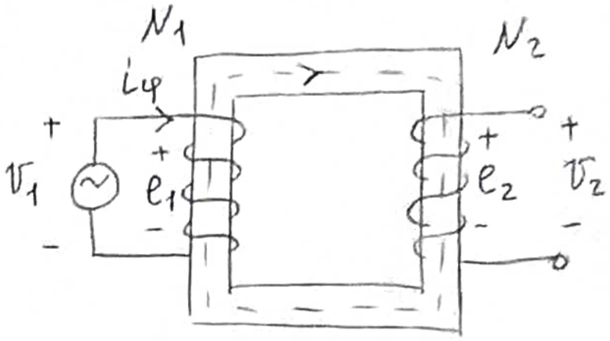
Robt 304 Zarema Balgabekova HW 2

Exercise 1



Given: $N_1 = 1200$ turns

$N_2 = 75$ turns

$A_c = 42\ cm^2 = 42 \cdot 10^{-4}\ m^2$

$B_{max-rms} = 1.45\ T$

a) $f = 60\ Hz$

From Gauss law, $\varphi_{max} = B_{max} A_c$

The waveform of flux is sinusoidal; $\varphi = \varphi_{max} \sin \omega t$

From Faraday Law, $e_1 = N_1 \dfrac{d\varphi}{dt}$

$e_1 = \omega N_1 \varphi_{max} \cos \omega t = 2\pi f N_1 B_{max} A_c \cos \omega t =$

$E_{max1} \cos \omega t$

$\Rightarrow E_{max1} = 2\pi f N_1 B_{max} A_c$

$E_{max-rms1} = \dfrac{E_{max1}}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} \pi f N_1 B_{max} A_c = 2\pi f N_1 B_{max\_rms} A_c$

Primary voltage:

$$V_{max-rms\,1} = E_{max-rms\,1} = 2\pi f N_1 B_{max-rms} A_c =$$

$$2\pi \cdot 60 \cdot 1200 \cdot 1.45 \cdot 42 \cdot 10^{-4} = 2755 \ V\ rms$$

Secondary voltage:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_{max-rms\,2} = \frac{N_2}{N_1} V_{max-rms\,1} = \frac{75}{1200} \cdot 2755 = 172 \ V\ rms$$

B) $f = 50\ Hz$

Primary voltage:

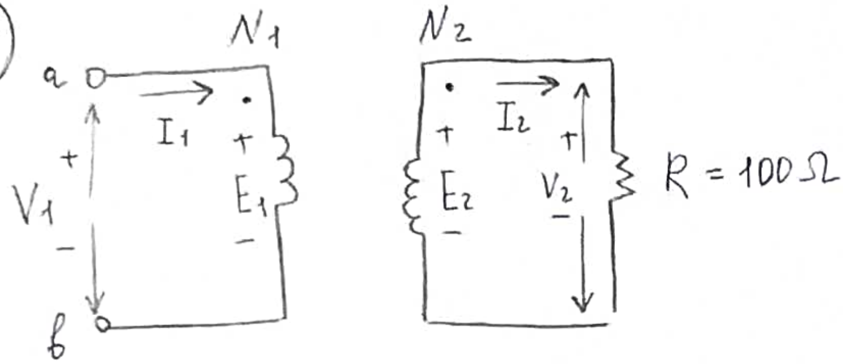
$$V_{max-rms\,1} = 2\pi \cdot 50 \cdot 1200 \cdot 1.45 \cdot 42 \cdot 10^{-4} = 2296 \ V\ rms$$

Secondary voltage:

$$V_{max-rms\,2} = \frac{75}{1200} \cdot 2296 = 143.5 \ V\ rms$$

Exercise 2

a)



$R = 100\,\Omega$

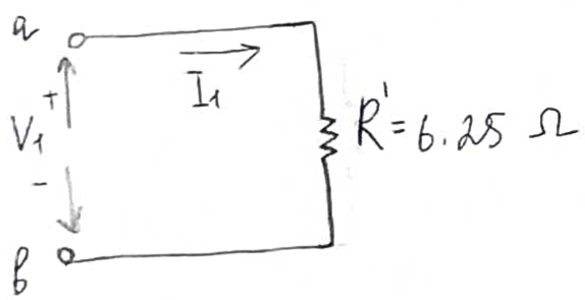
$\frac{N_1}{N_2} = \frac{1}{4}$

$V_1 = 10\ V\ rms$

$f = 1\ kHz$

Referring the resistance to the primary circuit,

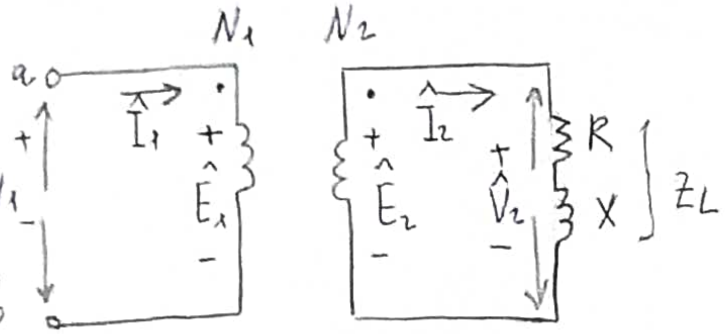
$$R' = \left(\frac{N_1}{N_2}\right)^2 \cdot R = \left(\frac{1}{4}\right)^2 \cdot 100 = 6.25\,\Omega$$



