Robt 303 Zarema Balgabekova HW 4

1. $G_c(s) = K \qquad G_p(s) = \frac{1}{s(s+8)}$

$$L(s) = G_c(s) \cdot G_p(s) = K \frac{1}{s(s+8)}$$

$N(s) = 1$

$D(s) = s(s+8)$

$\Rightarrow$ $n = 2$, $m = 0$, $r = n - m = 2$ $\Rightarrow$ 2 asymptotes

No zeros. Poles at 0 and -8.

$$x_a = \frac{0 - (0+8)}{2} = -4$$

$$2\psi_a = \begin{cases} (2h+1)\pi & h = 0, 1 \quad PL \\ 2h\pi & h = 0, 1 \quad NL \end{cases}$$

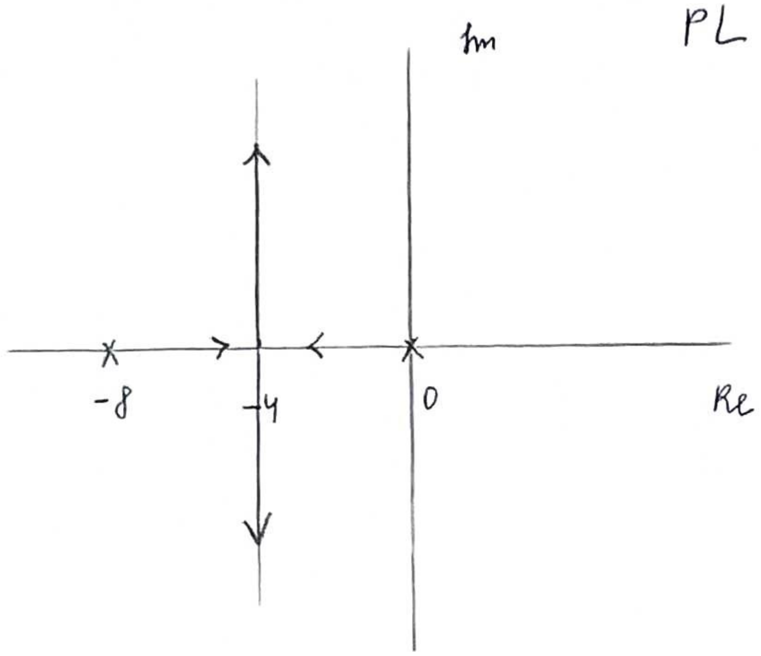
$$\Rightarrow \psi_a = \begin{cases} \frac{\pi}{2}, \frac{3\pi}{2} & PL \\ 0, \pi & NL \end{cases}$$

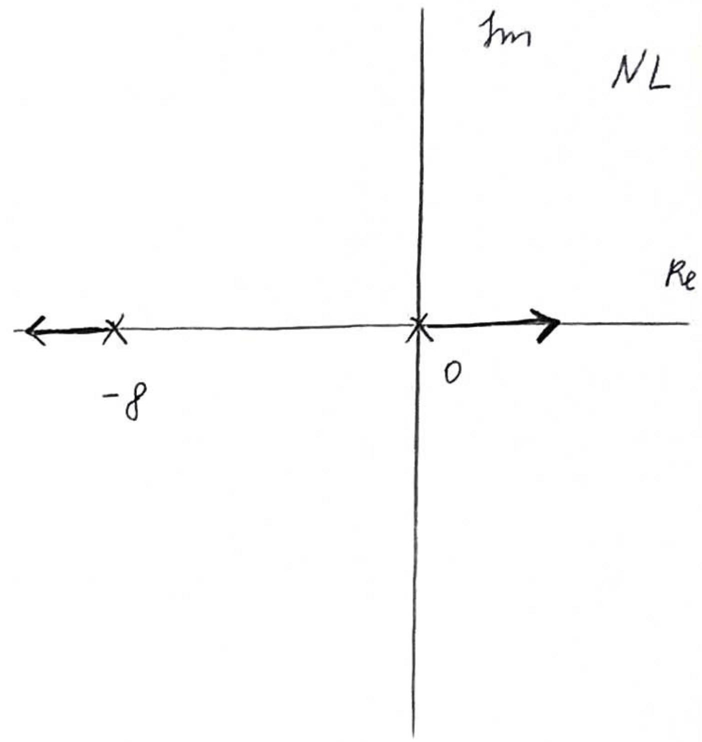
Real axis:

$(-\infty, -8)$ 2 poles $\in NL$

$(-8, 0)$ 1 pole $\in PL$

$(0, +\infty)$ 0 pole $\in NL$





Design requirements: $M_p \leq 20\%$

$T_r \leq 0.333\,s$

$T_{s,2\%} \leq 2s$

$$T_{s,2\%} = \frac{4}{\zeta \omega_n} \leq 2 \quad \Rightarrow \quad \zeta \omega_n \geq 2$$

$$\zeta = -\frac{\ln(M_p/100)}{\sqrt{\pi^2 + \ln^2(M_p/100)}} = \frac{-\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} = 0.456$$

$$\Rightarrow \quad \zeta \geq 0.456$$

$$T_r = \frac{1.8}{\omega_n} \leq 0.333 \quad \Rightarrow \quad \omega_n \geq 5.4054$$

all requirements can be satisfied with a proportional controller.

let $\zeta\omega_n = 4$, therefore, the real part of a test point is equal to -4.

