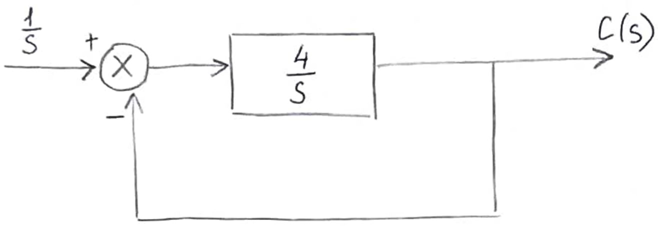
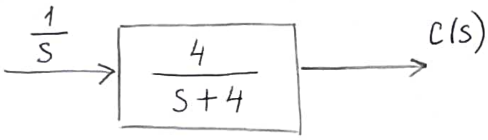
Robt 303 Zarema Balgabekova HW2

1. $c(t)$, time constant, rise time, settling time -?





$$C(s) = \frac{4}{s+4} \cdot \frac{1}{s} = \frac{1}{s} - \frac{1}{s+4}$$

$$\Rightarrow \boxed{c(t) = (1 - e^{-4t})\, u(t)}$$

Time constant: $T = \frac{1}{a} = \frac{1}{4}\,s$ $\boxed{T = \frac{1}{4}\,s}$

Rise time: $T_r = 2.2T = \frac{2.2}{4} = 0.55s$ $\boxed{T_r = 0.55s}$

Settling time: $T_{s,5\%} = 3T = \frac{3}{4}\,s$ $T_{s,2\%} = 4T = 1\,s$

$$\boxed{T_{s,5\%} = \frac{3}{4}\,s \qquad T_{s,2\%} = 1\,s}$$

MATLAB code:

```
A = [0 4];
B = [1 4];
sys1 = tf(A, B)
figure
step(sys1)
```

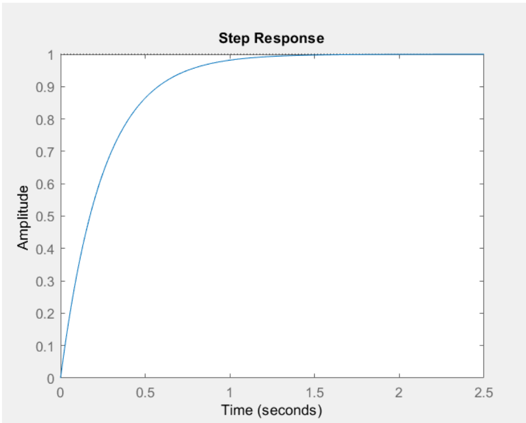


Figure 1: Step Response for G(s) = $\frac{4}{s+4}$ (sys1)

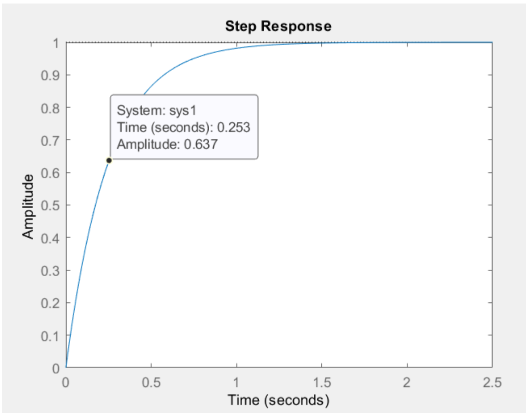


Figure 2: Measure of Time Constant for sys1

From Figure 2, time constant can be approximated as 0.253 s, which agrees with the computed value - 0.25 s.

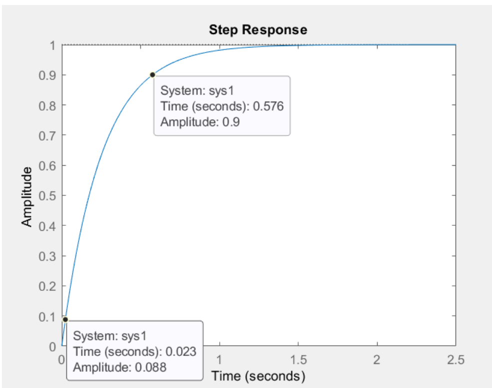


Figure 3: Measure of Rise Time for sys1

From Figure 3, rise time can be approximated as 0.553 s, which agrees with the computed value – 0.55 s.

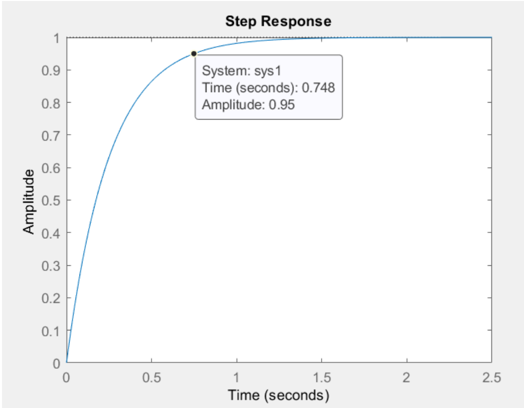


Figure 4: Measure of Settling Time to 5% for sys1

From Figure 4, settling time to 5% is 0.748 s, which is almost the same as the calculated value – 0.75 s.

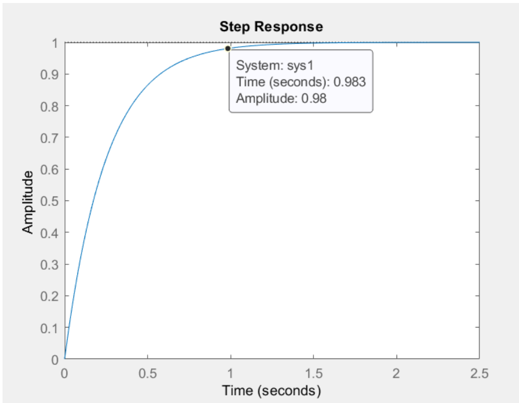
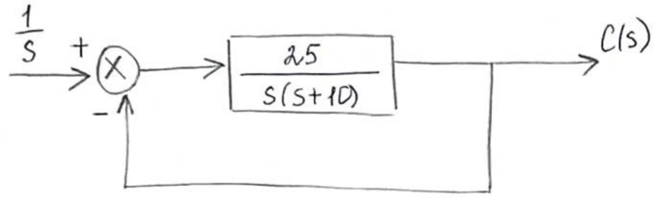
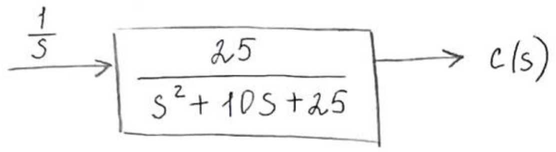


Figure 5: Measure of Settling Time to 2% for sys1

From Figure 5, settling time to 2% is 0.983 s, which is almost the same as the estimated value – 1 s.





