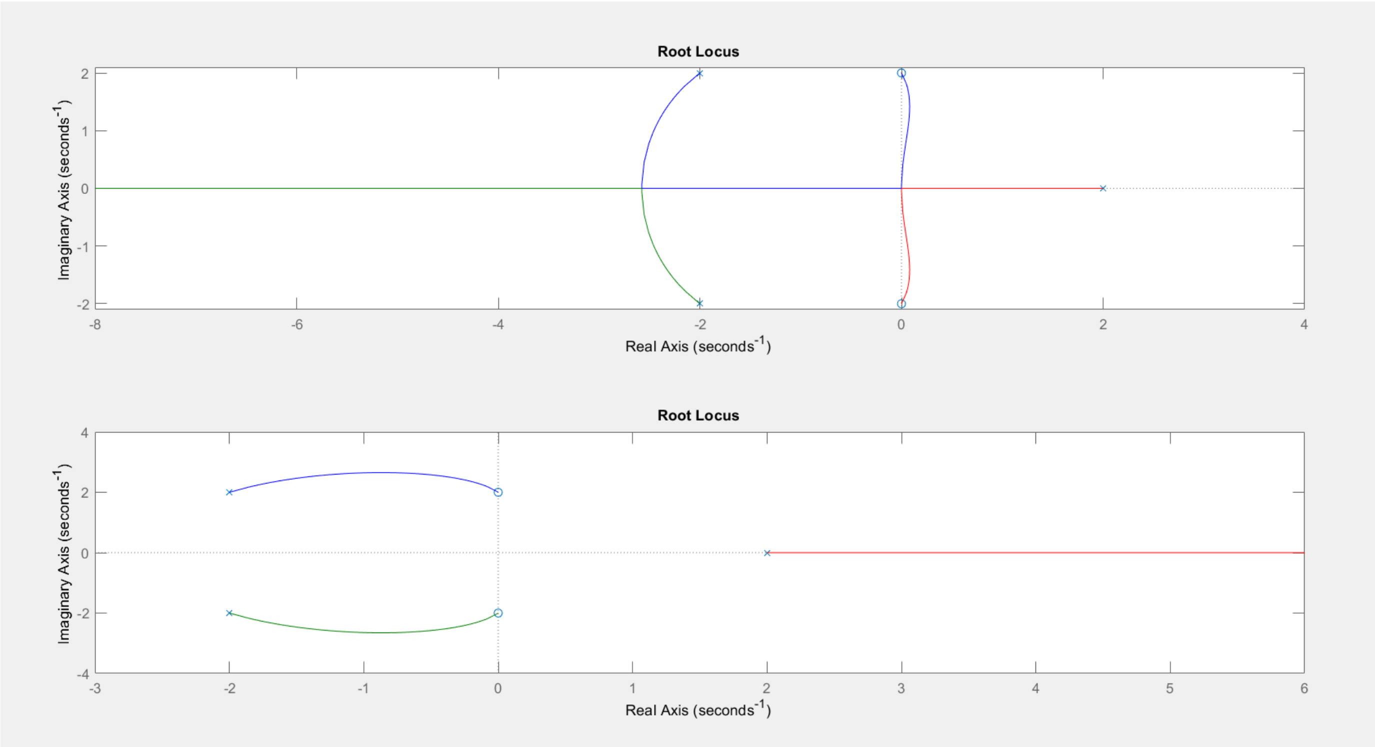
$4k-16 > 0 \qquad k > 4$

$\frac{4(4-k)}{2+k} > 0 \qquad k < 4$

$\Rightarrow$ The closed system is unstable for any k.

MATLAB code:

```
sys2 = tf([1 0 4], [1 2 0 -16]);
figure
subplot(2, 1, 1)
rlocus(sys2)
subplot(2, 1, 2)
rlocus(-sys2)
```



The obtained sketch is pretty similar to the MATLAB plot.

3. $G_{p3}(s) = \dfrac{s^2+4}{s^2+25}$

$$L(s) = G_c(s) \cdot G_{p3}(s) = k \cdot \frac{s^2+4}{s^2+25} = \rho \frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s^2+4$ $\Rightarrow$ $n = 2$