We need to find the imaginary part of the test point, knowing that $\omega_n = \sqrt{[Re(s^*)]^2 + [Im(s^*)]^2}$

(s^* - a test point)

$\omega_n \geq 5.4054$ Choose $\omega_n = 7$

$$7 = \sqrt{(-4)^2 + [Im(s^*)]^2}$$

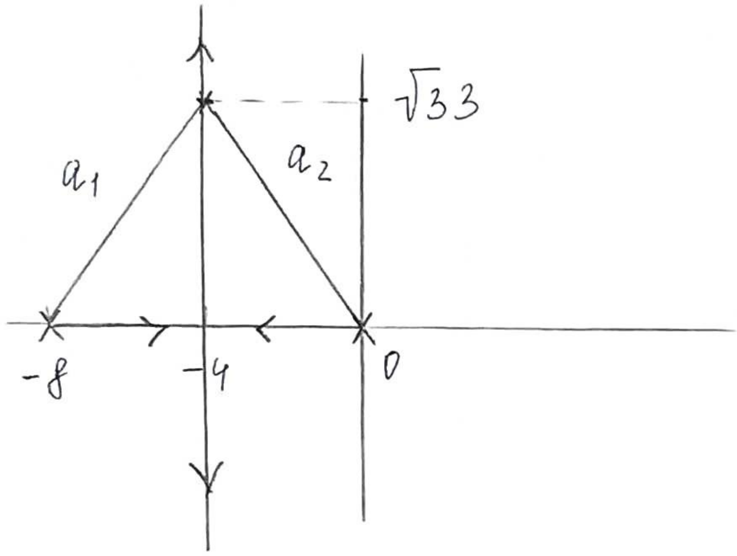
$$\Rightarrow Im(s^*) = \sqrt{33} \approx 5.745$$

If $\omega_n = 7$ and $\zeta\omega_n = 4$, then $\zeta = \frac{4}{7} \approx 0.571$

Therefore, all requirements are satisfied if the test point is: $s^* = -4 + \sqrt{33}\,j$

To find K , magnitude condition is used :

$$\left| \frac{N(s)}{D(s)} \right|_{s*} = \left| -\frac{1}{K} \right| = \frac{1}{K}$$



$$\frac{1}{a_1 \, a_2} = \frac{1}{K} \qquad a_1 = a_2 = \sqrt{4^2 + 33} = 7$$

$$\frac{1}{7 \cdot 7} = \frac{1}{K} \Rightarrow \boxed{K = 49}$$

$$\Rightarrow G_c(s) = 49$$

MATLAB verification is done with the help of *rltool(Gp)*.
Design requirements are set in Root Locus Editor window (Figure 1).

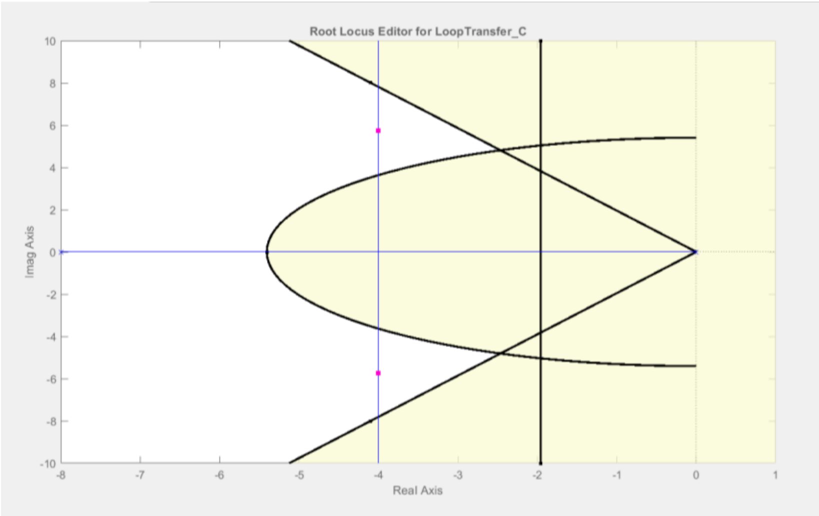


Figure 1: Setting Design Requirements

Equating the gain of a compensator to 49, we obtain the following step response and its parameters (Figure 2).

