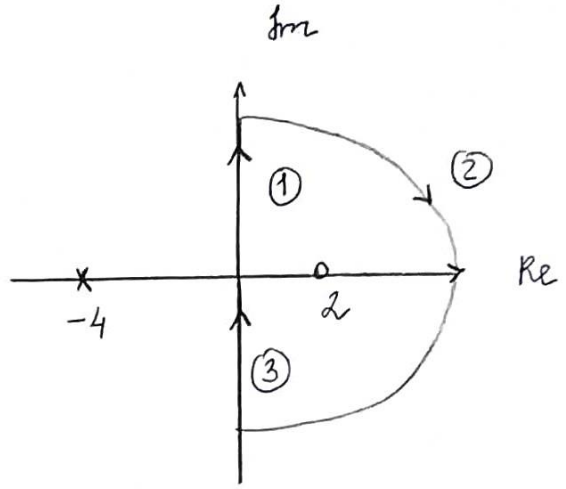
Robt 303 Zarema Balgabekova HW 5

1)

1. $L(s) = k \dfrac{s-2}{s+4}$

$k=1$ $\quad L(s) = \dfrac{s-2}{s+4}$



1 $\quad s = j\omega \qquad \omega : 0^+ \to \infty$

$$L(j\omega) = \frac{j\omega - 2}{j\omega + 4}$$

$$|L(j\omega)| = \frac{\sqrt{\omega^2 + 4}}{\sqrt{\omega^2 + 16}} \qquad \angle L(j\omega) = \left(\tan^{-1}\left(-\frac{\omega}{2}\right) + \pi\right) - \tan^{-1}\left(\frac{\omega}{4}\right) =$$

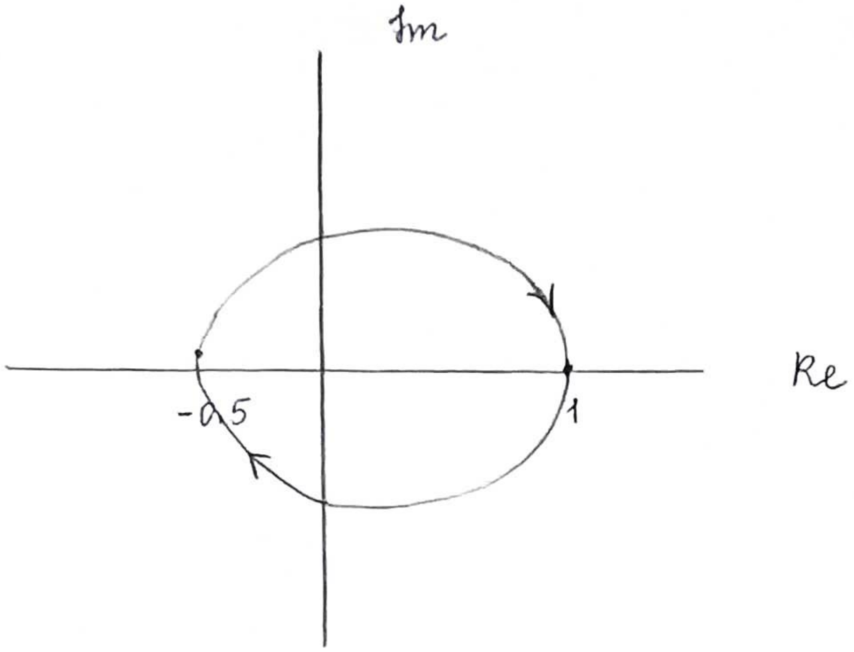
$$\pi - \left(\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{4}\right)\right)$$

$\omega \to 0^+ = \varepsilon_1 \quad \Rightarrow |L(j\omega)| = 0.5 \qquad \angle L(j\omega) = \pi - \varepsilon_1$

$\omega \to \infty \quad \Rightarrow |L(j\omega)| = 1 \qquad \angle L(j\omega) = \pi - \left(\frac{\pi}{2} + \frac{\pi}{2}\right) = 0$

2 $\quad s = Re^{j\theta} \qquad R \to \infty \qquad \theta = \frac{\pi}{2} \xrightarrow{cw} -\frac{\pi}{2}$

$$L(s) = \frac{Re^{j\theta} - 2}{Re^{j\theta} + 4} = \frac{Re^{j\theta}}{Re^{j\theta}} = 1$$



$-\infty < -\frac{1}{k} < -0.5$ $\quad P=0 \quad N=0 \quad Z=0$

$0 < k < 2$ $\quad \Rightarrow$ closed-loop system is stable

$-0.5 < -\frac{1}{k} < 1$ $\quad P=0 \quad N=1 \quad Z=1$

$k > 2 \quad k < -1$ $\quad \Rightarrow$ closed-loop system is unstable with 1 RHP pole

