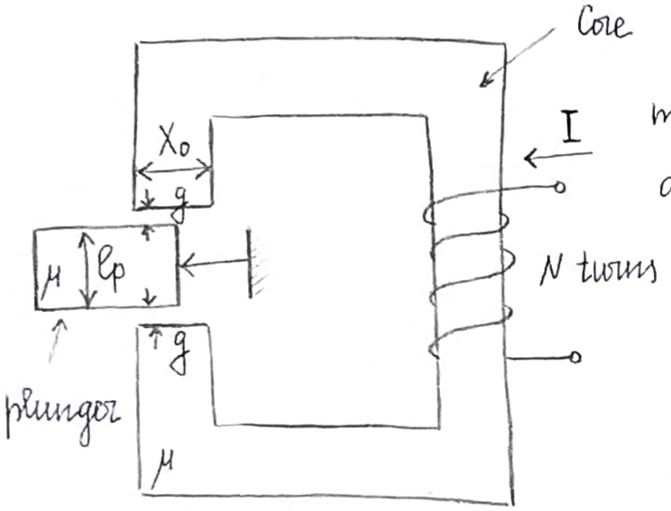
RoBt 304 Zarema Balgabekova HW 1

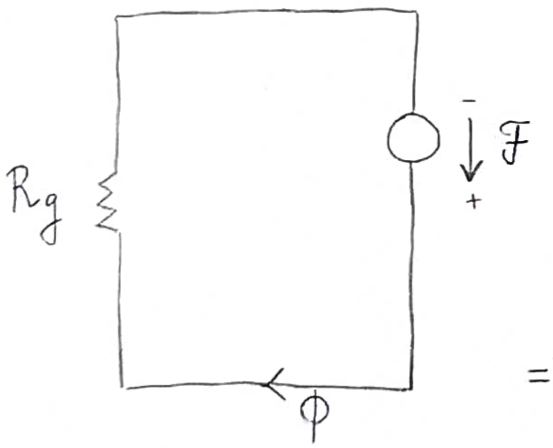
Exercise 1



$$A_g = A_c\left(1 - \frac{x}{X_0}\right)$$

neglect any fringing phenomenon

a) $\mu \to \infty$ $\quad 0 < x < 0.9 X_0$ $\quad$ As $\mu \to \infty$, $R_{c+p} = 0$



$$R_g = \frac{2g}{\mu_0 A_g} = \frac{2g}{\mu_0 A_c\left(1 - \frac{x}{X_0}\right)}$$

$$\left[\frac{A \cdot turns}{Wb}\right]$$

$$\Rightarrow \quad \frac{2g}{\mu_0 A_c} < R_g < \frac{2g}{\mu_0 A_c \cdot 0.1}$$

$$\frac{2g}{\mu_0 A_c} < R_g < \frac{20g}{\mu_0 A_c} \quad \left[\frac{A\ turns}{Wb}\right]$$

From Gauss Law, $\phi = B_g A_g = B_c A_c$

Also, $\phi = \frac{\mathcal{F}}{R_g}$

where $\mathcal{F} = NI$ from Ampere's law

$$\Rightarrow \quad \frac{NI}{2g}\,\mu_0 \cancel{A_c}\cancel{\left(1-\frac{x}{X_0}\right)} = B_g \cancel{A_c}\cancel{\left(1-\frac{x}{X_0}\right)}$$

$$B_g = \frac{NI\mu_0}{2g} \quad [T]$$

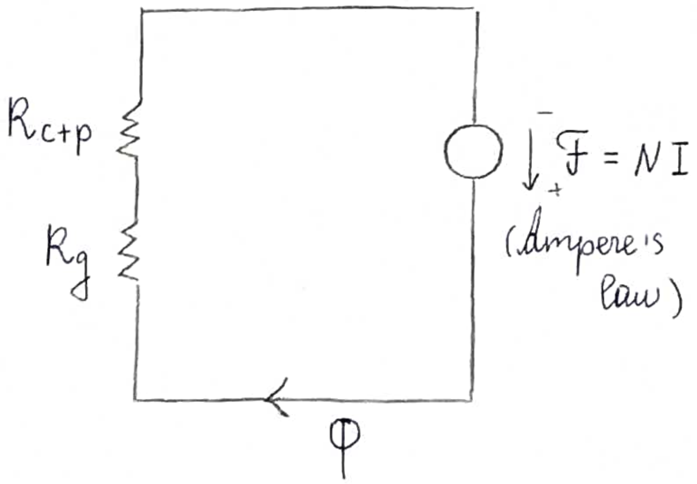
$$B_c = B_g \frac{A_g}{A_c} = \left(\frac{NI\mu_0}{2g}\right)\frac{\cancel{A_c}\left(1-\frac{x}{X_0}\right)}{\cancel{A_c}} =$$