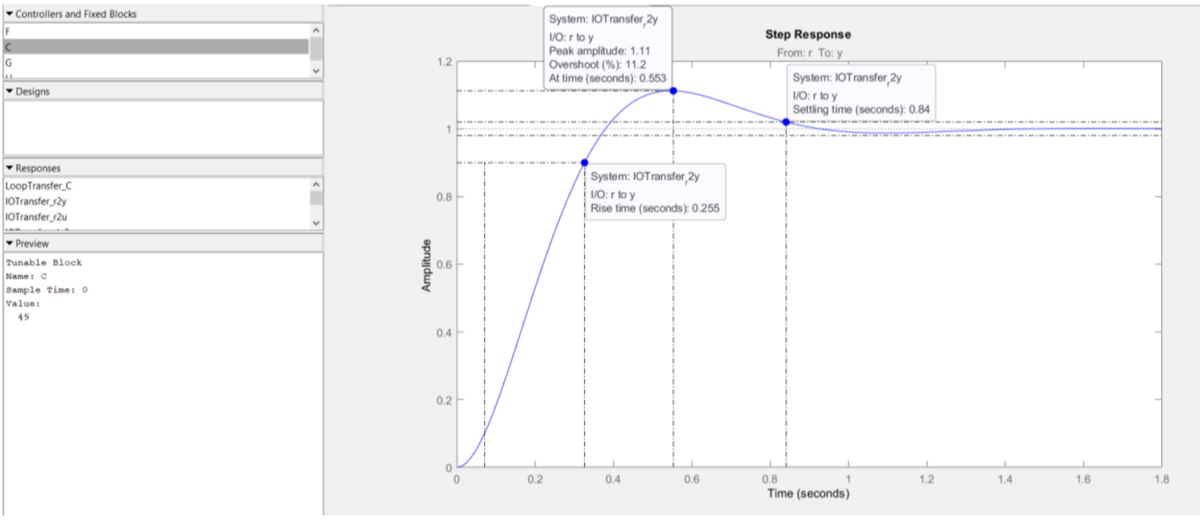


Figure 2: Step Response and Its Parameters for *Gc(s) = 49*

It is seen from Figure 2 that all design requirements are satisfied, therefore, the design is verified.

2. $G_p(s) = \frac{10}{s^2}$

Design a practically realizable compensator $G_c(s)$ to locate the dominant poles of the closed loop system at

$s = -1 \pm 13j$

$L(s) = G_c(s) \cdot G_p(s) = 10K \frac{1}{s^2}$

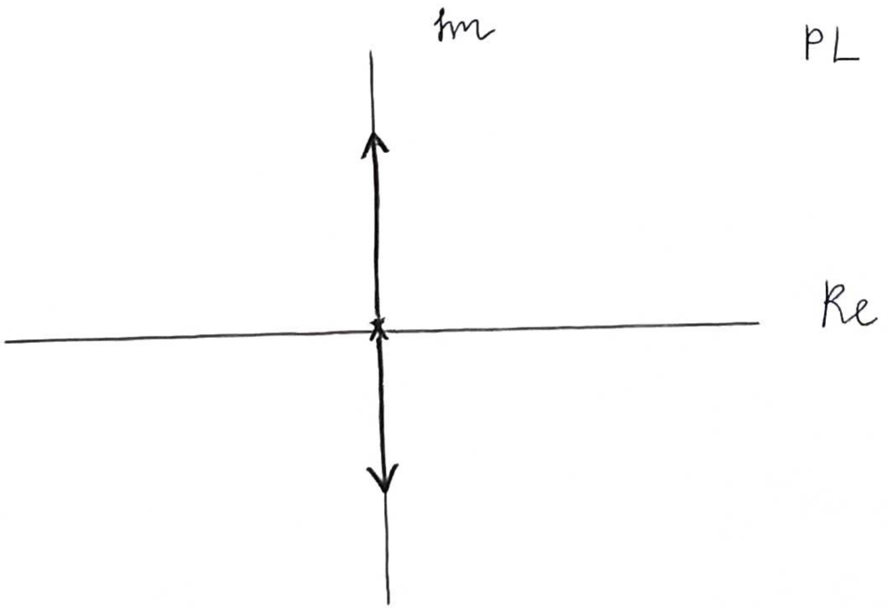
$N(s) = 1$ $\Rightarrow$ $n = 2$

$D(s) = s^2$ $\quad$ $m = 0$

$r = 2 \Rightarrow 2$ asymptotes

$x_a = 0$ $\qquad \psi_a = \frac{\pi}{2}, \frac{3\pi}{2}$ (PL)

No zeros, $\qquad$ Poles ; 2 poles at 0,



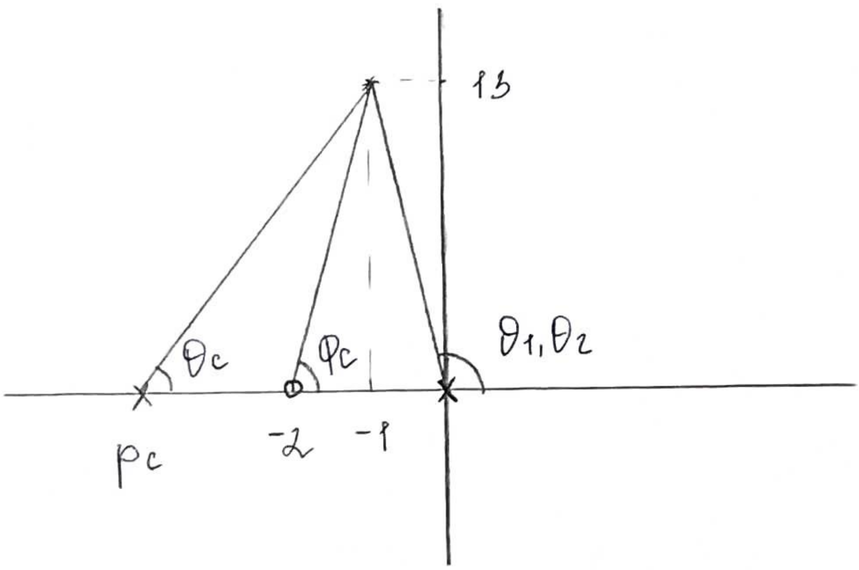
To have the dominant poles at $s = -1 \pm 13j$,
we need to change the root locus plot.