$$C(s) = \frac{25}{s^2+10s+25} \cdot \frac{1}{s} = \frac{25}{s(s+5)^2} = \frac{1}{s} - \frac{1}{s+5} - \frac{5}{(s+5)^2}$$

$$\Rightarrow \boxed{c(t) = \left(1 - e^{-5t} - 5te^{-5t}\right) u(t)}$$

This is a critically damped system $\Rightarrow$

Time constant: $\boxed{T = \frac{1}{5} s}$

Rise time: $T_r = 3.36T = \frac{3.36}{5} = 0.672 s$ $\boxed{T_r = 0.672 s}$

Settling time: $T_{s,5\%} = 4.74T = \frac{4.74}{5} = 0.948 s$

$T_{s,2\%} = 5.84T = \frac{5.84}{5} = 1.168 s$

$$\boxed{T_{s,5\%} = 0.948 s \qquad T_{s,2\%} = 1.168 s}$$

MATLAB code:

```
C = [0 25];
D = [1 10 25];
sys2 = tf(C, D)
figure
step(sys2)
```

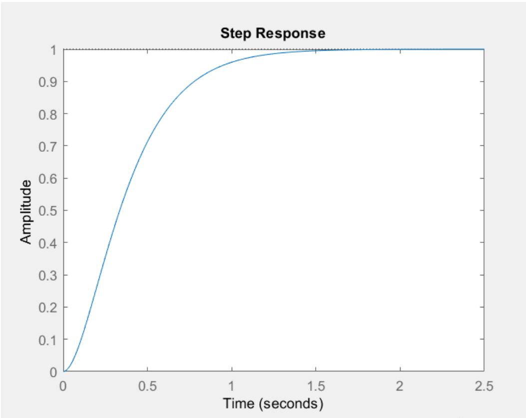


Figure 6: Step Response for G(s) = $\frac{25}{s^2+10s+25}$ (sys2)

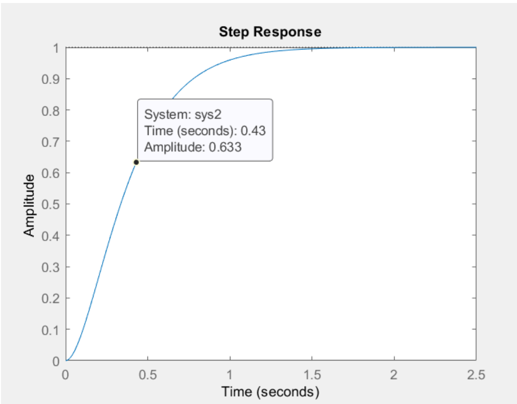


Figure 7: Measure of Time Constant for sys2

From Figure 7, time constant can be approximated as 0.43 s, which is around two times larger than the computed value – 0.2 s.

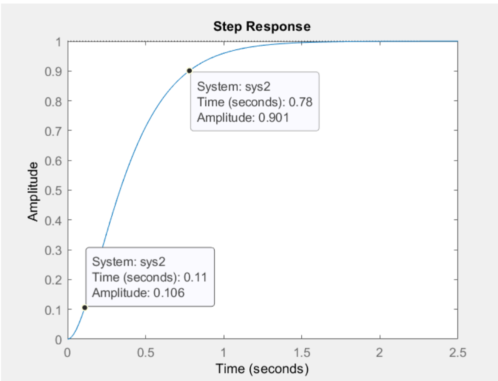


Figure 8: Measure of Rise Time for sys2

From Figure 8, Rise time can be approximated as 0.67 s, which agrees with the calculated value – 0.672 s.

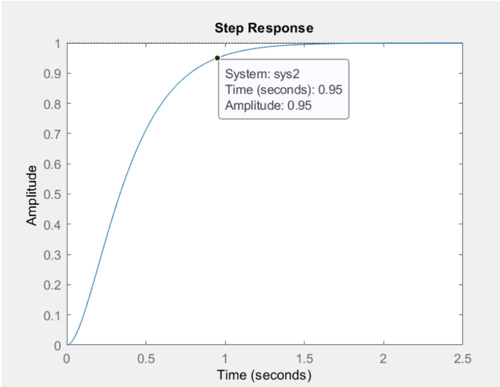


Figure 9: Measure of Settling Time to 5% for sys2

From Figure 9, settling time to 5% is 0.95 s, which agrees with the estimated value – 0.948 s.

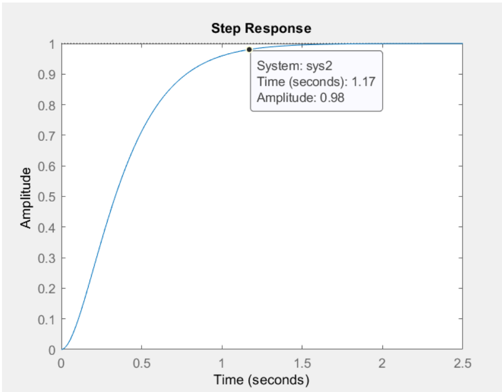


Figure 10: Measure of Settling Time to 2% for sys2

From Figure 10, settling time to 2% is 1.17 s, which is almost the same as the calculated value – 1.168 s.

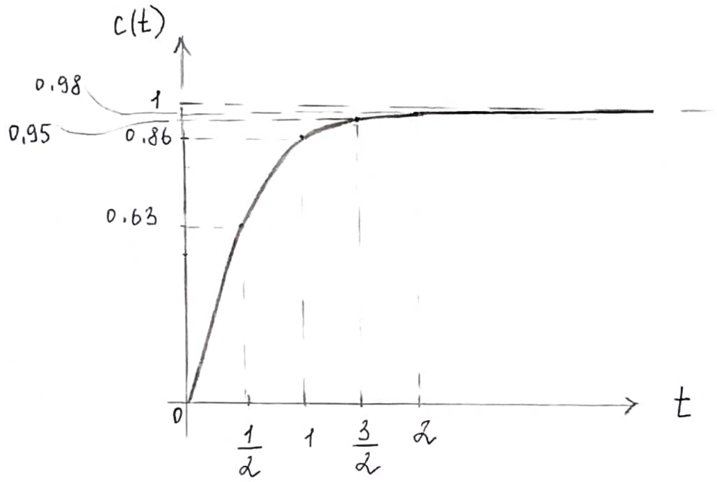
Therefore, though the estimated time constant differs from the value from the graph, all other calculations that used this value are almost the same as the values from the graph.

