Robt 304 Zarema Balgabekova HW1

Exercise 1

The First Surgical Robot

Though the earliest industrial robots were created in the early 1950s [1], they have been used in medicine only since 1980s. The reason for this could be that additional improvements were needed to ensure the safety of patients. In general, surgical robots are used not only to reduce the amount of work done by surgeons but also to minimize the number of invasive incisions, eliminate undesired motion, and improve surgeons' dexterity during medical operations, as well as to allow remote surgery [2].

Currently, there are three main types of surgery robotics systems: active, semi-active, and master-slave systems. Active systems are autonomous systems that perform preprogrammed tasks under the supervision of a surgeon. Semi-active systems combine surgeon-driven and preprogrammed tasks, whereas master-slave systems entirely depend on a surgeon's hand movements and lack any autonomous component [3].

In the 1970s, the U.S. National Aeronautics and Space Administration (NASA) and Defense Advanced Research Project Agency (DARPA) started to explore the idea of remote surgery (telesurgery) and its further application for astronauts and soldiers, respectively [2]. Though these projects have been never fully implemented, they laid the foundation for surgery robotics. As a result, the first use of a robot in a surgical operation was documented in 1985. It was the PUMA 560 (Programmable Universal Machine for Assembly or Programmable Universal Manipulation arm), a standard industrial robotic arm developed at Unimation. After developing a computer program for the arm, Dr. Kwoh from Memorial Medical Center, Long Beach, CA successfully implemented the placement of a needle into the human brain for biopsy with the guidance of Computed Tomography (CT) [4]. This operation was previously exposed to errors because of surgeons' hand tremors [2]. The PUMA 560 was subsequently used by Davies et al to implement a transurethral resection of the prostate (TURP) [3].

Puma 560

Fig. 1. The PUMA 560 industrial robotic arm [5]

The success of the PUMA 560 launched the development of other surgical robots. For example, the ROBODOC system created to improve the precision of hip replacement surgical operation became the first active surgery robotics system approved by the U.S. Food and Drug Administration (FDA) [3]. By the late 1990s, the da Vinci surgical system for minimally invasive surgery was tested, which then became the dominant surgery robotics system worldwide for almost a decade [2] [3]. At present time, robotics surgery is continuing to develop, and the COVID-19 pandemic can also contribute to its development as surgical robots can mitigate the spread of virus [6]. Overall, the future of robotics surgery is promising.

References

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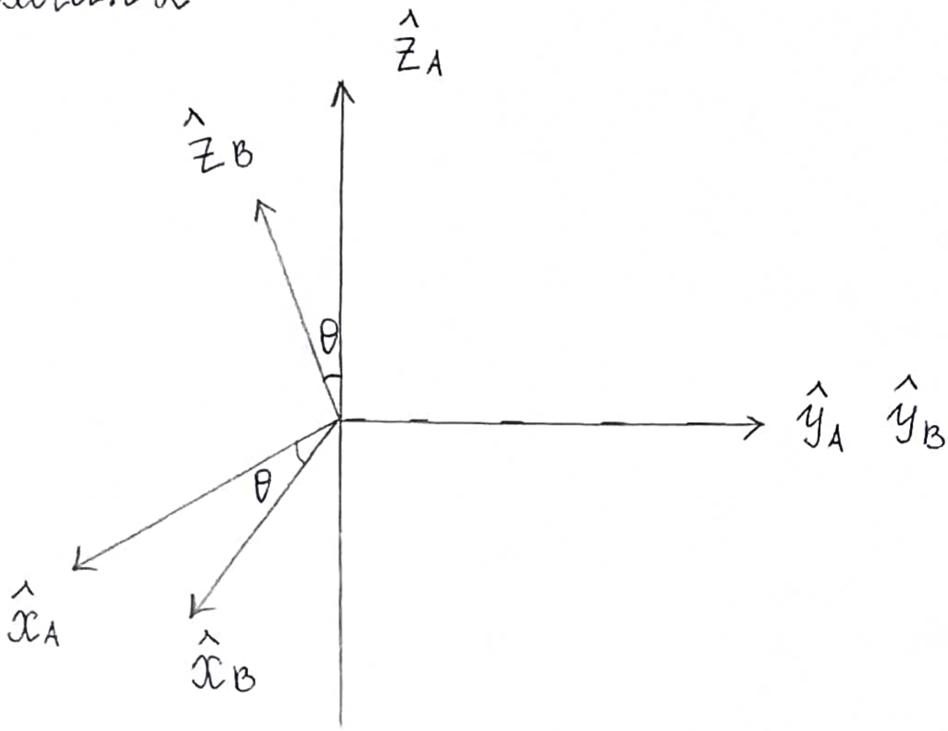
[6] A. Zemmar, A. M. Lozano, B. J. Nelson, "The rise of robots in surgical environments during COVID-19", *Nature Machina Intelligence,* vol. 2, pp. 566-572, 2020. Accessed on: Jan. 31, 2021 [Online]. Available doi: org/10.1038/s42256-020-00238-2