$D(s) = s^2+25$

$m = 2$

$r = n - m = 0$ asymptotes

Zeros: $s^2 + 4 = 0$

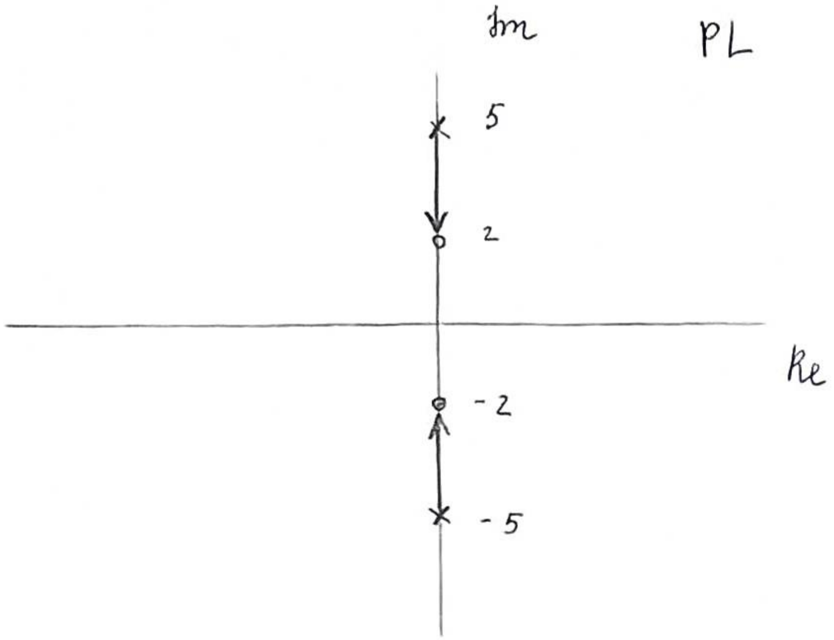
$s_1 = 2j \qquad s_2 = -2j$

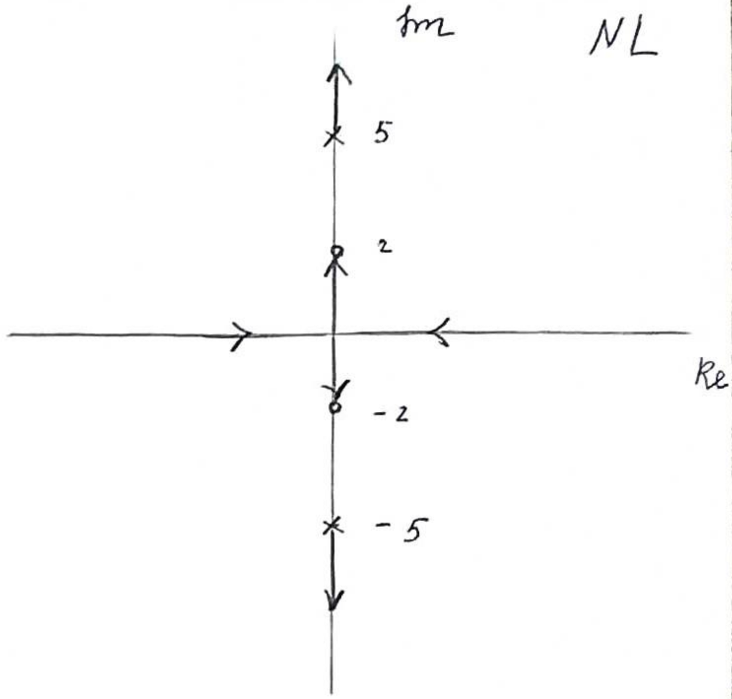
Poles: $s^2 + 25 = 0$

$s_1 = 5j \qquad s_2 = -5j$

Real axis: $(-\infty, 0)$ 4 poles/zeros $\in NL$

$(0, +\infty)$ 0 poles/zeros $\in NL$

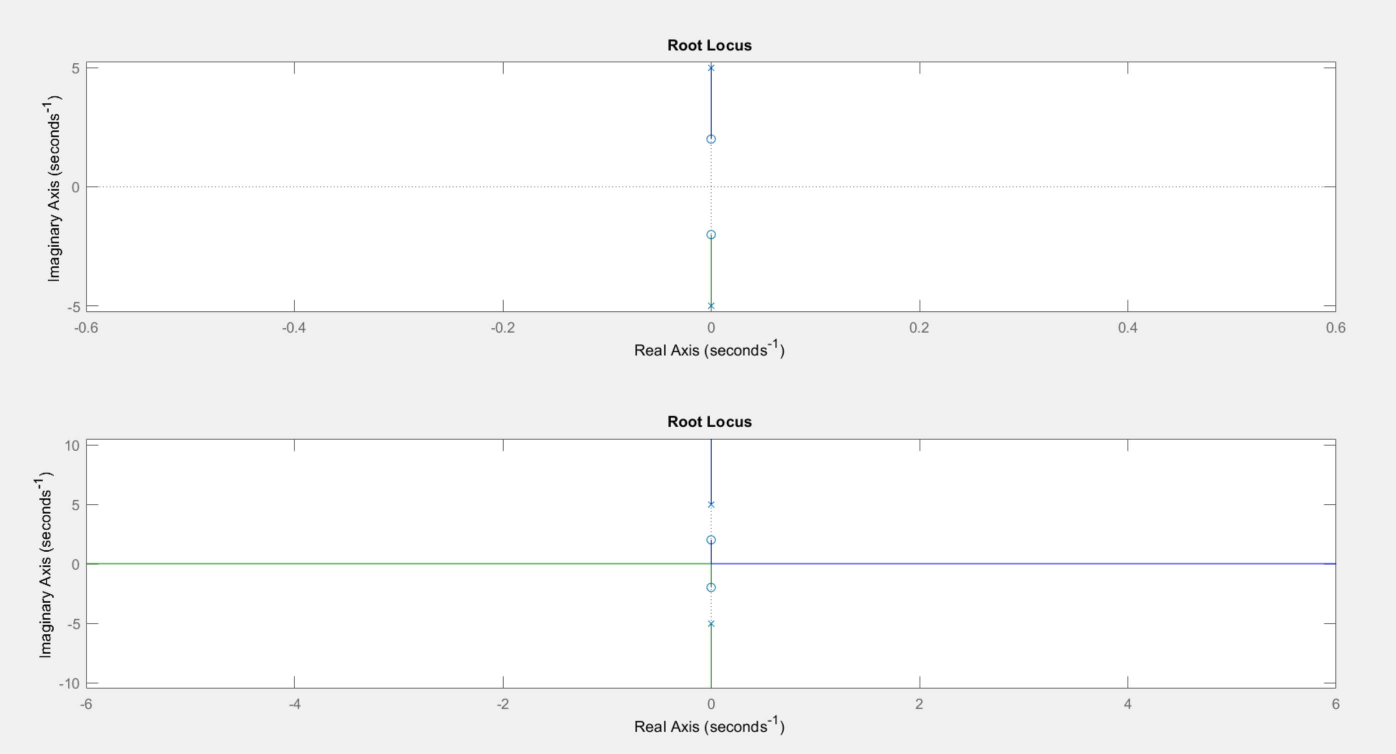




As PL is on the imaginary axis, the closed loop system is unstable for any K.

MATLAB code:

```
sys3 = tf([1 0 4], [1 0 25]);
figure
subplot(2, 1, 1)
rlocus(sys3)
subplot(2, 1, 2)
rlocus(-sys3)
```



The obtained sketch is pretty similar to the MATLAB plot.

4. $G_{p_4}(s) = \dfrac{s^2+2s+4}{s^2-2s+1}$

$$L(s) = G_c(s)\,G_{p_4}(s) = k\,\frac{s^2+2s+4}{s^2-2s+1} = \rho\,\frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s^2+2s+4$

$D(s) = s^2-2s+1$

$\Rightarrow$ $n = 2$

$m = 2$

$r = n - m = 0 \Rightarrow$ 0 asymptotes

Zeros: $s^2+2s+4 = 0$

$s_1 = -1+\sqrt{3}j \qquad s_2 = -1-\sqrt{3}j$

Poles: $s^2-2s+1 = 0$

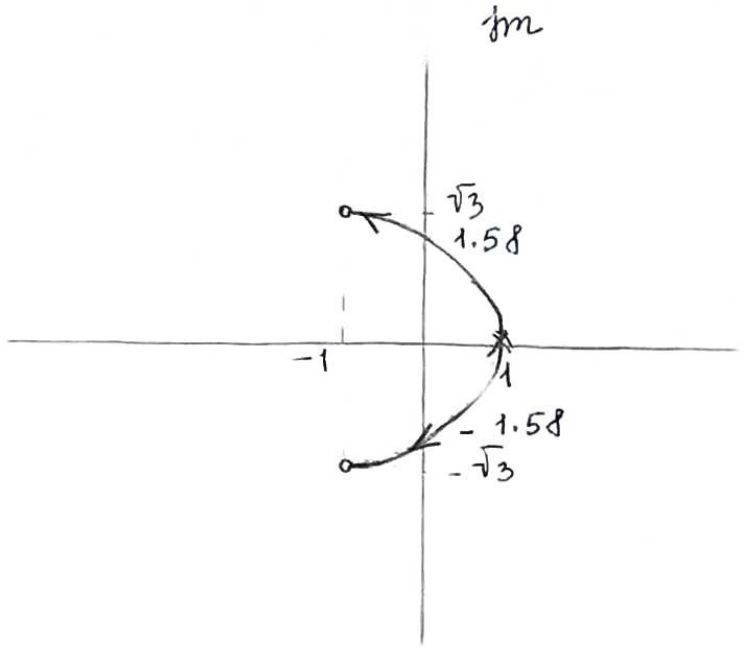
$(s-1)^2 = 0$

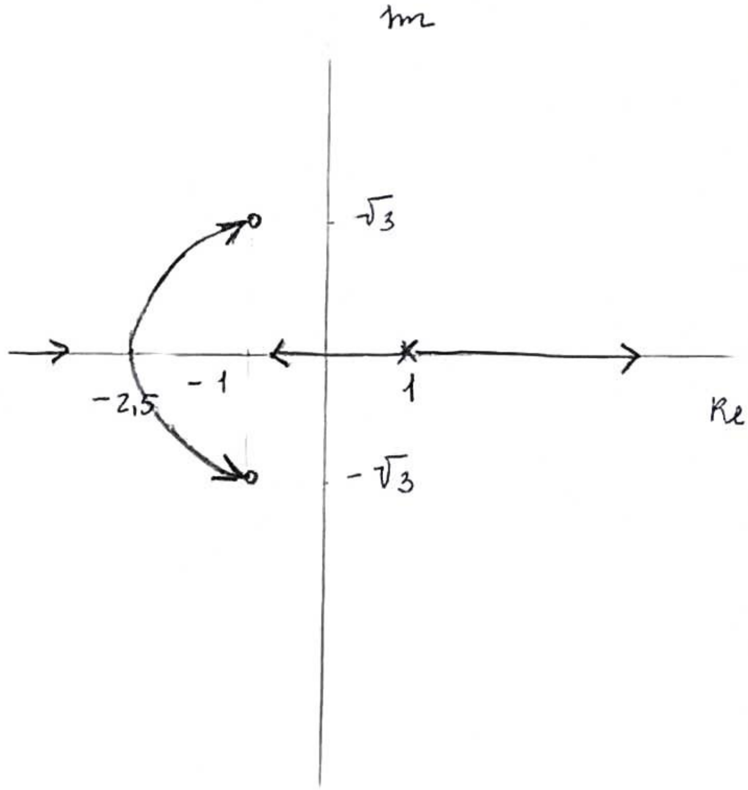
$s_{1,2} = 1$

Real axis: $(-\infty, -1)$ 4 poles/zeros $\in NL$

$(-1, 1)$ 2 poles $\in NL$

$(1, +\infty)$ 0 poles/zeros $\in NL$





Break-in/away.

$$N(s)\,D'(s) - N'(s)\,D(s) = 0$$

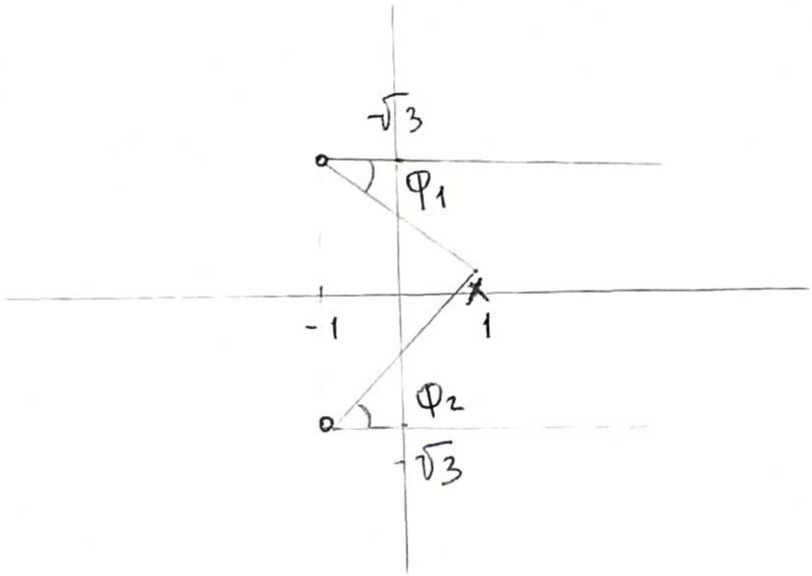
$$(s^2+2s+4)\cdot(2s-2) - (2s+2)(s^2-2s+1) = 0$$

