The code is not reported to save the space, but it can be seen in the HW2.m file.

Exercise 1

The matrix representing a rotation about a generic axis K passing through the origin looks as follows:

$$R_K(\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}$$

, where $v\theta = 1 - c\theta$

We can denote the columns of the matrix with versors:

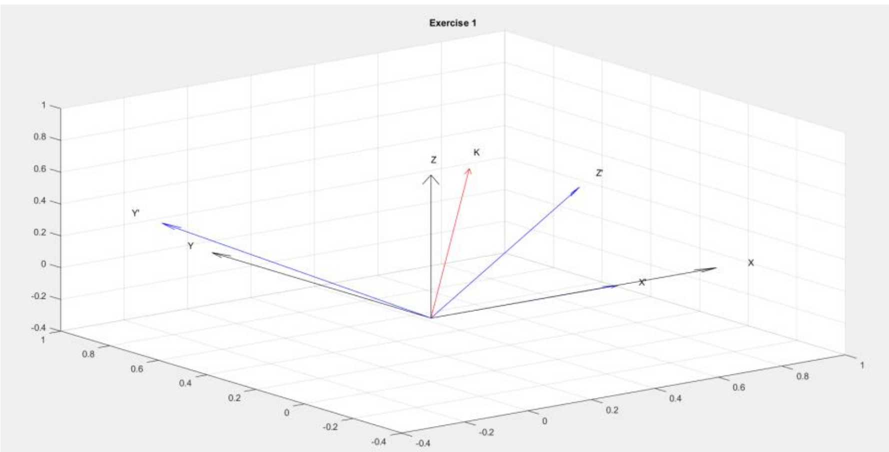
$$R = \begin{bmatrix} \hat{X} & \hat{Y} & \hat{Z} \end{bmatrix}$$

To be orthonormal, the matrix must satisfy the following condition:

$$\begin{aligned} &\|\hat{X}\| = 1, && \hat{X} \cdot \hat{Y} = 0, \\ &\|\hat{Y}\| = 1, && \hat{X} \cdot \hat{Z} = 0, \\ &\|\hat{Z}\| = 1, && \hat{Y} \cdot \hat{Z} = 0. \end{aligned}$$

MATLAB was chosen to implement the task. In MATLAB, the *norm()* function was used to check the unit length condition, and the *dot()* function was used to check the orthogonality condition. Please run the code to see that the conditions were verified.

Fixed frame, moving frame, and the rotation axis were visualized with the *quiver3()* function; and the axes were labeled with the *text()* function. In the visualization, the moving frame is rotated by 30° and colored in blue. The rotation axis is shown in red, and the fixed frame – in black.



Exercise 2

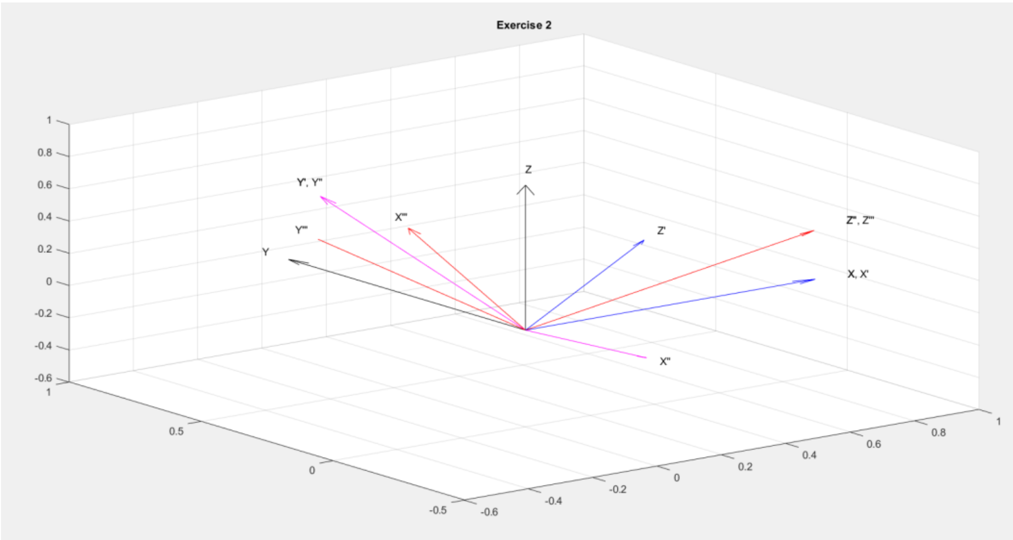
Since we use the X-Y-Z Euler angles convention, the post-multiplication rule was applied with respect to the three principal rotation matrices. Again, the functions *norm()* and *dot()* were used to verify that the obtained compound matrix is orthonormal. Please run the code to see that this was verified.