$1 < -\frac{1}{k} < \infty$ $\quad P=0 \quad N=0 \quad Z=0$

$\Rightarrow$ closed-loop system is stable

$-1 < k < 0$

MATLAB code:

```
L = zpk(2, -4, 1);
figure
rlocus(L)
figure
rlocus(-L)
```

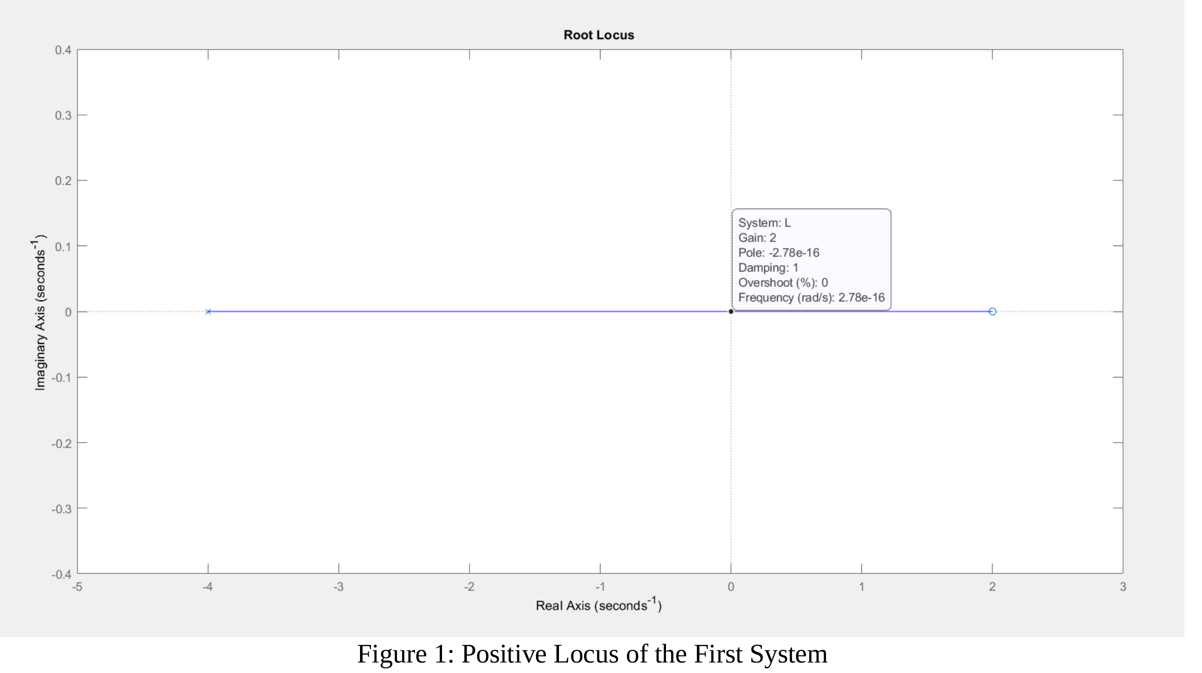


Figure 1: Positive Locus of the First System

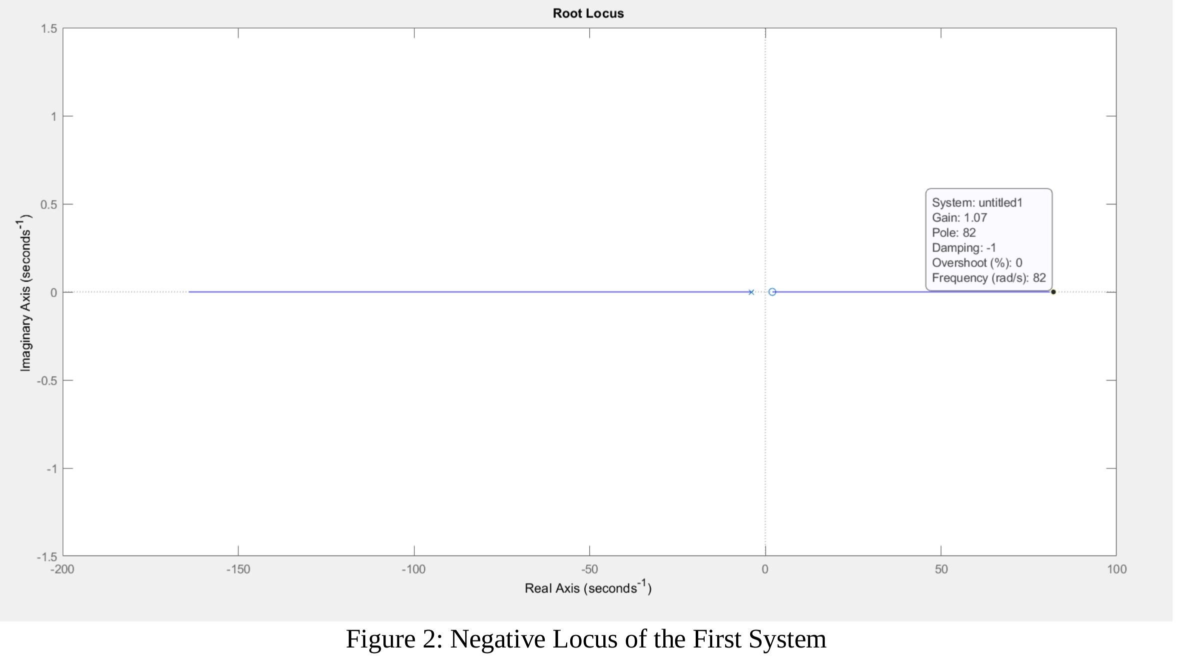
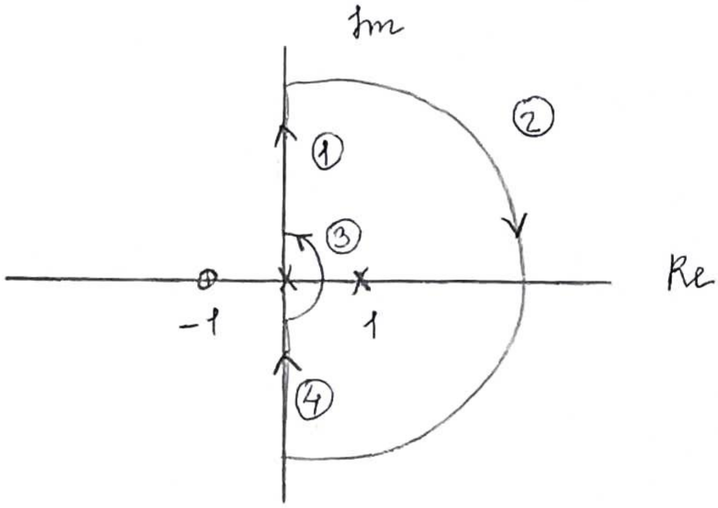


Figure 2: Negative Locus of the First System

It is seen from Figures 1 and 2 that the root locus plot verifies our findings from the Nyquist plot.

2. $L(s) = k \dfrac{s+1}{s(s-1)}$

$k = 1$ $\qquad L(s) = \dfrac{s+1}{s(s-1)}$



① $s = j\omega$ $\qquad \omega: 0^+ \to \infty$

$$L(j\omega) = \frac{1+j\omega}{j\omega(j\omega - 1)}$$

$$|L(j\omega)| = \frac{\sqrt{1+\omega^2}}{\omega\sqrt{1+\omega^2}} = \frac{1}{\omega}$$

$$\angle L(j\omega) = \tan^{-1}\omega - \left(\frac{\pi}{2} + \tan^{-1}(-\omega) + \pi\right) =$$

$$2\tan^{-1}\omega - \frac{3\pi}{2}$$

$\omega \to 0^+ = \xi$ $\quad \Rightarrow \quad |L(j\omega)| = \infty$

$$\angle L(j\omega) = \xi - \frac{3\pi}{2}$$

$\omega \to \infty \quad \Rightarrow \quad |L(j\omega)| = 0$

$$\angle L(j\omega) = 2 \cdot \frac{\pi}{2} - \frac{3\pi}{2} = -\frac{\pi}{2}$$

$\omega = 1 \quad \Rightarrow \quad |L(j\omega)| = 1 \qquad \angle L(j\omega) = -\pi$

② $s = Re^{j\theta} \qquad R \to \infty \qquad \theta = \frac{\pi}{2} \xrightarrow{cw} -\frac{\pi}{2}$

$$L(s) = \frac{Re^{j\theta}+1}{Re^{j\theta}(Re^{j\theta}-1)} = \frac{Re^{j\theta}}{(Re^{j\theta})^2} = 0$$

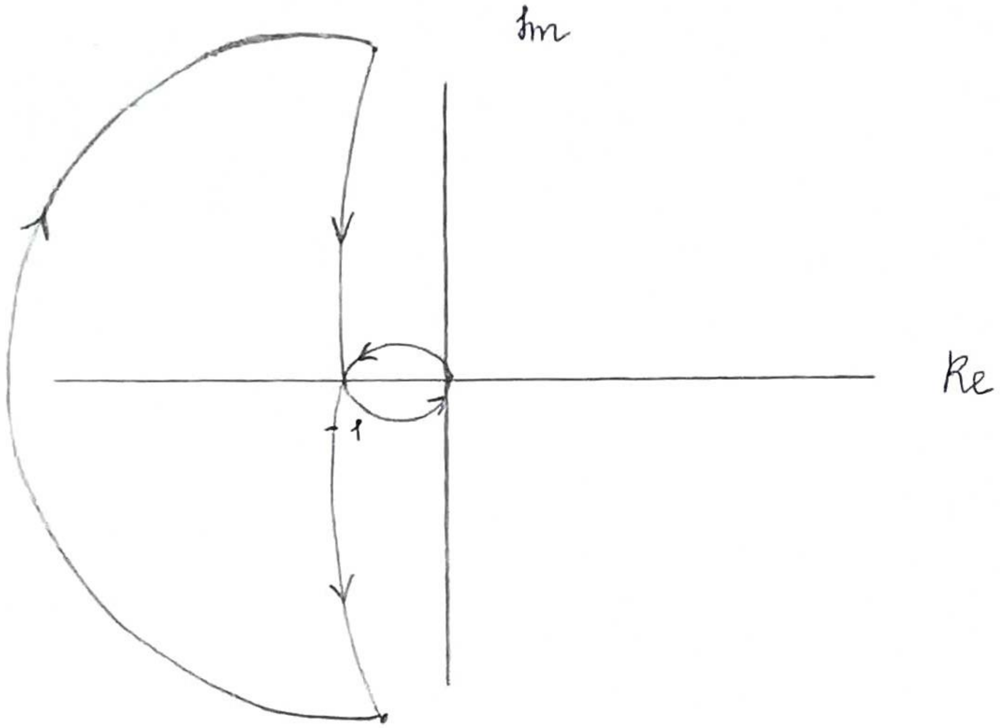
③ $s = re^{j\theta} \qquad r \to 0 \qquad \theta = -\frac{\pi}{2} \xrightarrow{ccw} \frac{\pi}{2}$

