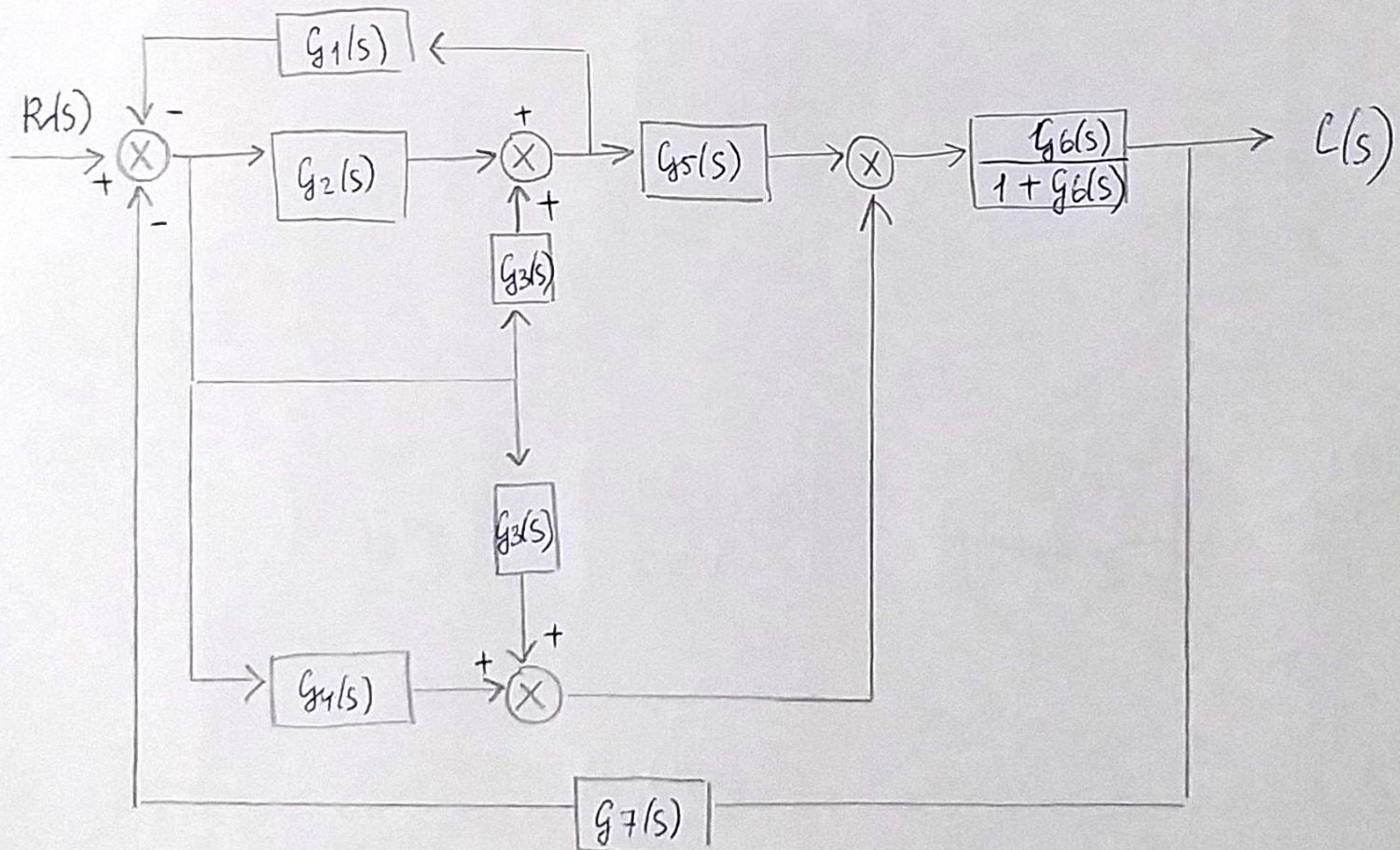
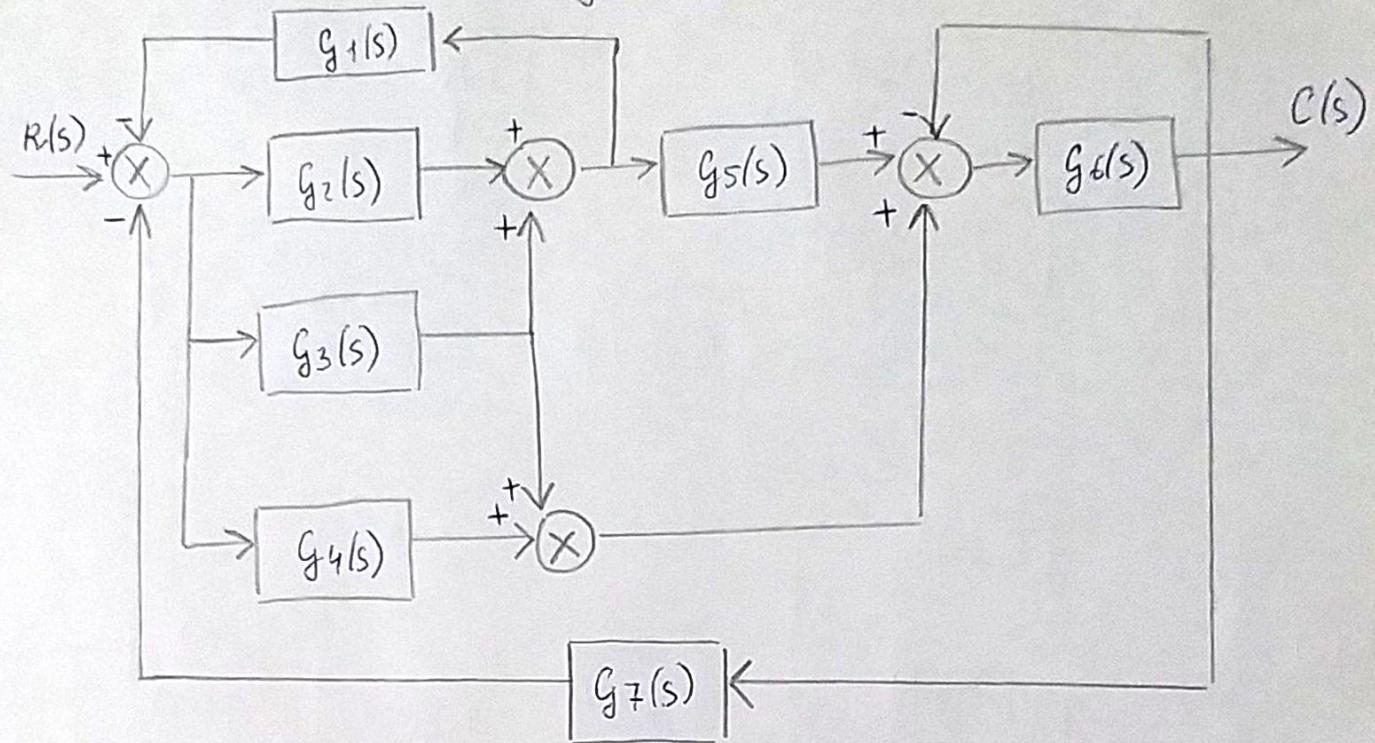
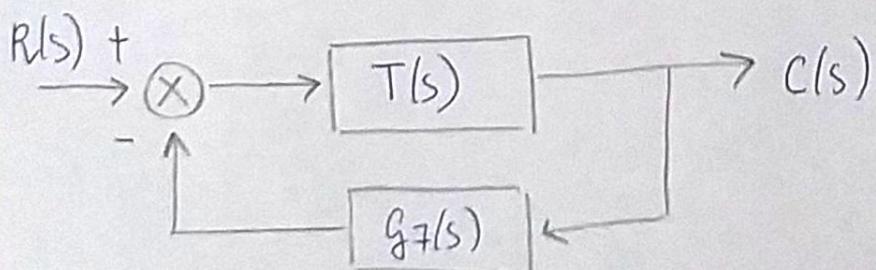
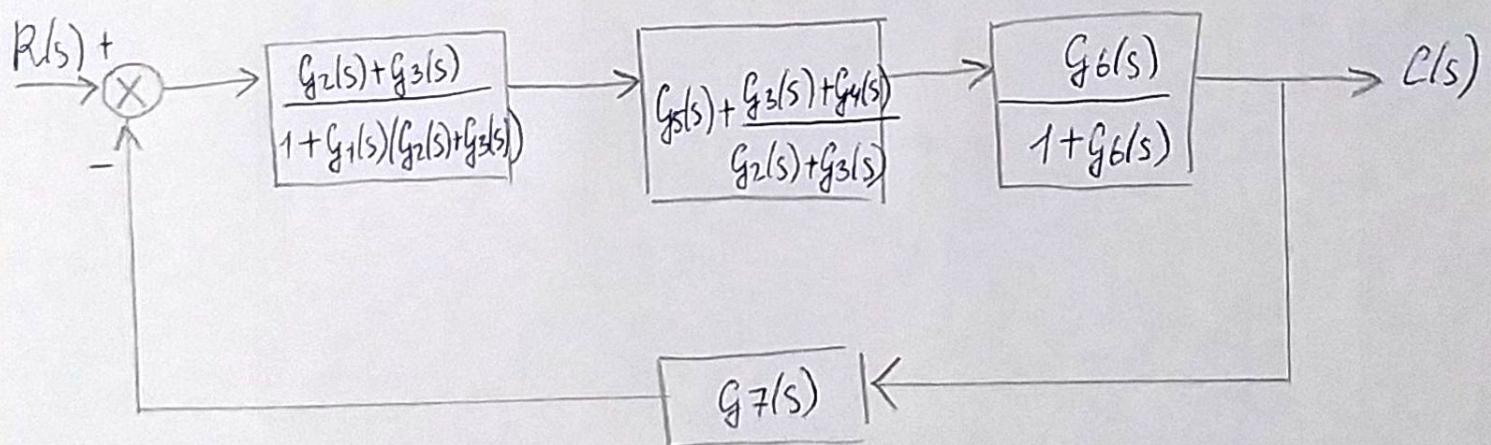
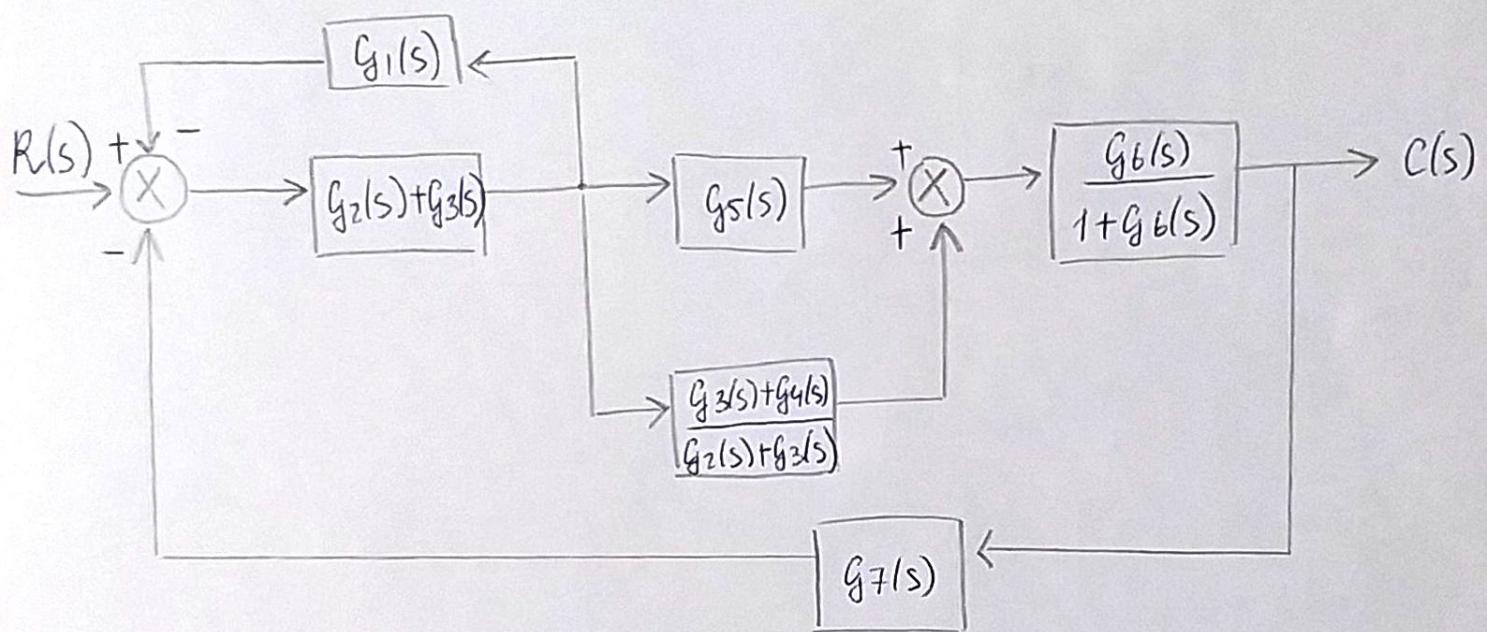
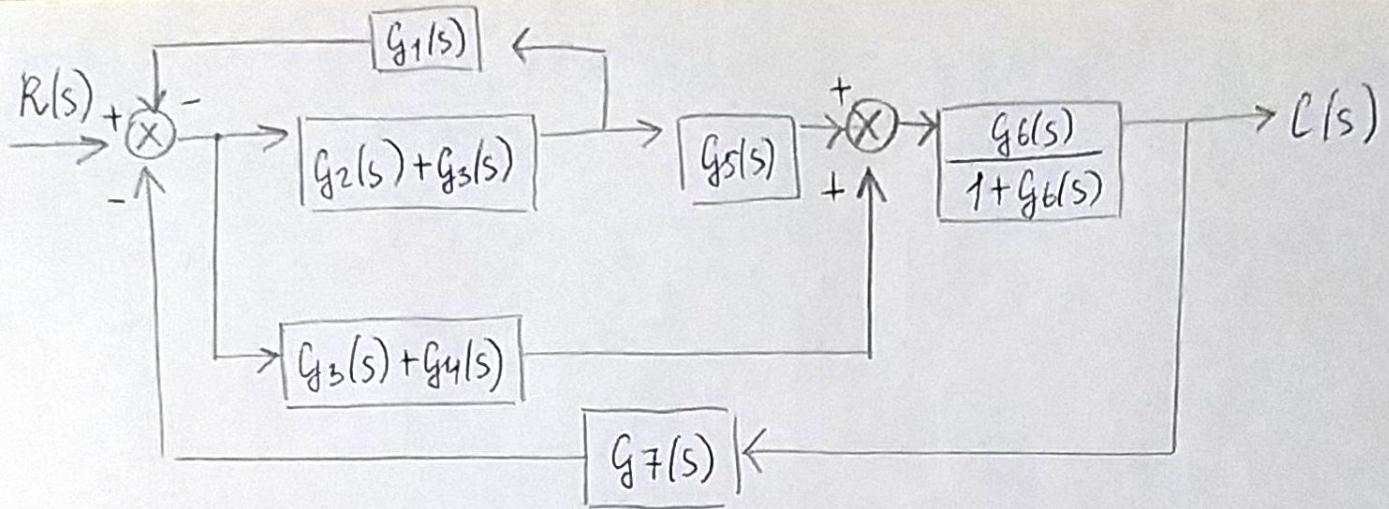