The function *quiver3()* was used to visualize the fixed frame and the moving frames that represent three consecutive rotations.

In the visualization, the fixed frame is colored in black, the rotation by an angle $\alpha = 30^{\circ}$ about axis X is colored in blue, the rotation by an angle $\beta = 45^{\circ}$ about axis Y' is colored in magenta, and the rotation by an angle $\gamma = 60^{\circ}$ about axis Z'' is colored in red.

Axes X and X', Y' and Y'', and Z'' and Z''' coincide because the principal rotations are implemented about these axes.

The function *text()* was used to label the axes.



Exercise 3

After obtaining the compound rotation using the post-multiplication rule, it was possible to calculate the screw axis and the rotation angle with the following formulas:

$$\hat{K} = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \qquad \theta = \mathrm{Acos}\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right)$$

, where r_{ij} represents the element of the compound rotation matrix in the i^th^ row and the j^th^ column

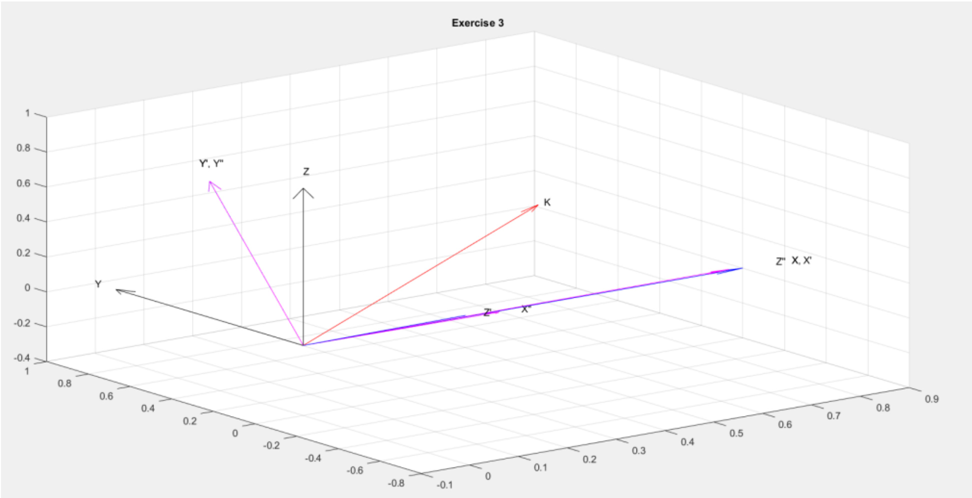
As a result, $\theta = 73.7201°$ and $K = \begin{matrix} 0.7701 \\ 0.5525 \\ 0.3190 \end{matrix}$

The length of a screw axis = 1 (calculated with the *norm()* function).