$$L(s) = \frac{re^{j\theta}+1}{re^{j\theta}(re^{j\theta}-1)} = \frac{1}{-re^{j\theta}} = \frac{e^{-j\pi}}{re^{j\theta}} =$$

$$\frac{1}{r} e^{-j(\pi+\theta)} = Re^{j\alpha} \qquad R \to \infty$$

$$\alpha = -(\pi+\theta)$$

$$\alpha = -\frac{\pi}{2} \xrightarrow{CW} -\frac{3\pi}{2}$$



$$-\infty < -\frac{1}{K} < -1$$

$$0 < K < 1$$

$P = 1 \quad N = 1 \quad Z = 2$

$\Rightarrow$ closed-loop system is unstable with 2 RHP poles

$$-1 < -\frac{1}{K} < 0$$

$$k > 1$$

$P = 1 \quad N = -1 \quad Z = 0$

$\Rightarrow$ closed-loop system is stable

$$0 < -\frac{1}{k} < \infty$$

$$k < 0$$

$P = 1 \quad N = 0 \quad Z = 1$

$\Rightarrow$ closed-loop system is unstable with 1 RHP pole

MATLAB code:

```
L = zpk(-1, [0 1], 1);
figure
rlocus(L)
figure
rlocus(-L)
```

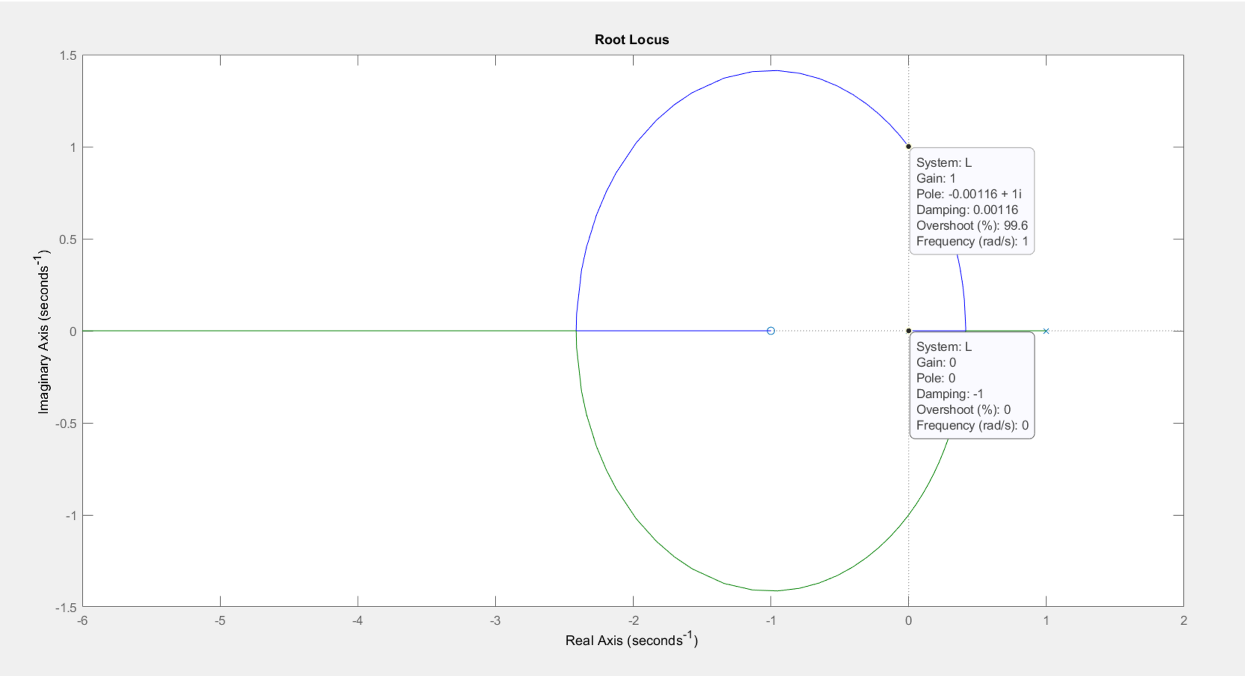


Figure 3: Positive Root Locus of the Second System

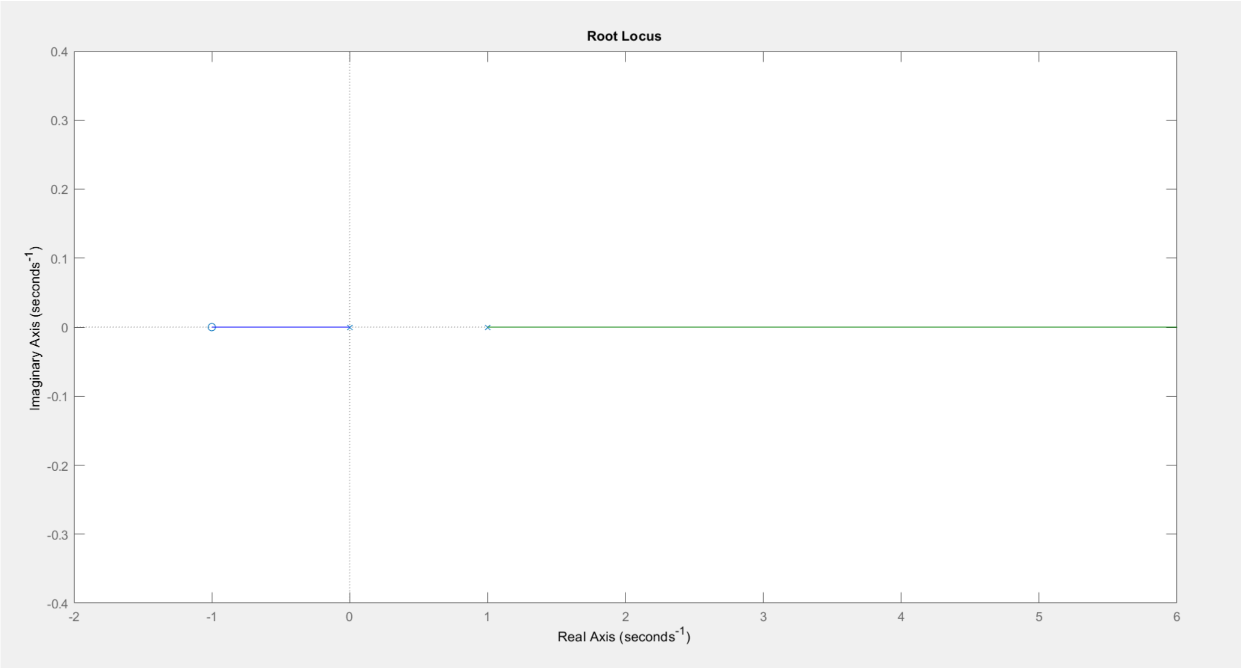
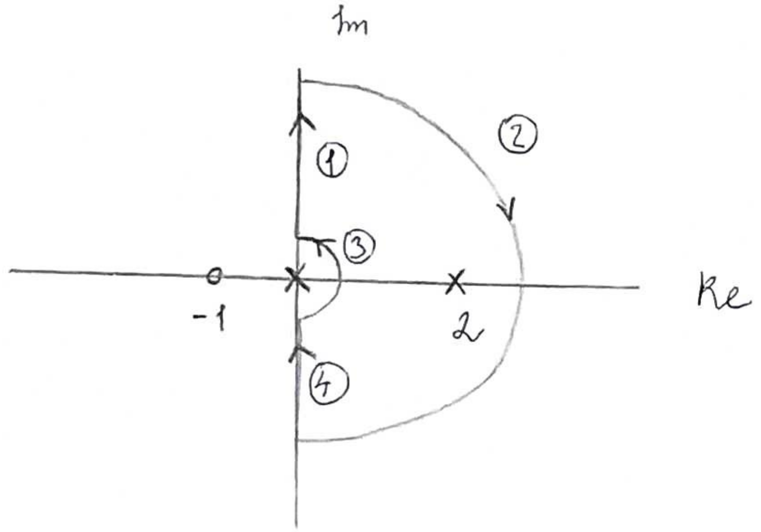