This could be done with a lead compensator:

$$G_c(s) = K \frac{s+z_c}{s+p_c}$$ (practically realizable)

Choose arbitrarily: $z_c = 2$

Find p_c using the phase condition



$$\theta_1 = \theta_2 = 180° - \tan^{-1}(13) = 94.4°$$

$$\varphi_c = \tan^{-1}(13) = 85.6°$$

$$\varphi_c - (\theta_c + \theta_1 + \theta_2) = -180°$$

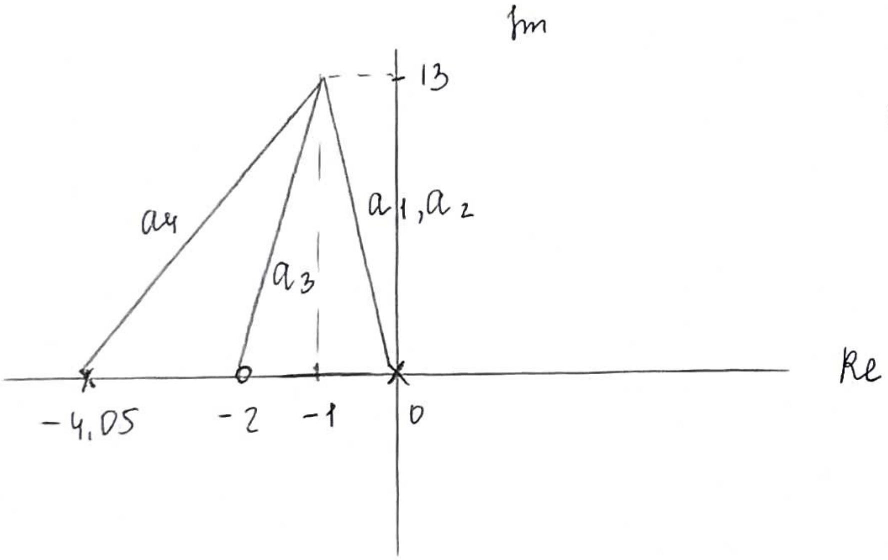
$$\theta_c = 180° + \varphi_c - \theta_1 - \theta_2 = 180° + 85.6° - 2 \cdot 94.4° = 76.8°$$

$$\tan(76.8°) = \frac{13}{p_c - 1}$$

$$p_c = 4.05$$

To find K, use the magnitude condition:

$$\left|\frac{N(s)}{D(s)}\right|_{s^*} = \left|-\frac{1}{P}\right| = \frac{1}{10K}$$



$a_1 = a_2 = a_3$

$$a_1 = \sqrt{13^2 + 1^2} = \sqrt{170}$$

$$a_4 = \sqrt{13^2 + 3.05^2} \approx 13.35$$

$$\frac{\cancel{a_3}}{a_1 \cancel{a_2} a_4} = \frac{1}{10K}$$

$$\frac{1}{\sqrt{170} \cdot 13.35} = \frac{1}{10K}$$

$$\Rightarrow K = 17.4 \quad \Rightarrow \boxed{G_c(s) = 17.4 \frac{(s+2)}{(s+4.05)}}$$

$$\Rightarrow L(s) = G_c(s) \cdot G_p(s) = 17{,}4 \frac{(s+2)}{(s+4{,}05)} \cdot \frac{10}{s^2} =$$

$$\frac{174\ (s+2)}{s^2\,(s+4{,}05)}$$

$$\Rightarrow T(s) = \frac{L(s)}{1+L(s)} = \frac{174\ (s+2)}{s^2\,(s+4{,}05) + 174(s+2)} =$$

$$\frac{174(s+2)}{s^3 + 4{,}05\,s^2 + 174\,s + 348}$$

$$s^3 + 4{,}05\,s^2 + 174\,s + 348 = 0$$

$$s_1 \approx -2{,}048 \qquad s_{2,3} = -1{,}001 \pm 12{,}996j$$

Therefore, the dominant poles are indeed at

$s = -1 \pm 13j$ (approximately)

As the third pole and zero almost coincide, the second-order approximation is valid

The second-order approximation:

$$T(s) = \frac{174}{s^2+2s+170}$$

$\omega_n^2 = 170$

$2\zeta\omega_n = 2$

$$T_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{170}} \approx 0.138\ s$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1} = 4s$$

$$M_p = 100 \cdot e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-\frac{1}{\sqrt{170}}\pi/\sqrt{1-\frac{1}{170}}}$$

$$\approx 78.5\%$$

MATLAB code:

```
L = zpk(-2, [0 0 -4.05], 174)
T = feedback(L, 1)
figure
step(T)
stepinfo(T)
```

The transfer function of the open loop-system is defined using *zpk()*, and the closed-loop system is defined using *feedback()*. Then, step response of the closed-loop system is plotted (Figure 3), and its parameters are measured using *stepinfo()* (Figure 4).

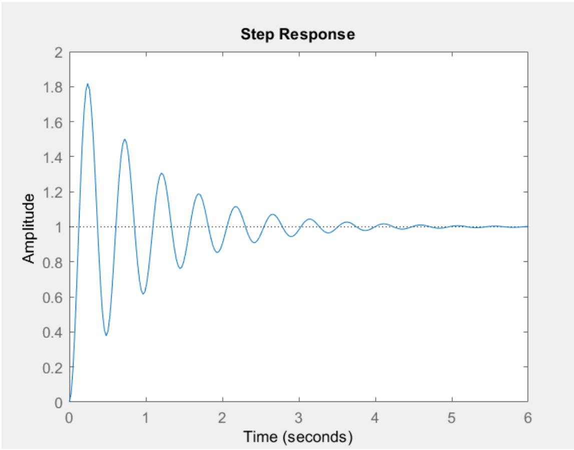


Figure 3: Step Response of the Closed-Loop System

```
  struct with fields:

        RiseTime: 0.0836
    SettlingTime: 3.8923
     SettlingMin: 0.3778
     SettlingMax: 1.8188
       Overshoot: 81.8751
      Undershoot: 0
            Peak: 1.8188
        PeakTime: 0.2410
```

Figure 4: Parameters of Step Response

Comparing the calculated values of the parameters with those obtained via MATLAB, it is seen the percent overshoot and settling time values are almost the same in both cases. However, the rising time values differ more, therefore, the approximation of rising time is not good enough.

