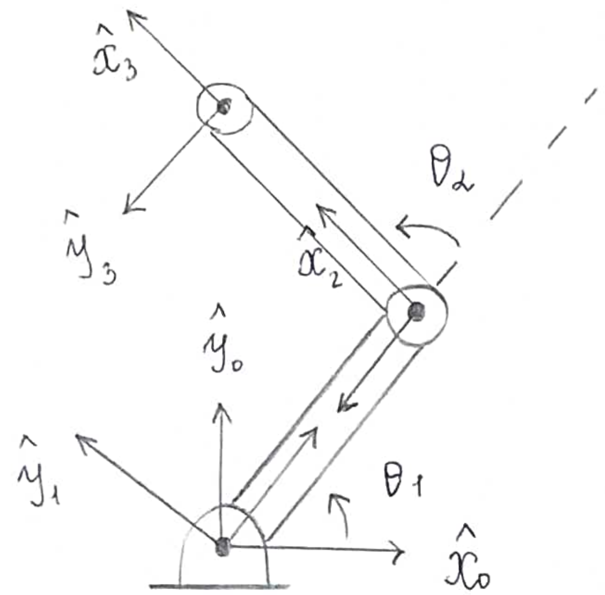
Robt 312 Zarema Balgabekova HW 4

Exercise 1



$L_1 = 1\,m$

$L_2 = 0.8\,m$

$\dot{\theta} = [40 \quad 30]^T \; [°/s]$

$\theta = [45 \quad 45]^T$

$\theta = [0 \quad 0]^T$

From the manipulator, transformation ${}^0_1T$ consists of rotation about Z-axis and no translation; ${}^1_2T$ consists of rotation about Z-axis and translation along X-axis; ${}^2_3T$ consists of no rotation and translation along X-axis

$$\Rightarrow {}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We know that

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\, {}^{i}\omega_i + \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}$$

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R\, ({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1})$$

$$^{0}\omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^{0}v_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{i = 0}$

$$^{1}\omega_1 = {}^{1}_{0}R\, {}^{0}\omega_0 + \dot{\theta}_1\, {}^{1}\hat{Z}_1 = {}^{0}_{1}R^{T}\, {}^{0}\omega_0 + \dot{\theta}_1\, {}^{1}\hat{Z}_1 =$$

$$\begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$^{1}v_1 = {}^{1}_{0}R\, ({}^{0}v_0 + {}^{0}\omega_0 \times {}^{0}P_1) =$$

$$\begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{i = 1}$

$$^{2}\omega_{2} = {}^{2}_{1}R\, {}^{1}\omega_{1} + \dot{\theta}_{2}\, {}^{2}\hat{Z}_{2} = {}^{1}_{2}R^{T}\, {}^{1}\omega_{1} + \dot{\theta}_{2}\, {}^{2}\hat{Z}_{2} =$$

$$\begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$^{2}v_{2} = {}^{2}_{1}R \left({}^{1}v_{1} + {}^{1}\omega_{1} \times {}^{1}P_{2} \right) = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \right.$$

$$\left. \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_1\dot{\theta}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} L_1\dot{\theta}_1 \sin\theta_2 \\ L_1\dot{\theta}_1 \cos\theta_2 \\ 0 \end{bmatrix}$$

$\boxed{i=2}$

$${}^{3}w_{3} = {}^{3}_{2}R\,{}^{2}w_{2} + \dot{\theta}_{3}\,{}^{3}\hat{Z}_{3} = {}^{2}_{3}R^{T}\,{}^{2}w_{2} + \dot{\theta}_{3}\,{}^{3}\hat{Z}_{3} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$${}^{3}v_{3} = {}^{3}_{2}R\left({}^{2}v_{2} + {}^{2}w_{2} \times {}^{2}P_{3}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} L_1 \dot{\theta}_1 S_2 \\ L_1 \dot{\theta}_1 C_2 \\ 0 \end{bmatrix} + \right.$$