Figure 4: Negative Root Locus of the Second System

It is seen from Figures 3 and 4 that the root locus plot verifies our findings from the Nyquist plot.

3. $$L(s) = \frac{k\,(s+1)}{s^2(s-2)}$$

$k = 1$ $$L(s) = \frac{s+1}{s^2(s-2)}$$



① $s = j\omega$ $\qquad \omega : 0^+ \to +\infty$

$$L(j\omega) = \frac{1+j\omega}{(j\omega)^2\,(j\omega-2)} = \frac{1+j\omega}{-\omega^2\,(j\omega-2)}$$

$$|L(j\omega)| = \frac{\sqrt{1+\omega^2}}{\omega^2\sqrt{\omega^2+4}}$$

$$\angle L(j\omega) = \tan^{-1}(\omega) - \left(\pi + \tan^{-1}\left(-\frac{\omega}{2}\right) + \pi\right) =$$
$$\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) - 2\pi$$

$\omega \to 0^+ = \varepsilon \qquad \Rightarrow |L(j\omega)| = \infty$

$$\angle L(j\omega) = \varepsilon - 2\pi$$

$$\omega \to \infty \quad \Rightarrow \quad |L(j\omega)| = 0$$

$$\angle L(j\omega) = \frac{\pi}{2} + \frac{\pi}{2} - 2\pi = -\pi$$

(2) $s = Re^{j\theta}$ $\qquad R \to \infty \qquad \theta \ \frac{\pi}{2} \xrightarrow{cw} -\frac{\pi}{2}$

$$L(s) = \frac{Re^{j\theta} + 1}{(Re^{j\theta})^2 (Re^{j\theta} - 2)} = \frac{1}{(Re^{j\theta})^2} = 0$$

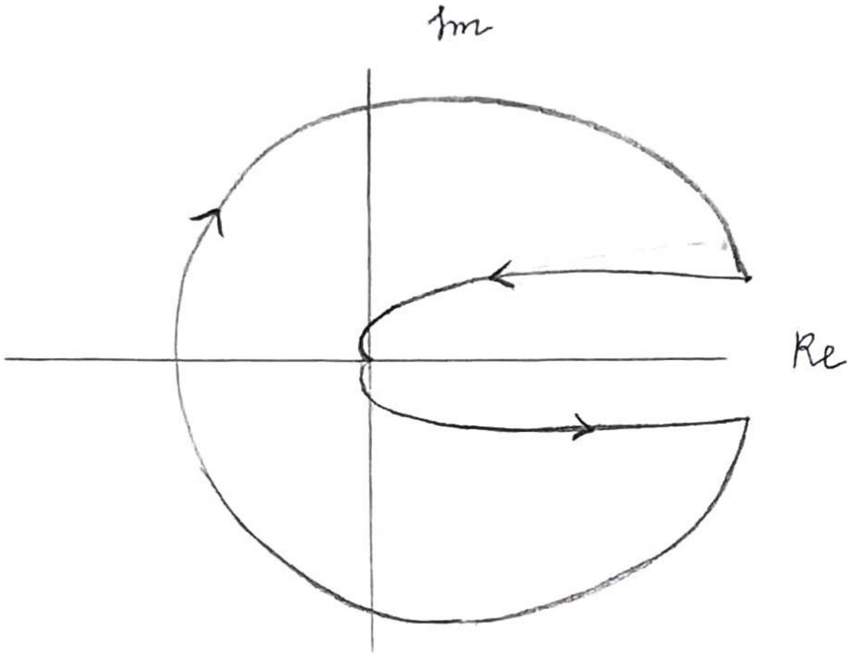
(3) $s = re^{j\theta}$ $\qquad r \to 0 \qquad \theta = -\frac{\pi}{2} \xrightarrow{ccw} \frac{\pi}{2}$

$$L(s) = \frac{re^{j\theta} + 1}{(re^{j\theta})^2 (re^{j\theta} - 2)} = \frac{1}{-2(re^{j\theta})^2} = \frac{e^{-j\pi}}{2r^2 e^{j2\theta}} =$$

$$Re^{-j(\pi + 2\theta)} = Re^{j\alpha}$$

$R \to \infty$

$\alpha = 0 \xrightarrow{cw} -2\pi$



$-\infty < -\frac{1}{k} < 0$ $\quad$ $P = 1 \quad N = 1 \quad Z = 2$

$\Rightarrow$ closed-loop system is unstable with 2 RHP poles

$k > 0$

$0 < -\frac{1}{k} < \infty$ $\quad$ $P = 1 \quad N = 0 \quad Z = 1$

$\Rightarrow$ closed-loop system is unstable with 1 RHP pole

$k < 0$

MATLAB code:

```
L = zpk(-1, [0 0 2], 1);
figure
rlocus(L)
figure
rlocus(-L)
```