$$\frac{NI\mu_0\left(1-\frac{x}{X_0}\right)}{2g} \quad [T]$$

$$\Rightarrow \quad \frac{\mu_0 NI \cdot 0.1}{2g} < B_c < \frac{NI\mu_0}{2g}$$

$$\frac{NI\mu_0}{20g} < B_c < \frac{NI\mu_0}{2g} \quad [T]$$

b) μ is const



R_g is the same as in a) :

$$R_g = \frac{2g}{\mu_0 A_c \left(1 - \frac{x}{X_0}\right)} \quad \left[\frac{A \cdot turns}{Wb}\right]$$

$$\frac{2g}{\mu_0 A_c} < R_g < \frac{20g}{\mu_0 A_c} \quad \left[\frac{A \cdot turns}{Wb}\right]$$

$$R_{c+p} = \frac{l_c + l_p}{\mu A_c} \quad \left[\frac{A \cdot turns}{Wb}\right]$$

From Gauss law: $\varphi = B_g \cdot A_g = B_c A_c$

Also, $$\varphi = \frac{\mathcal{F}}{R_g + R_{c+p}} = \frac{NI}{\frac{2g}{\mu_0 A_c \left(1 - \frac{x}{X_0}\right)} + \frac{l_c + l_p}{\mu A_c}} =$$

$$\frac{NI}{\frac{2g\mu + (l_c + l_p)\mu_0 \left(1 - \frac{x}{X_0}\right)}{\mu_0 \mu A_c \left(1 - \frac{x}{X_0}\right)}} = \frac{NI \mu_0 \mu A_c \left(1 - \frac{x}{X_0}\right)}{2g\mu + (l_c + l_p)\mu_0 \left(1 - \frac{x}{X_0}\right)}$$

$$\Rightarrow \quad B_g \cancel{A_c \left(1 - \frac{x}{X_0}\right)} = \frac{NI \mu_0 \mu \cancel{A_c \left(1 - \frac{x}{X_0}\right)}}{2g\mu + (l_c + l_p)\mu_0 \left(1 - \frac{x}{X_0}\right)}$$

$$B_g = \frac{N I \mu_0}{2g + \frac{\mu_0}{\mu}(l_c + l_p)\left(1 - \frac{x}{X_0}\right)} \quad [T]$$

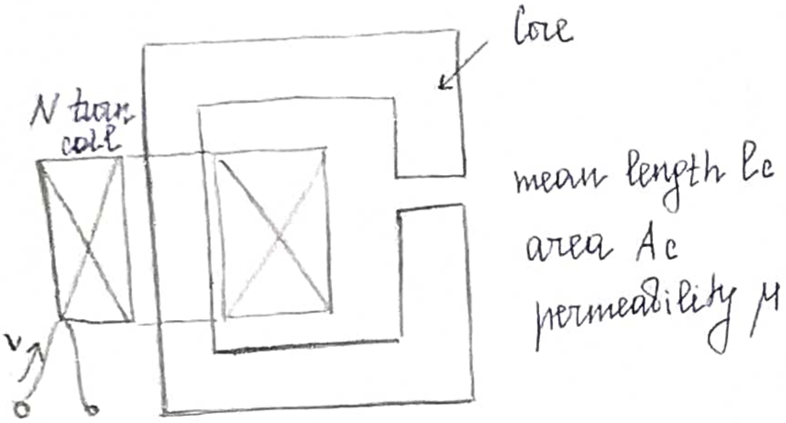
$$0 < x < 0.9 X_0 \quad \Rightarrow \quad \frac{N I \mu_0}{2g + \frac{\mu_0}{\mu}(l_c + l_p)} < B_g < \frac{N I \mu_0}{2g + 0.1\frac{\mu_0}{\mu}(l_c + l_p)} \quad [T]$$

$$B_c = B_g \frac{A_g}{A_c} = B_g \left(1 - \frac{x}{X_0}\right) =$$

$$\frac{N I \mu_0 \left(1 - \frac{x}{X_0}\right)}{2g + \frac{\mu_0}{\mu}(l_c + l_p)\left(1 - \frac{x}{X_0}\right)} = \frac{N I \mu_0}{\frac{2g}{\left(1 - \frac{x}{X_0}\right)} + \frac{\mu_0}{\mu}(l_c + l_p)} \quad [T]$$

$$\Rightarrow \quad \frac{N I \mu_0}{20g + \frac{\mu_0}{\mu}(l_c + l_p)} < B_c < \frac{N I \mu_0}{2g + \frac{\mu_0}{\mu}(l_c + l_p)} \quad [T]$$