$$2(s-1)(s^2+2s+4) - 2(s+1)(s-1)^2 = 0$$

$$2(s-1)\left(\cancel{s^2}+2s+4-\cancel{s^2}+1\right) = 0$$

$s_1 = 1$ $\qquad$ $s_2 = -2{,}5$

$\varphi_1 = -\varphi_2$



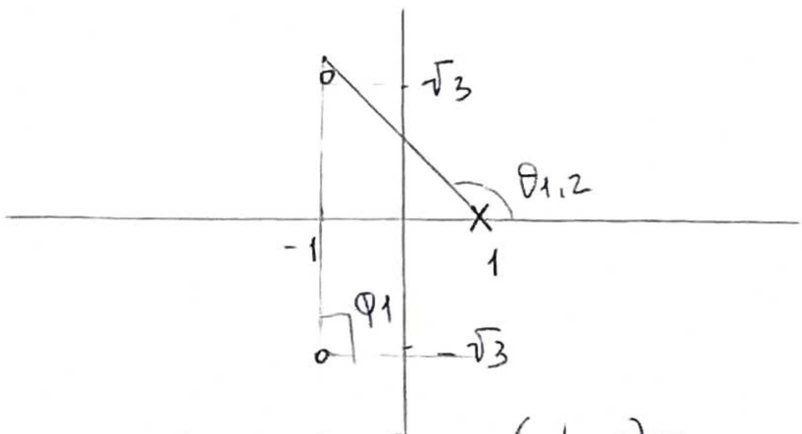
$$q \alpha_{dep} = \begin{cases} \varphi_1 + \varphi_2 + (2k+1)\pi & k = 0,1 \quad PL \\ \varphi_1 + \varphi_2 + 2k\pi & k = 0,1 \quad NL \end{cases}$$

$\overset{\shortparallel}{2}$

$$\alpha_{dep} = \begin{cases} \frac{\pi}{2}, \ \frac{3\pi}{2} \\ 0, \ \pi \end{cases}$$

$\varphi_1 = 90°$

$\theta_1 = \theta_2 = 180° - \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = 139°$



$$\beta_{arr} = \begin{cases} -\varphi_1 + \theta_1 + \theta_2 + (2k+1)\pi & k = 0 \quad PL \\ -\varphi_1 + \theta_1 + \theta_2 + (2k+1)\pi & k = 0 \quad NL \end{cases}$$

$$\beta_{arr} = \begin{cases} 8° & PL \\ 188° & NL \end{cases}$$

$D(s) + \rho N(s) = 0$

$s^2 - 2s + 1 + k\,(s^2 + 2s + 4) = 0$

$(k+1)s^2 + (2k-2)s + (4k+1) = 0$

$$\begin{array}{c|cc} s^2 & k+1 & 4k+1 \\ s^1 & 2k-2 & 0 \\ s^0 & 4k+1 & \end{array}$$

$k+1 > 0 \qquad k > -1$

$2k-2 > 0 \qquad k > 1$

$4k+1 > 0 \qquad k > -\frac{1}{4}$

if $k = 1$

$2s^2 + 5 = 0$

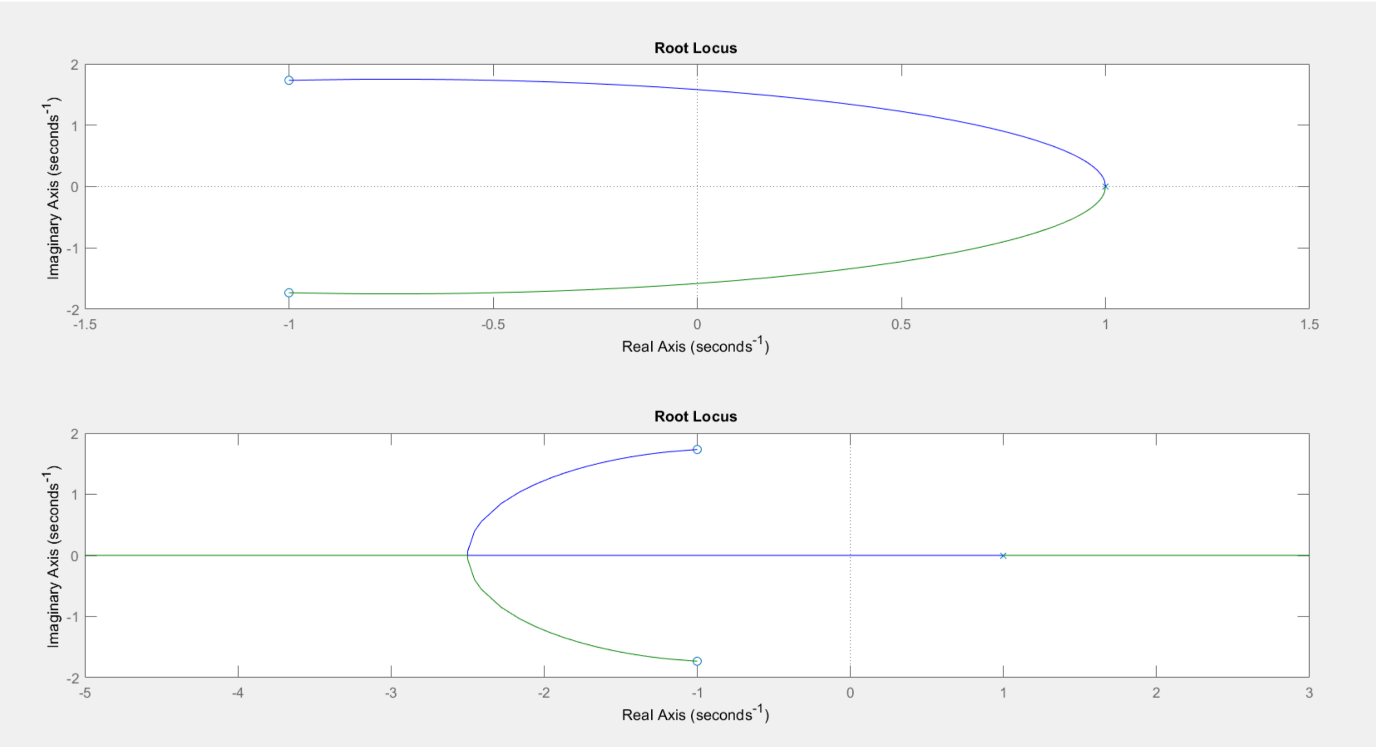
$s^2 = -\frac{5}{2}$

$s = \pm 1.58$

$\Rightarrow$ The closed loop system is stable for $k > 1$

MATLAB code:

```
sys4 = tf([1 2 4], [1 -2 1]);
figure
subplot(2, 1, 1)
rlocus(sys4)
subplot(2, 1, 2)
rlocus(-sys4)
```



The obtained sketch is pretty similar to the MATLAB plot.

5. $G_{p5}(s) = \dfrac{s^2 - 2s + 4}{(s-1)(s^2+2s+1)}$

$$L(s) = G_c(s)\, G_{p5}(s) = k \cdot \frac{s^2-2s+4}{(s-1)(s^2+2s+1)} = \rho \frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s^2 - 2s + 4$ $\Rightarrow$ $n = 3$, $m = 2$

$D(s) = s^3 + s^2 - s - 1$

$r = 3 - 2 = 1 \Rightarrow$ 1 asymptote

Zeros: $s^2 - 2s + 4 = 0$

$s_1 = 1 + \sqrt{3}j \qquad s_2 = 1 - \sqrt{3}j$

Poles: $(s-1)(s^2+2s+1) = 0$

$(s-1)(s+1)^2 = 0$

$s_1 = 1 \qquad s_{2,3} = -1$

$$x_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{(-1-\cancel{\sqrt{3}j} - 1 + \cancel{\sqrt{3}j}) - (\cancel{-1} + \cancel{1} + 1)}{1} = -3$$

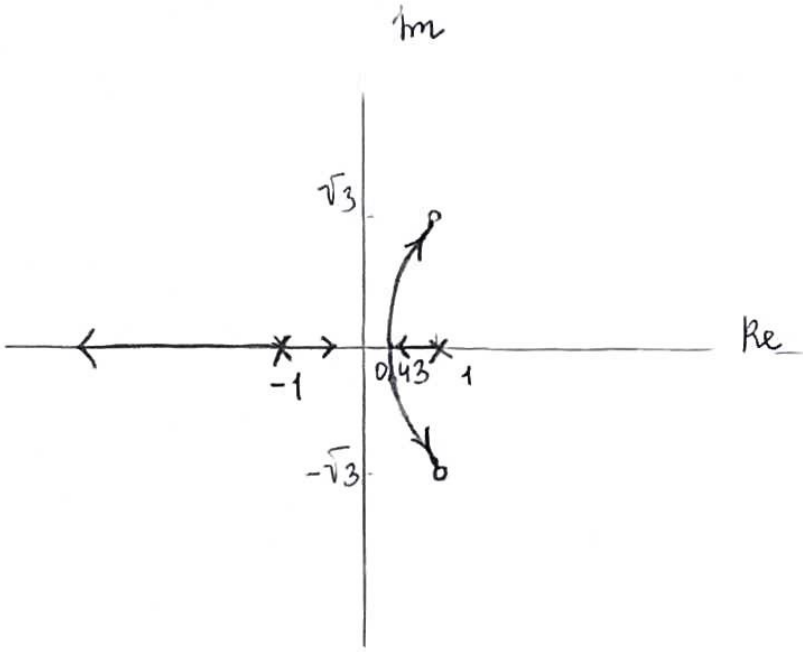
$$r\psi_a = \begin{cases} (2h+1)\pi & h = 0 \quad PL \\ 2h\pi & h = 0 \quad NL \end{cases}$$