Exercise 2

1)



$${}^A_B R = [\,{}^A\hat{x}_B \quad {}^A\hat{y}_B \quad {}^A\hat{z}_B\,] = \begin{bmatrix} \hat{x}_B\cdot\hat{x}_A & \hat{y}_B\cdot\hat{x}_A & \hat{z}_B\cdot\hat{x}_A \\ \hat{x}_B\cdot\hat{y}_A & \hat{y}_B\cdot\hat{y}_A & \hat{z}_B\cdot\hat{y}_A \\ \hat{x}_B\cdot\hat{z}_A & \hat{y}_B\cdot\hat{z}_A & \hat{z}_B\cdot\hat{z}_A \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \cos 90^\circ & \sin\theta \\ \cos 90^\circ & \cos 0^\circ & \cos 90^\circ \\ -\sin\theta & \cos 90^\circ & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

2) ${}^A P = [2 \quad 1 \quad 0]^T$ $\qquad \theta = +30^\circ$

We know that ${}^A p = {}^A_B R \; {}^B p$

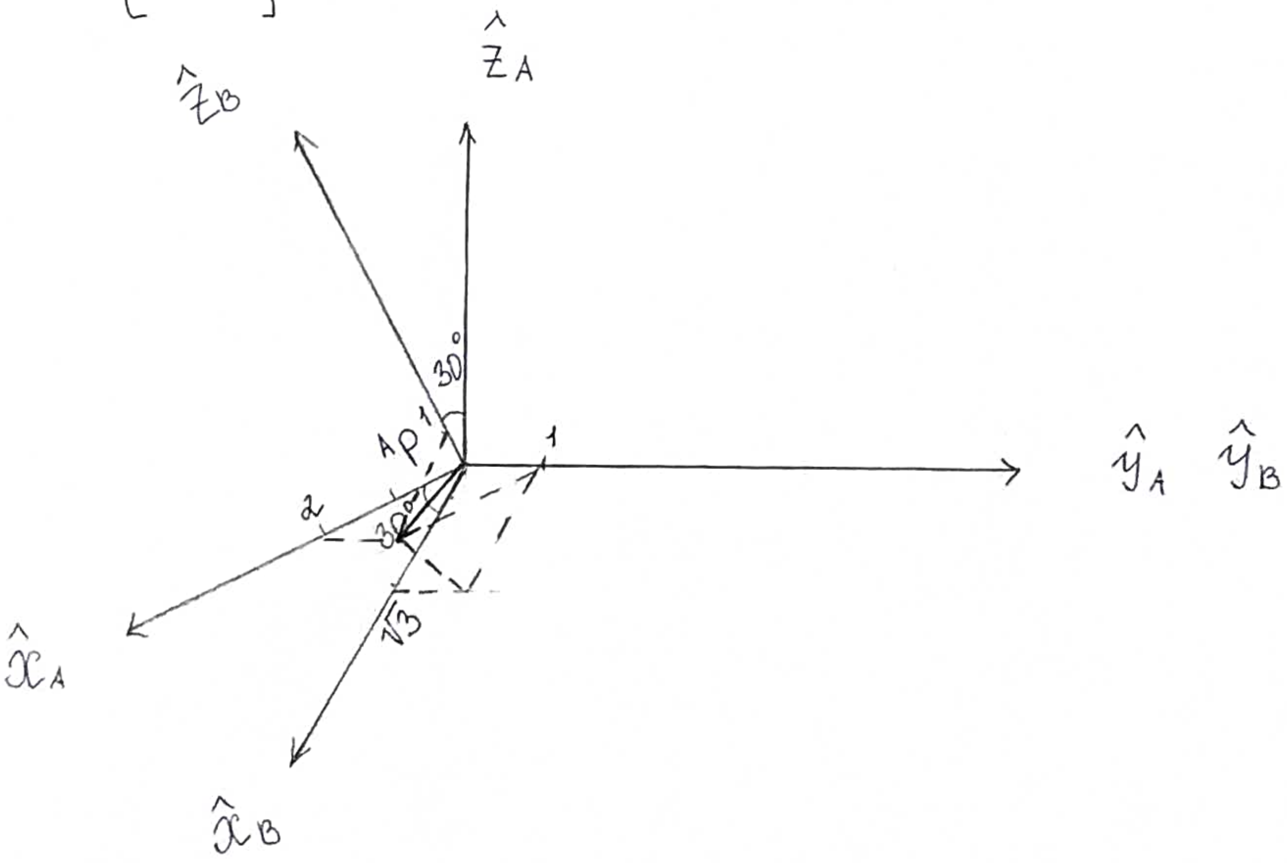
$\Rightarrow {}^B P = {}^A_B R^{-1} \cdot {}^A p = {}^A_B R^T \cdot {}^A p$