Exercise 2

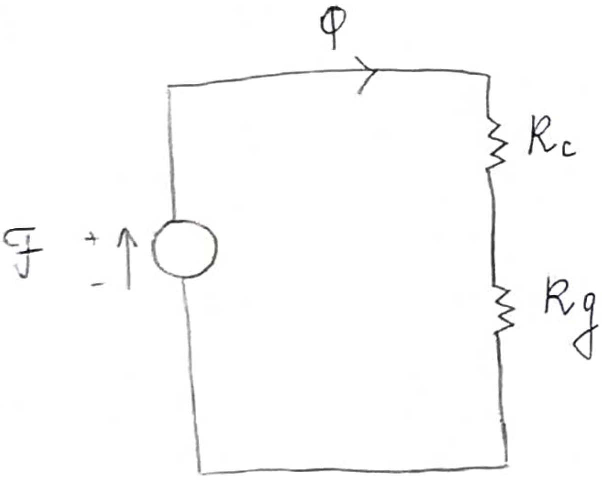


$A_c = 5.0\ cm^2$

$l_c = 25\ cm$

a) $L = 14\ mH$ $\quad I = 6\,A$ $\quad B_{saturation} = 1.7\ T$

$\mu = 3200\,\mu_0$



From Gauss law,

$$\Phi_{sat.} = B_{sat.} \cdot A_c = 1.7 \cdot 5 \cdot 10^{-4} = 8.5 \cdot 10^{-4}\ [WB]$$

$$L = \frac{\lambda}{I} = \frac{N\,\Phi_{sat.}}{I} \Rightarrow N = \frac{LI}{\Phi_{sat.}} = \frac{14 \cdot 10^{-3} \cdot 6}{8.5\ 10^{-4}} = 98.8$$

$$= 99 \text{ turns}$$

$$R_{tot} = R_c + R_g = \frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} = \frac{l_c}{3200\mu_0 A_c} + \frac{g}{\mu_0 A_c}$$

$$= \frac{l_c + 3200g}{3200\,\mu_0 A_c}$$

$$L = \frac{N^2}{R_{tot}} = \frac{3200 N^2 \mu_0 A_c}{l_c + 3200 g}$$

$$\Rightarrow g = \frac{1}{3200}\left(\frac{3200 N^2 \mu_0 A_c}{L} - l_c\right) =$$

$$\frac{1}{3200}\left(\frac{3200 \cdot 99^2 \cdot 4\pi \cdot 10^{-7} \cdot 5 \cdot 10^{-4}}{14 \cdot 10^{-3}} - 0.25\right) =$$

$$3.62 \cdot 10^{-4}\ m = 0.36\ mm$$

b) $I = 6A$ Equation 2.1 : $W = \int_V \left(\frac{B^2}{2\mu}\right) dV$

From equation 2.1, $W_g = \frac{B_{sat}^2}{2\mu_0} V_g = \frac{B_{sat}^2}{2\mu_0} A_g \cdot g =$

$$\frac{B_{sat}^2}{2\mu_0} A_c \cdot g = \frac{1.7^2}{2 \cdot 4\pi \cdot 10^{-7}} 5 \cdot 10^{-4}\ 3.6 \cdot 10^{-4}$$