3. $G_p(s) = \dfrac{1}{s(s+2)(s+4)}$

Requirements; $T_s < 3s$

$M_p < 20\%$

$e_{ss} < 0.1$ to a unit ramp

$$L(s) = G_c(s)\, G_p(s) = K \frac{1}{s(s+2)(s+4)}$$

$N(s) = 1$

$D(s) = s(s+2)(s+4)$

$n = 3$

$m = 0$

$r = 3 \Rightarrow 3$ asymptotes

$$x_a = \frac{0 - (0+2+4)}{3} = -2$$

$3\psi_a = (2h+1)\pi \quad h = 0, 1, 2 \quad (PL)$

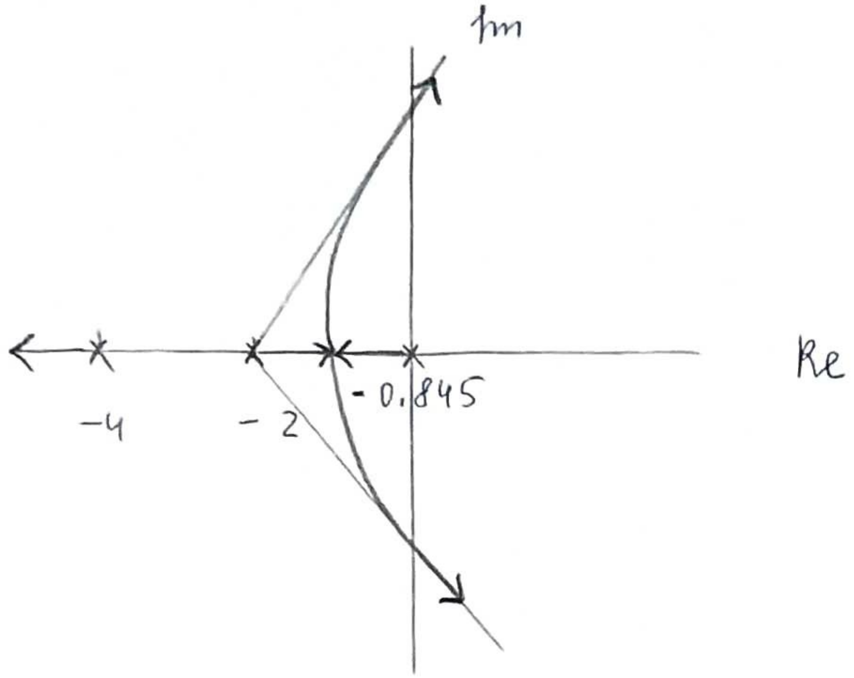
$\psi_a = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

Real axis: $(-\infty, -4)$ 3 poles $\in PL$

$(-4, -2)$ 2 poles

$(-2, 0)$ 1 pole $\in PL$

$(0, +\infty)$ 0 poles



Break in / away points:

$$N(s)D'(s) - N'(s)D(s) = 0 \qquad D(s) = s^3 + 6s^2 + 8s$$

$$3s^2 + 12s + 8 = 0$$

$$s_1 = -0.845 \qquad s_2 = -3.155$$

$$T_s = \frac{4}{\zeta \omega_n} < 3 \qquad \zeta \omega_n > \frac{4}{3} \approx 1.333$$

$$\zeta = \frac{-\ln(M_p/100)}{\sqrt{\pi^2 + \ln^2(M_p/100)}} = -\frac{\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} \approx 0.456$$

$$\Rightarrow \zeta > 0.456$$

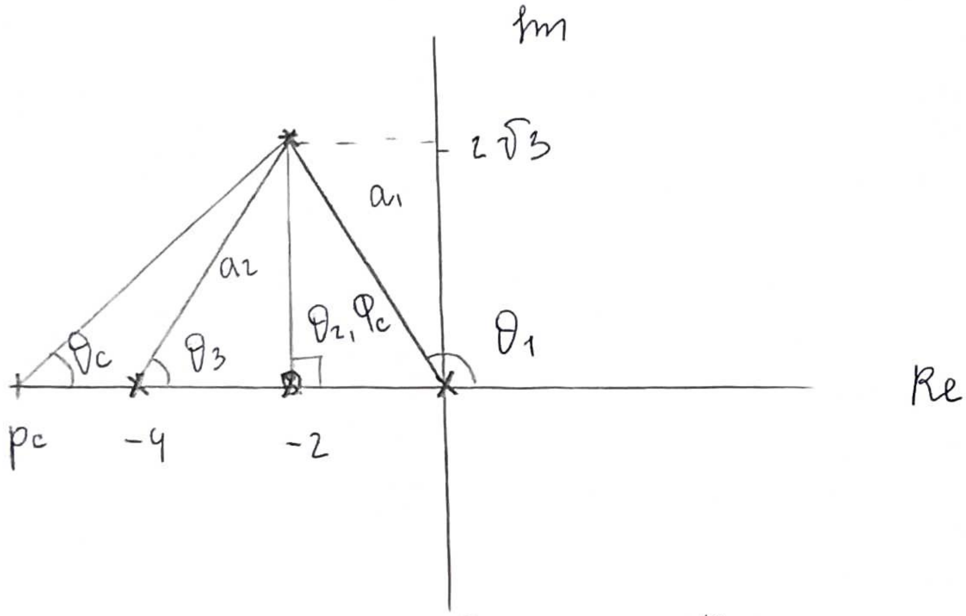
Choose $\zeta = 0.5$, $\zeta \omega_n = 2$