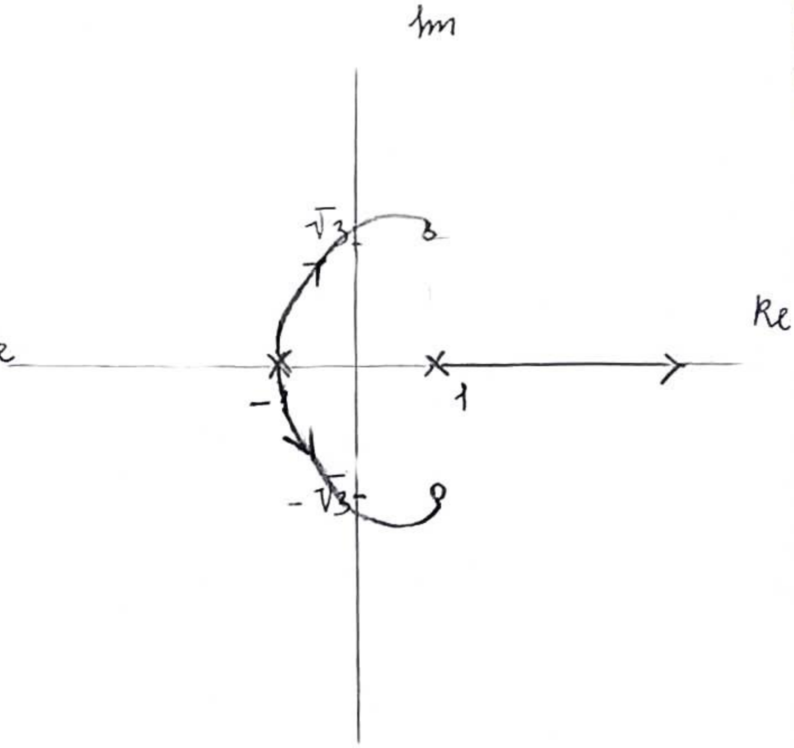
$$\psi_a = \begin{cases} \pi & PL \\ 0 & NL \end{cases}$$

Real axis, $(-\infty, -1)$ 5 poles/zeros $\in PL$

$(-1, 1)$ 3 poles/zeros $\in PL$

$(1, +\infty)$ 0 poles/zeros $\in NL$





Break-in/away:

$$N(s)\,D'(s) - N'(s)\,D(s) = 0$$

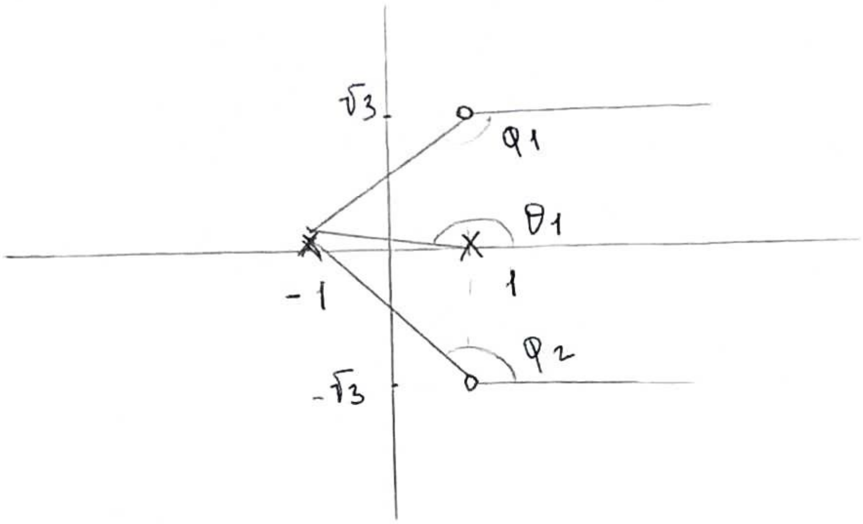
$$(s^2-2s+4)\,(3s^2+2s-1) - (2s-2)\cdot(s-1)(s+1)^2 = 0$$

$$(s^2-2s+4)\,(3s-1)(s+1) - 2\,(s-1)^2(s+1)^2 = 0$$

$$(s+1)\left[(s^2-2s+4)\,(3s-1) - 2\,(s^2-2s+1)(s+1)\right] = 0$$

$$(s+1)\,(s^3-5s^2+16s-6) = 0$$

$s = -1$ $\qquad$ $s = 0.42715 \approx 0.43$



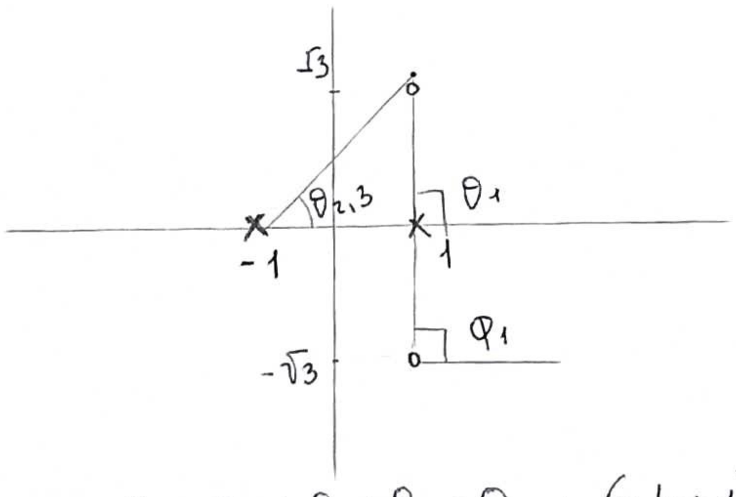
$\varphi_1 = -\varphi_2$

$\theta_1 = \pi$

$$q\,\alpha_{dep} = \begin{cases} \varphi_1 + \varphi_2 - \theta_1 + (2k+1)\pi & k = 0,1 \quad PL \\ \varphi_1 + \varphi_2 - \theta_1 + 2k\pi & k = 0,1 \quad NL \end{cases}$$

$2''$

$$\alpha_{dep} = \begin{cases} 0,\ \pi & PL \\ -\frac{\pi}{2},\ \frac{\pi}{2} & NL \end{cases}$$



$\varphi_1 = 90°$

$\theta_1 = 90°$

$\theta_2 = \theta_3 = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = 41°$

$$\beta_{arr} = \begin{cases} -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + (2k+1)\pi & k = 0 \quad PL \\ -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + 2k\pi & k = 0 \quad NL \end{cases}$$