$$I_1 = \frac{V_1}{R'} = \frac{10}{6.25} = 1.6\ A\ rms$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad V_2 = \frac{N_2}{N_1} V_1 = \frac{4}{1} \cdot 10 = 40\ V\ rms$$

b)



$Z_L = 100 + j\,10\ \Omega$

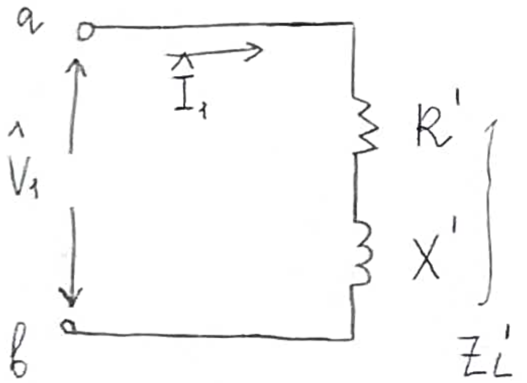
$R = Re(Z_L) = 100\,\Omega$

$X = Im(Z_L) = 10\,\Omega$

Referring the impedance to the primary circuit,

$$Re(Z_L') = R' = \left(\frac{N_1}{N_2}\right)^2 R = \left(\frac{1}{4}\right)^2 \cdot 100 = 6.25\ \Omega$$

$$Im(Z_L') = X' = \left(\frac{N_1}{N_2}\right)^2 X = \left(\frac{1}{4}\right)^2 \cdot 10 = 0.625\ \Omega$$



$$\hat{I}_1 = \frac{\hat{V}_1}{Z_L'} = \frac{\hat{V}_1}{R' + jX'} =$$

$$\frac{10}{6.25 + j\,0.625} = \frac{10\,(6.25 - j\,0.625)}{6.25^2 + 0.625^2} =$$

$$1.584 - 0.1584\,j = 1.5919 \angle -5.71^\circ \text{ A rms}$$

$$P = |\hat{I}_1|^2 R' = 1.5919^2 \cdot 6.25 = 15.84\ W$$

$$Q = |\hat{I}_1|^2 X' = 1.5919^2 \cdot 0.625 = 1.584\ VAR$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{15.84^2 + 1.584^2} = 15.92\ VA$$

Power factor: $\cos(\theta) = \frac{P}{S} = \frac{15.84}{15.92} = 0.995$ lagging

(The system is inductive)

Exercise 3:

$V_1 : V_2 = 120\,V : 2400\,V \quad \Rightarrow \quad \frac{N_1}{N_2} = \frac{120}{2400}$

$f = 60\ Hz$

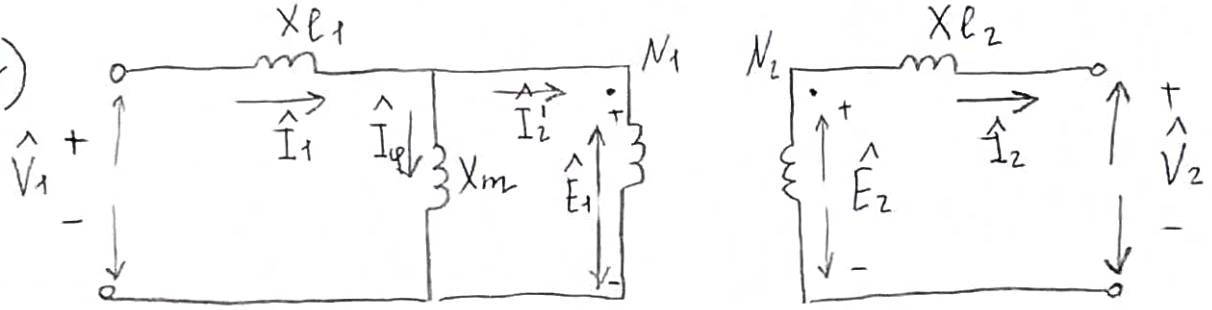
$S = 50\ kVA$

$X_m = 34.6\ \Omega$ (from 120V terminals)