$(\theta = 60°)$

$\Rightarrow$ A test point is : $s^* = -2 + 2\sqrt{3}\,j$

Choose arbitrarily : $z_c = 2$

Find p_c using the phase condition :



φ_c and θ_2 cancel each other

$$\theta_1 = 180 - \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 120°$$

$$\theta_3 = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 60°$$

$$-\theta_1 - \theta_3 - \theta_c = -180°$$

$$\theta_c = 180° - 120° - 60° = 0$$

$\Rightarrow$ p_c is at infinity and can be neglected

To find K, use the magnitude condition

$$\left|\frac{N(s)}{D(s)}\right|_{s^*} = \left|-\frac{1}{K}\right| = \frac{1}{K}$$

$$\frac{1}{a_1 \cdot a_2} = \frac{1}{K} \qquad a_1 = a_2 = \sqrt{2^2 + (2\sqrt{3})^2} = 4$$

$$\frac{1}{4 \cdot 4} = \frac{1}{K} \quad \Rightarrow \quad K = 16$$

$$L(s) = G_c(s)\, G_p(s) = 16\,(s+2) \cdot \frac{1}{s(s+2)(s+4)}$$

$$K_v = \lim_{s \to 0} s\, L(s) = 4$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{4} = 0.25$$ to a unit ramp

$\Rightarrow$ steady-state error requirement is not satisfied

But introducing a PI controller with arbitrarily chosen zero: $z_c = -0.01$,

we increase the system's type $\Rightarrow$

$e_{ss} = 0$

$$\Rightarrow \boxed{G_c(s) = \frac{16\,(s+2)(s+0.01)}{s}}$$

MATLAB verification is done with the help of *rltool(Gp)*.
Design requirements are set in Root Locus Editor window (Figure 5).

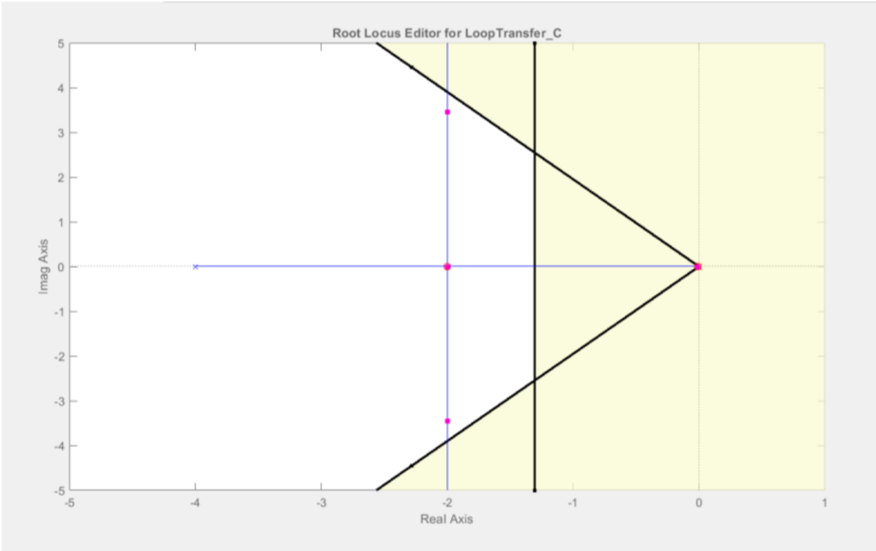


Figure 5: Setting Design Requirements

Equating the compensator to the designed one, we obtain the following step response and its parameters (Figure 6).

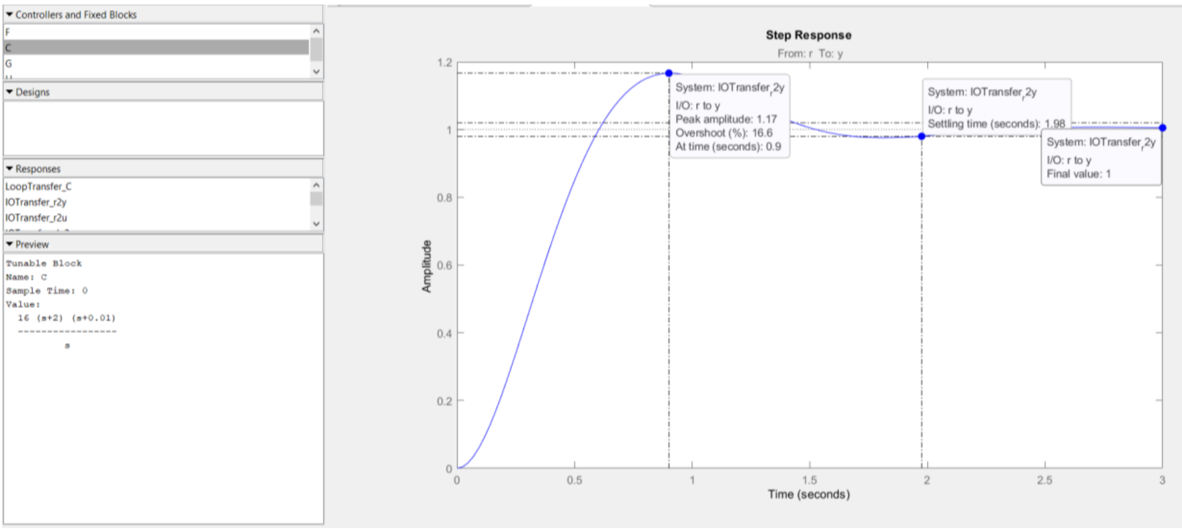


Figure 6: Step Response and Its Parameters

It is seen from Figure 5 that percent overshoot and settling time requirements are satisfied. The steady-state error requirement for a unit ramp is also satisfied because the open-loop system is a Type-2 system. Therefore, the design is verified.