$$\beta_{arr} = \begin{cases} 262° & PL \\ 82° & NL \end{cases}$$

$D(s) + P N(s) = 0$

$s^3 + s^2 - s - 1 + k(s^2 - 2s + 4) = 0$

$s^3 + (k+1)s^2 + (-2k-1)s + (4k-1) = 0$

s^3	1	$-2k-1$
s^2	$k+1$	$4k-1$
s^1	$-\frac{(2k^2+7k)}{k+1}$	
s^0	$4k-1$	

$k+1 > 0 \qquad k > -1$

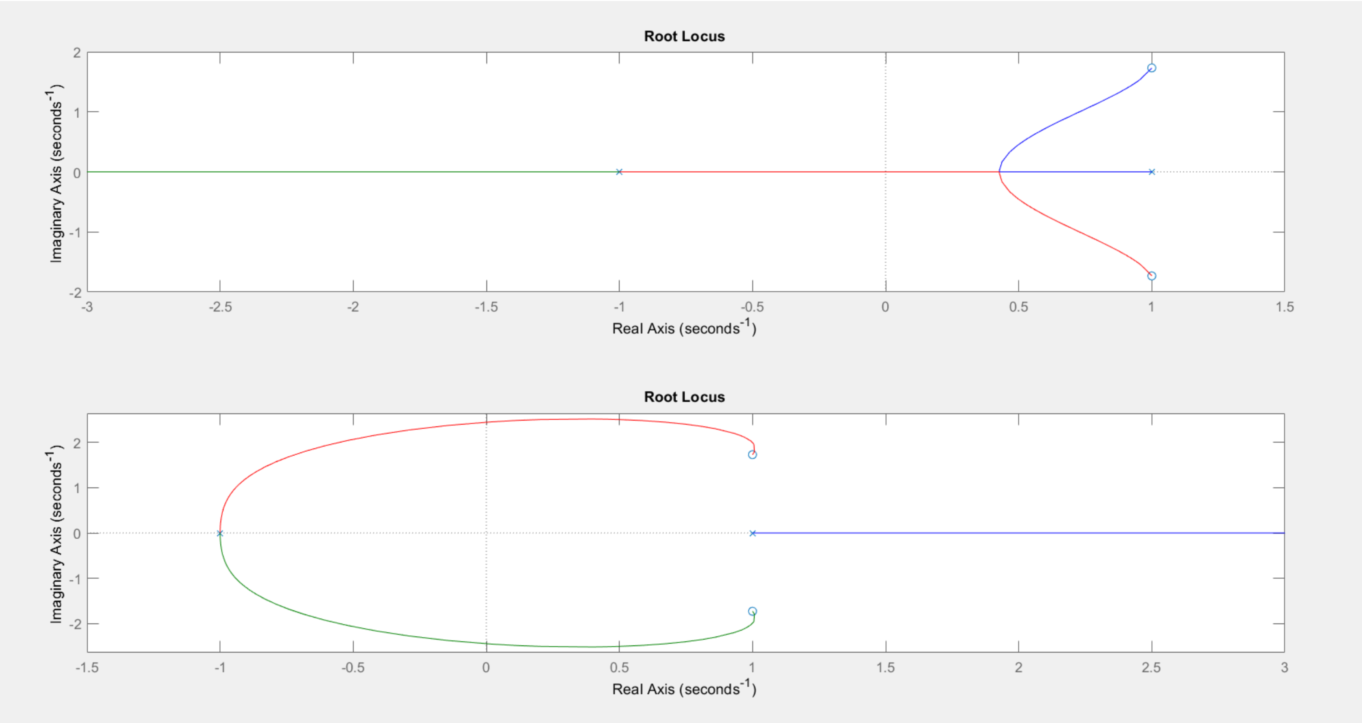
$4k-1 > 0 \qquad k > \frac{1}{4}$

$-(2k^2+7k) > 0 \qquad -\frac{7}{2} < k < 0$

$\Rightarrow$ The closed loop system is unstable for any k.

MATLAB code:

```
sys5 = tf([1 -2 4], [1 1 -1 -1]);
figure
subplot(2, 1, 1)
rlocus(sys5)
subplot(2, 1, 2)
rlocus(-sys5)
```



The obtained sketch is pretty similar to the MATLAB plot.

6. $$G_{p6}(s) = \frac{s^2+2s+2}{(s+3)^4}$$

$$L(s) = G_c(s)\cdot G_{p6}(s) = k \cdot \frac{s^2+2s+2}{(s+3)^4} = \rho\,\frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s^2+2s+2$ $\Rightarrow$ $n = 4$, $m = 2$

$D(s) = (s+3)^4$

$r = n-m = 2 \Rightarrow 2$ asymptotes

Zeros: $s^2+2s+2=0$

$s_1 = -1+j \qquad s_2 = -1-j$

Poles: $(s+3)^4 = 0$

$S_1 = S_2 = S_4 = S_4 = -3$

$$x_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{(1-\cancel{j}+1+\cancel{j}) - (3+3+3+3)}{2} = -5$$

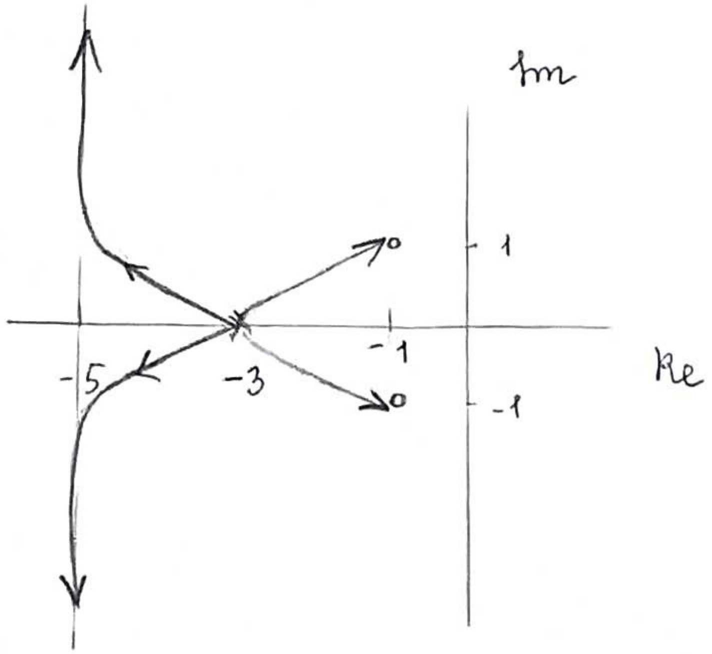
$$r\psi_a = \begin{cases} (2h+1)\pi & h=0,1 \quad PL \\ 2h\pi & h=0,1 \quad NL \end{cases}$$

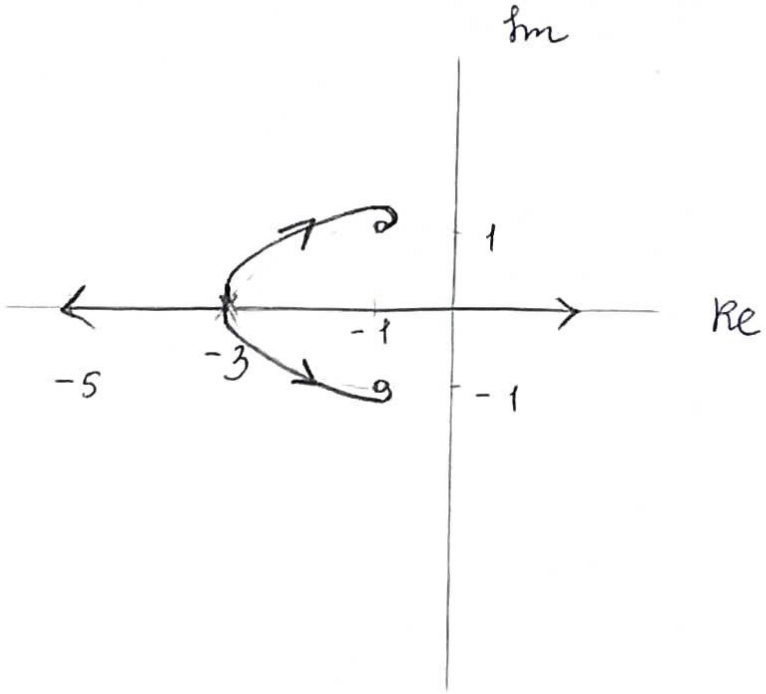
$$\psi_a = \begin{cases} \frac{\pi}{2},\ \frac{3\pi}{2} & PL \\ 0,\ \pi & NL \end{cases}$$

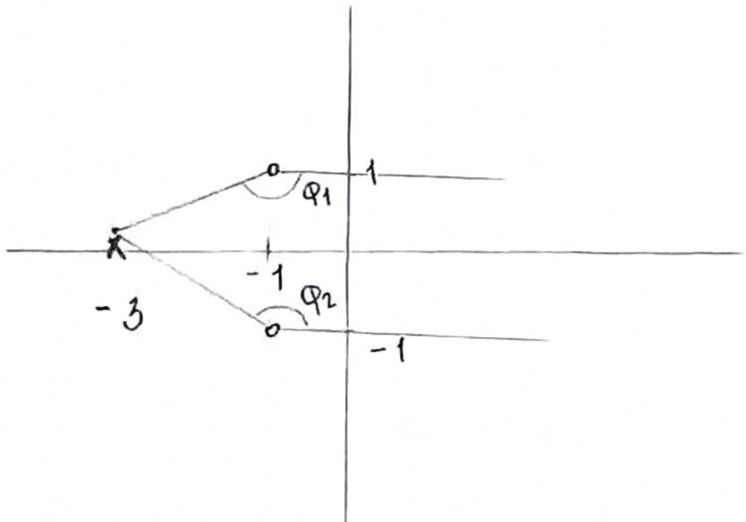
Real axis: $(-\infty, -3)$ 6 poles/zeros $\in NL$

$(-3, -1)$ 2 zeros $\in NL$

$(-1, +\infty)$ 0 poles/zeros $\in NL$



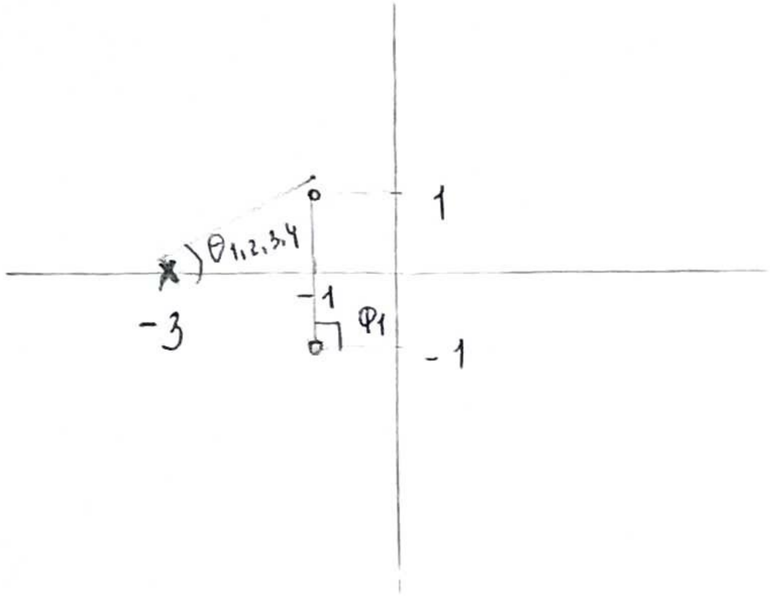




$\varphi_1 = -\varphi_2$

$$4\alpha_{dep} = \begin{cases} \varphi_1 + \varphi_2 + (2k+1)\pi & k = 0, 1, 2, 3 \quad PL \\ \varphi_1 + \varphi_2 + 2k\pi & k = 0, 1, 2, 3 \quad NL \end{cases}$$

$$\alpha_{dep} = \begin{cases} \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} & PL \\ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \end{cases}$$