$X_{\ell_1} = 27.4\ m\Omega$

$X_{\ell_2} = 11.2\ \Omega$

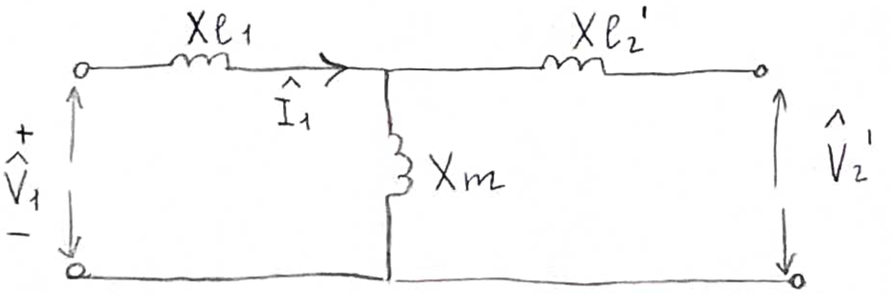
a)



$V_1 = 120V$

Referring the reactance to the primary side:

$$X_{\ell_2}' = \left(\frac{N_1}{N_2}\right)^2 X_{\ell_2} = \left(\frac{120}{2400}\right)^2 \cdot 11.2 = 0.028\ \Omega$$



$$\hat{I}_1 = \frac{\hat{V}_1}{(X_{\ell_1} + X_m)j} = \frac{120}{(27.4 \cdot 10^{-3} + 34.6)j} = -3.465j = 3.465\angle{-90^\circ}\ A$$

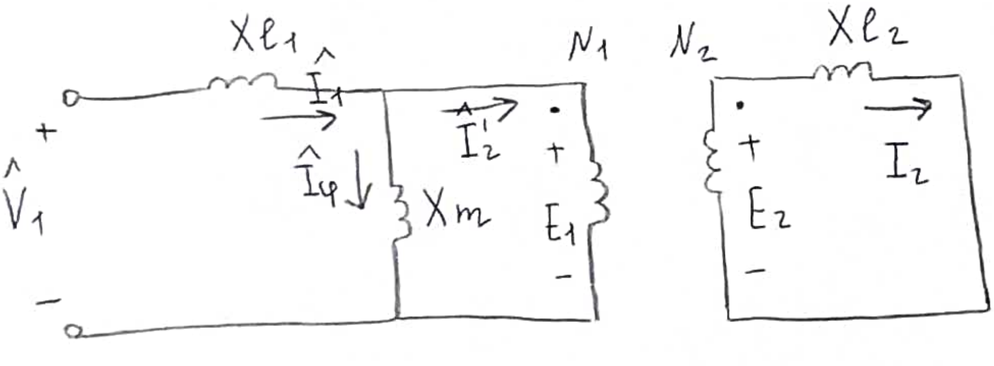
$$\hat{V}_2' = \hat{I}_1 j X_m = \frac{\hat{V}_1}{(X_{\ell_1} + X_m)\cancel{j}} \cancel{j} X_m = \frac{X_m}{X_{\ell_1} + X_m} \hat{V}_1$$

Using $\frac{V_1}{V_2} = \frac{N_1}{N_2}$,

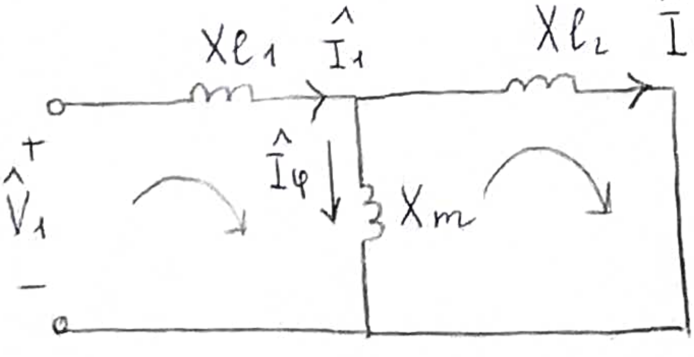
$$\hat{V}_2 = \frac{N_2}{N_1} \hat{V}_2' = \frac{N_2}{N_1} \frac{X_m}{X_{\ell_1} + X_m} \hat{V}_1 =$$

$$\frac{2400}{120} \frac{34.6}{27.4 \cdot 10^{-3} + 34.6} 120 = 2398 \text{ V}$$

b)



Equivalent circuit referred to the primary:



$$\hat{I}_1 = \frac{S}{120} = \frac{50 \cdot 10^3}{120} = 416.67 \text{ A}$$

$$X_{tot} = X_{\ell_1} + X_m \parallel X'_{\ell_2} = X_{\ell_1} + \frac{X_m X'_{\ell_2}}{X_m + X'_{\ell_2}} =$$

$$27.4 \cdot 10^{-3} + \frac{34.6 \cdot 0.028}{34.6 + 0.028} = 0.05538 \ \Omega$$

$$\hat{V}_1 = \hat{I}_1 j X_{tot} = 416.67 j 0.05538 = 23.1 j = 23.1 \angle 90^\circ \text{ V}$$