2. a) $T(s) = \frac{2}{s+2}$

Pole at $s=-2$ and no zeros

$$C(s) = \frac{2}{s+2} \cdot \frac{1}{s} = \frac{1}{s} - \frac{1}{s+2}$$

$\Rightarrow$ Step response: $c(t) = (1-e^{-2t})\,u(t)$



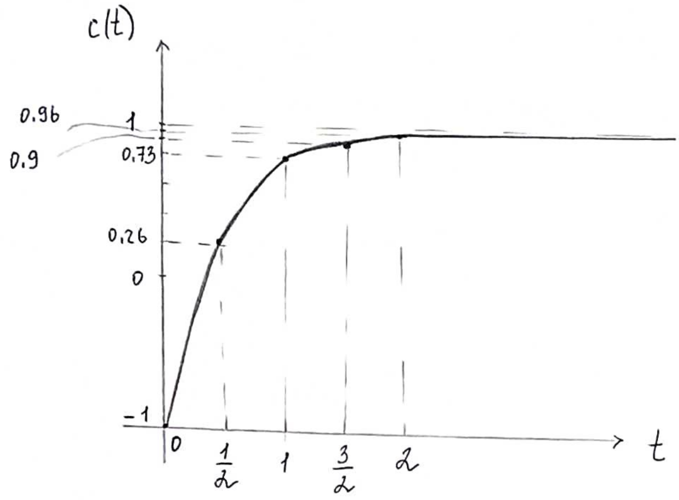
All MATLAB plots are presented after manual plots

b) $T(s) = \frac{-(s-2)}{s+2}$

Pole at $s=-2$ and zero at $s=2$

$$C(s) = \frac{-(s-2)}{s+2} \cdot \frac{1}{s} = \frac{1}{s} - \frac{2}{s+2}$$

$\Rightarrow$ Step response: $c(t) = (1 - 2e^{-2t})\, u(t)$

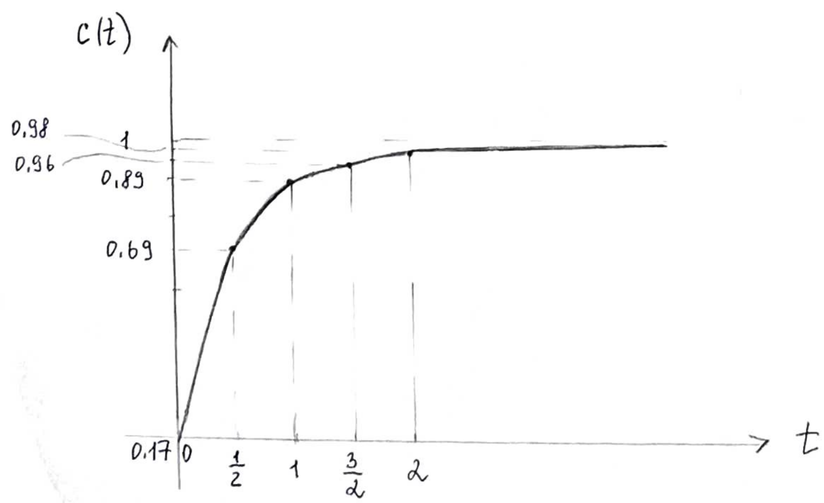


c) $T(s) = \dfrac{s+12}{6(s+2)}$

Pole at $s = -2$ and zero at $s = -12$

$$C(s) = \frac{s+12}{6(s+2)} \cdot \frac{1}{s} = \frac{1}{s} - \frac{5}{6} \cdot \frac{1}{s+2}$$

$\Rightarrow$ Step response: $c(t) = \left(1 - \frac{5}{6} e^{-2t}\right) u(t)$

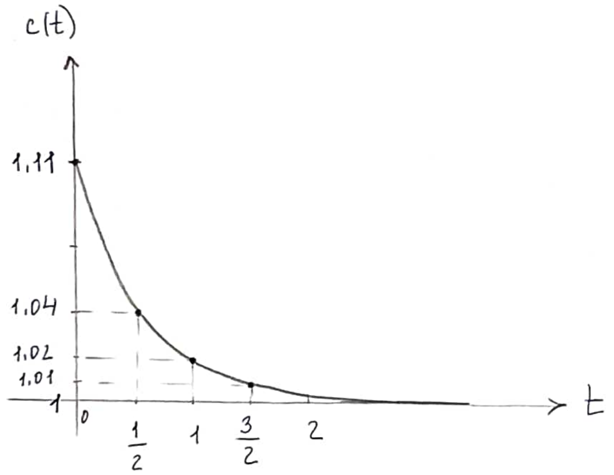


d) $T(s) = \dfrac{s+1.8}{0.9(s+2)}$

Pole at $s = -2$ and zero at $s = -1.8$

$$C(s) = \frac{s+1.8}{0.9(s+2)} \cdot \frac{1}{s} = \frac{1}{s} + \frac{1}{9} \cdot \frac{1}{s+2}$$

$\Rightarrow$ Step response: $c(t) = \left(1 + \frac{1}{9} e^{-2t}\right) u(t)$



MATLAB code:

```
sys1 = tf([0 2], [1 2])
figure
step(sys1)
sys2 = tf([-1 2], [1 2])
figure
step(sys2)
sys3 = tf([1 12], [6 12])
figure
step(sys3)
sys4 = tf([1 1.8], [0.9 1.8])
figure
step(sys4)
```