$\varphi_1 = 90°$

$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \tan^{-1}\left(\frac{1}{2}\right) =$

$26.565°$

$$\beta_{arr} = \begin{cases} -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + (2k+1)\pi & k=0 \quad PL \\ -\varphi_1 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + 2k\pi & k=0 \quad NL \end{cases}$$

$$\beta_{arr} = \begin{cases} 196.5° & PL \\ 16.5° & NL \end{cases}$$

$$\left| \frac{N(s)}{D(s)} \right|_{s=0} = \left| -\frac{1}{\rho} \right|$$

$$\left| \frac{2}{81} \right| = \left| -\frac{1}{\rho} \right|$$

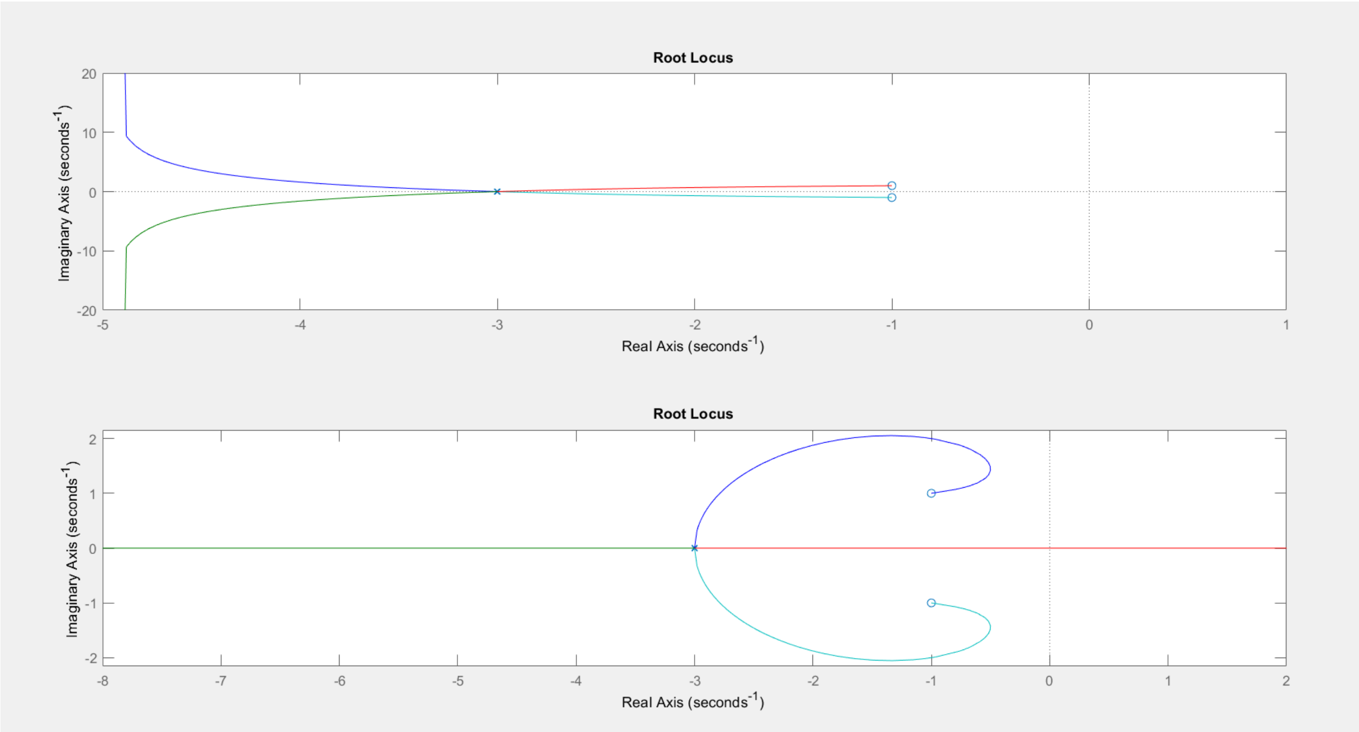
$|\rho| = 40.5 \qquad \rho = -40.5 \quad \Rightarrow k = -40.5$

$\Rightarrow$ The closed loop system is stable for

$k > -40.5$

MATLAB code:

```
sys6 = tf([1 2 2], [1 12 54 108 81]);
figure
subplot(2, 1, 1)
rlocus(sys6)
subplot(2, 1, 2)
rlocus(-sys6)
```



The obtained sketch is pretty similar to the MATLAB plot.

7. $G_{p7}(s) = \dfrac{1}{(s^2+2s+2)^2}$

$$L(s) = G_c(s) \cdot G_{p7}(s) = k \cdot \frac{1}{(s^2+2s+2)^2} = \rho \cdot \frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = 1$

$D(s) = (s^2+2s+2)^2$

$n = 4$

$m = 0$

$r = n - m = 4 \Rightarrow$ 4 asymptotes

No zeros

Poles: $(s^2+2s+2)(s^2+2s+2) = 0$

$$s_{1,2} = -1+j \qquad s_{3,4} = -1-j$$

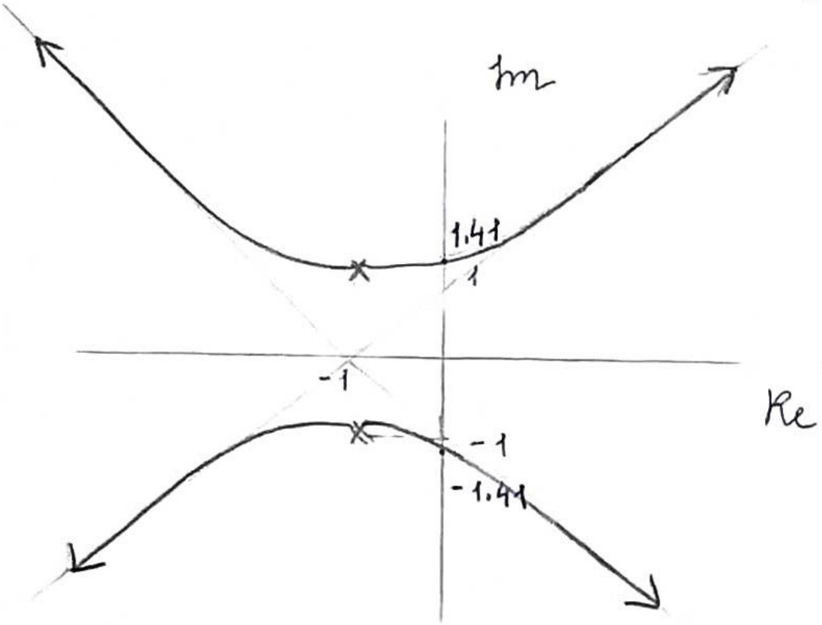
$$x_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{0-(1+1+1+1)}{4} = -1$$

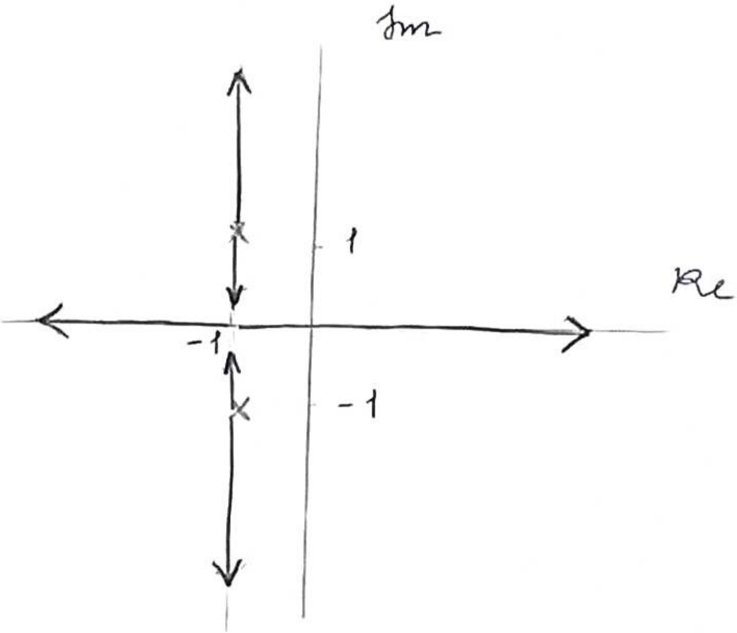
$$r\,\psi_a = \begin{cases} (2h+1)\pi & h = 0,1,2,3 \quad PL \\ 2h\pi & h = 0,1,2,3 \quad NL \end{cases}$$