1. a)





$$T(s) = \frac{(g_2(s) + g_3(s)) \cdot (g_3(s) + g_4(s) + g_2(s)g_5(s) + g_3(s)g_5(s)) \cdot g_6(s)}{(1 + g_1(s)g_2(s) + g_1(s)g_3(s)) (g_2(s) + g_3(s)) \cdot (1 + g_6(s))}$$

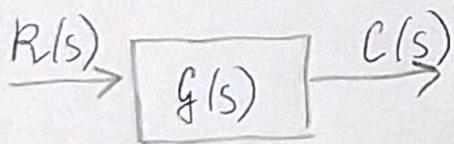

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$$\frac{g_6(s) \cdot (g_3(s) + g_4(s) + g_2(s)g_5(s) + g_3(s)g_5(s))}{(1 + g_6(s)) (1 + g_1(s)g_2(s) + g_1(s)g_3(s))}$$

$$y(s) = \frac{C(s)}{R(s)} = \frac{T(s)}{1 + g_7(s)T(s)} =$$

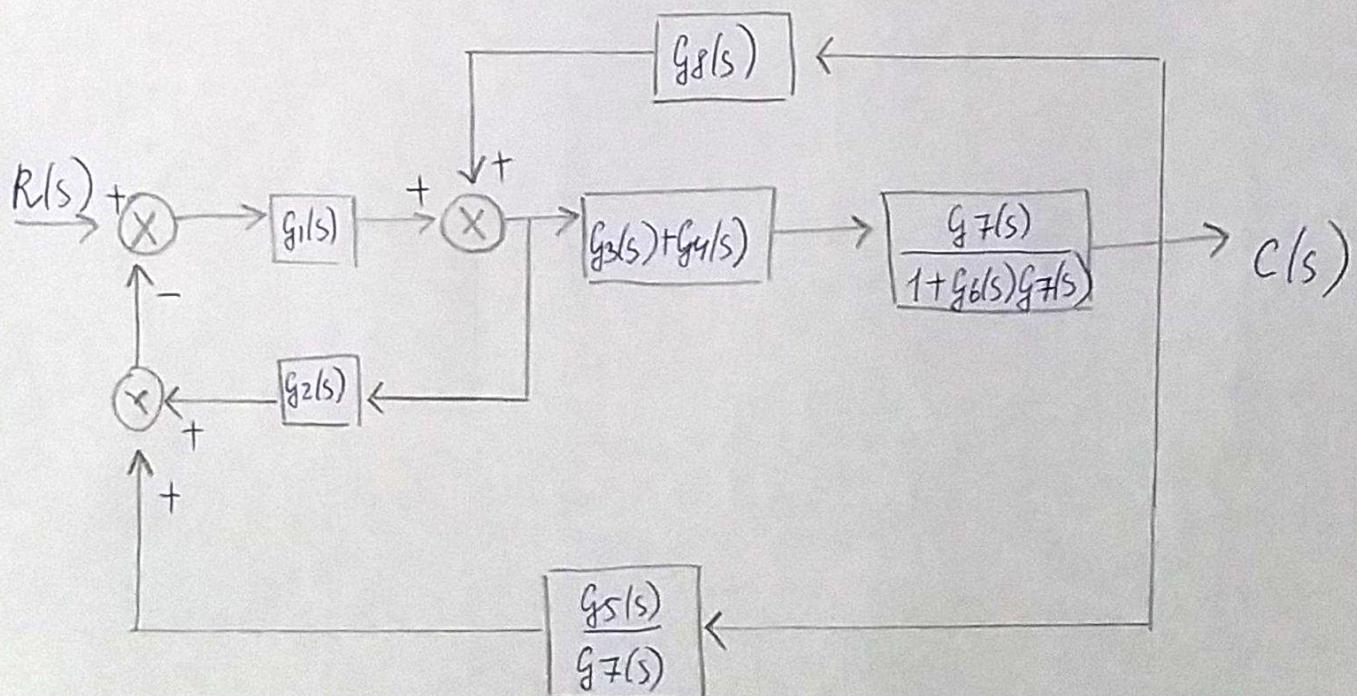
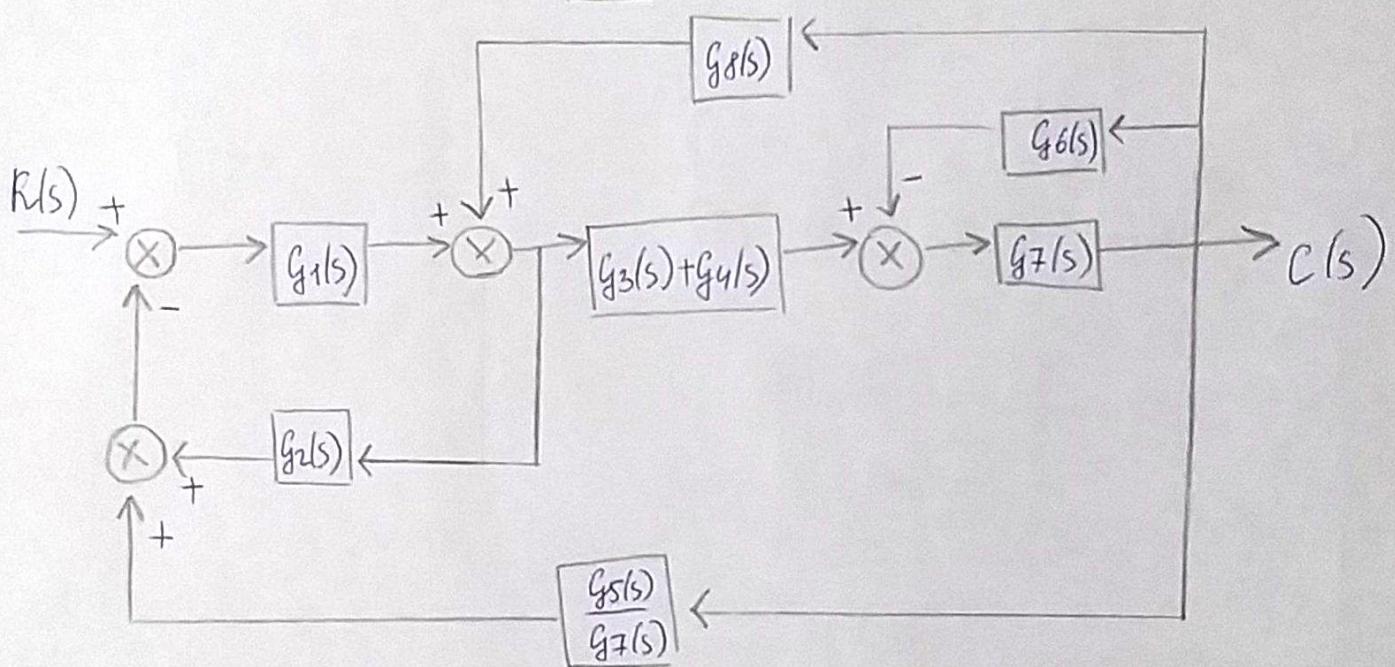
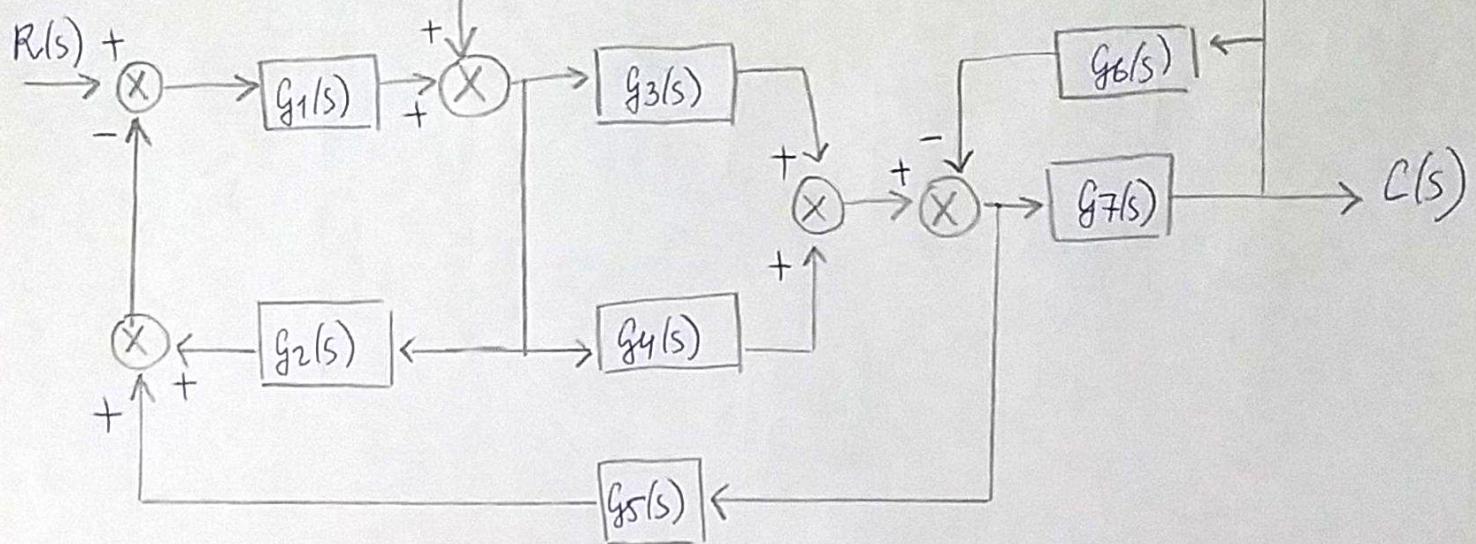
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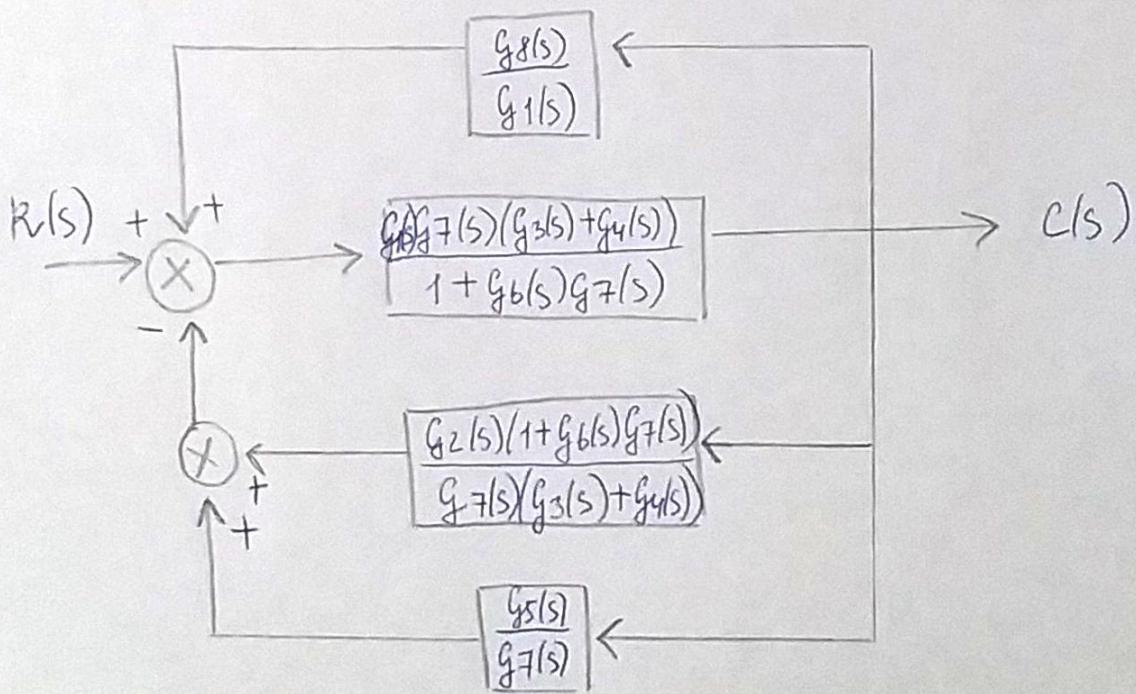
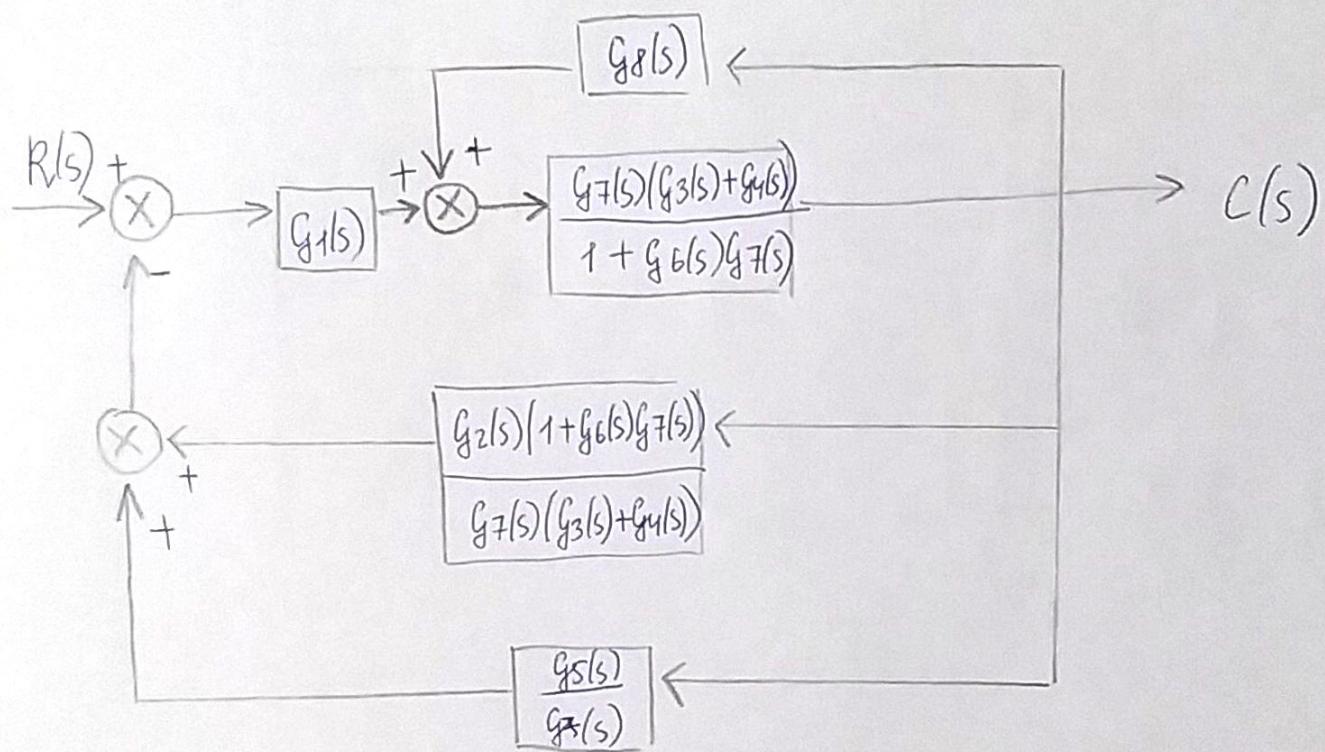
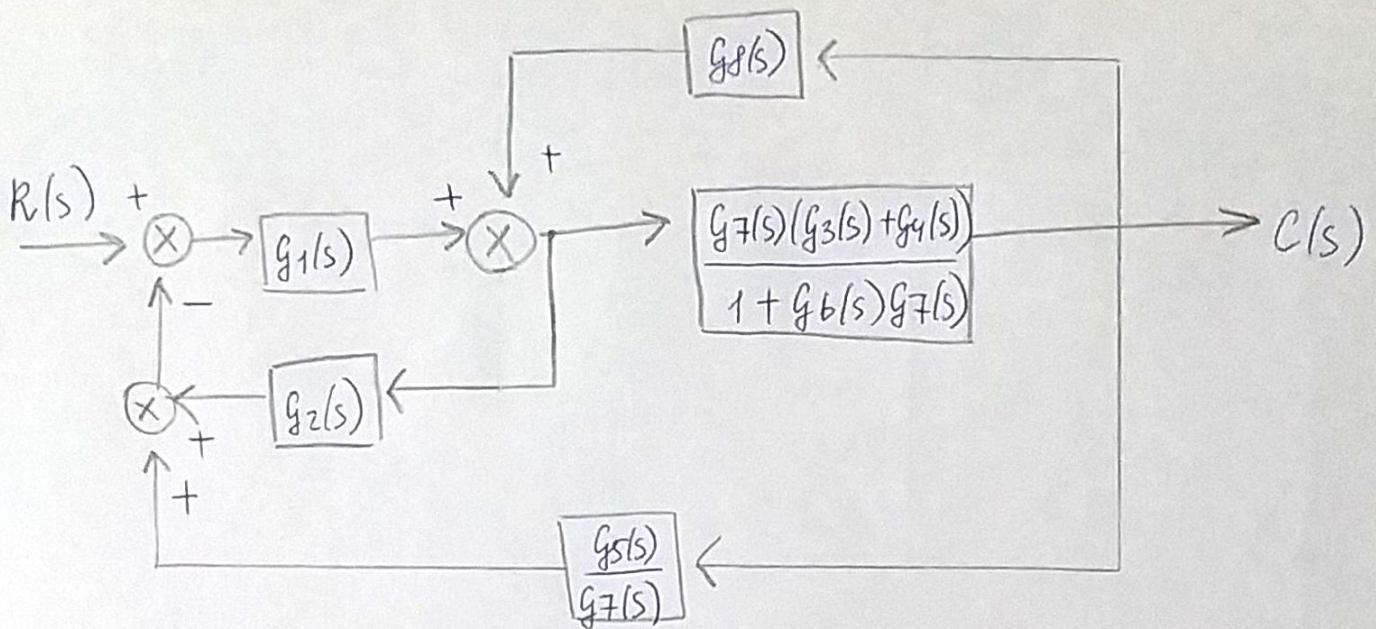