4. $$G_p(s) = \frac{1600}{(s+2)(s+4)(s^2+s+200)}$$

Requirements: $e_{ss} < 0.1$ for a unit step

$T_{s,2\%} < 2s$

$M_p < 20\%$

$$L(s) = G_c(s)\,G_p(s) = K \frac{1600}{(s+2)(s+4)(s^2+s+200)}$$

$N(s) = 1$ $\quad\quad$ $n = 4$

$D(s) = (s+2)(s+4)(s^2+s+200)$ $\quad\quad$ $m = 0$

$r = 4 \Rightarrow 4$ asymptotes

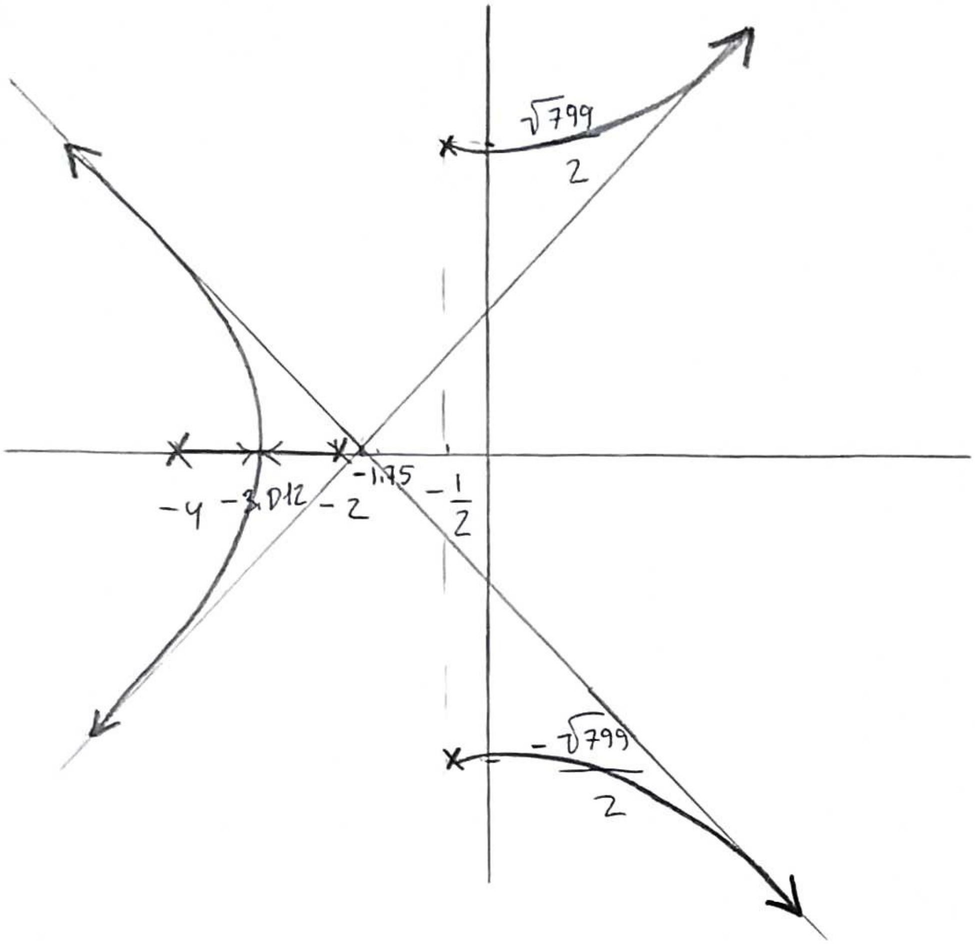
No zeros $\quad\quad$ Poles: $(s+2)(s+4)(s^2+s+200) = 0$

$s_1 = -2 \quad s_2 = -4 \quad s_{3,4} = -\frac{1}{2} \pm \frac{\sqrt{799}}{2} j$

$$x_a = \frac{0 - \left(2+4+\frac{1}{2}+\frac{1}{2}\right)}{4} = -\frac{7}{4} = -1.75$$

$4\psi_a = (2h+1)\pi \quad h = 0, 1, 2, 3 \quad (PL)$

$\psi_a = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \quad (PL)$



Real axis:

$(-\infty, -4)$ 4 poles $\in NL$

$(-4, -2)$ 3 poles $\in PL$

$(-2, -\frac{1}{2})$ 2 poles $\in NL$

$(-\frac{1}{2}, +\infty)$ 0 poles $\in NL$

Break-in/away:

$N(s)D'(s) - N'(s)D(s) = 0$

$D(s) = s^4 + 7s^3 + 214s^2 + 1208s + 1600$

$$4s^3 + 21s^2 + 428s + 1208 = 0$$

$$s \approx -3.012$$

The requirements can be met if a notch filter is used.