$$\left. \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} L_1 \dot{\theta}_1 S_2 \\ L_1 \dot{\theta}_1 C_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} L_1 \dot{\theta}_1 S_2 \\ L_1 \dot{\theta}_1 C_2 + L_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix}$$

$${}^{0}V_3 = {}^{0}_{3}R\,{}^{3}V_3 = {}^{0}_{1}R\,{}^{1}_{2}R\,{}^{2}_{3}R\,{}^{3}V_3$$

$${}^{0}_{1}R\,{}^{1}_{2}R = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C2 & -S2 & 0 \\ S2 & C2 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C1C2 - S1S2 & -C1S2 - S1C2 & 0 \\ S1C2 + C1S2 & -S1S2 + C1C2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C12 & -S12 & 0 \\ S12 & C12 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C12 = \cos(\theta_1 + \theta_2)$

$S12 = \sin(\theta_1 + \theta_2)$

$${}^{0}_{1}R\,{}^{1}_{2}R\,{}^{2}_{3}R = \begin{bmatrix} C12 & -S12 & 0 \\ S12 & C12 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C12 & -S12 & 0 \\ S12 & C12 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{0}v_3 = \begin{bmatrix} c12 & -s12 & 0 \\ s12 & c12 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \dot{\theta}_1 s_2 \\ L_1 \dot{\theta}_1 c_2 + L_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} c12 L_1 \dot{\theta}_1 s_2 - s12 (L_1 \dot{\theta}_1 c_2 + L_2 (\dot{\theta}_1 + \dot{\theta}_2)) \\ s12 L_1 \dot{\theta}_1 s_2 + c12 (L_1 \dot{\theta}_1 c_2 + L_2 (\dot{\theta}_1 + \dot{\theta}_2)) \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} -s_1 L_1 \dot{\theta}_1 - s12 L_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ c_1 L_1 \dot{\theta}_1 + c12 L_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix}$$

$$\boxed{\text{For} \quad \theta = [45 \quad 45]^T}$$

$$^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 40 \end{bmatrix} [°/s] \qquad ^{1}v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} [m/s]$$

$$^{2}\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 40+30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 70 \end{bmatrix} [°/s] \qquad ^{2}v_2 = \begin{bmatrix} 1 \cdot 40 \cdot \frac{\sqrt{2}}{2} \\ 1 \cdot 40 \cdot \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 20\sqrt{2} \\ 20\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 28.3 \\ 28.3 \\ 0 \end{bmatrix}$$

$[m/s]$

$${}^{3}\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 70 \end{bmatrix} [°/s] \quad {}^{3}v_3 = \begin{bmatrix} 1 \cdot 40 \cdot \frac{\sqrt{2}}{2} \\ 1 \cdot 40 \cdot \frac{\sqrt{2}}{2} + 0.8 \cdot 70 \\ 0 \end{bmatrix} = \begin{bmatrix} 20\sqrt{2} \\ 20\sqrt{2} + 56 \\ 0 \end{bmatrix} = \begin{bmatrix} 28.3 \\ 84.3 \\ 0 \end{bmatrix}$$

$[m/s]$

$${}^{0}v_3 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \cdot 1 \cdot 40 - 1 \cdot 0.8 \cdot 70 \\ \frac{\sqrt{2}}{2} \cdot 1 \cdot 40 + 0 \cdot 0.8 \cdot 70 \\ 0 \end{bmatrix} = \begin{bmatrix} -20\sqrt{2} - 56 \\ 20\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} -84.3 \\ 28.3 \\ 0 \end{bmatrix}$$

$[m/s]$

$\boxed{\text{For } \theta = [0 \;\; 0]^T}$

$${}^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 40 \end{bmatrix} [°/s] \qquad {}^{1}v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} [m/s]$$

$${}^{2}\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 70 \end{bmatrix} [°/s] \qquad {}^{2}v_2 = \begin{bmatrix} 1 \cdot 40 \cdot 0 \\ 1 \cdot 40 \cdot 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 0 \end{bmatrix} [m/s]$$

$$ {}^3\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 70 \end{bmatrix} [°/s] \qquad {}^3v_3 = \begin{bmatrix} 1 \cdot 40 \cdot 0 \\ 1 \cdot 40 \cdot 1 + 0.8 \cdot 70 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 96 \\ 0 \end{bmatrix} [m/s] $$

$$ {}^0v_3 = \begin{bmatrix} -0 \cdot 1 \cdot 40 - 0 \cdot 0.8 \cdot 70 \\ 1 \cdot 1 \cdot 40 + 1 \cdot 0.8 \cdot 70 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 96 \\ 0 \end{bmatrix} [m/s] $$