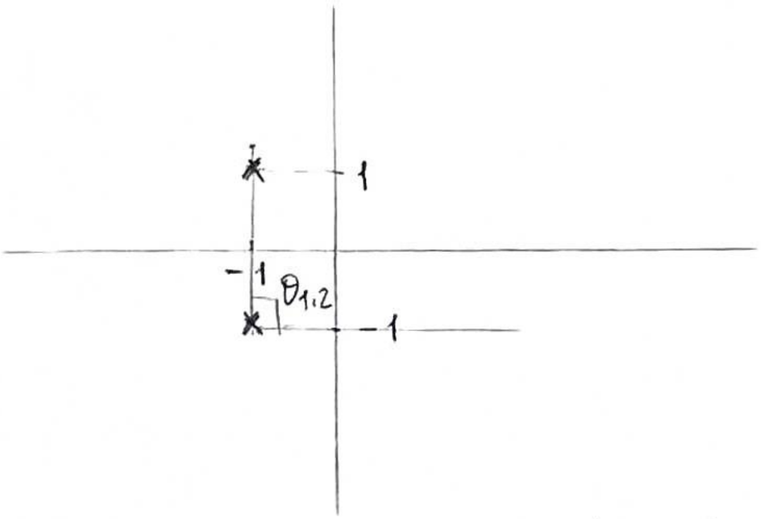
$$\psi_a = \begin{cases} \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} & PL \\ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \end{cases}$$

Real axis $(-\infty, -1)$ 4 poles $\in NL$

$(-1, +\infty)$ 0 poles/zeros $\in NL$







$\theta_1 = \theta_2 = 90°$

$q = 2$

$$q\,\alpha_{dep} = \begin{cases} -90° - 90° + (2k+1)\pi & k = 0,1 \quad PL \\ -90° - 90° + 2k\pi & k = 0,1 \quad NL \end{cases}$$

$$\alpha_{dep} = \begin{cases} 0,\ \pi & PL \\ -\frac{\pi}{2},\ \frac{\pi}{2} & \end{cases}$$

$$D(s) + P\,N(s) = 0$$

$$s^4 + 4s^3 + 8s^2 + 8s + (4 + k) = 0$$

s^4	1	8	$4+k$
s^3	4	8	
s^2	6	$4+k$	
s^1	$\frac{32-4k}{6}$		
s^0	$4+k$		

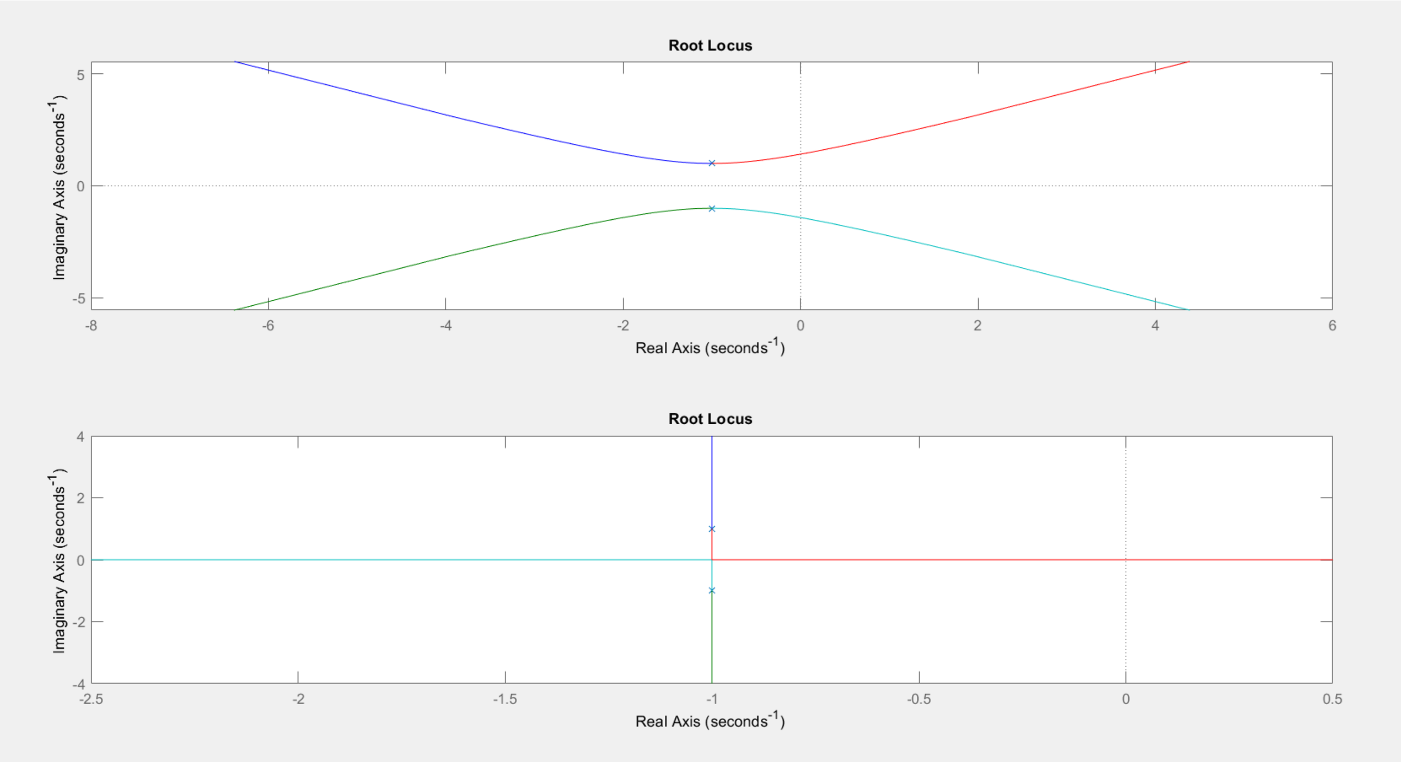
$\frac{32-4k}{6} > 0$ $\quad$ $k < 8$ $\quad$ if $k = 8$ $\quad$ $s = \pm 1{,}41\,i$

$4+k > 0$ $\quad$ $k > -4$

$\Rightarrow$ The closed loop system is stable for $-4 < k < 8$

MATLAB code:

```
sys7 = tf([0 1], [1 4 8 8 4]);
figure
subplot(2, 1, 1)
rlocus(sys7)
subplot(2, 1, 2)
rlocus(-sys7)
```



The obtained sketch is pretty similar to the MATLAB plot.

8. $G_{p8}(s) = \dfrac{(s+6)}{(s^2+2s+2)^2}$

$$L(s) = G_c(s) \cdot G_{p8}(s) = k \cdot \frac{(s+6)}{(s^2+2s+2)^2} = \rho \frac{N(s)}{D(s)}$$

$\rho = k$

$N(s) = s+6$ $\Rightarrow$ $n = 4$, $m = 1$

$D(s) = (s^2+2s+2)^2$

$r = n - m = 3 \Rightarrow$ 3 asymptotes

Zeros: $s + 6 = 0$

$s = -6$

Poles: $(s^2+2s+2)(s^2+2s+2) = 0$

$s_{1,2} = -1+j \qquad s_{3,4} = -1-j$

$$x_a = \frac{\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i}{r} = \frac{6 - (1+1+1+1)}{3} = \frac{2}{3} \approx 0.67$$