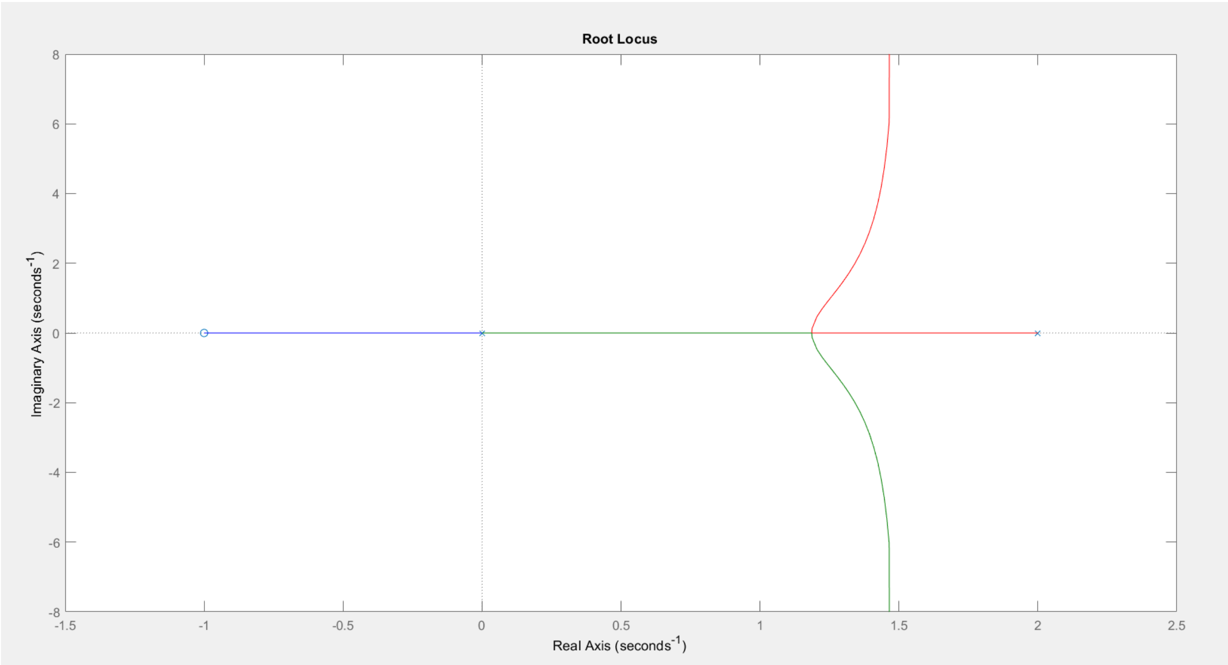


Figure 5: Positive Locus of the Third System

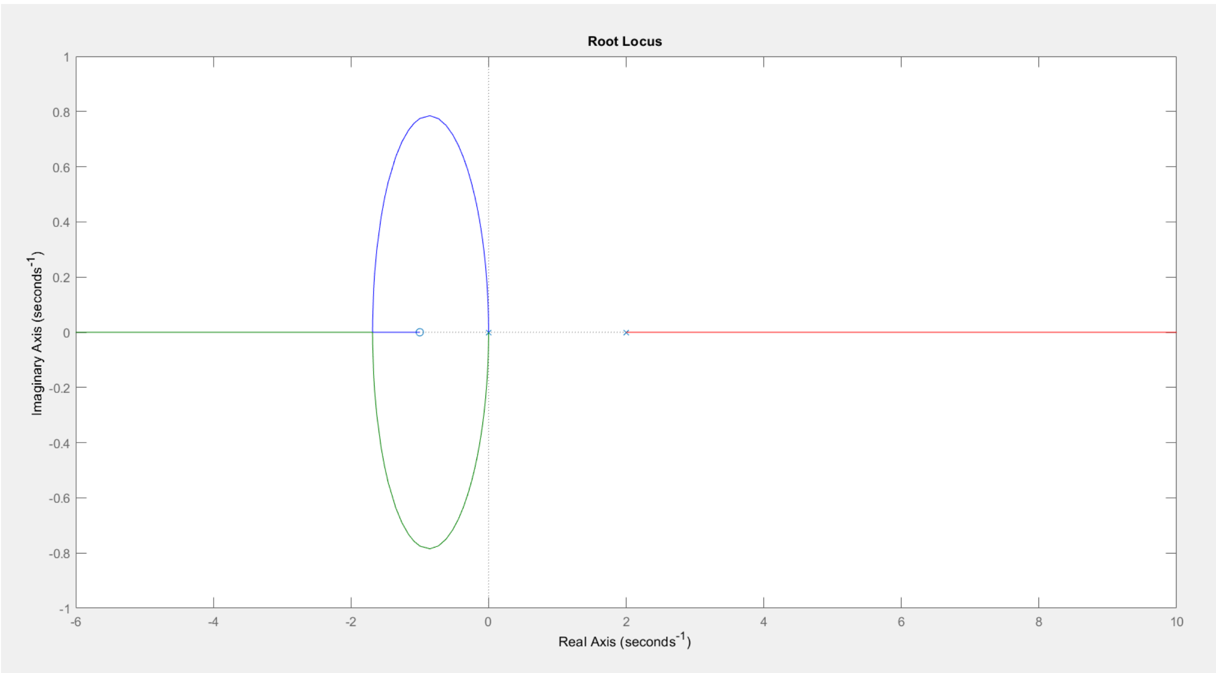
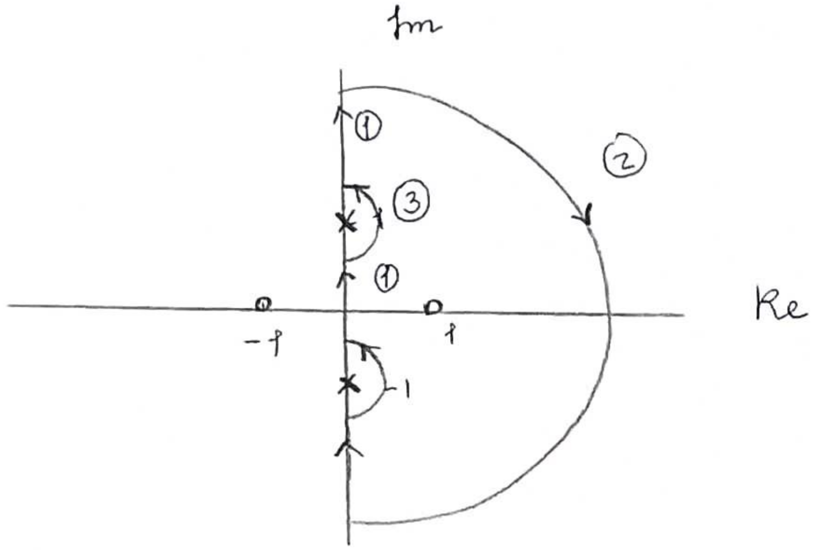


Figure 6: Negative Locus of the Third System

It is seen from Figures 5 and 6 that the root locus plot verifies our findings from the Nyquist plot.

4. $L(s) = k\,\dfrac{s^2-1}{s^2+1}$

$k=1$ $\qquad L(s) = \dfrac{s^2-1}{s^2+1} = \dfrac{(s-1)(s+1)}{s^2+1}$



① $s = j\omega$ $\qquad \omega:\ 0^+ \to 1^-$

$\qquad\qquad 1^+ \to +\infty$

$$L(j\omega) = \frac{(j\omega)^2-1}{(j\omega)^2+1} = \frac{-\omega^2-1}{-\omega^2+1} = \frac{-(1+\omega^2)}{1-\omega^2}$$

$$|L(j\omega)| = \frac{1+\omega^2}{|1-\omega^2|}$$

$$\angle L(j\omega) = \begin{cases} \pi & 1-\omega^2 > 0 \quad (0<\omega<1) \\ 0 & 1-\omega^2 < 0 \quad (\omega > 1) \end{cases}$$

$\omega \to 0^+ = \xi \Rightarrow |L(j\omega)| = 1 \qquad \angle L(j\omega) = \pi$