Considerations about the results:

${}^1\omega_1$, ${}^2\omega_2$, ${}^3\omega_3$ are the same for both configurations because they depend only on $\dot{\theta}$, not θ.

${}^1v_1$ is always zero because ${}^0v_0$ and ${}^0\omega_0$ are zero.

x component of ${}^2v_2$, ${}^3v_3$, and ${}^0v_3$ depends on sine functions, and y component – on cosine functions. Therefore, for $\theta = [0 \ 0]^T$, we have zero x component and maximum y component

Exercise 2.

$$
{}^{0}_{4}T = \begin{bmatrix} * & * & * & L_3(c_1c_2c_3 - c_1s_2s_3) + L_2c_1c_2 \\ * & * & * & L_3(s_1c_2c_3 - s_1s_2s_3) + L_2s_1c_2 \\ * & * & * & L_3(s_2c_3 - c_2s_3) + L_2s_2 + L_1 \\ * & * & * & 1 \end{bmatrix}
$$

$${}^{0}F = [F_x \quad F_y \quad F_z]^T \qquad \tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T$$

We know that $\tau = {}^{0}J^T(\theta)\,{}^{0}F$

$\Rightarrow$ We have to find ${}^{0}J^T(\theta)$

Also, ${}^{0}v = {}^{0}J(\theta)\,\dot{\theta}$

let ${}^{0}P_{4x} = L_3(c_1c_2c_3 - c_1s_2s_3) + L_2c_1c_2$

$${}^{0}P_{4y} = L_3(s_1c_2c_3 - s_1s_2s_3) + L_2s_1c_2$$

$${}^{0}P_{4z} = L_3(s_2c_3 - c_2s_3) + L_2s_2 + L_1$$

$${}^{0}\dot{P}_{4x} = {}^{0}v_{4x} = L_3(-s_1c_2c_3\dot{\theta}_1 + s_1s_2s_3\dot{\theta}_1) + L_3(-c_1s_2c_3\dot{\theta}_2 - c_1c_2s_3\dot{\theta}_2) +$$
$$L_3(-c_1c_2s_3\dot{\theta}_3 - c_1s_2c_3\dot{\theta}_3) - L_2s_1c_2\dot{\theta}_1 - L_2c_1s_2\dot{\theta}_2$$

$$ {}^0\dot{P}_{4y} = {}^0V_{4y} = L_3\left(c_1c_2c_3\dot{\theta}_1 - c_1s_2s_3\dot{\theta}_1\right) + L_3\left(-s_1s_2c_3\dot{\theta}_2 - s_1c_2s_3\dot{\theta}_2\right) + $$

$$ L_3\left(-s_1c_2s_3\dot{\theta}_3 - s_1s_2c_3\dot{\theta}_3\right) + L_2c_1c_2\dot{\theta}_1 - L_2s_1s_2\dot{\theta}_2 $$

$$ {}^0\dot{P}_{4z} = {}^0V_{4z} = L_3\left(c_2c_3\dot{\theta}_2 + s_2s_3\dot{\theta}_2\right) + L_3\left(-s_2s_3\dot{\theta}_3 - c_2c_3\dot{\theta}_3\right) + $$