$$ {}^{A}_{B}R^{T} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} $$

If $\theta = +30^\circ$, $$ {}^{A}_{B}R^{T} = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix} $$

$$ {}^{B}P = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 \cdot 2 + 0 + 0 \\ 0 + 1 + 0 \\ 1/2 \cdot 2 + 0 + 0 \end{bmatrix} = $$

$$ \begin{bmatrix} \sqrt{3} \\ 1 \\ 1 \end{bmatrix} $$



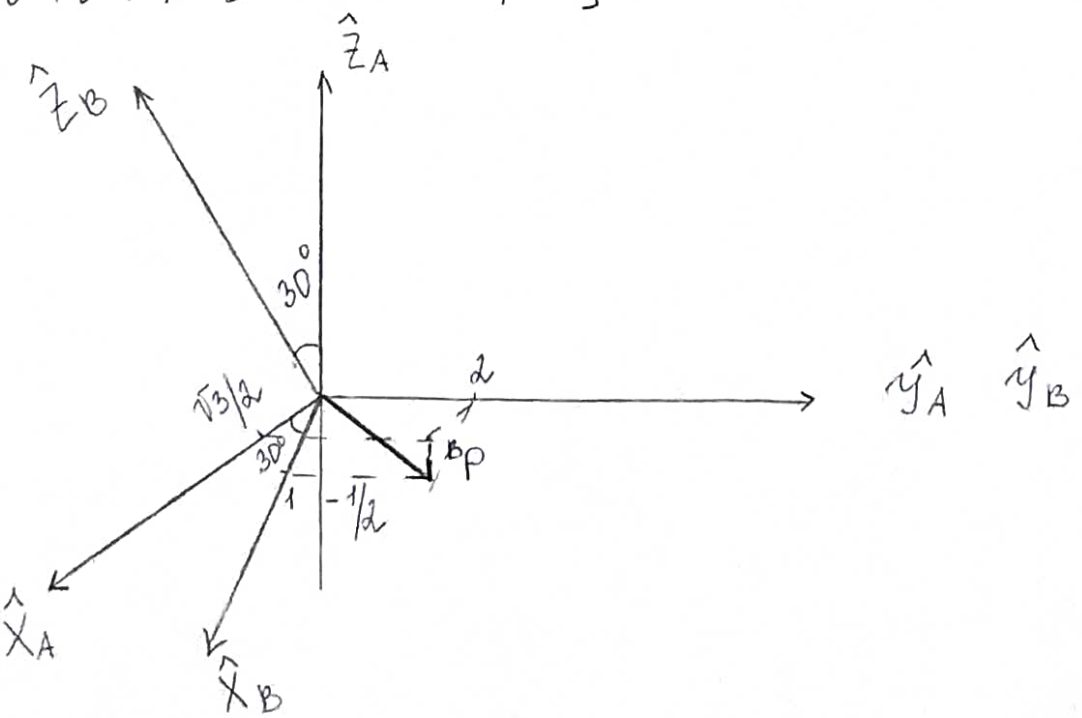
3) $${}^{A}_{B}T = \left[\begin{array}{ccc|c} & {}^{A}_{B}R & & 0 \\ & & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If $\theta = +30^\circ$, $${}^{A}_{B}T = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) $${}^{B}P = [1 \quad 2 \quad 0 \quad 1]^{T}$$

$${}^{A}P = {}^{A}_{B}T \cdot {}^{B}P = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \sqrt{3}/2 + 0 + 0 + 0 \\ 0 + 2 + 0 + 0 \\ -\frac{1}{2} + 0 + 0 + 0 \\ 0 + 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 \\ 2 \\ -1/2 \\ 1 \end{bmatrix}$$