$$= 0.207\ J$$

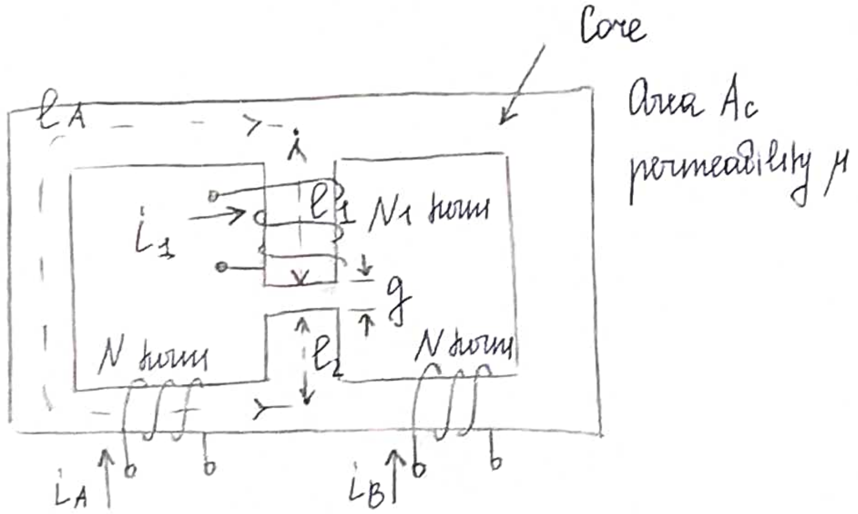
$$W_c = \frac{B_{sat}^2}{2\mu} V_c = \frac{B_{sat}^2}{2 \cdot 3200 \mu_0} A_c \cdot l_c = \frac{1.7^2}{2 \cdot 3200 \cdot 4\pi \cdot 10^{-7}} 5 \cdot 10^{-4} \cdot 0.25$$

$$= 0.045\ J$$

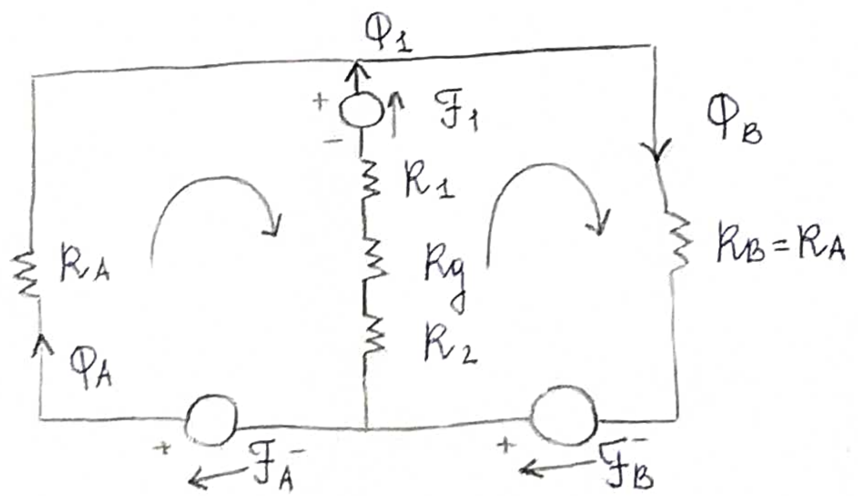
$W_{tot} = W_g + W_c = 0.207 + 0.045 = 0.252 \text{ J}$

W_g is much higher than W_c because the permeability of the core is much higher than free space permeability ($\mu = 3200 \mu_0$)

Exercise 3



a)



From Ampere's law, $\mathcal{F}_A = N i_A$ $\quad \mathcal{F}_B = N i_B$ $\quad \mathcal{F}_1 = N_1 i_1$

$$R_A = \frac{l_A}{\mu A_c} \qquad R_1 = \frac{l_1}{\mu A_c} \qquad R_g = \frac{g}{\mu_0 A_c} \qquad R_2 = \frac{l_2}{\mu A_c}$$

Kirchhoff's law

loop 1: $-N i_A + \varphi_A R_A + (R_1 + R_g + R_2)(\varphi_A - \varphi_B) + N_1 i_1 = 0$

$$\varphi_A (R_A + R_1 + R_g + R_2) - \varphi_B (R_1 + R_g + R_2) = N i_A - N_1 i_1$$

$$\varphi_A = \frac{N i_A - N_1 i_1 + \varphi_B (R_1 + R_g + R_2)}{R_A + R_1 + R_g + R_2} \qquad [1]$$

loop 2: $(R_1 + R_g + R_2)(\varphi_B - \varphi_A) - N_1 i_1 + \varphi_B R_A - N i_B = 0$

$$\varphi_B (R_1 + R_g + R_2 + R_A) - \varphi_A (R_1 + R_g + R_2) = N_1 i_1 + N i_B \qquad [2]$$