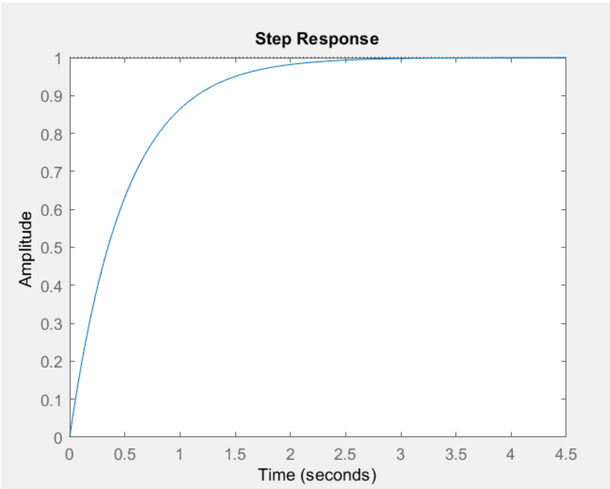


Figure 11: Step Response for T(s) = $\frac{2}{s+2}$

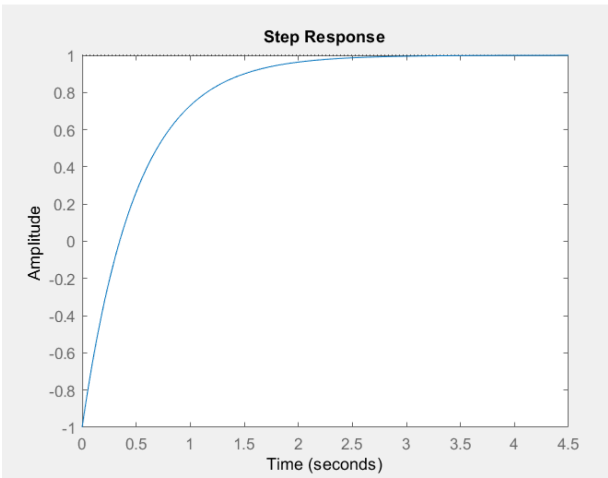


Figure 12: Step Response for T(s) = $\frac{-(s-2)}{s+2}$

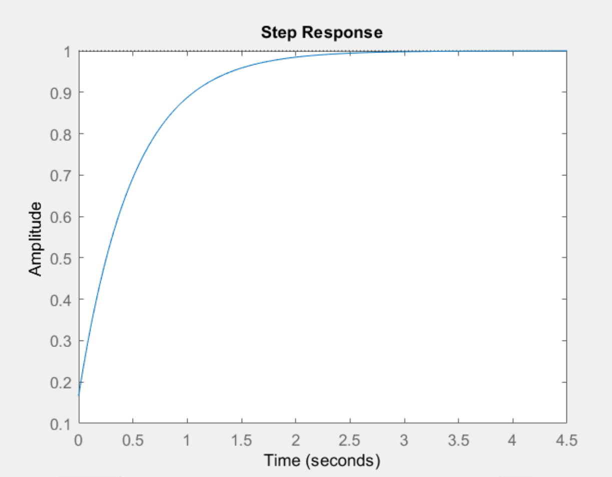


Figure 13: Step Response for T(s) = $\frac{s+12}{6(s+2)}$

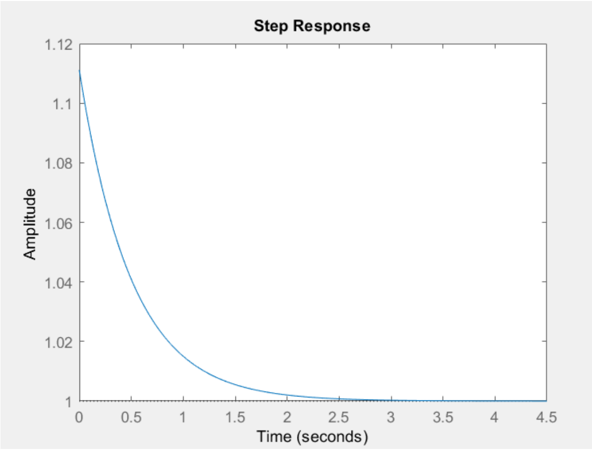


Figure 14: Step Response for T(s) = $\frac{s+1.8}{0.9(s+2)}$

From both manual and MATLAB plots, it is seen that the addition of zeros to the first order system changes the initial value of the response. The steady-state value is one for all cases.

Comparing systems a and b, it can be concluded that the addition of a positive zero changes the initial value of the response from zero to a negative value.

Comparing systems a and c, it can be concluded that the addition of a negative zero that differs from the pole changes the initial value of the response to a positive value that is less than one.

Comparing systems a and d, it can be concluded that the addition of a negative zero that is almost equal to the pole changes the initial value of the response to a positive value that is slightly larger than one.

3. a) $T(s) = \dfrac{5}{(s+2)(s+4)}$

Poles at -2 and -4. No zeros

General form of step response: $c(t) = (A + Be^{-2t} + Ce^{-4t})u(t)$

$\Rightarrow$ Overdamped response

b) $T(s) = \dfrac{5(s+3.8)}{(s+2)(s+4)}$

Poles at -2 and -4. Zero at -3.8

General form of step response: $c(t) = (A + Be^{-2t} + Ce^{-4t})u(t)$

$\Rightarrow$ Overdamped response

c) $T(s) = \dfrac{10}{s^2 + 2.4s + 144}$

$D = 1.2^2 - 144 = -142.56$

$s_1 = -1.2 + \dfrac{18\sqrt{11}}{5} i \qquad s_2 = -1.2 - \dfrac{18\sqrt{11}}{5} i$

Poles at $-1.2 \pm \dfrac{18\sqrt{11}}{5} i$. No zeros

General form of step response:

$$c(t) = \left[A + Be^{-1.2t}\sin\left(\frac{18\sqrt{11}}{5}t\right) + C\cdot e^{-1.2t}\cdot\cos\left(\frac{18\sqrt{11}}{5}t\right)\right]u(t)$$

$\Rightarrow$ Underdamped response

d) $T(s) = \dfrac{10}{(s+10)(s^2+2.4s+144)}$

Poles at -10, $-1.2 \pm \dfrac{18\sqrt{11}}{5}i$. No zeros.

General form of step response:

$$c(t) = \left[A + Be^{-10t} + Ce^{-1.2t}\sin\left(\frac{18\sqrt{11}}{5}t\right) + De^{-1.2t}\cos\left(\frac{18\sqrt{11}}{5}t\right)\right]u(t)$$

$\Rightarrow$ Underdamped response

e) $T(s) = \dfrac{4}{s^2+4}$

Poles at $\pm 2j$. No zeros

General form of step response:

$$c(t) = [A + B\cos(2t)]\,u(t)$$

$\Rightarrow$ undamped response

f) $T(s) = \dfrac{s+3}{s^2+4}$

Poles at $\pm 2j$. Zero at -3.

General form of step response:

$c(t) = [A + B\cos(2t) + C\sin(2t)]\,u(t)$

$\Rightarrow$ undamped response

4. Natural frequency, damping ratio -?

a) $T(s) = \dfrac{5}{s^2+6s+8}$

$\omega_n = \sqrt{8} = 2\sqrt{2}$ $\qquad \zeta = \dfrac{6}{2 \cdot 2\sqrt{2}} = \dfrac{3\sqrt{2}}{4} \approx 1.06 > 1$

$\Rightarrow$ overdamped response

All MATLAB plots are provided later

b) ω_n and ζ are the same as in a)

$\omega_n = 2\sqrt{2}$ $\qquad \zeta = \dfrac{3\sqrt{2}}{4} > 1$

$\Rightarrow$ overdamped response

c) $w_n = \sqrt{144} = 12$ $\qquad \zeta = \frac{2 \cdot 4}{2 \cdot 12} = 0.1 \in (0, 1)$

$\Rightarrow$ underdamped response

d) w_n and ζ are the same as in c)

$w_n = 12$ $\qquad \zeta = 0.1 \in (0, 1)$

$\Rightarrow$ underdamped response

e) $w_n = \sqrt{4} = 2$ $\qquad \zeta = 0$

$\Rightarrow$ undamped response

f) w_n and ζ are the same as in e)

$w_n = 2$ $\qquad \zeta = 0$

$\Rightarrow$ undamped response

MATLAB code:

```
sys1 = tf([0 5], [1 6 8])
figure
step(sys1)
sys2 = tf([5 19], [1 6 8])
figure
step(sys2)
sys3 = tf([0 10], [1 2.4 144])
figure
step(sys3)
sys4 = tf([0 10], [1 12.4 168 1440])
figure
step(sys4)
sys5 = tf([0 4], [1 0 4])
figure
step(sys5, 10)
sys6 = tf([1 3], [1 0 4])
figure
step(sys6, 10)
```