$$ L_2c_2\dot{\theta}_2 $$

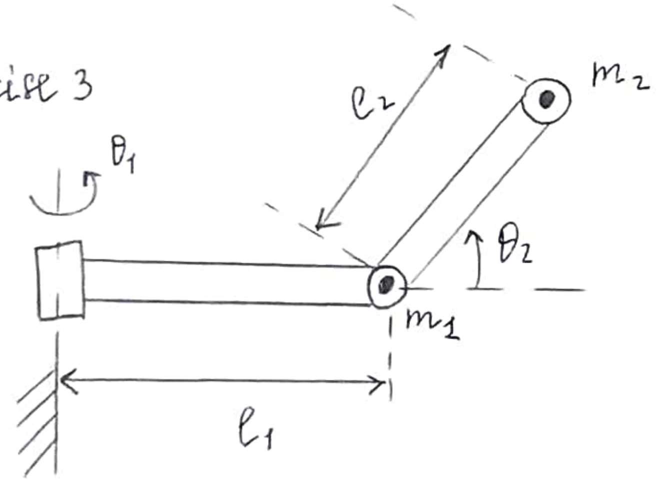
$$ \Rightarrow {}^0V_4 = \overbrace{\begin{bmatrix} L_3(-s_1c_2c_3 + s_1s_2s_3) - L_2s_1c_2 & L_3(-c_1s_2c_3 - c_1c_2s_3) - L_2c_1s_2 & L_3(-c_1c_2s_3 - c_1s_2c_3) \\ L_3(c_1c_2c_3 - c_1s_2s_3) + L_2c_1c_2 & L_3(-s_1s_2c_3 - s_1c_2s_3) - L_2s_1s_2 & L_3(-s_1c_2s_3 - s_1s_2c_3) \\ 0 & L_3(c_2c_3 + s_2s_3) + L_2c_2 & L_3(-s_2s_3 - c_2c_3) \end{bmatrix}}^{{}^0J(\theta)} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

$\Rightarrow$

$$\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} = \overbrace{\begin{bmatrix} L_3(-s_1c_2c_3+s_1s_2s_3)-L_2s_1c_2 & L_3(c_1c_2c_3-c_1s_2s_3)+L_2c_1c_2 & 0 \\ L_3(-c_1s_2c_3-c_1c_2s_3)-L_2c_1s_2 & L_3(-s_1s_2c_3-s_1c_2s_3)-L_2s_1s_2 & L_3(c_2c_3+s_2s_3)+L_2c_2 \\ L_3(-c_1c_2s_3-c_1s_2c_3) & L_3(-s_1c_2s_3-s_1s_2c_3) & L_3(-s_2s_3-c_2c_3)\end{bmatrix}}^{{}^0J^T(\theta)}$$

$$= \underbrace{\begin{bmatrix} -s_1L_3c_{23}-L_2s_1c_2 & c_1L_3c_{23}+L_2c_1c_2 & 0 \\ -c_1L_3s_{23}-L_2c_1s_2 & -s_1L_3s_{23}-L_2s_1s_2 & L_3\cos(\theta_2-\theta_3)+L_2c_2 \\ -c_1L_3s_{23} & -s_1L_3s_{23} & -L_3\cos(\theta_2-\theta_3)\end{bmatrix}}_{{}^0J^T(\theta)} \begin{bmatrix}F_x\\ F_y\\ F_z\end{bmatrix}$$

Exercise 3



We know that

$$k_i = \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} {}^i\omega_i^T \, {}^{ci}I_i \, {}^i\omega_i$$

$$u_i = -m_i {}^0g^T \, {}^0P_{ci} + u_{ref\,i}$$

For this problem, it is easier to calculate kinetic energy as ${}^{C_i}I_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Kinetic energy:

$$k_1 = \frac{1}{2} m_1 \|v_{c_1}\|^2 = \frac{1}{2} m_1 (l_1 \dot{\theta}_1)^2 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2$$

$\|v_{c_2}\|^2$ depends on both $\dot{\theta}_1$ and $\dot{\theta}_2$:

$$\|v_{c_2}\|^2 = (l_1 + l_2 \cos\theta_2)^2 \cdot \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2$$

$$\Rightarrow k_2 = \frac{1}{2} m_2 \left[(l_1 + l_2 \cos\theta_2)^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 \right]$$