$$\frac{g_6(s) \cdot (g_3(s) + g_4(s) + g_2(s)g_5(s) + g_3(s)g_5(s))}{(1 + g_6(s))(1 + g_1(s)g_2(s) + g_1(s)g_3(s)) + g_6(s)g_7(s)(g_3(s) + g_4(s) + g_2(s)g_5(s) + g_3(s)g_5(s))}$$

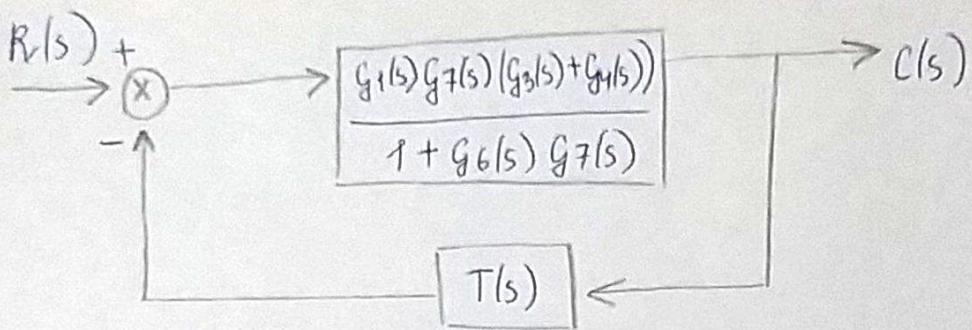


, where  $G(s)$  is given above

b)







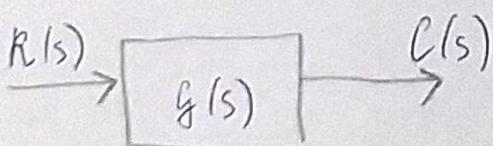
$$T(s) = \frac{G_5(s)}{G_7(s)} + \frac{G_2(s)(1 + G_6(s)G_7(s))}{G_7(s)(G_3(s) + G_4(s))} - \frac{G_8(s)}{G_1(s)} =$$

$$\frac{G_1(s)G_5(s)(G_3(s) + G_4(s)) + G_1(s)G_2(s)(1 + G_6(s)G_7(s)) - G_7(s)G_8(s)(G_3(s) + G_4(s))}{G_1(s)G_7(s)(G_3(s) + G_4(s))}$$