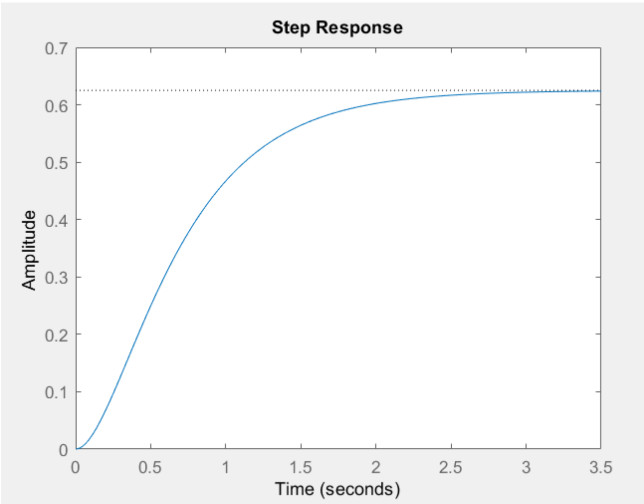


Figure 15: Step Response for T(s) = $\frac{5}{(s+2)(s+4)}$

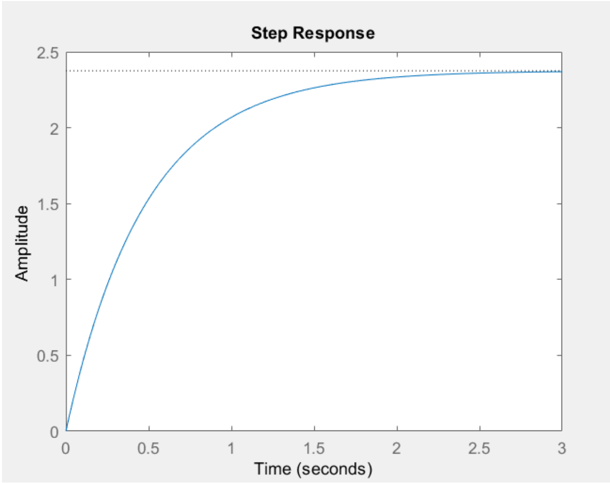


Figure 16: Step Response for T(s) = $\frac{5(s+3.8)}{(s+2)(s+4)}$

Comparing systems a and b, it is seen that the addition of a negative zero slightly changes the shape of the response, which allows system b to reach the steady-state value faster than system a reaches it. Also, the steady-state value of system b is larger because its DC gain has increased. The initial value of the response is zero for both systems.

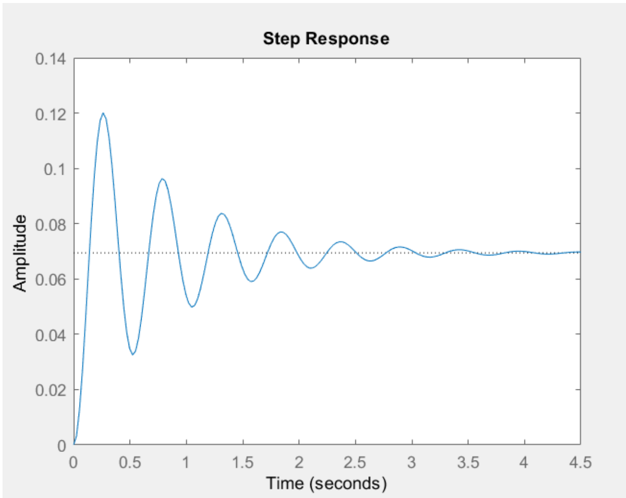


Figure 17: Step Response for T(s) = $\frac{10}{s^2+2.4s+144}$

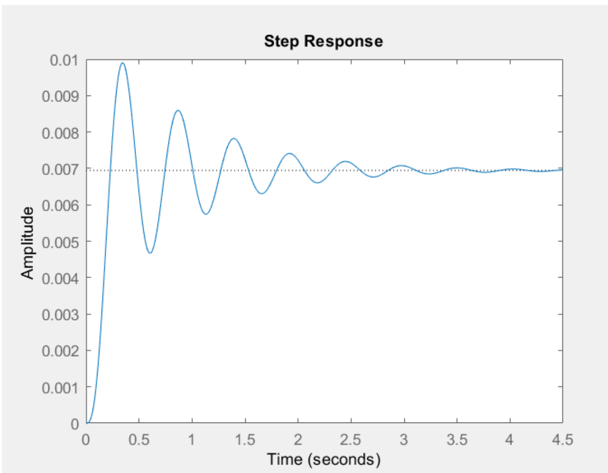


Figure 18: Step Response for T(s) = $\frac{10}{(s+10)(s^2+2.4s+144)}$

Comparing systems c and d, it is seen that the addition of a negative pole decreases the steady-state value because DC gain decreases by ten times. Peak values also decrease, but both responses start from zero.

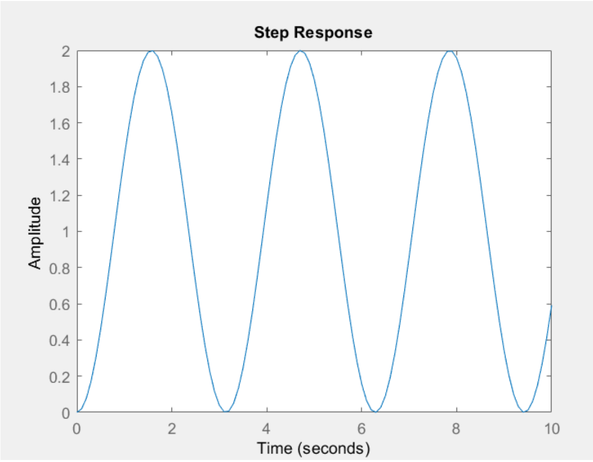


Figure 19: Step Response for T(s) = $\frac{4}{s^2+4}$

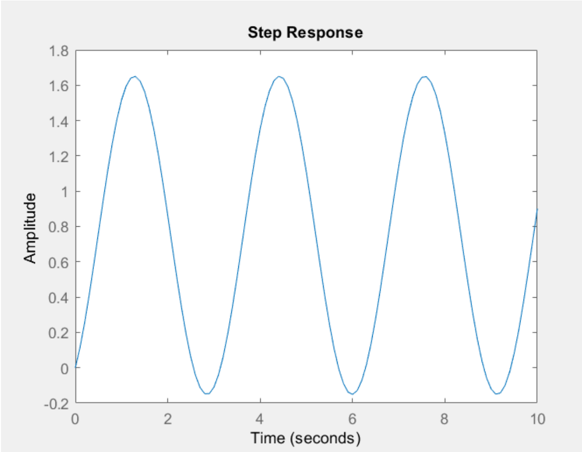


Figure 20: Step Response for T(s) = $\frac{s+3}{s^2+4}$

Comparing systems e and f, it is seen that the addition of a negative zero slightly decreases the amplitude of oscillations and vertically shifts the response. Both responses start from zero.