Exercise 3

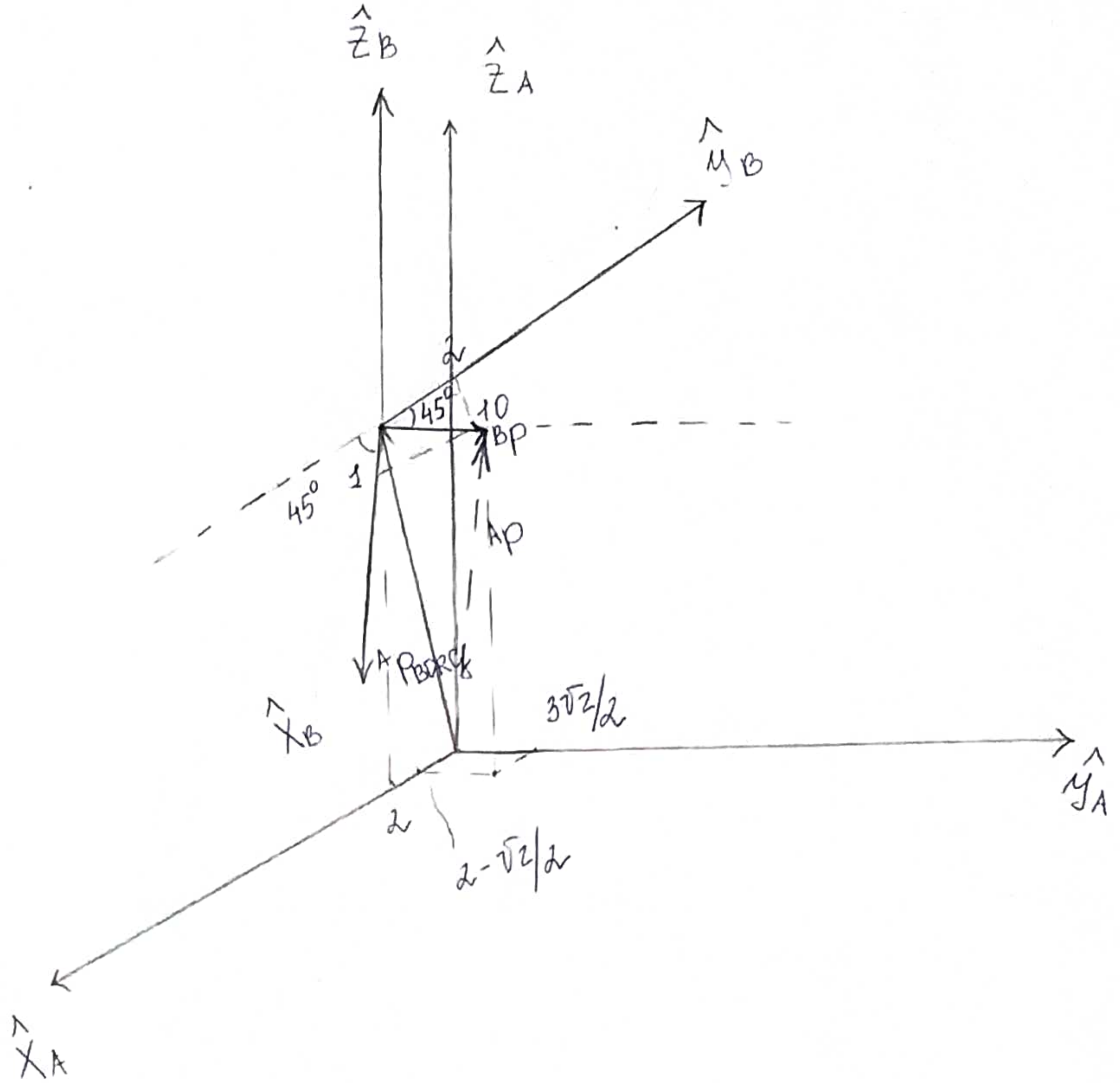
$$^{B}P_{H} = [1 \ 2 \ 0 \ 1]^{T} \qquad \theta = +45^{\circ}$$