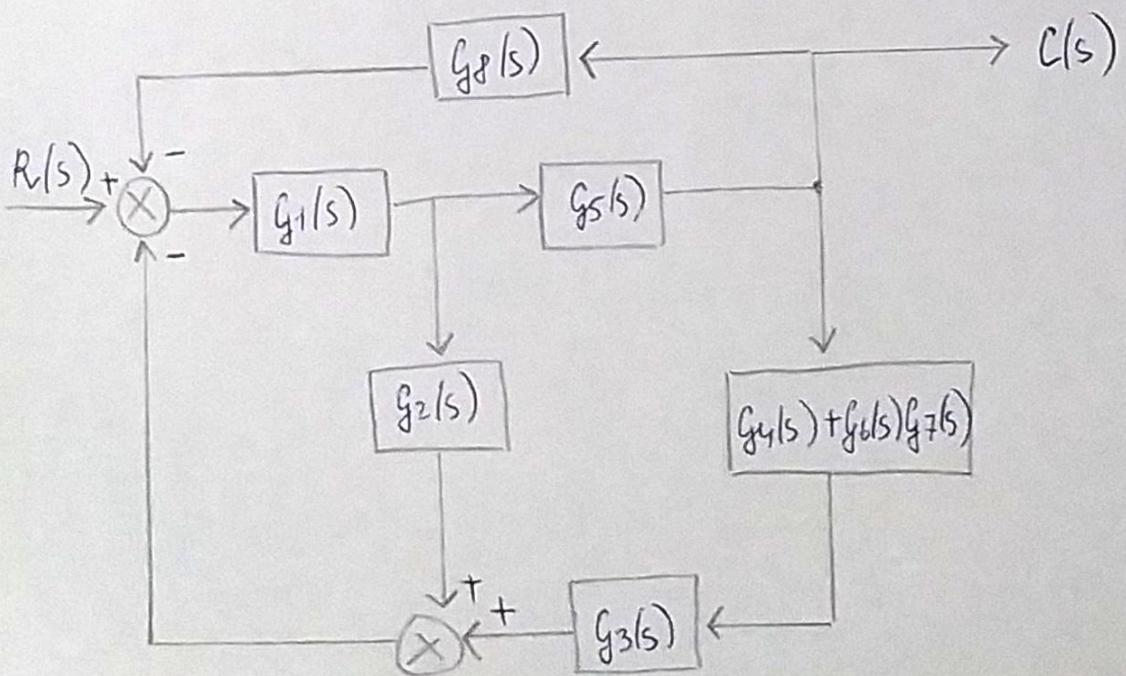
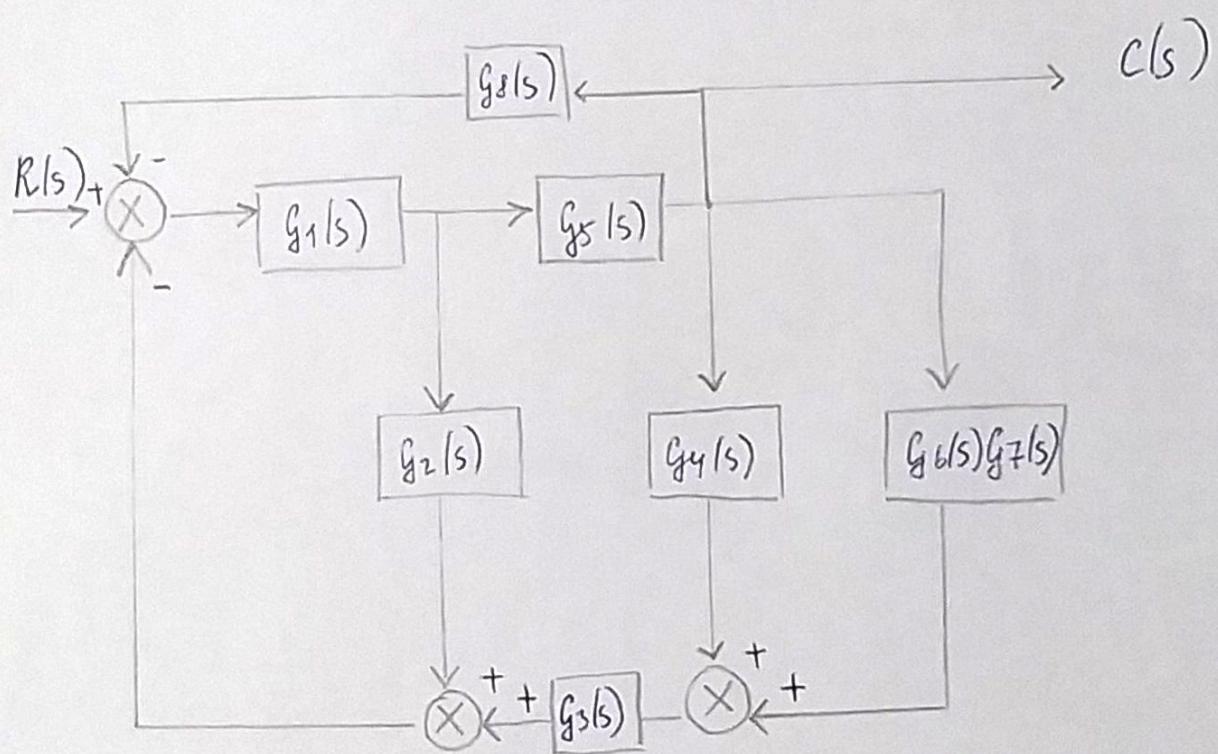
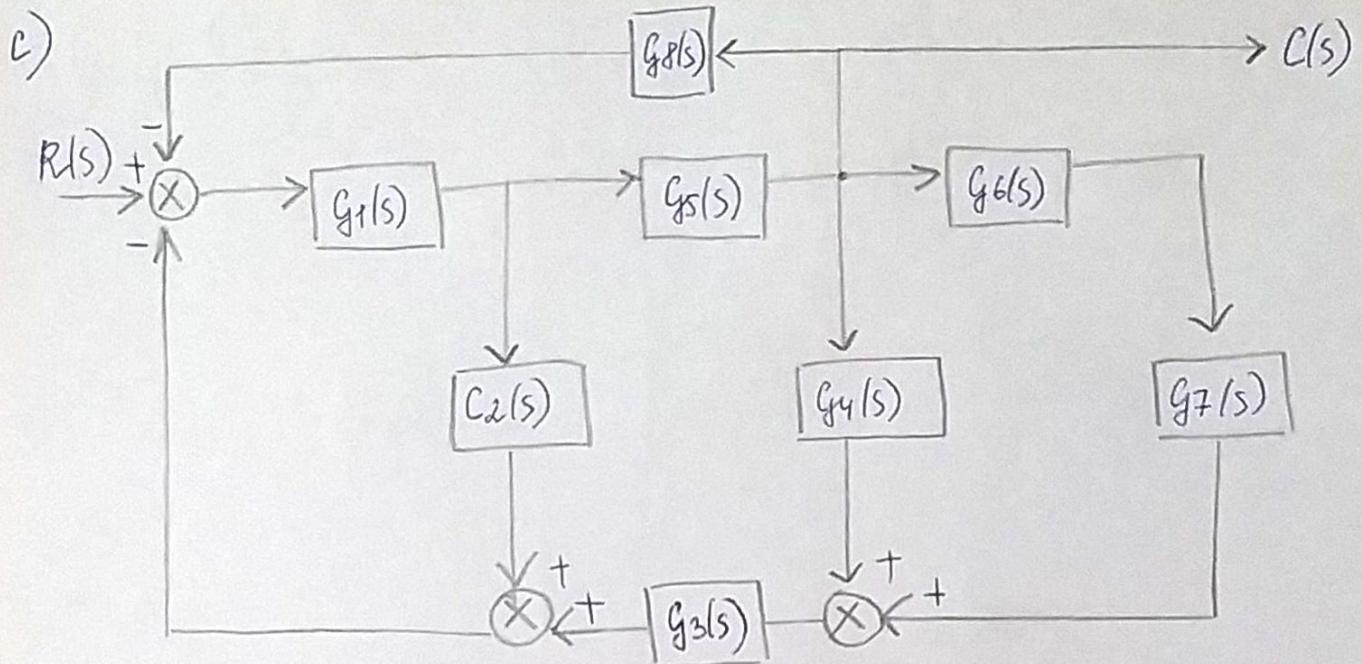
$$G(s) = \frac{C(s)}{R(s)} = \frac{\frac{G_1(s)G_7(s)(G_3(s) + G_4(s))}{1 + G_6(s)G_7(s)}}{1 + \frac{G_1(s)G_7(s)(G_3(s) + G_4(s))}{1 + G_6(s)G_7(s)} \cdot T(s)} =$$

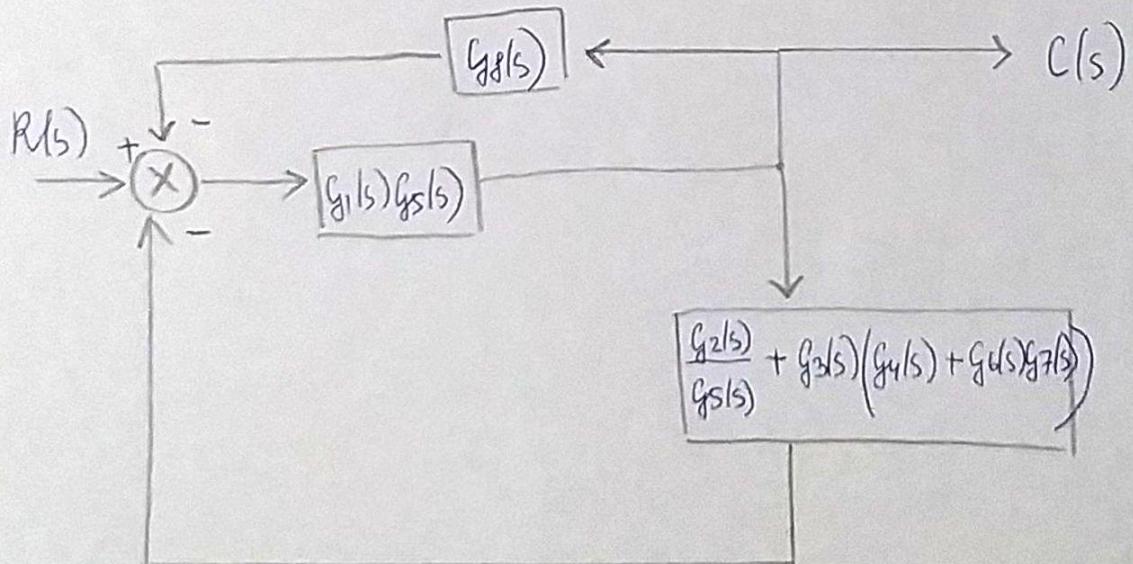
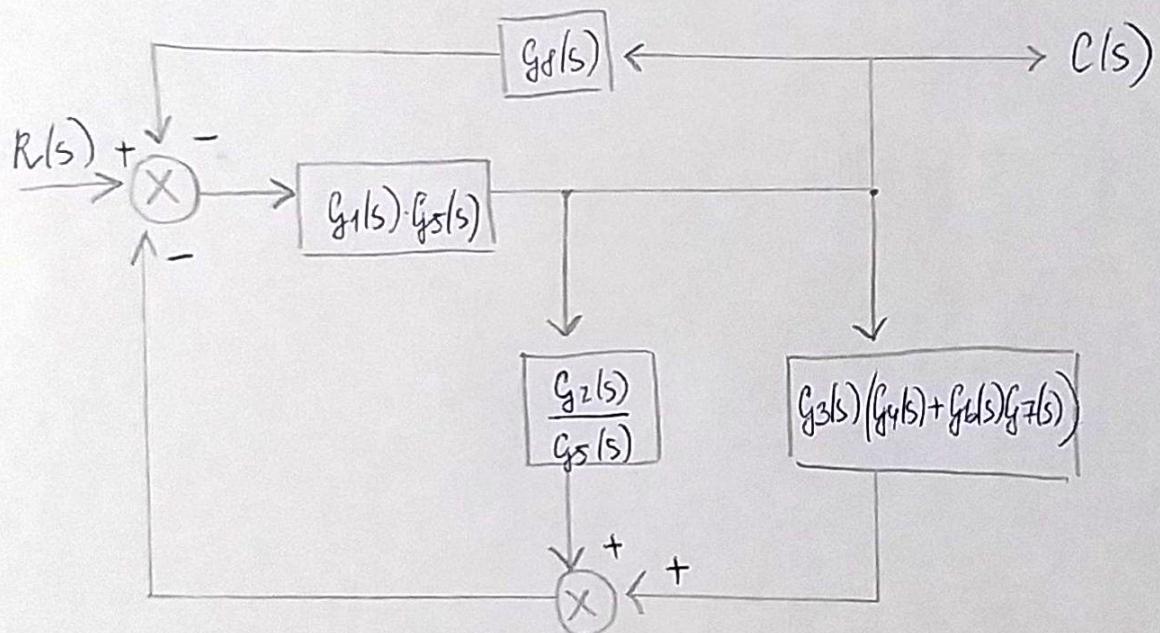
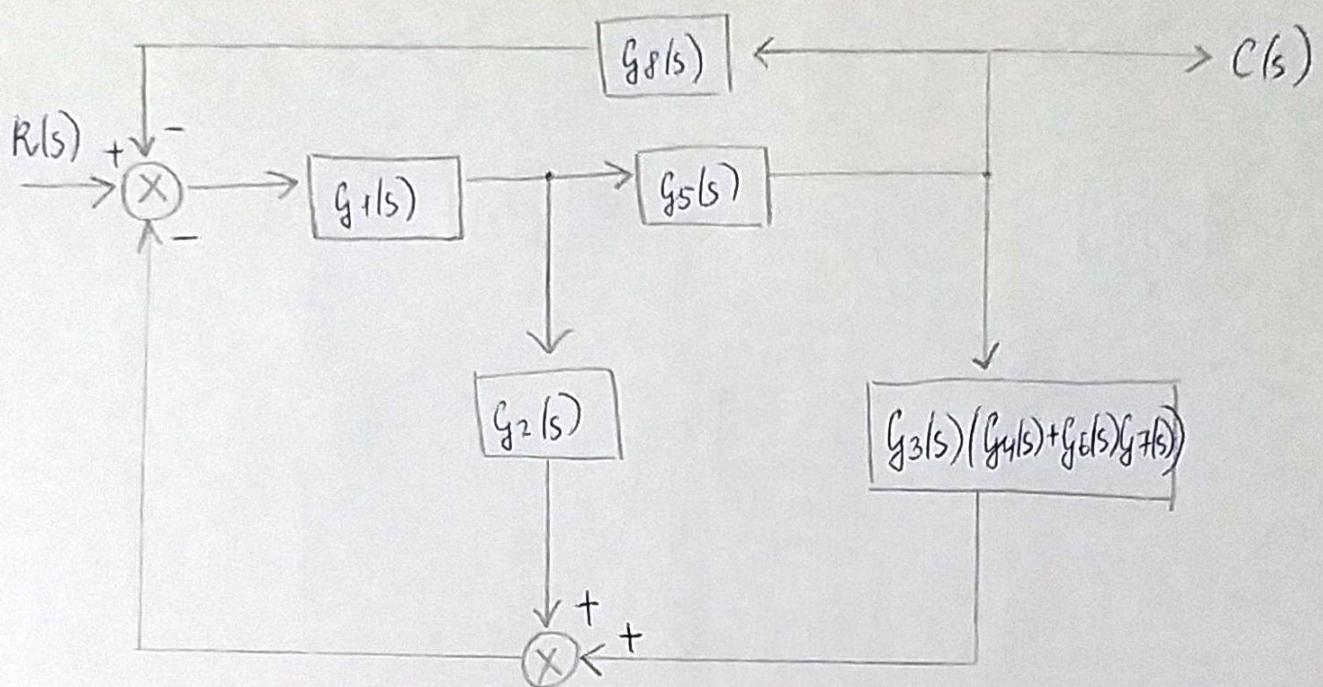
$$\frac{G_1(s)G_7(s)(G_3(s) + G_4(s))}{1 + G_6(s)G_7(s)(G_3(s) + G_4(s))}$$