$\omega = 1^- \Rightarrow |L(j\omega)| = \infty \qquad \angle L(j\omega) = \pi$

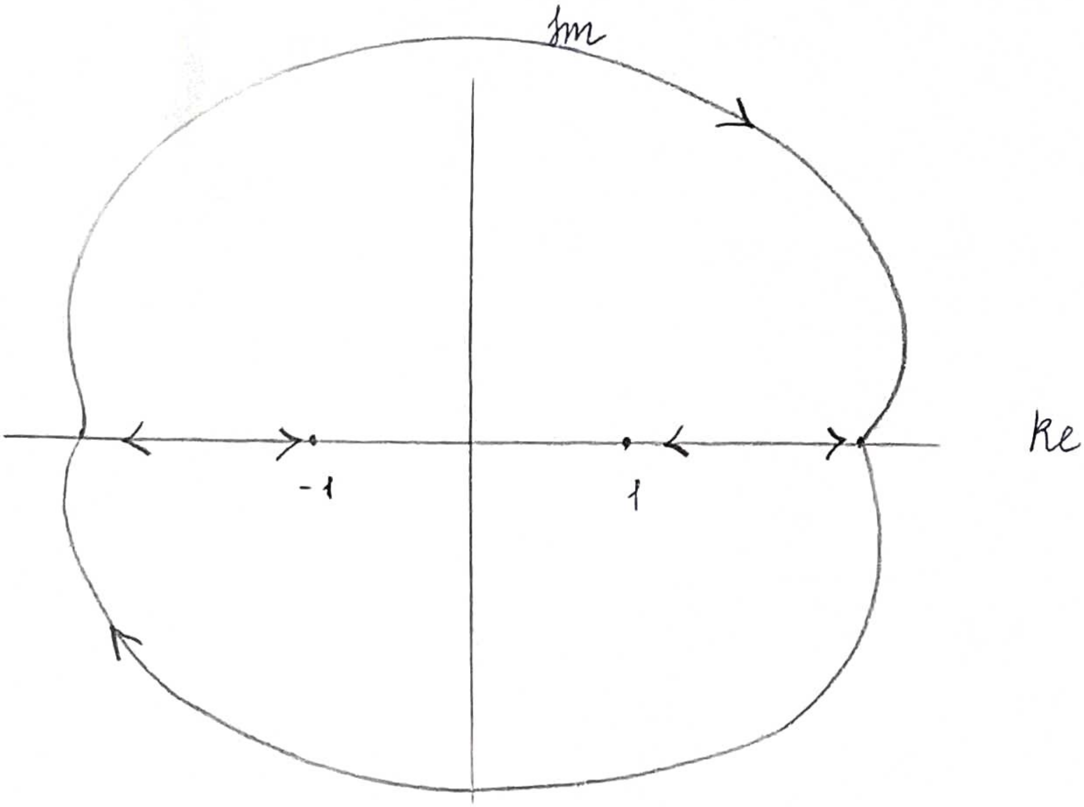
$\omega = 1^+ \Rightarrow |L(j\omega)| = \infty \qquad \angle L(j\omega) = 0$

$\omega \to \infty \Rightarrow |L(j\omega)| = 1 \qquad \angle L(j\omega) = 0$

(2) $s = Re^{j\theta} \qquad R \to \infty \qquad \theta : \frac{\pi}{2} \xrightarrow{CW} -\frac{\pi}{2}$

$$L(s) = \frac{(Re^{j\theta})^2 - 1}{(Re^{j\theta})^2 + 1} = \frac{(Re^{j\theta})^2}{(Re^{j\theta})^2} = 1$$

(3) $\omega : 1^- \to 1^+ \quad (\pi \Rightarrow CW)$



$-1 < -\frac{1}{k} < 1$ $\qquad$ $P = 0 \quad N = 1 \quad Z = 1$

$k < -1 \qquad k > 1$ $\Rightarrow$ closed-loop system is unstable with 1 RHP pole

$$\left.\begin{array}{c} -\infty < -\frac{1}{k} < -1 \\ 0 < k \; 1 \\ 1 < -\frac{1}{k} < \infty \\ -1 < k < 0 \end{array}\right\}$$

closed-loop system is unstable with 2 poles on the imaginary axis

MATLAB code:

```
L = tf([1 0 -1], [1 0 1]);
figure
rlocus(L)
figure
rlocus(-L)
```

