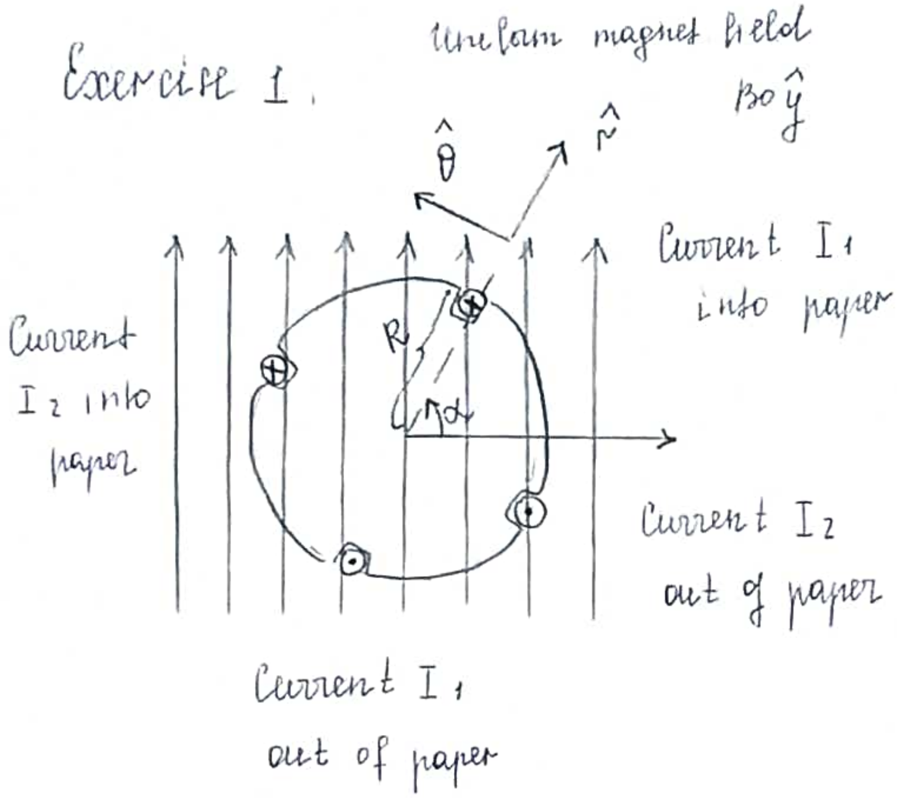
Robt 304 Zarema Balgabekova HW 3

Exercise 1.



$l = 0.30\, m$

$R = 0.13\, m$

$B_0 = 0.85\, T$

a) $I_1 = 0 \quad I_2 = 5\, A$

b) $I_1 = 5\, A \quad I_2 = 0$

c) $I_1 = 8\, A \quad I_2 = 8\, A$

From lorentz Force law,

$$F_{1in} = F_{1out} = - I_1 B_0 l \sin\alpha$$

$$F_{2in} = F_{2out} = - I_2 B_0 l \sin(90-\alpha) = - I_2 B_0 l \cos\alpha$$

$$T = - I_1 B_0 l \sin\alpha \cdot 2R - I_2 B_0 l \cos\alpha \cdot 2R =$$

$$- B_0 l\, 2R \,(I_1 \sin\alpha + I_2 \cos\alpha) =$$

$$- 0.85 \cdot 0.3 \cdot 2 \cdot 0.13 \,(I_1 \sin\alpha + I_2 \cos\alpha) =$$

$$- 0.0663\,(I_1 \sin\alpha + I_2 \cos\alpha)\ N \cdot m$$

a) $T = -0.0663(0 \cdot \sin\alpha + 5\cos\alpha) = -0.3315\cos\alpha \ N \cdot m$

b) $T = -0.0663(5 \cdot \sin\alpha + 0 \cdot \cos\alpha) = -0.3315\sin\alpha \ N \cdot m$

c) $T = -0.0663(8\sin\alpha + 8\cos\alpha) = -0.5304(\sin\alpha + \cos\alpha) N \cdot m$

Exercise 2.