Total kinetic energy:

$$k = k_1 + k_2 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left[(l_1 + l_2 \cos\theta_2)^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 \right] =$$

$$\frac{1}{2} \dot{\theta}_1^2 \left(m_1 l_1^2 + m_2 (l_1 + l_2 \cos\theta_2)^2 \right) + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2$$

$$\Rightarrow k = k(\theta, \dot{\theta})$$

Potential energy:

It is seen from the manipulator that

$$u_1 = 0$$

$$u_2 = m_2 g l_2 \sin\theta_2 + c \qquad (c - \text{constant})$$

Total potential energy:

$$u = u_1 + u_2 = m_2 g l_2 \sin\theta_2 + c$$

$$\Rightarrow u = u(\theta)$$

We know that

$$\frac{d}{dt}\frac{\partial k}{\partial \dot\theta} - \frac{\partial k}{\partial \theta} + \frac{\partial u}{\partial \theta} = \tau$$

$$\frac{\partial k}{\partial \dot\theta} = \begin{bmatrix} \dfrac{\partial k}{\partial \dot\theta_1} \\ \dfrac{\partial k}{\partial \dot\theta_2} \end{bmatrix} = \begin{bmatrix} \dot\theta_1 (m_1 l_1^2 + m_2 (l_1 + l_2 \cos\theta_2)^2) \\ \dot\theta_2 m_2 l_2^2 \end{bmatrix}$$

$$\frac{\partial K}{\partial \theta} = \begin{bmatrix} \frac{\partial K}{\partial \theta_1} \\ \frac{\partial K}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} 0 \\ -\dot{\theta}_1^2 m_2 (l_1 + l_2 \cos\theta_2) l_2 \sin\theta_2 \end{bmatrix}$$

$$\frac{\partial U}{\partial \theta} = \begin{bmatrix} \frac{\partial U}{\partial \theta_1} \\ \frac{\partial U}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} 0 \\ m_2 g l_2 \cos\theta_2 \end{bmatrix}$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\theta}} = \begin{bmatrix} \ddot{\theta}_1 (m_1 l_1^2 + m_2 (l_1 + l_2 \cos\theta_2)^2) - 2\dot{\theta}_1 m_2 (l_1 + l_2 \cos\theta_2) l_2 \sin\theta_2 \, \dot{\theta}_2 \\ \ddot{\theta}_2 m_2 l_2^2 \end{bmatrix}$$

$$\Rightarrow \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 (m_1 l_1^2 + m_2 (l_1 + l_2 \cos\theta_2)^2) - 2 m_2 l_2 (l_1 + l_2 \cos\theta_2) \sin\theta_2 \, \dot{\theta}_1 \dot{\theta}_2 \\ \ddot{\theta}_2 m_2 l_2^2 + \dot{\theta}_1^2 m_2 (l_1 + l_2 \cos\theta_2) l_2 \sin\theta_2 + m_2 g l_2 \cos\theta_2 \end{bmatrix}$$

Exercise 4.

State-space matrix equation:

$$\tau = \mathcal{M}(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$$

$$\Rightarrow \mathcal{M}(\theta) = \begin{bmatrix} m_1 l_1^2 + m_2(l_1 + l_2\cos\theta_2)^2 & 0 \\ 0 & m_2 l_2^2 \end{bmatrix}$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -2m_2 l_2(l_1 + l_2\cos\theta_2)\sin\theta_2\ \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_1^2\ m_2 l_2\ (l_1 + l_2\cos\theta_2)\sin\theta_2 \end{bmatrix}$$

$$G(\theta) = \begin{bmatrix} 0 \\ m_2 g\, l_2\cos\theta_2 \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} m_1 l_1^2 + m_2 (l_1 + l_2 \cos\theta_2)^2 & 0 \\ 0 & m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} +$$

$$\begin{bmatrix} -2m_2 l_2 (l_1 + l_2 \cos\theta_2) \operatorname{sen}\theta_2 \, \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1^2 \, m_2 l_2 (l_1 + l_2 \cos\theta_2) \operatorname{sen}\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ m_2 g \, l_2 \cos\theta_2 \end{bmatrix}$$

The dynamic system is not coupled because

its mass matrix is diagonal