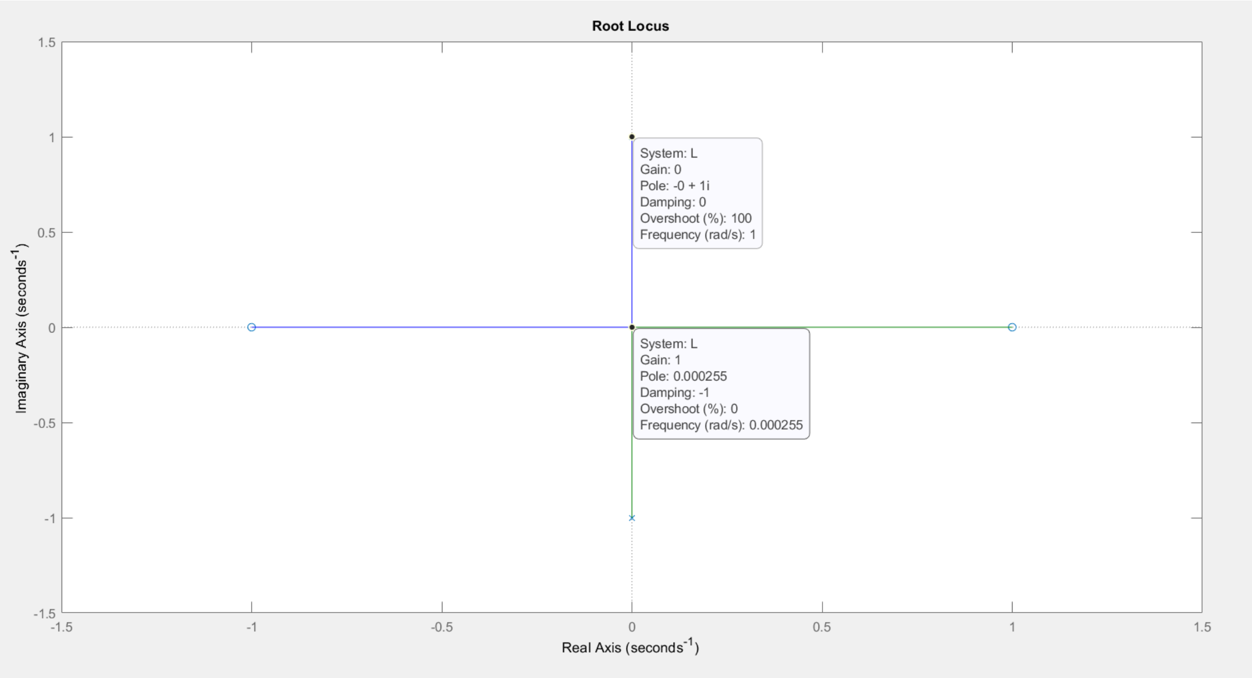


Figure 7: Positive Locus of the Fourth System

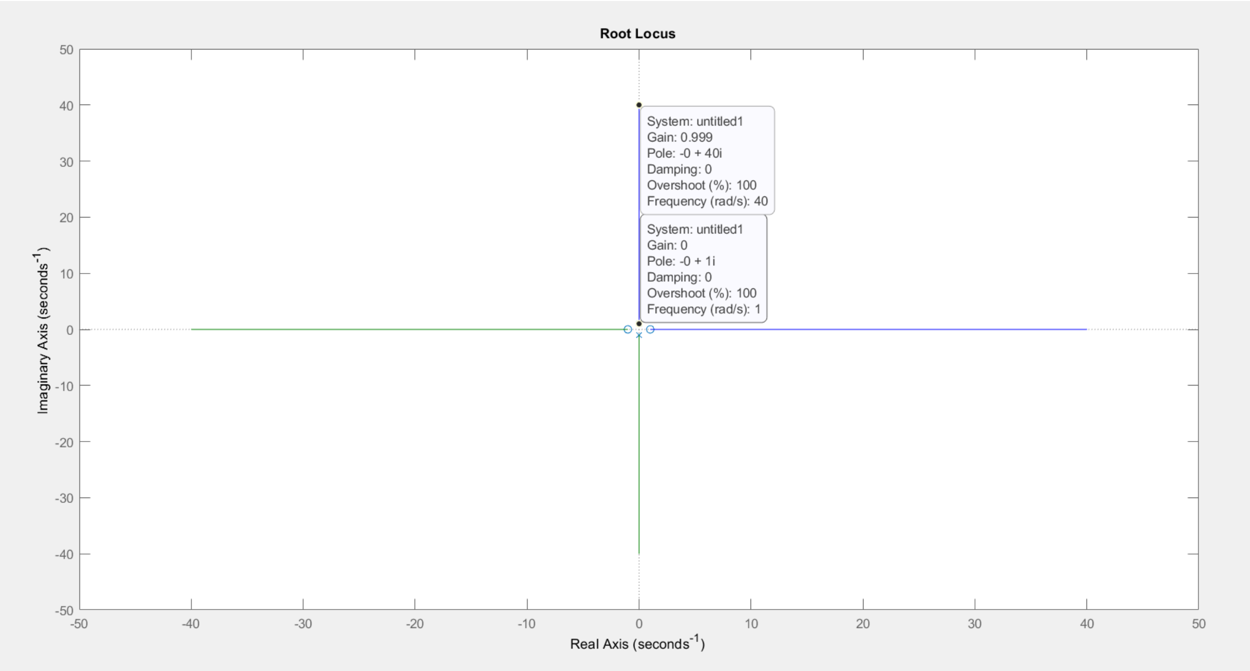


Figure 8: Negative Locus of the Fourth System

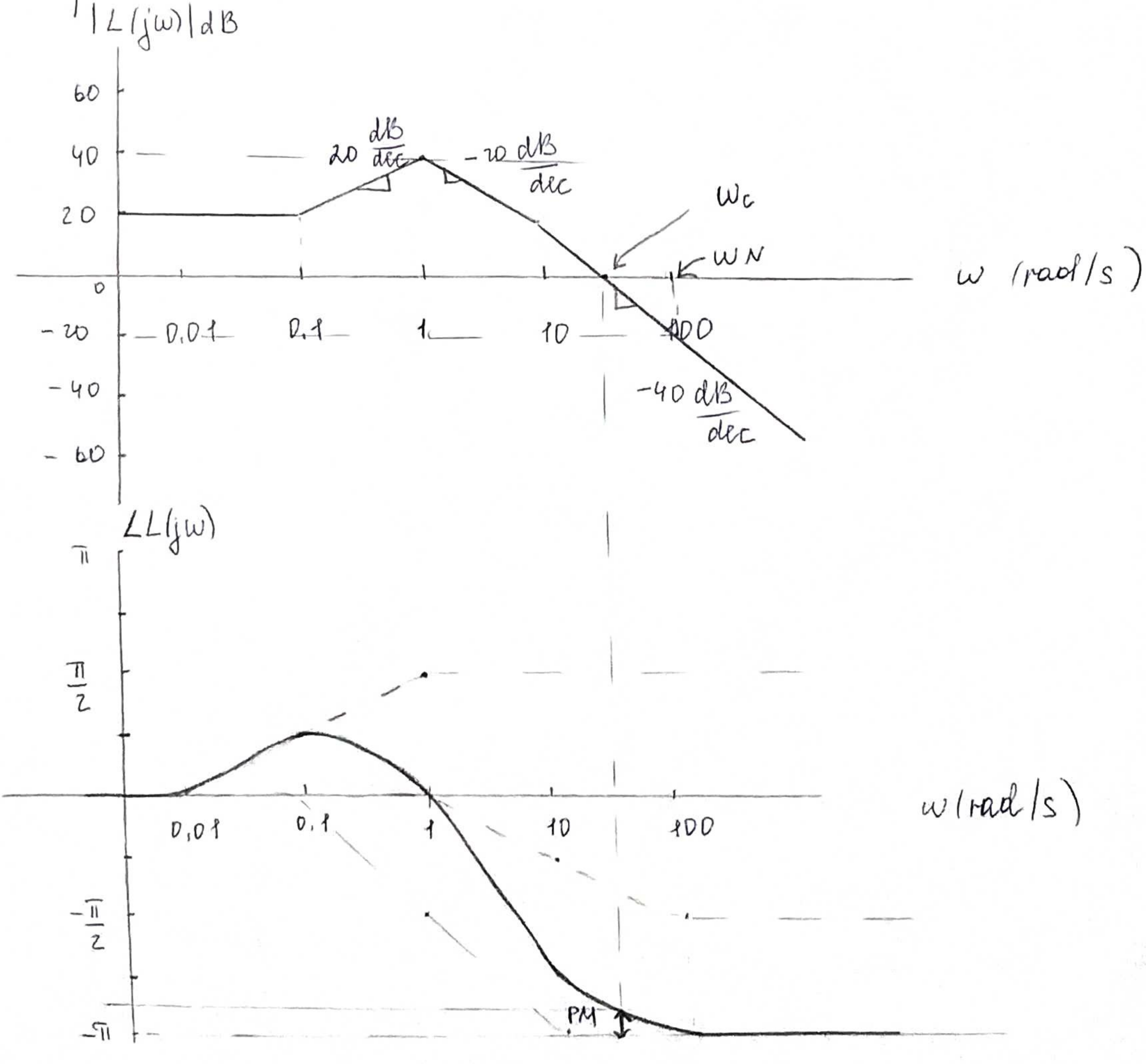
It is seen from Figures 7 and 8 that the root locus plot verifies our findings from the Nyquist plot.

2) $G_c(s) = 1 \quad G_p(s) = 10 \dfrac{1+10s}{(1+s)^2 (1+0.1s)}$

$$L(s) = G_c(s)\, G_p(s) = \frac{10(1+10s)}{(1+s)^2 (1+0.1s)}$$

Zero at -0.1

2 poles at -1 and 1 pole at -10



From the drawing,

$\omega_c \approx 30$ rad/s $\qquad PM \approx 22.5°$

$PM > 0° \quad \Rightarrow T(s)$ is BIBO stable

$\gamma_N = 0.1 \quad \Rightarrow |L(j\omega)| < 20\log 0.1 = -20\,dB$

$\Rightarrow$ Minimum frequency : $\omega_N = 100$ rad/s

$\gamma_D = 0.01 \quad \Rightarrow |L(j\omega)| > -20\log 0.01 = 40\,dB$

However, $|L(j\omega)|$ is never greater than 40 dB $\Rightarrow$

γ_D cannot be equat to 0.01 $\Rightarrow$

maximum frequency ω_D cannot be found

$0° < PM < 75° \quad \Rightarrow$ second-order approxim

$\omega_n = \omega_c = 30$ rad/s $\qquad \zeta = \dfrac{PM}{100} = \dfrac{22.5}{100} = 0.225$

$$T_{s,2\%} \approx \frac{4}{\zeta\omega_n} = \frac{4}{0.225 \cdot 30} \approx 0.593\,s$$

$$M_p = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-0.225\pi/\sqrt{1-0.225^2}} \approx 48.4\,\%$$

Asymptotic Bode diagram gives only approximated calculations, Better calculations can be done with MatLab

