

Task 1

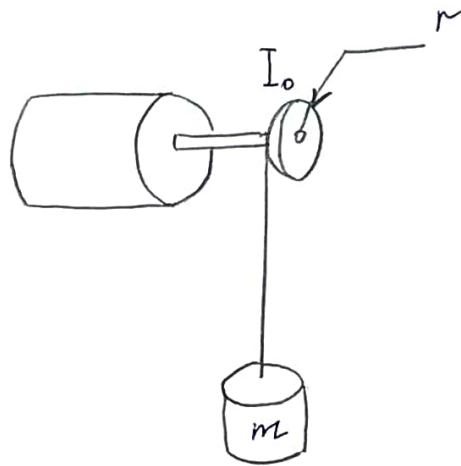
Given: $m = 10 \text{ kg}$

$$r = 0.035 \text{ m}$$

$$I_0 = 0.005 \text{ kg} \cdot \text{m}^2$$

$$T = 5 - 0.05 \omega \text{ Nm}$$

$$\omega(0) = 0$$



Find: t for $\omega = 10 \text{ 1/sec}$

$$h(t)$$

Solution:

Total moment of inertia:

$$I = I_0 + mr^2 = 0.005 + 10 \cdot 0.035^2 = 0.01725 \text{ kg} \cdot \text{m}^2$$

Total torque: $T_{\text{total}} = T - mgr = 5 - 0.05 \omega - 10 \cdot 9.8 \cdot 0.035$

$$T_{\text{total}} = 1.57 - 0.05 \omega \text{ Nm}$$

Newton's second law:

$$\alpha = \frac{T_{\text{total}}}{I}$$

$$\dot{\omega} = \frac{1.57}{0.01725} - \frac{0.05}{0.01725} \omega$$

$$\dot{\omega} + 2.9 \omega = 91, \quad \omega(0) = 0$$

$$w = A e^{-2.9t} + \frac{91}{2.9}$$

$$w = A e^{-2.9t} + 31$$

$$0 = A + 31 \quad (w(0) = 0)$$

$$A = -31$$

$$\Rightarrow w = -31 e^{-2.9t} + 31 \quad 1/\text{sec}$$

$$\Rightarrow t = -\frac{1}{2.9} \ln \left(\frac{w-31}{-31} \right)$$

$$t = -\frac{1}{2.9} \ln \left(\frac{10-31}{-31} \right) \quad (w = 10 \quad 1/\text{sec})$$

$$t \approx 0.134 \text{ sec}$$

We know that $w = \frac{d\theta}{dt} = -31 e^{-2.9t} + 31$

$$\begin{aligned} \Rightarrow \theta &= \int_0^t (-31 e^{-2.9t} + 31) dt = \frac{31}{2.9} e^{-2.9t} + 31t - \frac{31}{2.9} \\ &= 10.7 e^{-2.9t} + 31t - 10.7 \end{aligned}$$

$$\begin{aligned} \theta &= 10.7 e^{-2.9 \cdot 0.134} + 31 \cdot 0.134 - 10.7 = 0.709 \text{ rad} = \\ (t = 0.134 \text{ sec}) & \quad 0.113 \text{ rev} \end{aligned}$$

$$h = \theta \cdot r = 0.113 \cdot 0.035 = 4 \cdot 10^{-3} \text{ m} = 4 \text{ mm}$$

Task 2

Given: $T_{\text{nom}} = 3.5 \text{ mNm}$

$$\omega = 5000 \text{ rpm}$$

$$V = 20 \text{ V}$$

Solution:

Convert rotational speed to angular speed:

$$\omega = \frac{5000 \cdot 2\pi}{60} \approx 523.6 \text{ rad/s}$$

Output power: $P_{\text{out}} = T \cdot \omega = 3.5 \cdot 10^{-3} \cdot 523.6 = 1.83 \text{ W}$

The motor should be rated at least 1.5 to 2 times the desired output power:

$$1.83 \cdot 1.5 = 2.745 \text{ W}$$
$$1.83 \cdot 2 = 3.66 \text{ W}$$

In the Maxon motor catalog, there are motors with powers of 3W and 3.5W, belonging to the required range.

Since the motors with power of 3W are more expensive and have smaller diameter (the motor should operate continuously in our case), I choose the motor with power of 3.5 W.

A DC supply voltage of 20V is available. Because operating a motor at voltage below nominal generally has no detrimental effect on its performance while higher voltage can cause motor coils to overheat, decreasing its life, I choose the motor with nominal voltage of 24V ($20V < 24V$).

Its no load speed is 6520 rpm, which is comparable with the desired speed of 5000 rpm

We also know that $P = VI$

$$\Rightarrow I = \frac{P}{V} = \frac{3.5}{20} = 0.175 A$$

(nominal current of a chosen motor is 0.177 A)

The motor's maximum winding temperature is $+85^{\circ}C$, which should be okay if it operates at room temperature and because supplied voltage is not higher or much less than nominal voltage and motor's size is not too small.

$\Rightarrow \boxed{\text{I choose the motor N 110140}}$