5. $T_{s,2\%}$, $T_{s,5\%}$, T_r, T_p, M_p - ?

a) $$T(s) = \frac{9}{s^2+6s+9} = \frac{9}{(s+3)^2}$$

This is a critically damped system : $T = \frac{1}{3}$ s

$$\Rightarrow T_{s,2\%} = 5.84\,T = \frac{5.84}{3} \simeq 1.947 \text{ s}$$

$$T_{s,5\%} = 4.74\,T = \frac{4.74}{3} = 1.58 \text{ s}$$

$$T_r = 3.36\,T = \frac{3.36}{3} = 1.12 \text{ s}$$

T_p and M_p are not applicable

All MATLAB plots are provided later

b) $$T(s) = \frac{22}{s^2+13s+22} = \frac{22}{(s+2)(s+11)} =$$

$$\frac{1}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{11}\right)}$$

This is an overdamped system with $T_1 = \frac{1}{2}$ and $T_2 = \frac{1}{11}$

As $T_1 \gg T_2$, it can be approximated as a first-order system ; $T = \frac{1}{2}$ s

$\Rightarrow \; T_{s,2\%} = 4T = \frac{4}{2} = 2 \text{ s}$

$T_{s,5\%} = 3T = \frac{3}{2} = 1.5 \text{ s}$

$T_r = 2.2T = \frac{2.2}{2} = 1.1 \text{ s}$

T_p and M_p are not applicable

c) $T(s) = \dfrac{0.01}{s^2 + 0.1s + 0.01}$

This is an underdamped system: $\omega_n = \sqrt{0.01} = 0.1$

$\zeta = \dfrac{0.1}{2 \cdot 0.1} = 0.5$

$$T_{s,2\%} = \frac{4}{\zeta \omega_n} = \frac{4}{0.5 \cdot 0.1} = 80 \text{ s}$$

$$T_{s,5\%} \approx \frac{3}{\zeta \omega_n} = \frac{3}{0.5 \cdot 0.1} = 60 \text{ s}$$

$$T_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{0.1} = 18 \text{ s}$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{0.1\sqrt{1-0.5^2}} \approx 36.28 \text{ s}$$

$$M_p = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100\, e^{-0.5\pi/\sqrt{1-0.5^2}} \approx 16.3\ \%$$

MATLAB code:

```
sys1 = tf([0 9], [1 6 9])
figure
step(sys1)
sys2 = tf([0 22], [1 13 22])
figure
step(sys2)
sys3 = tf([0 0.01], [1 0.1 0.01])
figure
step(sys3)
```

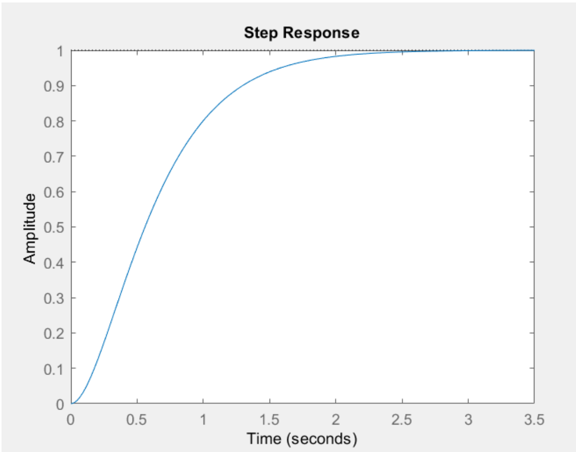


Figure 21: Step Response for T(s) = $\frac{9}{s^2+6s+9}$ (sys1)

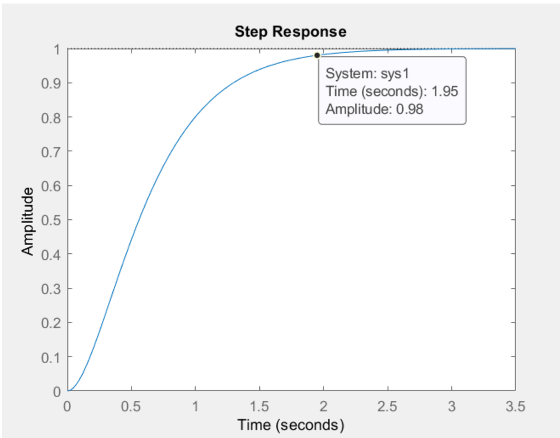


Figure 22: Measure of Settling Time to 2% for sys1

From Figure 22, settling time to 2% is 1.95 s, which agrees with the estimated value – 1.947s.

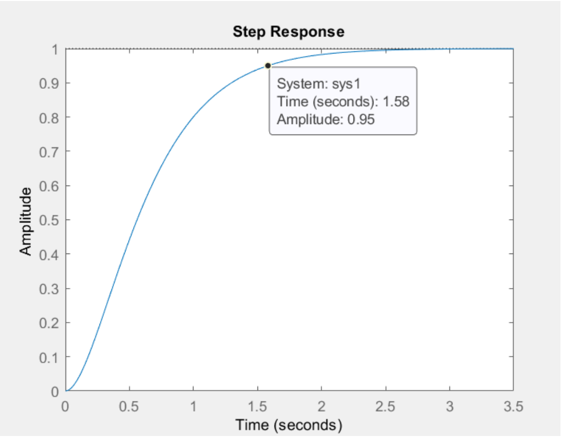


Figure 23: Measure of Settling Time to 5% for sys1

From Figure 23, settling time to 5% is 1.58 s, which is equal to the calculated value – 1.58 s.

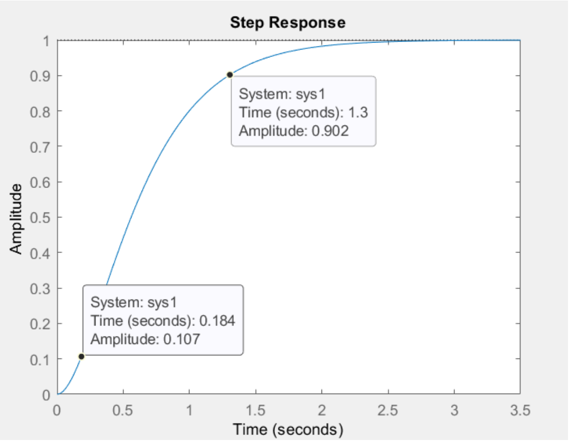


Figure 24: Measure of Rise Time for sys1

From Figure 24, rise time can be approximated as 1.116 s, which is almost the same as the computed value – 1.12 s.