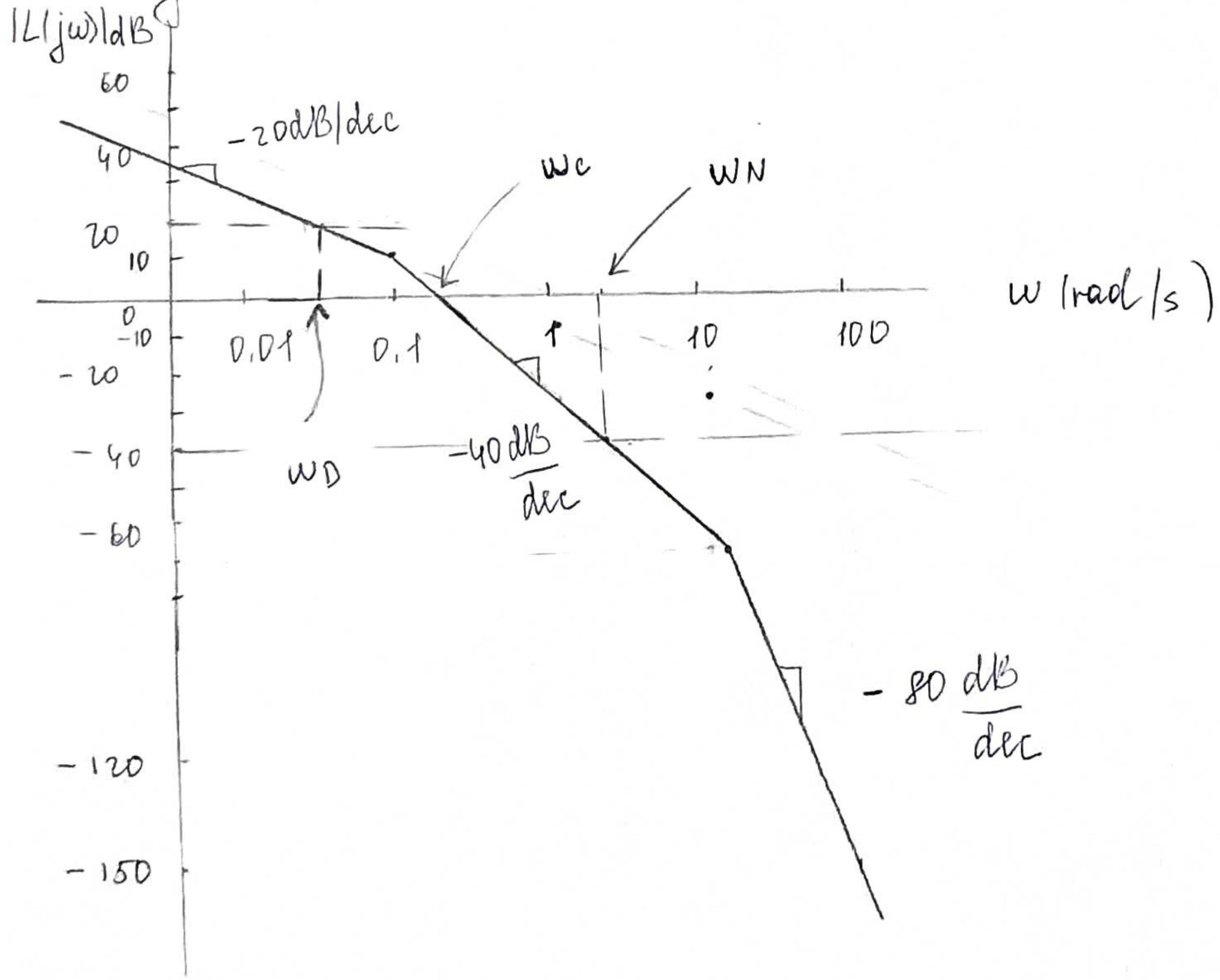
3) $G_c(s) = 1 \qquad G_p(s) = \dfrac{0.316}{s(1+10s)(1+0.1s)^2}$

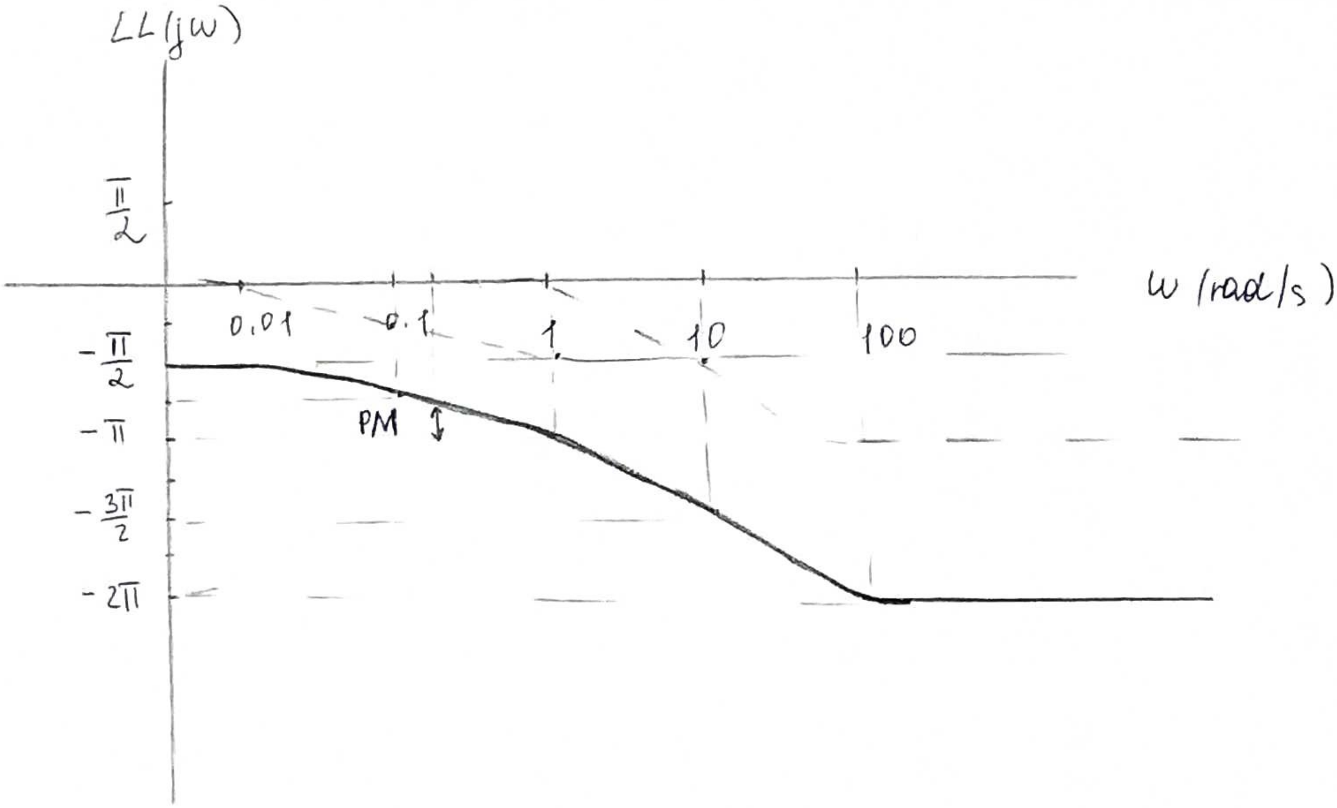
$$L(s) = G_c(s)\, G_p(s) = \frac{0.316}{s(1+10s)(1+0.1s)^2}$$

No zeros

Poles at 0, -0.1 and 2 poles at -10

$20\log(0.316) \approx -10$ dB





From the drawing, $\omega_c \approx 1.5$ rad/s

$PM \approx 30°$

$PM > 0° \Rightarrow T(s)$ is BIBO stable

$\gamma_N = 0.01 \Rightarrow |L(j\omega)| < 20 \log 0.01 = -40$ dB

$\Rightarrow$ Minimum frequency : $\omega_N \approx 2$ rad/s

$\gamma_D = 0.1 \Rightarrow |L(j\omega)| > -20 \log 0.1 = 20$ dB

$\Rightarrow$ Maximum frequency : $\omega_D \approx 0.03$ rad/s

$0° < PM < 75° \Rightarrow$ second-order approximation

$\omega = \omega_c = 1.5$ rad/s

$\zeta = \frac{PM}{100} = \frac{30}{100} = 0.3$

$$T_{s,2\%} \approx \frac{4}{\zeta \omega_n} = \frac{4}{0{,}3 \cdot 1{,}5} \approx 8{,}89\ s$$

$$M_p = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100\, e^{-0{,}3\pi/\sqrt{1-0{,}3^2}} \approx 37{,}2\%$$

Asymptotic Bode diagram gives only approximated calculations; Better calculations can be done with MatLab