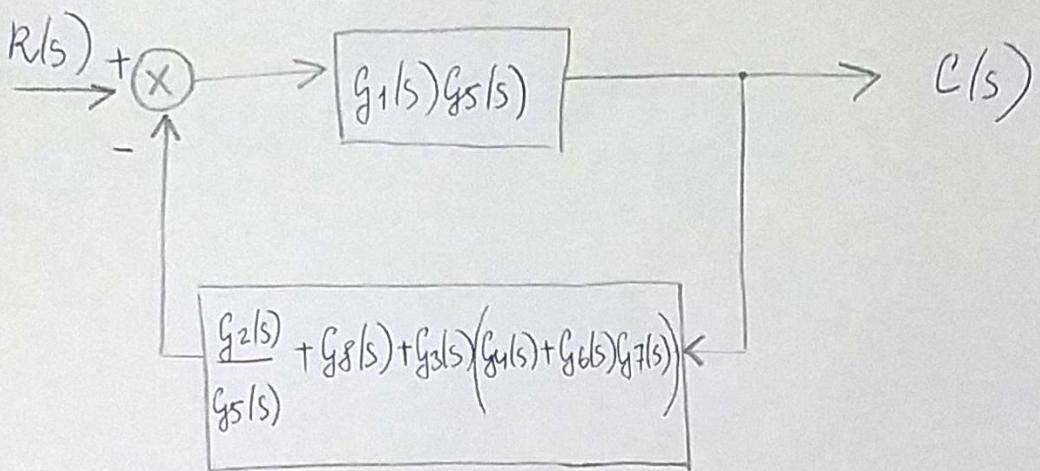
$$1 + G_6(s)G_7(s)(G_3(s) + G_4(s)) (G_1(s)G_5(s) - G_7(s)G_8(s)) + G_1(s)G_2(s)(1 + G_6(s)G_7(s))$$



, where  $G(s)$  is given above



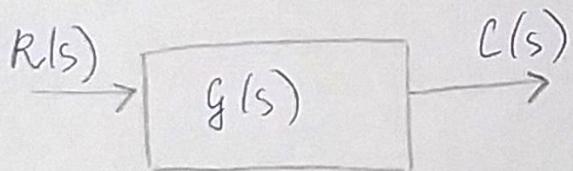




$$G(s) = \frac{C(s)}{R(s)} = \frac{G_1(s) G_5(s)}{1 + G_1(s) G_5(s) \left( \frac{G_2(s)}{G_5(s)} + G_8(s) + G_3(s)(G_4(s) + G_6(s) G_7(s)) \right)} =$$

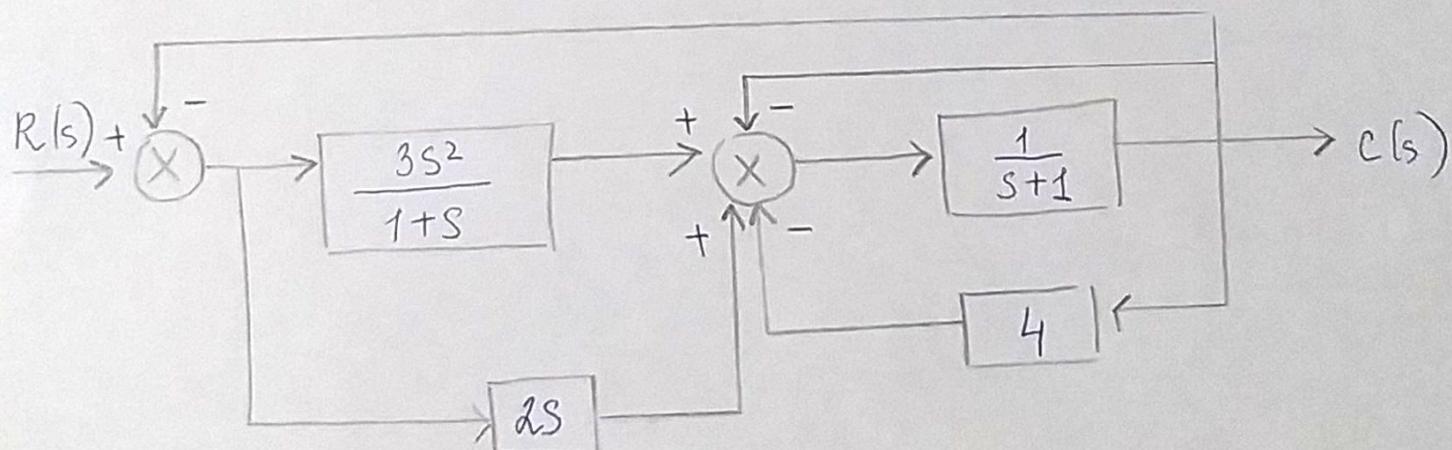
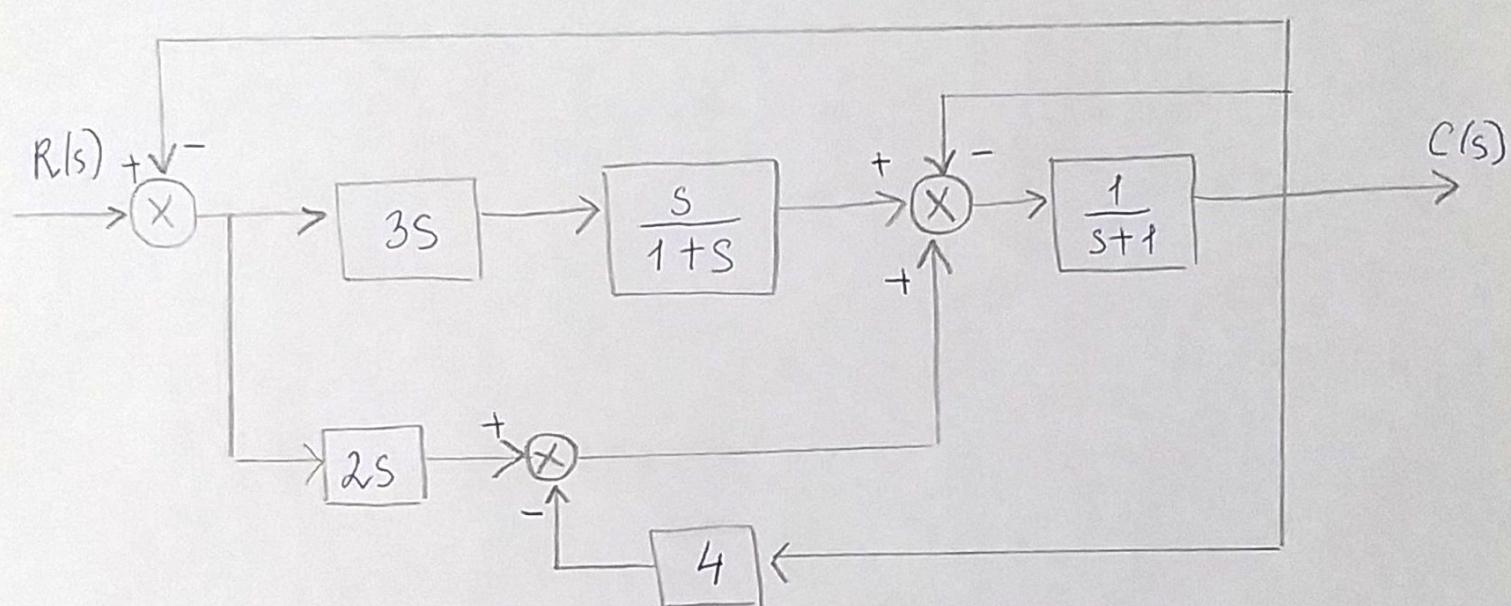
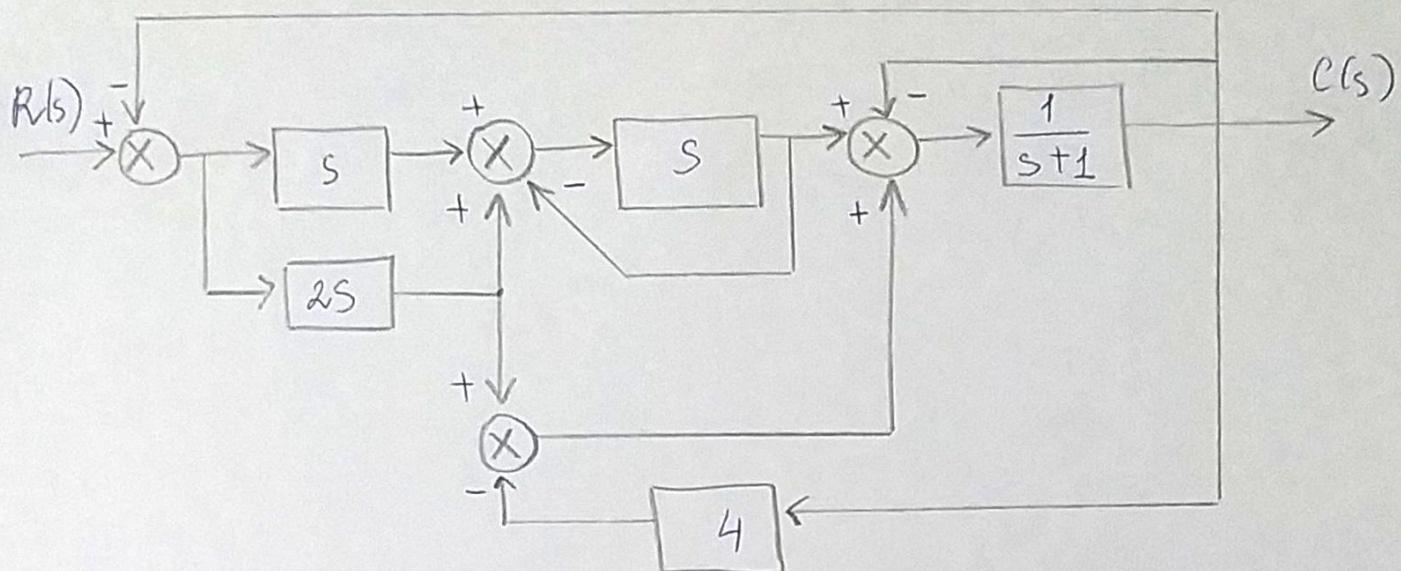