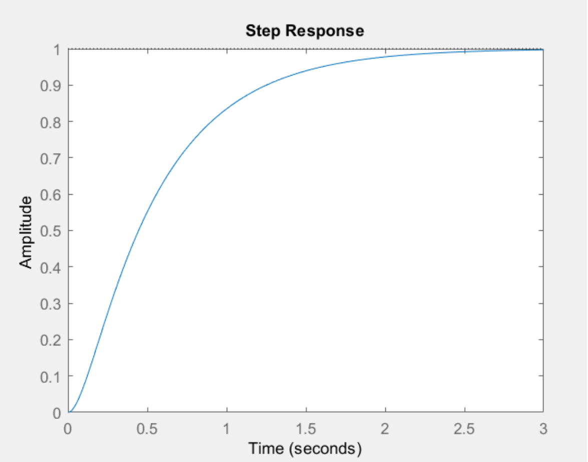


Figure 25: Step Response for T(s) = $\frac{22}{s^2+13s+22}$ (sys2)

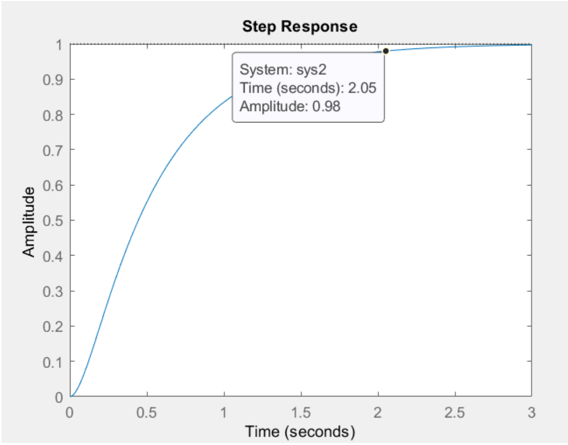


Figure 26: Measure of Settling Time to 2% for sys2

From Figure 26, settling time to 2% is 2.05 s, which is almost the same as the estimated value – 2 s.

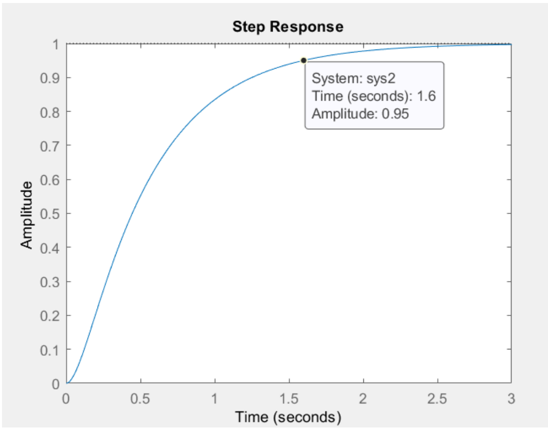


Figure 27: Measure of Settling Time to 5% for sys2

From Figure 27, settling time to 5% is 1.6 s, which agrees with the computed value – 1.5 s.

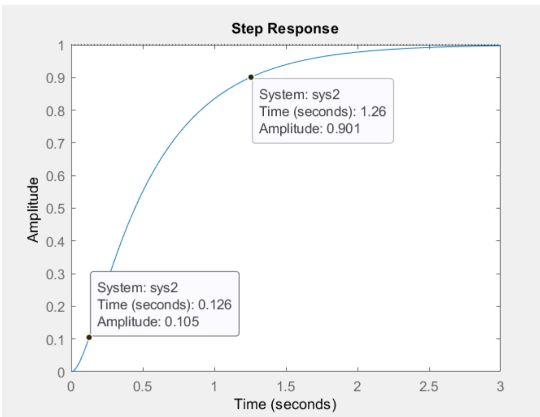


Figure 28: Measure of Rise Time for sys2

From Figure 28, rise time is approximately 1.134 s, which is almost equal to the computed value – 1.1 s.

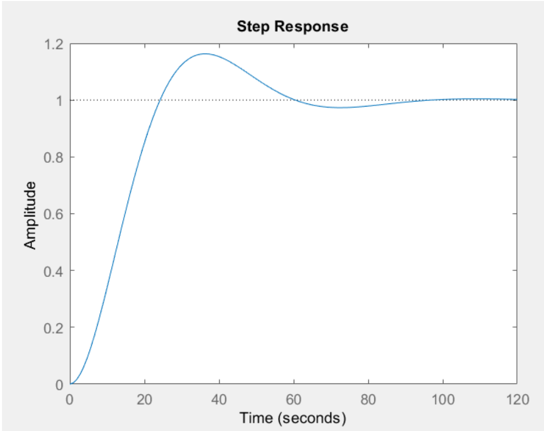


Figure 29: Step Response for T(s) = $\frac{0.01}{s^2+0.1s+0.01}$ (sys3)

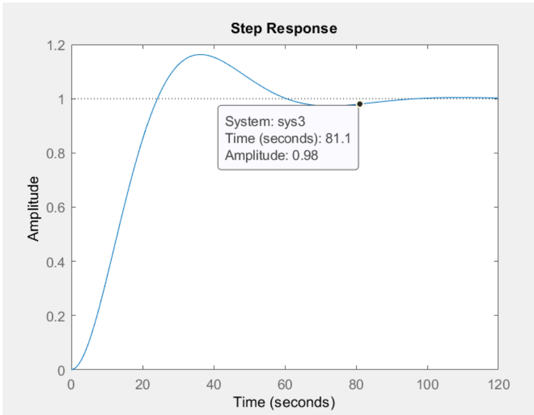


Figure 30: Measure of Settling Time to 2% for sys3

From Figure 30, settling time to 2% is 81.1 s, which agrees with the calculated value – 80 s.

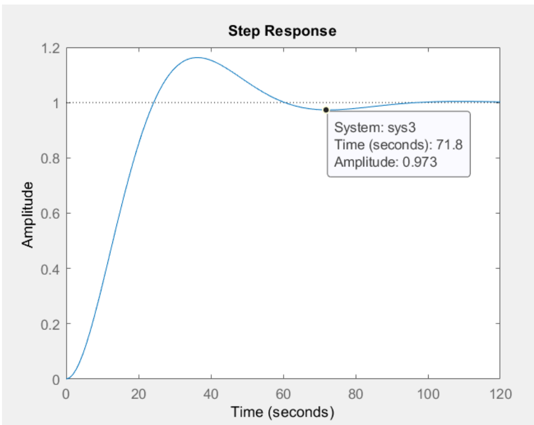


Figure 31: Measure of Settling Time to 5% for sys3

From Figure 31, it is seen that the response does not reach and stay within 5% of the steady-state value. Therefore, though settling time to 5% was estimated, it cannot be measured from the graph.

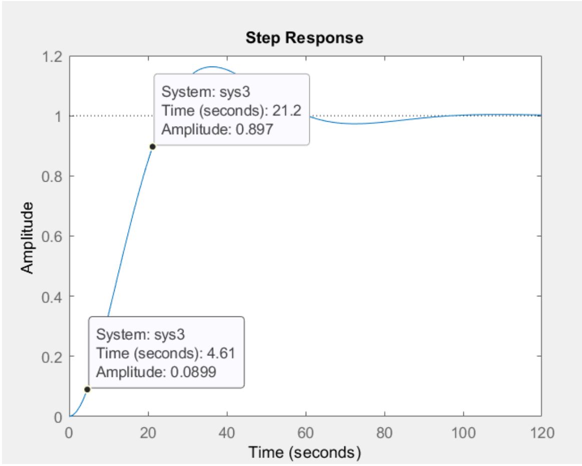


Figure 32: Measure of Rise Time for sys3

From Figure 32, rise time can be approximated as 16.59 s, which does not differ much from the estimated value – 18 s.