$I_1 = 8\sin\alpha \ A$ $\qquad$ $I_2 = 8\cos\alpha \ A$

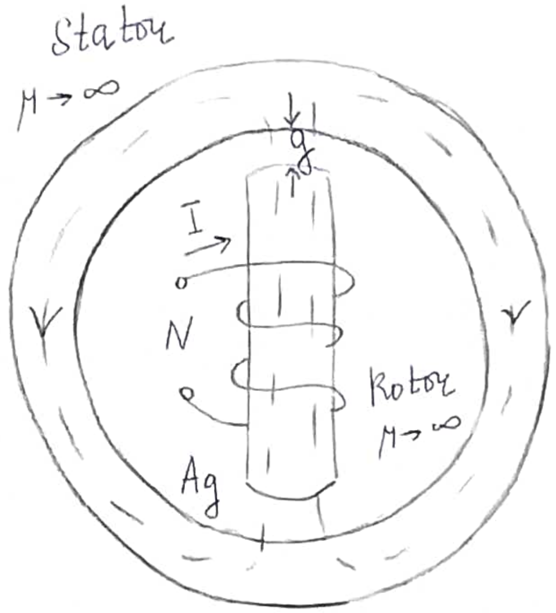
From Exercise 1,

$$T = -0.0663(I_1 \sin\alpha + I_2\cos\alpha) =$$

$$-0.0663(8\sin^2\alpha + 8\cos^2\alpha) =$$

$$-0.5304(\sin^2\alpha + \cos^2\alpha) = -0.5304 \ N \cdot m$$

Exercise 3



$I = 10\ A$

$N = 1000$ turns

$g = 1\ cm = 10^{-2}\ m$

$A_g = 2000\ cm^2 = 2000 \cdot 10^{-4}\ m^2$

$$L = \frac{N^2}{R_{tot}} = \frac{N^2}{\frac{2g}{\mu_0 A_g}} = \frac{N^2 \mu_0 A_g}{2g}$$

$$W_{fld} = \frac{1}{2} L I^2 = \frac{1}{2} \frac{N^2 \mu_0 A_g}{2g} I^2 =$$

$$\frac{1}{2} \frac{1000^2 \cdot 4\pi\, 10^{-7} \cdot 2000 \cdot 10^{-4}}{2 \cdot 10^{-2}} \cdot 10^2 = 200\pi = 628.3\ J$$

Exercise 4

$$L = \frac{2 l_0}{1 + x/x_0}$$

$l_0 = 30 \text{ mH} = 30 \cdot 10^{-3} \text{ H}$

$x_0 = 0.87 \text{ mm} = 0.87 \cdot 10^{-3} \text{ m}$

$R = 110 \text{ m}\Omega = 110 \cdot 10^{-3} \Omega$

a) $x = 0.90 \text{ mm} = 0.9 \cdot 10^{-3} \text{ m}$

current is increased from 0 to 6 A

b) $I = 6 \text{ A}$ displacement is increased to $1.8 \text{ mm} = 1.8 \cdot 10^{-3} \text{ m}$

a) For $x = 0.9 \text{ mm}$,

$$L = \frac{2 \cdot 30 \cdot 10^{-3}}{1 + \frac{0.9}{0.87}} = 29.5 \text{ mH}$$

$$W_{fld} = \frac{1}{2} L I^2 = \frac{1}{2} \cdot 29.5 \cdot 10^{-3} \cdot 6^2 = 0.531 \text{ J}$$

b) For $x = 1.8$ mm,

$$L = \frac{2 \cdot 30 \cdot 10^{-3}}{1 + \frac{1.8}{0.87}} = 19.55 \text{ mH}$$

$$W_{fld}^{new} = \frac{1}{2} L I^2 = \frac{1}{2} \cdot 19.55 \cdot 10^{-3} \cdot 6^2 = 0.352 \text{ J}$$

$$\Rightarrow \Delta W_{fld} = W_{fld}^{new} - W_{fld} = 0.352 - 0.531 = -0.179 \text{ J}$$

Exercise 5.

$$i(t) = I_0 \sin wt$$

$I_0 = 5.5$ A $w = 100\pi$ (50 Hz)

$x = x_0$

a) For $x = x_0$,

$$L = \frac{2 l_0}{2} = l_0 = 30 \text{ mH}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5.5}{\sqrt{2}} = 3.889 \text{ A}$$

$$W_{fld\_ave} = \frac{1}{2} L I_{rms}^2 = \frac{1}{2} \cdot 30 \cdot 10^{-3} \cdot 3{,}889^2 = 0.227\ J$$

b) $$P_{ave} = I_{rms}^2 R = 3{,}889^2 \cdot 110 \cdot 10^{-3} = 1.66\ W$$

Exercise 6.