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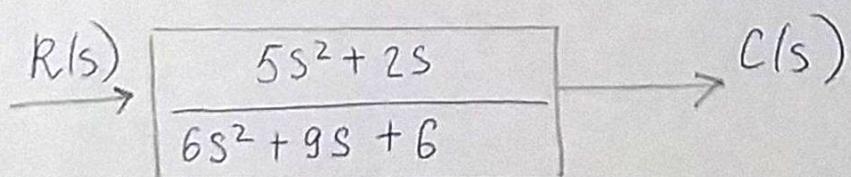
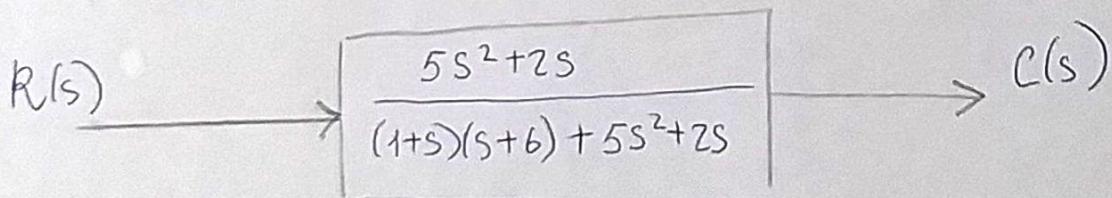
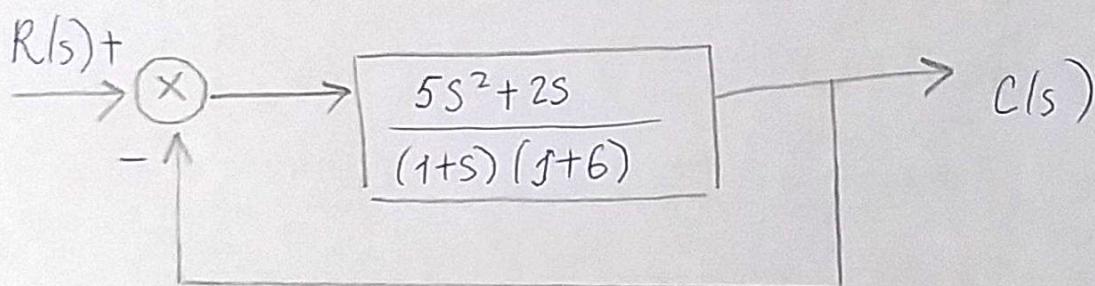
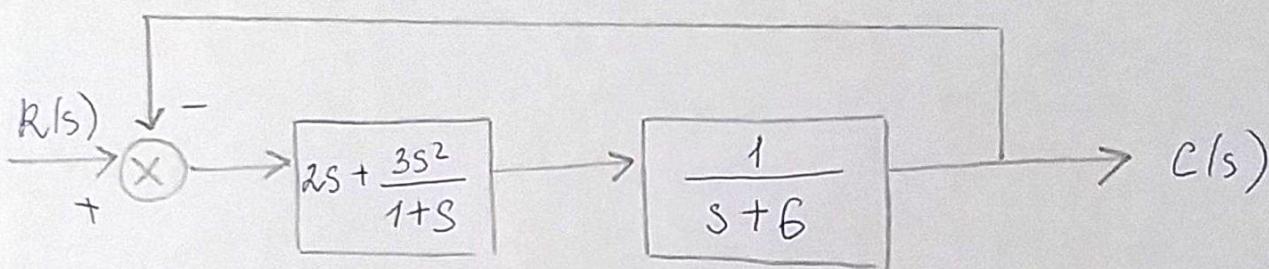
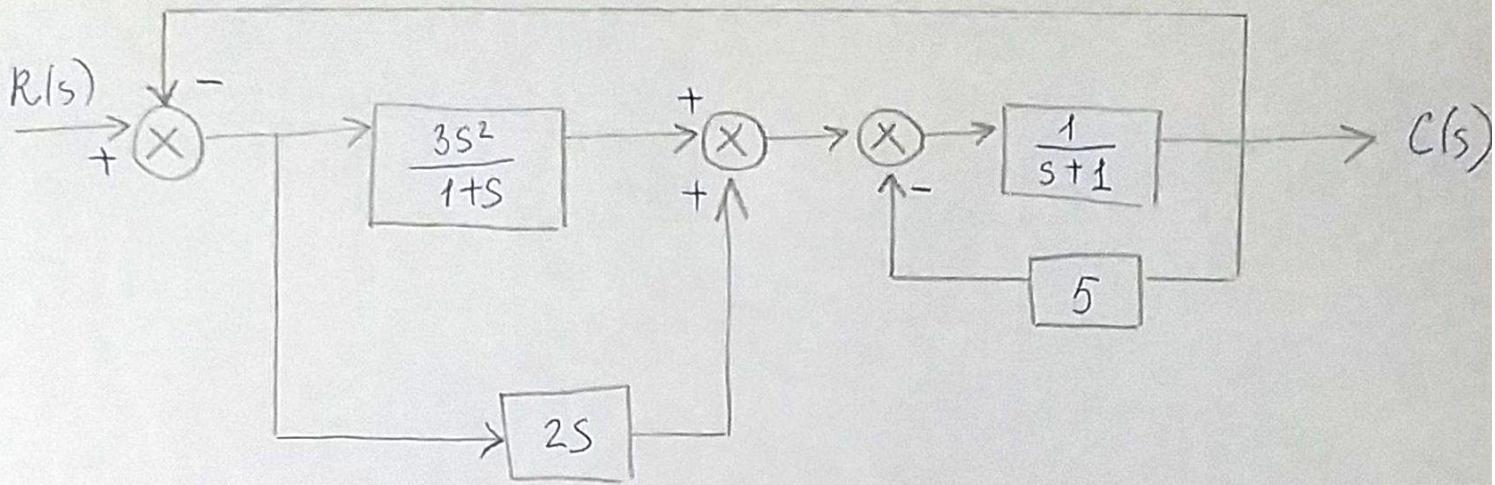

$$\frac{G_1(s) G_5(s)}{1 + G_1(s) \left[ G_2(s) + G_5(s) G_8(s) + G_3(s) G_5(s) (G_4(s) + G_6(s) G_7(s)) \right]}$$



, where  $G(s)$  is given above

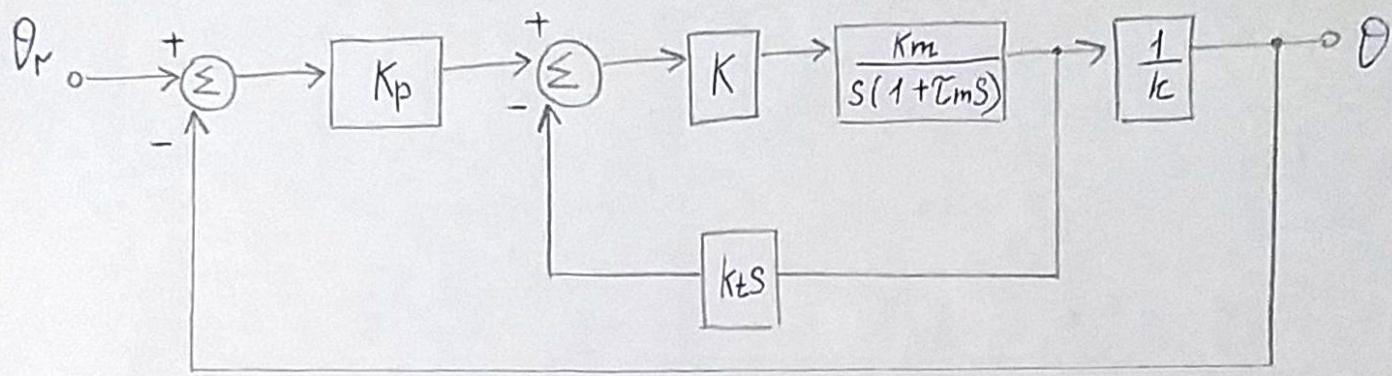
d)



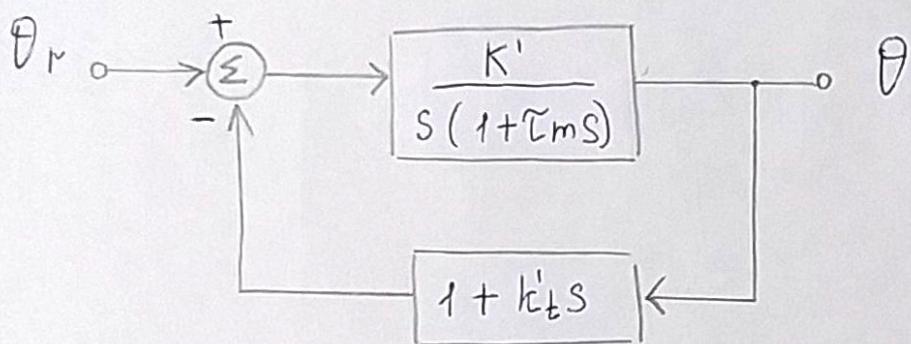


$$\Rightarrow f(s) = \frac{5s^2 + 2s}{6s^2 + 9s + 6}$$

2. Problem 4.6



a)



b)

a)  $K'$ ,  $k'_t$  - ?

b) system type,  $K_V$  - ?

c) Does the addition of tachometer feedback with positive  $k_t$  increase or decrease  $K_V$  - ?

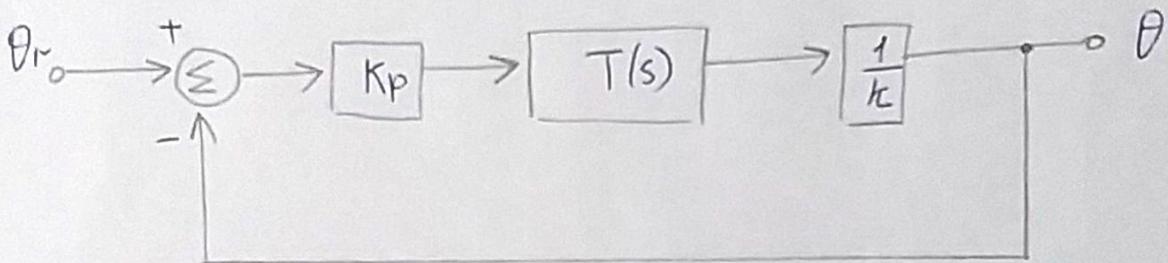
a) From Fig b),

$$\frac{\theta(s)}{\theta_r(s)} = \frac{\frac{K'}{s(1+\tau_ms)}}{1 + \frac{K'(1+k'_t s)}{s(1+\tau_ms)}} = \frac{K'}{s(1+\tau_ms) + K'(1+k'_t s)}$$

$$= \frac{K'}{s + \zeta_m s^2 + K' + K' k_t s} \quad [1]$$

From Fig a),

$$T(s) = \frac{\frac{KK_m}{s(1+\zeta_m s)}}{1 + \frac{KK_m k_t s}{s(1+\zeta_m s)}} = \frac{KK_m}{s(1+\zeta_m s) + KK_m k_t s}$$



$$\frac{\theta(s)}{\theta_r(s)} = \frac{\frac{K_p K K_m}{k(s(1+\zeta_m s) + K K_m k_t s)}}{1 + \frac{K_p K K_m}{k(s(1+\zeta_m s) + K K_m k_t s)}} =$$

$$\frac{K_p K K_m}{k(s(1+\zeta_m s) + K K_m k_t s) + K_p K K_m} = \frac{K_p K K_m}{ks + k\zeta_m s^2 + K K_m k_t s + K_p K K_m}$$

$$= \frac{\frac{K_p K K_m}{k}}{s + \zeta_m s^2 + \frac{K_p K K_m}{k} + K K_m k_t s} \quad [2]$$

Comparing [1] and [2],

$$K' = \frac{K_p K K_m}{k}$$

$$K' k_t' = K K_m k_t$$

$$\frac{K_p K K_m}{k} k_t' = K K_m k_t$$

$$k_t' = \frac{k k_t}{K_p}$$

$$K' = \frac{K_p K K_m}{k}, \quad k_t' = \frac{k k_t}{K_p}$$

b) let  $L(s)$  be an open-loop transfer function

$$L(s) = \frac{K_p T(s)}{k} = \frac{K_p K K_m}{k(s(1 + \zeta_m s) + K K_m k_t s)} = \frac{K_p K K_m}{ks + k\zeta_m s^2 + K K_m k_t k s} = \frac{\frac{K_p K K_m}{s(k\zeta_m s + K K_m k_t k + k)}}{}$$