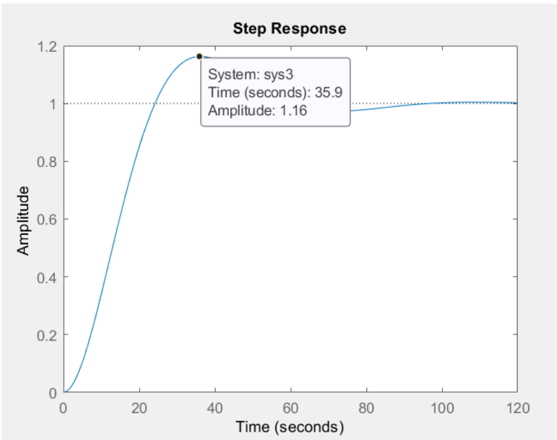
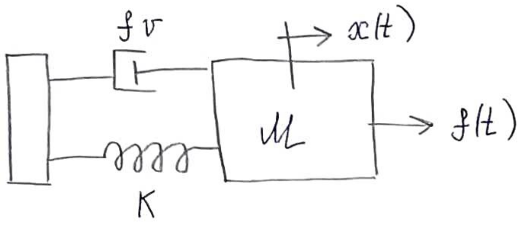


Figure 33: Measure of Peak Time and Percent Overshoot for sys3

From Figure 33, peak time is approximately 35.9 s and percent overshoot is 16 %, which agrees with the calculated values – 36.28 s and 16.3 %.

6.



$K = 1$, $f(t)$ - unit step

$M_p = 17\%$, $T_{s,2\%} = 5s$

M -? f_v -?

Newton's II law:

$$M\ddot{x}(t) = -Kx(t) - f_v\dot{x}(t) + f(t)$$

$$M\ddot{x}(t) + Kx(t) + f_v\dot{x}(t) = f(t)$$

Laplace transform:

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

$$X(s) = \frac{1}{Ms^2 + f_v s + K} \cdot F(s) = \frac{1}{Ms^2 + f_v s + 1} \cdot \frac{1}{s}$$

$$\Rightarrow G(s) \doteq \frac{1}{Ms^2 + f_v s + 1} = \frac{\frac{1}{M}}{s^2 + \frac{f_v}{M}s + \frac{1}{M}}$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{1}{M}}$$

$$2\zeta\omega_n = \frac{f_v}{M}$$

$$\zeta = \frac{f_v\sqrt{M}}{2M}$$

$$M_p = 100 \cdot e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 17$$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.17$$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln 0.17$$

$$-\zeta\pi = \ln 0.17 \cdot \sqrt{1-\zeta^2}$$

$$\zeta^2\pi^2 = (\ln 0.17)^2(1-\zeta^2)$$

$$\zeta^2(\pi^2 + (\ln 0.17)^2) = (\ln 0.17)^2$$

$$\zeta^2 = \frac{(\ln 0.17)^2}{\pi^2 + (\ln 0.17)^2}$$

$$\zeta = -\frac{\ln 0.17}{\sqrt{\pi^2 + (\ln 0.17)^2}} \approx 0.491$$

$$T_{s,2\%} = \frac{4}{\zeta\omega_n} = 5$$

$$\omega_n = \frac{4}{5\zeta} = \frac{4}{5 \cdot 0.491} \approx 1.629$$

$$M = \frac{1}{\omega_n^2} = \frac{1}{1.629^2} \approx 0.377$$

$$f_v = 2\zeta\sqrt{M} = 2 \cdot 0.491\sqrt{0.377} \approx 0.603$$

MATLAB code:

```
K = 1;
M = 0.377;
fv = 0.603;
sys = tf([0 1/M], [1 fv/M K/M])
step(sys)
```

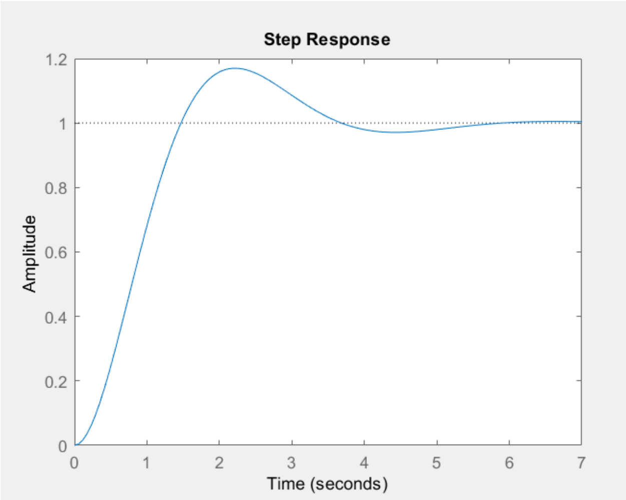


Figure 34: Step Response for T(s) = $\frac{1/M}{s^2+(fv/M)s+1/M}$ (sys)

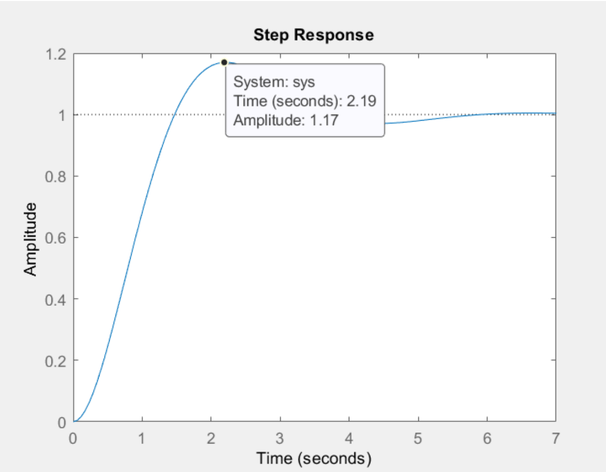


Figure 35: Measure of Percent Overshoot for sys

From Figure 35, percent overshoot is 17%, which matches the requirement.

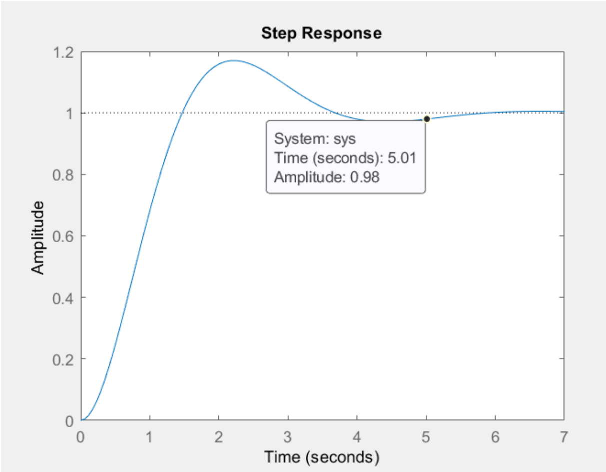


Figure 36: Measure of Settling Time to 2% for sys

From Figure 36, settling time to 2% is 5.01s, which also matches the requirement.