$$^{A}_{B}R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{A}P_{BORG} = \begin{bmatrix} 2 \\ 0 \\ 10 \end{bmatrix}$$

$$^{A}P_{H} = \left[\begin{array}{ccc|c} & ^{A}_{B}R & & ^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] {}^{B}P_{H} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 2 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 - \sqrt{2}/2 \cdot 2 + 0 + 2 \\ \sqrt{2}/2 + \sqrt{2}/2 \cdot 2 + 0 + 0 \\ 0 + 0 + 0 + 10 \\ 0 + 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 2 - \sqrt{2}/2 \\ 3\sqrt{2}/2 \\ 10 \\ 1 \end{bmatrix}$$



$\hat{Z}_B$
$\hat{Z}_A$
$\hat{y}_B$
2
45°
10
^BP
45°
1
^AP
$^AP_{BORG}$
$\hat{X}_B$
$3\sqrt{2}/2$
$\hat{y}_A$
2
$2-\sqrt{2}/2$
$\hat{X}_A$

Exercise 4

$$R_x(\theta) R_z(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \cos\theta\sin\alpha & \cos\theta\cos\alpha & -\sin\theta \\ \sin\theta\sin\alpha & \sin\theta\cos\alpha & \cos\theta \end{bmatrix}$$

Let's denote the columns of the matrix as

$$R_x(\theta) R_z(\alpha) = [\hat{x} \quad \hat{y} \quad \hat{z}]$$

For the columns of the matrix to be the versors of the frame {CS}, the following conditions must be true:

$$\|\hat{x}\| = \|\hat{y}\| = \|\hat{z}\| = 1$$

$$\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$$

$$\|\hat{x}\| = \sqrt{\cos^2\alpha + \cos^2\theta\sin^2\alpha + \sin^2\theta\sin^2\alpha} =$$

$$\sqrt{\cos^2\alpha + \sin^2\alpha\,\underbrace{(\cos^2\theta + \sin^2\theta)}_{1}} = \sqrt{\cos^2\alpha + \sin^2\alpha} =$$

$$\sqrt{1} = 1$$

$$\|\hat{y}\| = \sqrt{\sin^2\alpha + \cos^2\theta\cos^2\alpha + \sin^2\theta\cos^2\alpha} =$$

$$\sqrt{\sin^2\alpha + \cos^2\alpha(\cos^2\theta + \sin^2\theta)} = \sqrt{\sin^2\alpha + \cos^2\alpha} = 1$$

$$\|\hat{z}\| = \sqrt{0 + \sin^2\theta + \cos^2\theta} = \sqrt{1} = 1$$

$$\hat{x}\cdot\hat{y} = -\cos\alpha\sin\alpha + \cos^2\theta\sin\alpha\cos\alpha + \sin^2\theta\sin\alpha\cos\alpha =$$

$$\sin\alpha\cos\alpha(-1 + \underbrace{\cos^2\theta + \sin^2\theta}_{1}) = \sin\alpha\cos\alpha\cdot 0 = 0$$

$$\hat{x}\,\hat{z} = \cos\alpha\cdot 0 - \cancel{\cos\theta\sin\alpha\sin\theta} + \cancel{\sin\theta\sin\alpha\cos\theta} = 0$$

$$\hat{y}\,\hat{z} = -\sin\alpha\cdot 0 - \cancel{\cos\theta\cos\alpha\sin\theta} + \cancel{\sin\theta\cos\alpha\cos\theta} = 0$$

$\Rightarrow$ The obtained matrix contains as columns the versors of the frame $\{C\}$

$$R_z(\alpha)\,R_x(\theta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} =$$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha\cos\theta & \sin\alpha\sin\theta \\ \sin\alpha & \cos\alpha\cos\theta & -\cos\alpha\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow \quad R_x(\theta)\,R_z(\alpha) \neq R_z(\alpha)\,R_x(\theta)$$

If we invert the order of the two rotations, we will obtain another result as rotations don't compute in general.