The zeros of the notch filter must be placed close to the high-frequency poles: $-\frac{1}{2} \pm \frac{\sqrt{799}}{2} j \approx -\frac{1}{2} \pm 14.13j$

$\Rightarrow$ Place zeros at $-\frac{1}{2} \pm 14.2 j$

Also, two poles must be placed on the real axis, far from the origin. Choose: -50 and -55

Therefore, the notch filter is: $G_c(s) = \dfrac{K(s^2+s+201.89)}{(s+50)(s+55)}$

To find K, hard computations are needed

Therefore, the help of Matlab rltool() is used

Design requirements are set in Root Locus Editor window (Figure 7). The closed-loop dominant complex poles must be located in the white area. As a result, the obtained gain is 54200 (Figure 7).

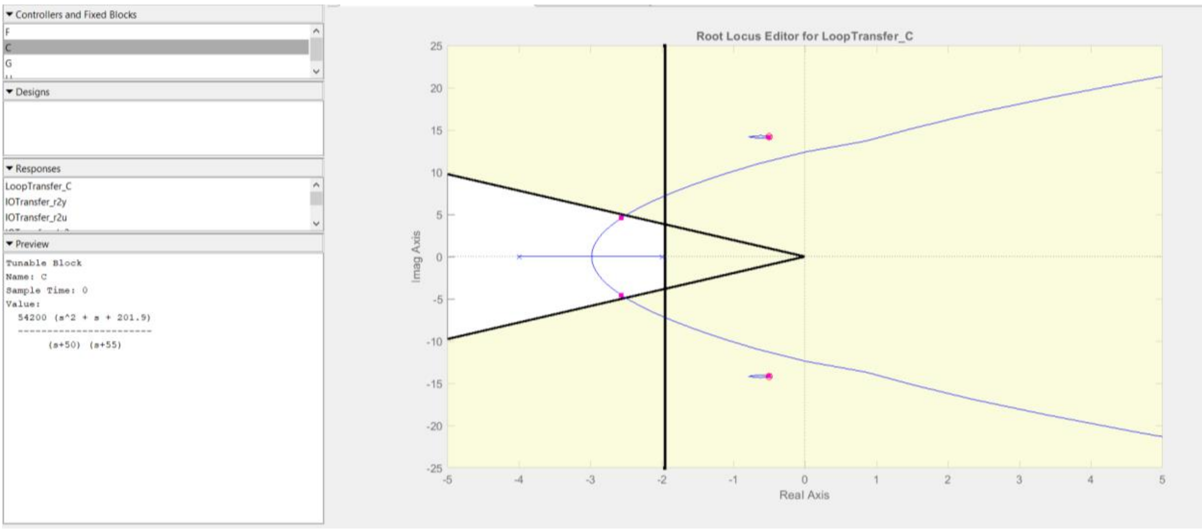


Figure 7: Setting Design Requirements and Obtaining Gain

With the obtained gain, the percent overshoot and settling time requirements are satisfied, whereas the steady-state error requirement is not (Figure 8).

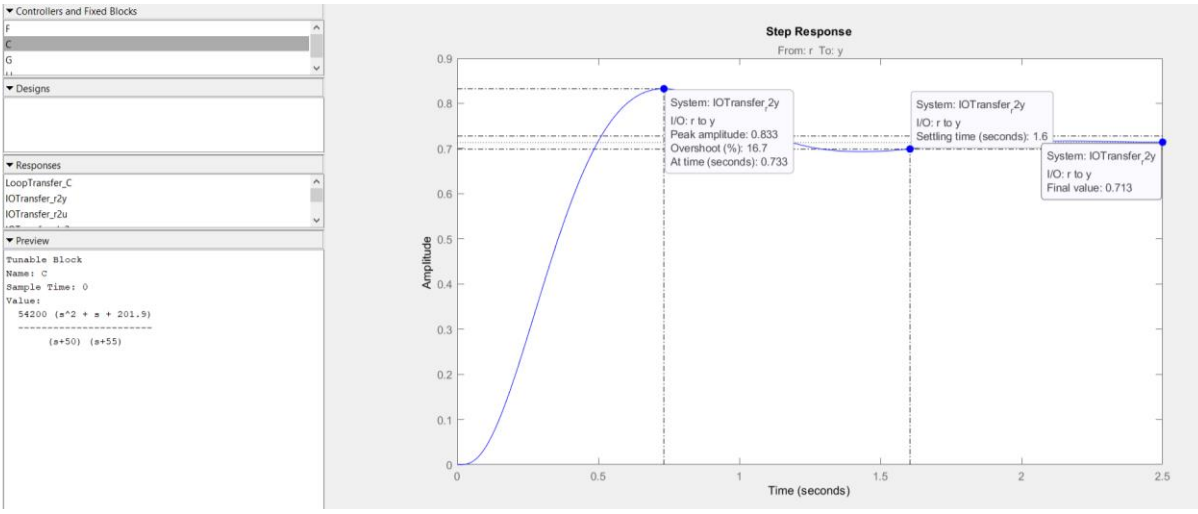


Figure 8: Step Response and Its Parameters

The steady-state error requirement can be satisfied by using a PI integrator with a zero placed close to the origin: $G_c(s) = \dfrac{s+0.01}{s}$
(system's type is increased to Type-1)

$\Rightarrow$ The designed compensator is:

$$G_c(s) = \frac{54200\,(s+0.01)\,/\,(s^2+s+201.89)}{s\,(s+50)\,(s+55)}$$