Substitute [1] into [2]:

$$\varphi_B (R_1 + R_g + R_2 + R_A) - \frac{[N i_A - N_1 i_1 + \varphi_B (R_1 + R_g + R_2)](R_1 + R_g + R_2)}{R_A + R_1 + R_g + R_2} = N_1 i_1 + N i_B$$

$$\varphi_B (R_1 + R_g + R_2 + R_A)^2 - \varphi_B (R_1 + R_g + R_2)^2 = N_1 i_1 R_A + N i_B (R_A + R_1 + R_g + R_2) + N i_A (R_1 + R_g + R_2)$$

$$\varphi_B \left[R_A (2R_1 + 2R_g + 2R_2 + R_A) \right] = N_1 i_1 R_A + N i_B (R_A + R_1 + R_g + R_2) + N i_A (R_1 + R_g + R_2)$$

$$\varphi_B = \frac{N_1 i_1}{2R_1 + 2R_g + 2R_2 + R_A} + \frac{N i_B (R_A + R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)} + \frac{N i_A (R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)}$$

Substituting φ_B into [1]:

$$\varphi_A = -\frac{N_1 i_1}{2R_1 + 2R_g + 2R_2 + R_A} + \frac{N i_A (R_A + R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)} + \frac{N i_B (R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)}$$

$$\varphi_1 = \varphi_B - \varphi_A = \frac{N_1 i_1}{R_1 + R_g + R_2 + R_A/2} + \frac{N i_B}{2R_1 + 2R_g + 2R_2 + R_A} - \frac{N i_A}{2R_1 + 2R_g + 2R_2 + R_A}$$

$$\lambda_B = N\varphi_B = \frac{N N_1 i_1}{2R_1 + 2R_g + 2R_2 + R_A} + \frac{N^2 i_B (R_A + R_1 + R_g + R_2)}{R_A(2R_1 + 2R_g + 2R_2 + R_A)}$$

$$+ \frac{N^2 i_A (R_1 + R_g + R_2)}{R_A(2R_1 + 2R_g + 2R_2 + R_A)} = L_{B1} i_1 + L_{BB} i_B + L_{BA} i_A$$

$$\lambda_A = N\varphi_A = \frac{-N N_1 i_1}{2R_1 + 2R_g + 2R_2 + R_A} + \frac{N^2 i_A (R_A + R_1 + R_g + R_2)}{R_A(2R_1 + 2R_g + 2R_2 + R_A)}$$

$$+ \frac{N^2 i_B (R_1 + R_g + R_2)}{R_A(2R_1 + 2R_g + 2R_2 + R_A)} = L_{A1} i_1 + L_{AA} i_A + L_{AB} i_B$$

$$\lambda_1 = N_1 \varphi_1 = \frac{N_1^2 i_1}{R_1 + R_g + R_2 + R_A/2} + \frac{N N_1 i_B}{2R_1 + 2R_g + 2R_2 + R_A}$$

$$- \frac{N N_1 i_A}{2R_1 + 2R_g + 2R_2 + R_A} = L_{11} i_1 + L_{1B} i_B + L_{1A} i_A$$

$$\Rightarrow L_{AA} = L_{BB} = \frac{N^2 (R_A + R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)} =$$

$$\frac{N^2 \left(\frac{l_A}{\mu A_c} + \frac{l_1}{\mu A_c} + \frac{l_2}{\mu A_c} + \frac{g}{\mu_0 A_c} \right)}{\frac{l_A}{\mu A_c} \left[2 \left(\frac{l_1}{\mu A_c} + \frac{l_2}{\mu A_c} + \frac{g}{\mu_0 A_c} \right) + \frac{l_A}{\mu A_c} \right]} =$$

$$\frac{N^2 \mu A_c \left(l_A + l_1 + l_2 + \frac{g\mu}{\mu_0} \right)}{l_A \left[2 \left(l_1 + l_2 + \frac{g\mu}{\mu_0} \right) + l_A \right]} \quad [H]$$

$$L_{11} = \frac{N_1^2}{R_1 + R_g + R_2 + R_A/2} = \frac{N_1^2 \mu A_c}{l_1 + l_2 + \frac{g\mu}{\mu_0} + l_A/2} \quad [H]$$

$$L_{AB} = L_{BA} = \frac{N^2 (R_1 + R_g + R_2)}{R_A (2R_1 + 2R_g + 2R_2 + R_A)} =$$

$$\frac{N^2 \mu A_c \left(l_1 + l_2 + \frac{g\mu}{\mu_0} \right)}{l_A \left[2 \left(l_1 + l_2 + \frac{g\mu}{\mu_0} \right) + l_A \right]} \quad [H]$$