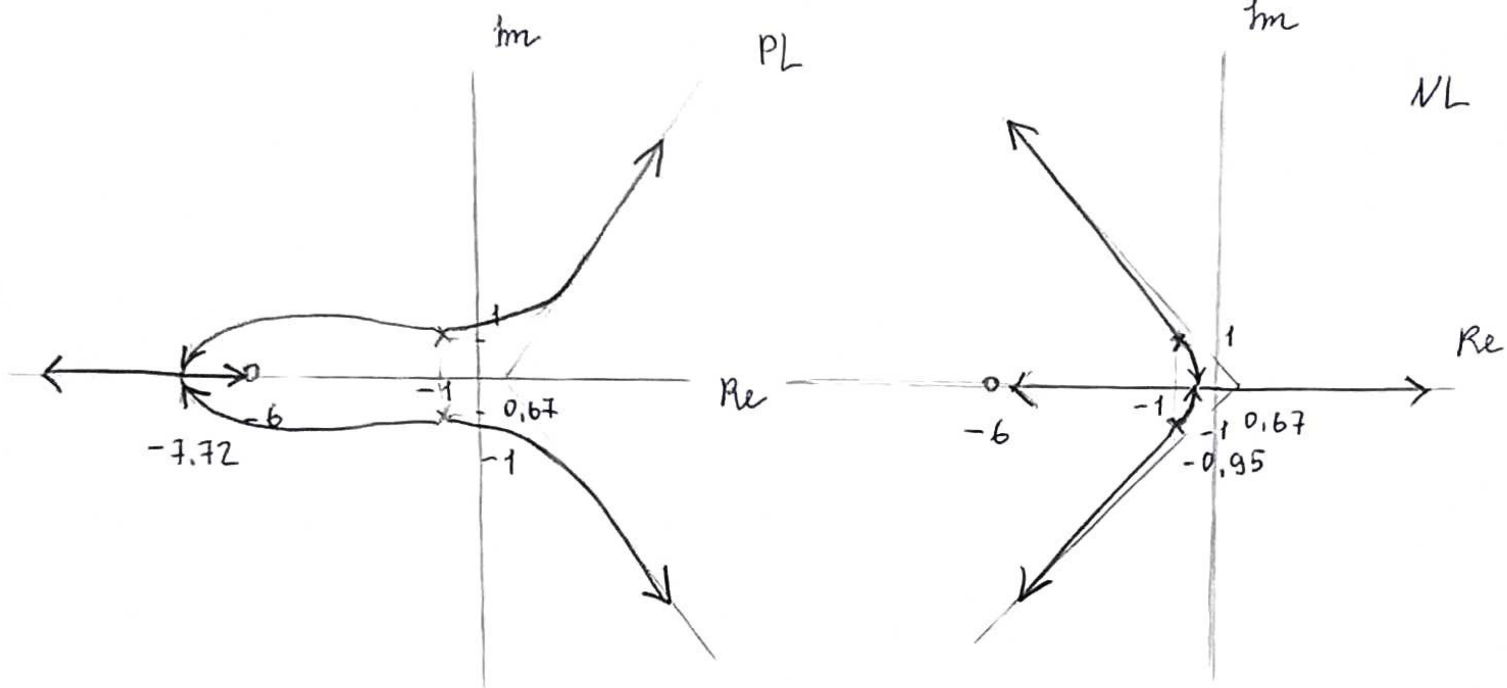
$$r\psi_a = \begin{cases} (2h+1)\pi & h = 0,1,2 \quad PL \\ 2h\pi & h = 0,1,2 \quad NL \end{cases}$$

$$\psi_a = \begin{cases} \frac{\pi}{3}, \pi, \frac{5\pi}{3} & PL \\ 0, \frac{2\pi}{3}, \frac{4\pi}{3} & NL \end{cases}$$

Real axis: $(-\infty, -6)$ 5 poles/zeros $\in$ PL

$(-6, -1)$ 4 poles $\in$ NL

$(-1, +\infty)$ 0 poles/zeros $\in$ NL



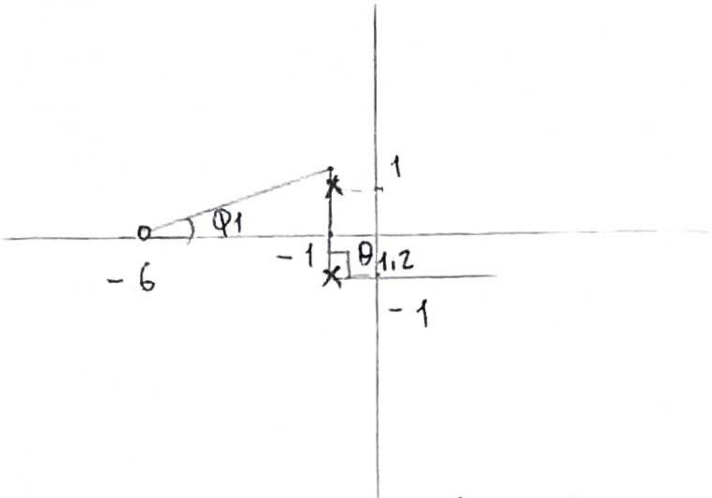
$$N(s)D'(s) - N'(s)D(s) = 0 \qquad D(s) = s^4 + 4s^3 + 8s^2 + 8s + 4$$

$$(s+6)\cdot(4s^3 + 12s^2 + 16s + 8) - (s^4 + 4s^3 + 8s^2 + 8s + 4) = 0$$

$$4s^4 + 36s^3 + 88s^2 + 104s + 48 - s^4 - 4s^3 - 8s^2 - 8s - 4 = 0$$

$$3s^4 + 32s^3 + 80s^2 + 96s + 44 = 0$$

$$s = -0,95037 \approx -0,95 \qquad s = -7,7163 \approx -7,72$$



$\varphi_1 = \tan^{-1}\left(\frac{1}{5}\right) = 11.3°$

$\theta_1 = \theta_2 = 90°$

$$q\alpha_{dep} = \begin{cases} 11.3° - 90° - 90° + (2k+1)\pi & k = 0, 1 \quad PL \\ 11.3° - 90° - 90° + 2k\pi & k = 0, 1 \quad NL \end{cases}$$

$q = 2$

$$\alpha_{dep} = \begin{cases} 5.65°,\ 185.65° & PL \\ -84.35°,\ 95.65° & NL \end{cases}$$

$D(s) + \beta N(s) = 0$

$s^4 + 4s^3 + 8s^2 + 8s + 4 + k(s+6) = 0$

s^4	1	8	$4+6k$
s^3	4	$8+k$	
s^2	$\frac{24-k}{4}$	$4+6k$	
s^1	$\frac{-0.25k^2-20k+32}{6-0.25k}$		
s^0	$4+6k$		

$\frac{24-k}{4} > 0 \quad k < 24$

$4 + 6k > 0 \quad k > -\frac{2}{3}$

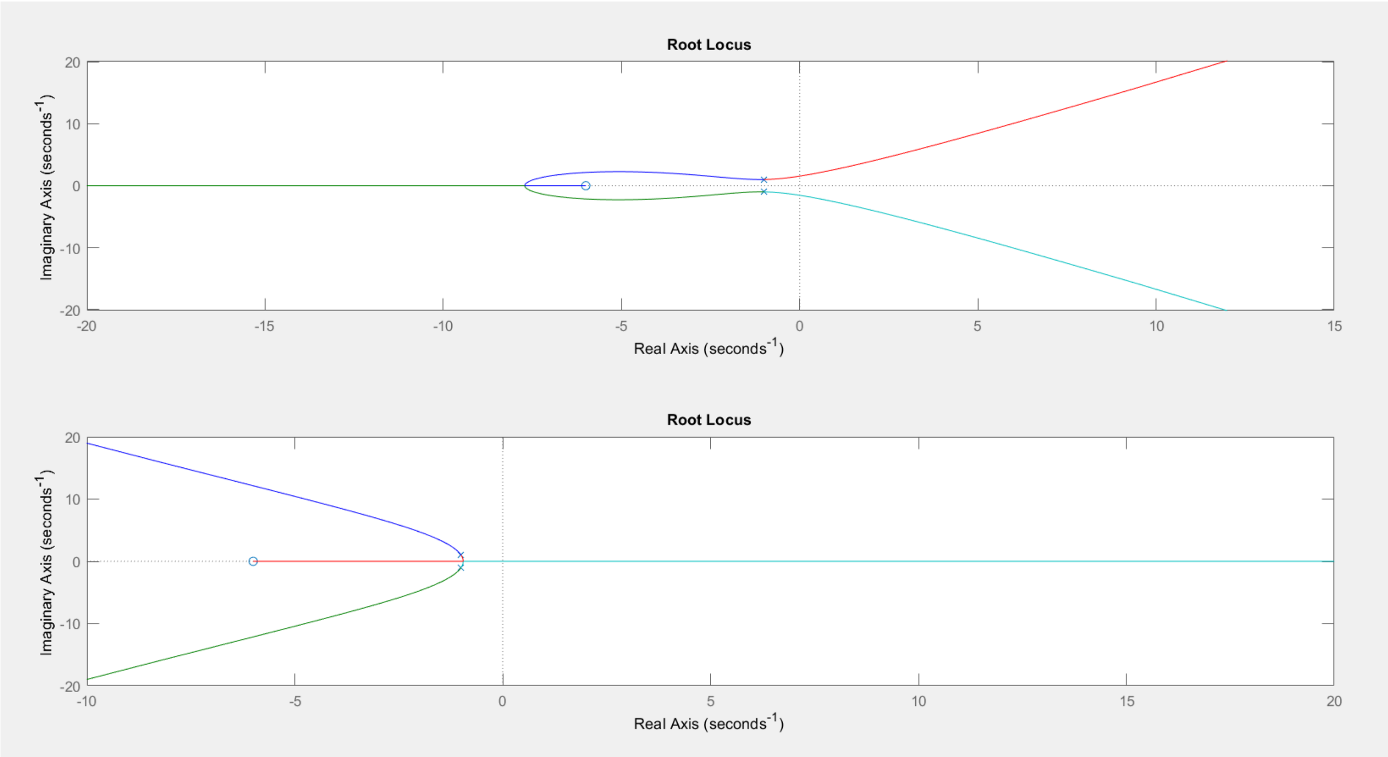
$-0.25k^2 - 20k + 32 > 0 \quad -81.6 < k < 1.57$

$\Rightarrow$ The closed loop system is stable for

$-\frac{2}{3} < k < 1.57$

MATLAB code:

```
sys8 = tf([1 6], [1 4 8 8 4]);
figure
subplot(2, 1, 1)
rlocus(sys8)
subplot(2, 1, 2)
rlocus(-sys8)
```



The obtained sketch is pretty similar to the MATLAB plot.