Kirchhoff's laws:

$$\hat{I}_\varphi = \hat{I}_1 - I'_2$$

loop 2: $$- \hat{I}_\varphi j X_m + \hat{I}'_2 j X'_{\ell_2} = 0$$

$$(\hat{I}'_2 - \hat{I}_1) j X_m + \hat{I}'_2 j X'_{\ell_2} = 0$$

$$\hat{I}'_2 (X_m + X'_{\ell_2}) \cancel{j} = \hat{I}_1 \cancel{j} X_m$$

$$\hat{I}'_2 = \frac{X_m}{X_m + X'_{\ell_2}} \hat{I}_1$$

Using $\frac{I_1}{I_2} = \frac{N_2}{N_1}$,

$$\hat{I}_2 = \frac{N_1}{N_2}\hat{I}_2' = \frac{N_1}{N_2}\frac{X_m}{X_m + X_{\ell_2}'}\hat{I}_1 =$$

$$\frac{120}{2400}\,\frac{34.6}{34.6 + 0.028}\,416.67 = 20.8\ A$$

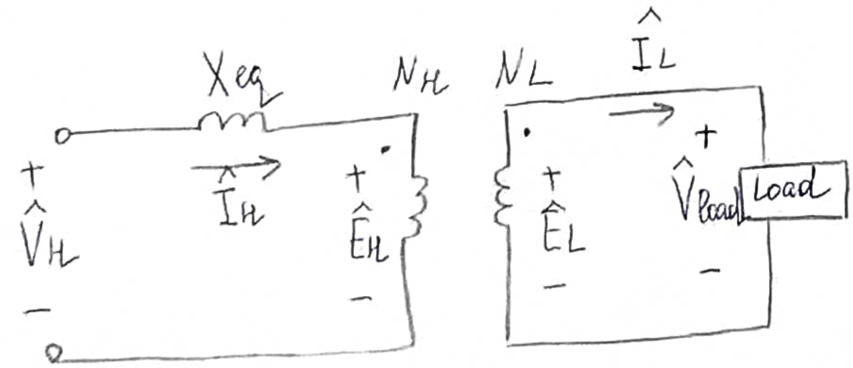
Exercise 4

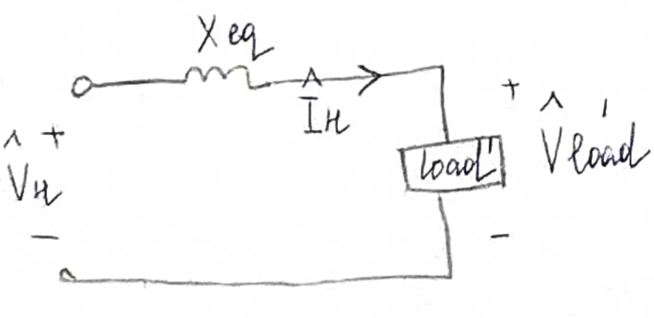
$V_L : V_H = 460\,V : 2400\,V \Rightarrow \frac{N_L}{N_H} = \frac{460}{2400}$

$X_{eq} = 37.2\,\Omega$ (high-voltage side)

$P_{load} = 25\ kW$ (unity power factor)

$V_{load} = 460\ V$





Equivalent circuit referred to the high-voltage side

$$I_L = \frac{P_{load}}{V_{load}} = \frac{25 \cdot 10^3}{460} = 54.35 \text{ A}$$

Using $\frac{I_1}{I_2} = \frac{N_2}{N_1}$,

$$I_H = \frac{N_L}{N_H} I_L = \frac{460}{2400} \cdot 54.35 = 10.417 \text{ A}$$

Using $\frac{V_1}{V_2} = \frac{N_1}{N_2}$, $V'_{load} = \frac{N_H}{N_L} V_{load} = \frac{2400}{460} 460 = 2400 \text{ V}$

$$\hat{V}_H = \hat{I}_H X_{eq} j + V'_{load} = 10.417 \cdot 37.2 j + 2400 =$$

$$387.5 j + 2400 \text{ V}$$

$$|V_H| = \sqrt{387.5^2 + 2400^2} = 2431 \text{ V}$$

$$\theta = \arctan\left(\frac{387.5}{2400}\right) = 9.17^\circ$$

$$\hat{V}_H = |V_H| \angle \theta = 2431 \angle 9.17^\circ \text{ V}$$

Power factor: $\cos\theta = \cos(9.17^\circ) = 0.987$ lagging

(the system is inductive)