$$A_g = A_0 \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) \qquad |\theta| \leq \frac{\pi}{6}$$

$g = 0{,}8$ mm $\qquad A_0 = 6$ mm $\qquad N = 650$ turns

a) $$W_{fld} = \int_V \left(\frac{B^2}{2\mu}\right) dV$$

$$W_{fld} = \frac{B_g^2}{2\mu_0} \cdot V_g = \frac{B_g^2}{\cancel{2}\mu_0} \cdot A_g \cdot \cancel{2}g = \frac{B_g^2 A_g \cdot g}{\mu_0}$$

From Ampere law, $\quad Ni = H_g \cdot 2g$

$$H_g = \frac{Ni}{2g}$$

$$B_g = \mu_0 H_g = \frac{\mu_0 Ni}{2g}$$

$$\Rightarrow W_{fld} = \frac{\cancel{\mu_0^2} N^2 i^2 A_g \cdot \cancel{g}}{4g^{\cancel{2}} \cancel{\mu_0}} = \frac{N^2 i^2 A_g \mu_0}{4g} =$$

$$\frac{\mu_0 N^2 i^2}{4g} \cdot A_0 \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) =$$

$$\frac{4\pi \cdot 10^{-7} \cdot 650^2}{4 \cdot 0{,}8 \cdot 10^{-3}} \cdot 6 \cdot 10^{-6} \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) i^2 =$$

$$9{,}95 \cdot 10^{-4} \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) i^2 \quad [J]$$

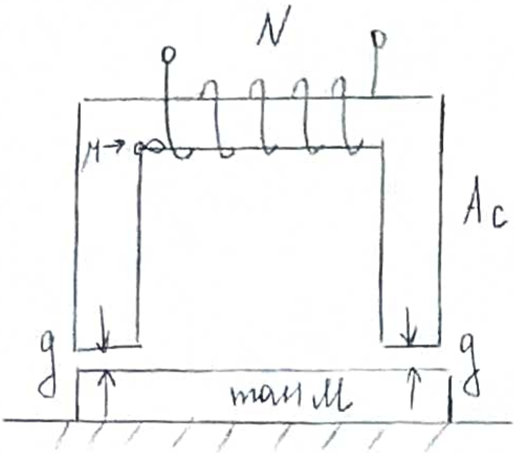
b) Also, $W_{fld} = \frac{1}{2} L i^2$

$$\Rightarrow L(\theta) = \frac{2 W_{fld}}{i^2} = \frac{\mu_0 N^2 A_0}{2g} \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) =$$

$$2 \cdot 9{,}95 \cdot 10^{-4} \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) = 1{,}99 \cdot 10^{-3} \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right)$$

$$= 1{,}99 \left(1 - \left(\frac{4\theta}{\pi}\right)^2\right) \quad [mH]$$

Exercise 7.



$N = 1000$ turns

$g_{min} = 0.8mm$

$A_c = 32\ cm^2$

$R = 2.8\ \Omega$

$M = 95\ kg$

$$F_g = Mg = 95 \cdot 9.8 = 931\ N$$

$$L(g) = \frac{N^2}{R_{tot}(g)} = \frac{N^2}{\frac{2g}{\mu_0 A_c}} = \frac{N^2 \mu_0 A_c}{2g}$$

$$f_{fld} = \frac{i^2}{2}\frac{dL(g)}{dg} = \frac{i^2}{2}\left(-\frac{N^2\mu_0 A_c}{2g^2}\right) =$$

$$-\frac{N^2 i^2 \mu_0 A_c}{4g^2}$$

To lift a slab, $f_{fld} = F_g = 931\ N$

$$i_{min} = \sqrt{\frac{4 f_{lld}\, g_{min}^2}{N^2 \mu_0 A_c}} = \frac{2 g_{min}}{N} \sqrt{\frac{f_{lld}}{\mu_0 A_c}} =$$

$$\frac{2 \cdot 0.8 \cdot 10^{-3}}{1000} \sqrt{\frac{931}{4\pi \cdot 10^{-7} \cdot 32 \cdot 10^{-4}}} = 0.77 \text{ A}$$

$$V_{min} = i_{min} \cdot R = 0.77 \cdot 2.8 = 2.156 \text{ V}$$