As  $L(s)$  has one integrator, the system type is Type-1

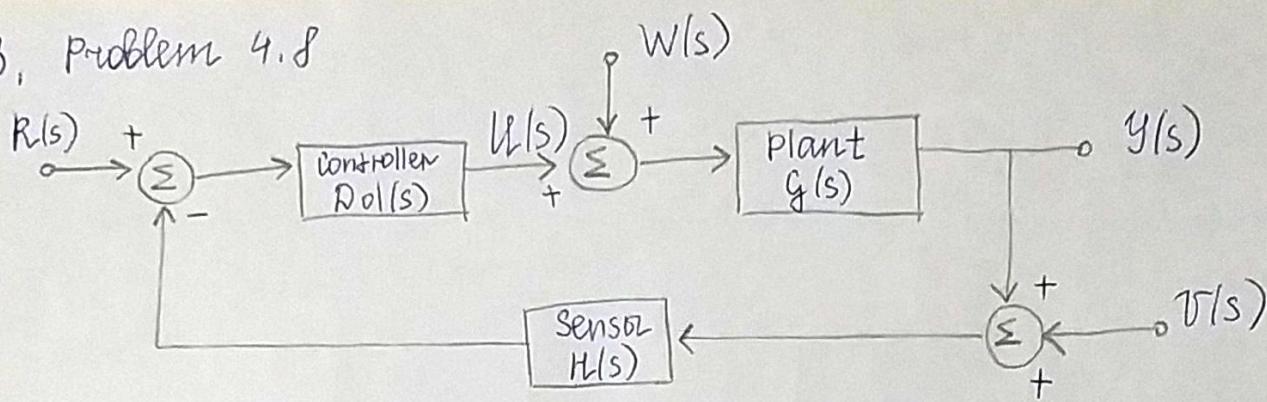
$$K_v = \lim_{s \rightarrow 0} s L(s) = \lim_{s \rightarrow 0} s \cdot \frac{K_p K K_m}{s(k\zeta_m s + K K_m k_t k + k)} = \frac{K_p K K_m}{K K_m k_t k + k} =$$

$$\frac{\frac{K_p K K_m}{k}}{k K_m k_t + 1} = \frac{K'}{K' k_t' + 1}$$

$$K_v = \frac{K'}{K' k_t' + 1}$$

c) As  $K_v = \frac{K_p K K_m}{K K_m k_t k + k}$ , the addition of tachometer feedback with positive  $k_t$  decreases  $K_v$ .

3, Problem 4.8



$$G(s) = \frac{1}{s} \quad D_C(s) = \frac{2(s+1)}{s} \quad H(s) = \frac{100}{s+100}$$

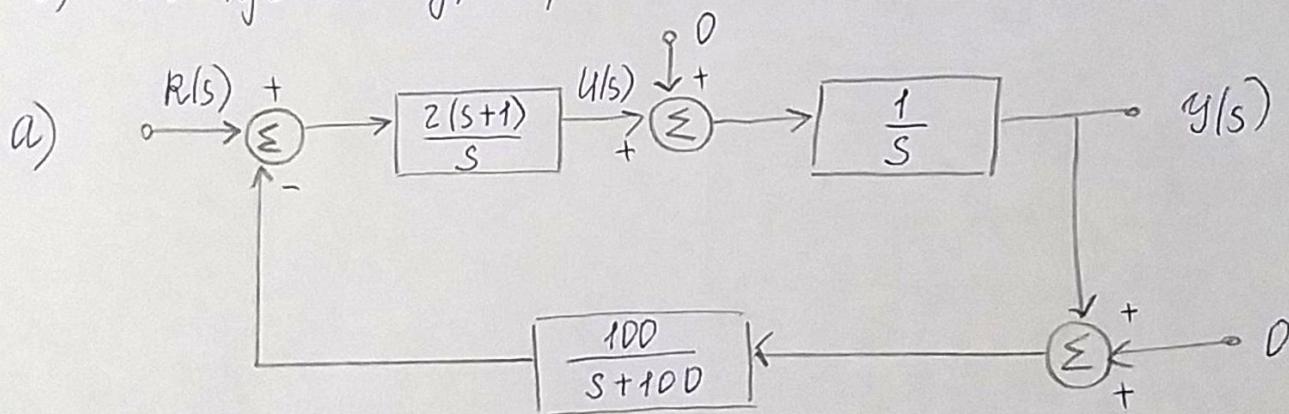
a)  $W=0 \quad \frac{Y(s)}{R(s)} - ?$

b)  $R=0 \quad \frac{Y(s)}{W(s)} - ?$

c)  $R$  is a unit-step input,  $W=0$  the tracking error -?

d)  $R$  is a unit-ramp input,  $W=0$  the tracking error -?

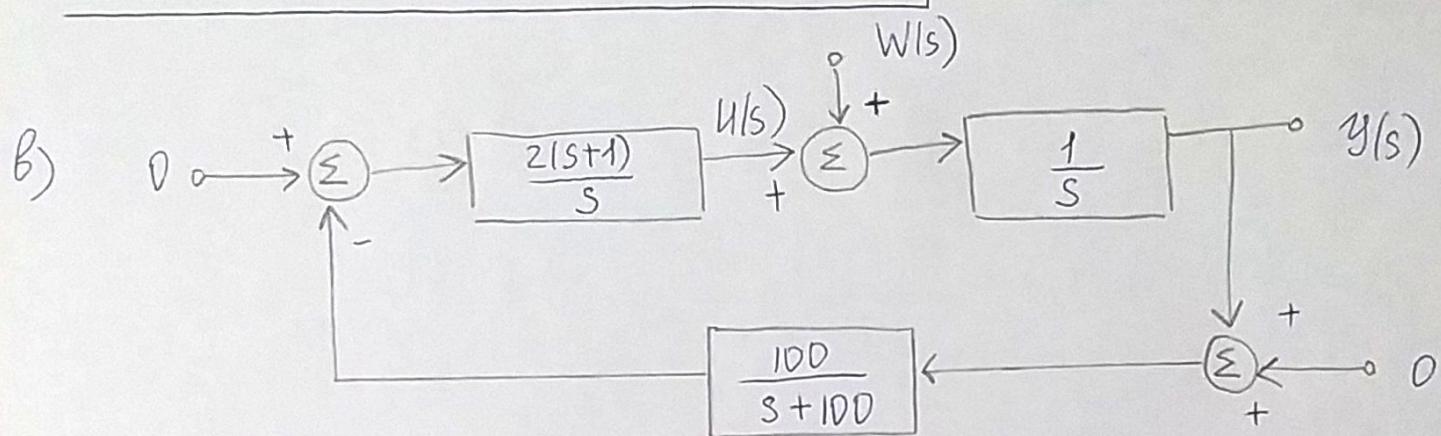
e) the system type, the error coefficient -?



$$\frac{Y(s)}{R(s)} = \frac{\frac{2(s+1)}{s} \cdot \frac{1}{s}}{1 + \frac{2(s+1)}{s} \cdot \frac{1}{s} \cdot \frac{100}{s+100}} = \frac{\frac{2(s+1)}{s^2}}{\frac{s^2(s+100) + 200(s+1)}{s^2(s+100)}} =$$

$$\frac{\frac{2(s+1) \cdot s^2 (s+100)}{s^2 (s^3 + 100s^2 + 200s + 200)}}{s^3 + 100s^2 + 200s + 200} = \frac{2(s+1)(s+100)}{s^3 + 100s^2 + 200s + 200}$$

$$\frac{y(s)}{R(s)} = \frac{2(s+1)(s+100)}{s^3 + 100s^2 + 200s + 200}$$



$$\frac{y(s)}{w(s)} = \frac{\frac{1}{s}}{1 - \left( -\frac{1}{s} \cdot \frac{100}{s+100} \cdot \frac{2(s+1)}{s} \right)} = \frac{\frac{1}{s}}{\frac{s^2(s+100) + 200(s+1)}{s^2(s+100)}} = \frac{\frac{1}{s}}{\frac{s(s+100)}{s^3 + 100s^2 + 200s + 200}} =$$

$$\frac{y(s)}{w(s)} = \frac{s(s+100)}{s^3 + 100s^2 + 200s + 200}$$

c)  $E(s) = R(s) - y(s) = R(s) - \frac{2(s+1)(s+100)}{s^3 + 100s^2 + 200s + 200} R(s) =$

$$\left( 1 - \frac{2(s+1)(s+100)}{s^3 + 100s^2 + 200s + 200} \right) R(s) = \left( \frac{s^3 + 98s^2 - 2s}{s^3 + 100s^2 + 200s + 200} \right) R(s)$$

$$\text{As } R(s) = \frac{1}{s}, \quad E(s) = \frac{s(s^2 + 98s - 2)}{s^3 + 100s^2 + 200s + 200} \cdot \frac{1}{s} =$$

$$\frac{s^2 + 98s - 2}{s^3 + 100s^2 + 200s + 200}$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \left( \frac{s^2 + 98s - 2}{s^3 + 100s^2 + 200s + 200} \right) = 0$$

$$\Rightarrow E_{ss} = 0$$

d)  $E(s) = \left( \frac{s^3 + 98s^2 - 2s}{s^3 + 100s^2 + 200s + 200} \right) \cdot R(s)$

$$\text{As } R(s) = \frac{1}{s^2}, \quad E(s) = \frac{s(s^2 + 98s - 2)}{s^3 + 100s^2 + 200s + 200} \left( \frac{1}{s^2} \right) =$$

$$\frac{s^2 + 98s - 2}{s(s^3 + 100s^2 + 200s + 200)}$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{s^2 + 98s - 2}{s(s^3 + 100s^2 + 200s + 200)} =$$

$$-\frac{2}{200} = -0.01$$

$$\Rightarrow E_{ss} = -0.01$$

e) As  $E_{ss}$  is a nonzero constant if  $R$  is a unit-step input, the system type is Type-1

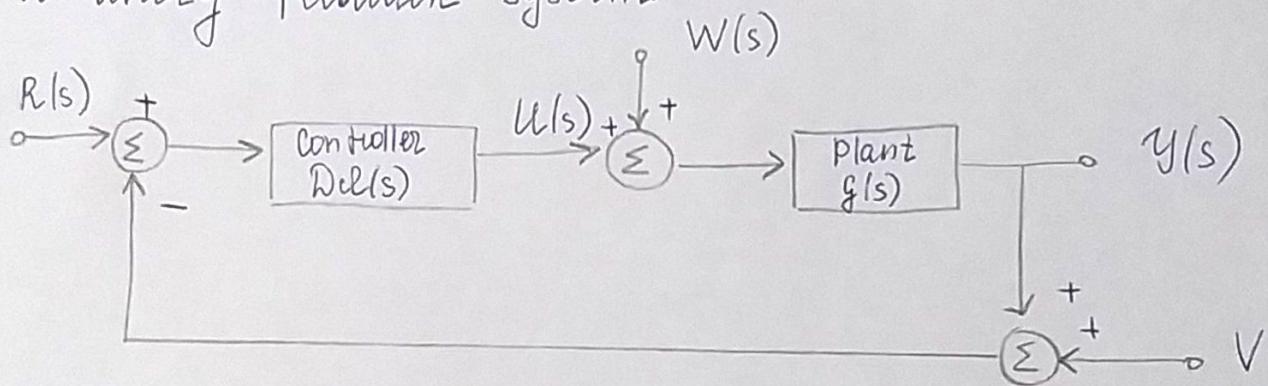
For a type-1 system, the error coefficient is the velocity error constant

$$e_{ss} = \frac{1}{K_v} \Rightarrow K_v = \frac{1}{e_{ss}} = \frac{1}{-0.01} = -100$$

$$K_v = -100$$

#### 4. Problem 4.12

A unity feedback system



$$\frac{Y(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

system type, error constant - ?

$$\frac{Y(s)}{R(s)} = \frac{G(s)Dcl(s)}{1 + G(s)Dcl(s)}$$

$$\frac{G(s)Dcl(s)}{1 + G(s)Dcl(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{1 + G(s)Dcl(s)}{G(s)Dcl(s)} = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2}$$

$$1 + \frac{1}{G(s)D(s)} = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2}$$

$$\frac{1}{G(s)D(s)} = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2} - 1$$

$$\frac{1}{G(s)D(s)} = \frac{s(s + 2\zeta\omega_n)}{\omega_n^2}$$

$$G(s)D(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$$L(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$L(s)$  has one integrator,  $\Rightarrow$  the system type is Type-1

For Type-1, corresponding error constant is velocity error constant

$$K_v = \lim_{s \rightarrow 0} s L(s) = \lim_{s \rightarrow 0} \frac{s \omega_n^2}{s(s + 2\zeta\omega_n)} = \frac{\omega_n^2}{2\zeta\omega_n}$$

$$K_v = \frac{\omega_n}{2\zeta}$$

5. Problem 4.13

$$G(s) = \frac{1}{s^2 + 2\zeta s + 1} \quad D_c(s) = \frac{K(s+a)}{(s+b)}$$

- a unity feedback structure

- a) Ignoring stability, what are the constraints on  $K, a, b$  so that the system is Type 1?
- b) What are the constraints placed on  $K, a, b$  so that the system is both stable and Type 1?
- c) What are the constraints on  $a, b$  so that the system is both Type 1 and remains stable for every positive value for  $K$ ?

$$a) L(s) = G(s)D_c(s) = \frac{K(s+a)}{(s+b)(s^2 + 2\zeta s + 1)}$$

Type-1 systems :  $L(s)$  has one integrator

$$\Rightarrow b=0$$

$a \neq 0$  - not to cancel pole at origin

$K \neq 0$  - so that  $L(s) \neq 0$

The constraints are  
 $b=0, a \neq 0, K \neq 0$

$$b) \text{ For Type-1, } L(s) = \frac{K(s+a)}{s(s^2 + 2\zeta s + 1)} \quad |B=0, a \neq 0, K \neq 0)$$

$$\text{For stability, } K(s+a) + s(s^2 + 2\zeta s + 1) = 0$$

must have roots with strictly negative real part

$$s^3 + 2\zeta s^2 + s + Ks + Ka = 0$$

$$s^3 + 2\zeta s^2 + (K+1)s + Ka = 0$$

Using Routh stability criterion,

$s^3$	1	$K+1$
$s^2$	$2\zeta$	$Ka$
$s^1$	$\frac{2\zeta(K+1)-Ka}{2\zeta}$	
$s^0$	$Ka$	

$$\Rightarrow 2\zeta > 0, \quad \frac{2\zeta(K+1)-Ka}{2\zeta} > 0, \quad Ka > 0 \quad \text{must be satisfied}$$

The constraints are :

$$B=0, \quad 2\zeta(K+1)-Ka > 0, \quad Ka > 0$$

c) If  $K > 0$ ,  $Ka > 0$  becomes  $a > 0$

$$2\zeta(K+1) - Ka > 0$$

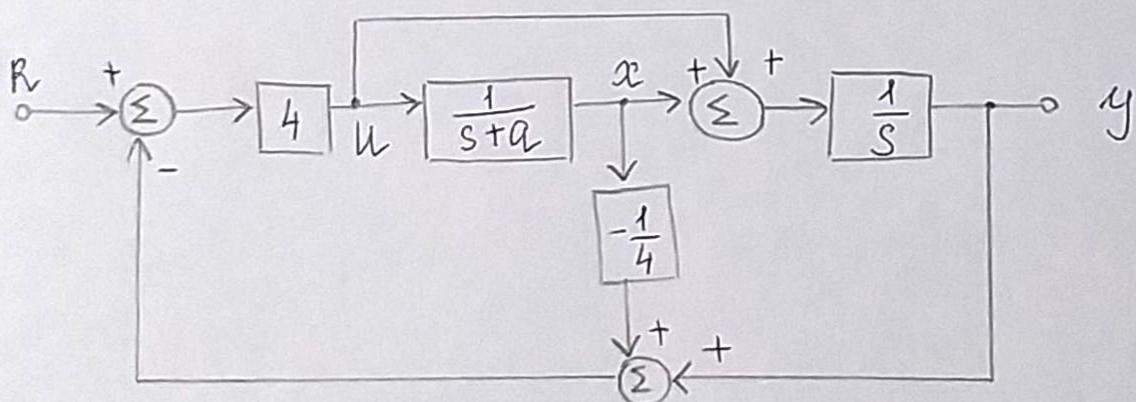
$$2\zeta K + 2\zeta - Ka > 0$$

$$2\zeta + K(2\zeta - a) > 0 \quad (2\zeta > 0, K > 0, a > 0)$$

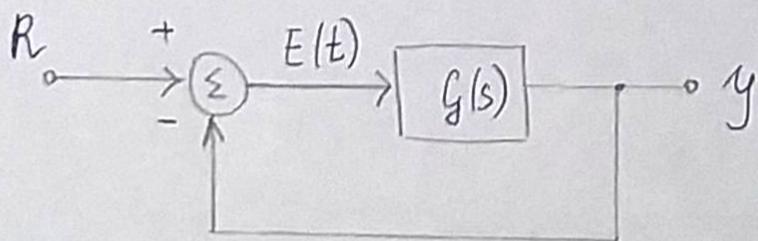
$\Rightarrow a < 2\zeta$  so that the system remains stable for every positive value for  $K$

The constraints are:  $B = 0$ ,  $D < a < 2\zeta$

### 6. Problem 4.2D



a)



b)

$$a) \quad g(s) - ?$$

b)  $a = 1$ , system type, error constant - ?

c)  $a = 1 + \delta a$ , system type, error constant - ?

a) From Fig. a),  $y(s) = \frac{1}{s} \left( 1 + \frac{1}{s+a} \right) u(s)$

$$u(s) = 4 \left[ R(s) - \left( y(s) - \frac{1}{4} x(s) \right) \right]$$

$$x(s) = \frac{1}{s+a} u(s)$$

$$\Rightarrow u(s) = 4R(s) - 4y(s) + \frac{1}{s+a} u(s)$$

$$\left( 1 - \frac{1}{s+a} \right) u(s) = 4 [R(s) - y(s)]$$

$$\frac{s+a-1}{s+a} u(s) = 4 [R(s) - y(s)]$$

$$u(s) = \frac{4(s+a)}{s+a-1} [R(s) - y(s)]$$

$$\Rightarrow y(s) = \frac{1}{s} \cdot \frac{s+a+1}{s+a} \cdot \left( \frac{4(s+a)}{s+a-1} [R(s) - y(s)] \right) =$$

$$\frac{4(s+a+1)}{s(s+a-1)} [R(s) - y(s)]$$

From Fig B),  $y(s) = G(s) \cdot [R(s) - y(s)]$

$$\Rightarrow G(s) = \frac{4(s+a+1)}{s(s+a-1)}$$

b) If  $a = 1$ ,  $G(s) = \frac{4(s+2)}{s \cdot s} = \frac{4(s+2)}{s^2}$

As open-loop transfer function has two integrators,

the system type is Type-2

Corresponding error constant for Type-2 is acceleration

error constant :  $K_a = \lim_{s \rightarrow 0} s^2 G(s) = \lim_{s \rightarrow 0} s^2 \cdot \frac{4(s+2)}{s^2} = 8$

$$K_a = 8$$

c) If  $a = 1 + \delta a$ ,  $G(s) = \frac{4(s+2+\delta a)}{s(s+\delta a)}$

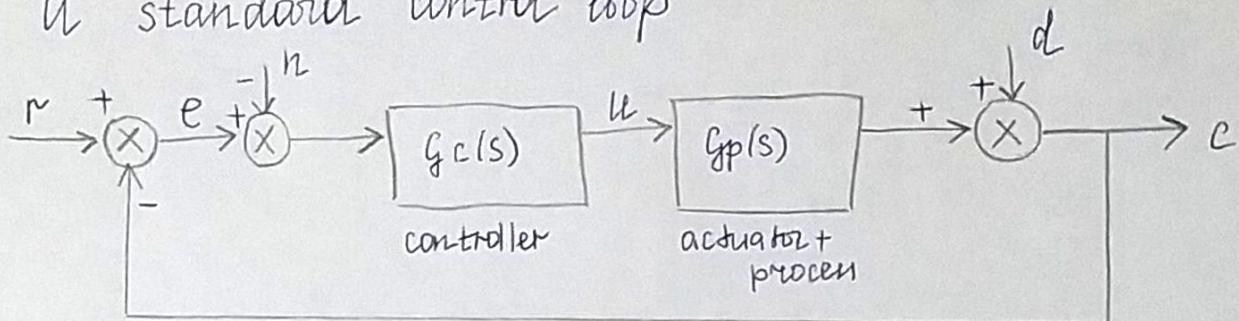
As  $G(s)$  has one integrator, the system type is Type-1

Corresponding error constant for Type-1 is velocity

error constant:  $K_v = \lim_{s \rightarrow 0} s G(s) = \lim_{s \rightarrow 0} \frac{s \cdot 4(s+2+\delta a)}{s(s+\delta a)} = \frac{4(2+\delta a)}{\delta a}$

$$K_v = \frac{4(2+\delta a)}{\delta a}$$

7. A standard control loop



$$G_c(s) = \frac{s+4}{s} \quad G_p(s) = \frac{250(s+6)(s+7)}{(s+3)(s^2+2s+12)} \quad (L(s) = G_c(s)G_p(s))$$

the steady-state errors for inputs  $2u(t)$ ,  $2tu(t)$ ,  $3t^2u(t)$  - ?

a)  $r(t) = 2u(t)$        $R(s) = \frac{2}{s}$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} sS(s)R(s) = \lim_{s \rightarrow 0} s \frac{1}{1+L(s)} \frac{2}{s} =$$

$$\lim_{s \rightarrow 0} \frac{2}{1 + \frac{250(s+4)(s+6)(s+7)}{s(s+3)(s^2+2s+12)}} = \lim_{s \rightarrow 0} \frac{2s(s+3)(s^2+2s+12)}{s(s+3)(s^2+2s+12) + 250(s+4)(s+6)(s+7)} =$$

0

$$\Rightarrow e_{ss} = 0 \quad \text{for } r(t) = 2u(t)$$

b)  $r(t) = 2tu(t)$        $R(s) = \frac{2}{s^2}$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{1}{1+L(s)} \frac{2}{s^2} = \lim_{s \rightarrow 0} \frac{2}{s + sL(s)} =$$

$$\lim_{s \rightarrow 0} \frac{2}{s + \frac{250(s+4)(s+6)(s+7)}{(s+3)(s^2+2s+12)}} = \lim_{s \rightarrow 0} \frac{2(s+3)(s^2+2s+12)}{s(s+3)(s^2+2s+12) + 250(s+4)(s+6)(s+7)} =$$

$$\frac{2 \cdot 3 \cdot 12}{250 \cdot 4 \cdot 6 \cdot 7} = \frac{3}{1750}$$

$$\Rightarrow e_{ss} = \frac{3}{1750} \quad \text{for } r(t) = 2t u(t)$$

c)  $r(t) = 3t^2 u(t)$        $R(s) = \frac{6}{s^3}$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{1}{1+L(s)} \frac{6}{s^3} = \lim_{s \rightarrow 0} \frac{6}{s^2 + L(s)s^2} =$$

$$\lim_{s \rightarrow 0} \frac{6}{s^2 + \frac{250s(s+4)(s+6)(s+7)}{(s+3)(s^2+2s+12)}} = \lim_{s \rightarrow 0} \frac{6(s+3)(s^2+2s+12)}{s^2(s+3)(s^2+2s+12) + 250(s+4)(s+6)(s+7)}$$

$$= \infty$$

$$\Rightarrow e_{ss} = \infty \quad \text{for } r(t) = 3t^2 u(t)$$

f. A standard control loop

$$G_c(s) = \frac{3}{s+7} \quad G_p(s) = \frac{10}{s^2+4s+7} \quad (L(s) = G_c(s) \cdot G_p(s))$$

the static error constants  $K_p, K_v, K_a - ?$

the steady-state errors for inputs  $5u(t), 10tu(t), 36t^2u(t)$

$$K_p = \lim_{s \rightarrow 0} L(s) = \lim_{s \rightarrow 0} \frac{30}{(s+7)(s^2+4s+7)} = \frac{30}{7 \cdot 7} = \frac{30}{49}$$

$$K_v = \lim_{s \rightarrow 0} sL(s) = \lim_{s \rightarrow 0} \frac{30s}{(s+7)(s^2+4s+7)} = 0$$

$$K_a = \lim_{s \rightarrow 0} s^2 L(s) = \lim_{s \rightarrow 0} \frac{30s^2}{(s+7)(s^2+4s+7)} = 0$$

$$\Rightarrow \left| K_p = \frac{3D}{4g}, \quad K_v = D, \quad K_a = D \right|$$

As the system is a type-0 system,

$$\text{for } r(t) = 5u(t) \quad e_{ss} = 5 \cdot \frac{1}{1+K_p} = 5 \cdot \frac{1}{1+\frac{3D}{4g}} = \frac{245}{79}$$

$$\text{for } r(t) = 10tu(t) \quad e_{ss} = \infty$$

$$\text{for } r(t) = 36t^2u(t) \quad e_{ss} = \infty$$

$$\Rightarrow \boxed{\begin{array}{ll} e_{ss} = \frac{245}{79} & \text{for } r(t) = 5u(t) \\ e_{ss} = \infty & \text{for } r(t) = 10tu(t) \\ e_{ss} = \infty & \text{for } r(t) = 36t^2u(t) \end{array}}$$