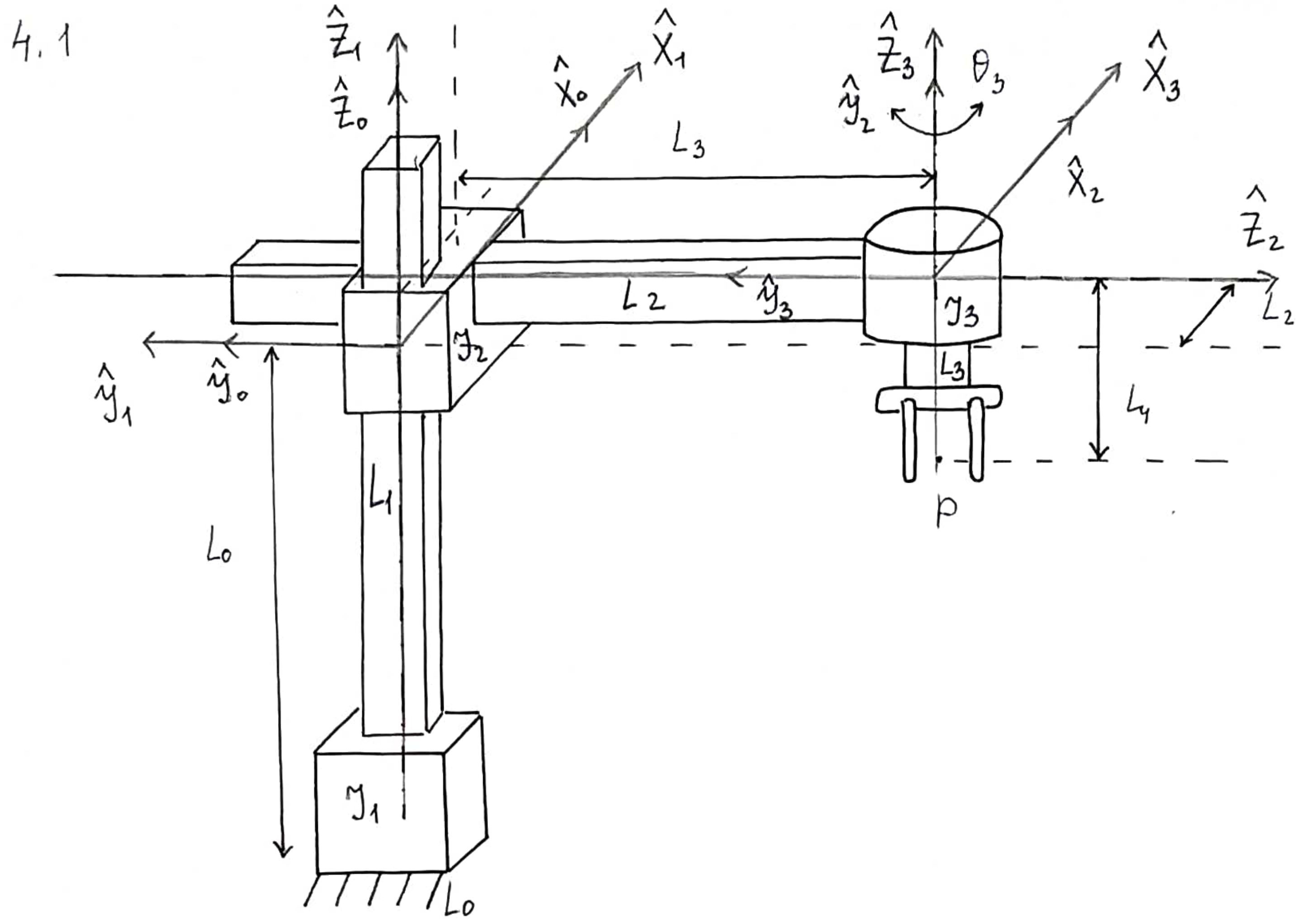
Visualization of frames and a screw axis was again obtained with the functions *quiver3()* and *text()*.

Axes X and X’, Y’ and Y’’ coincide because principal rotations are implemented about these axes.

The fixed frame is colored in black, the rotation by 60° about X axis is colored in blue, and the rotation by 45° about Y’ axis is colored in magenta. The screw axis is shown in red.



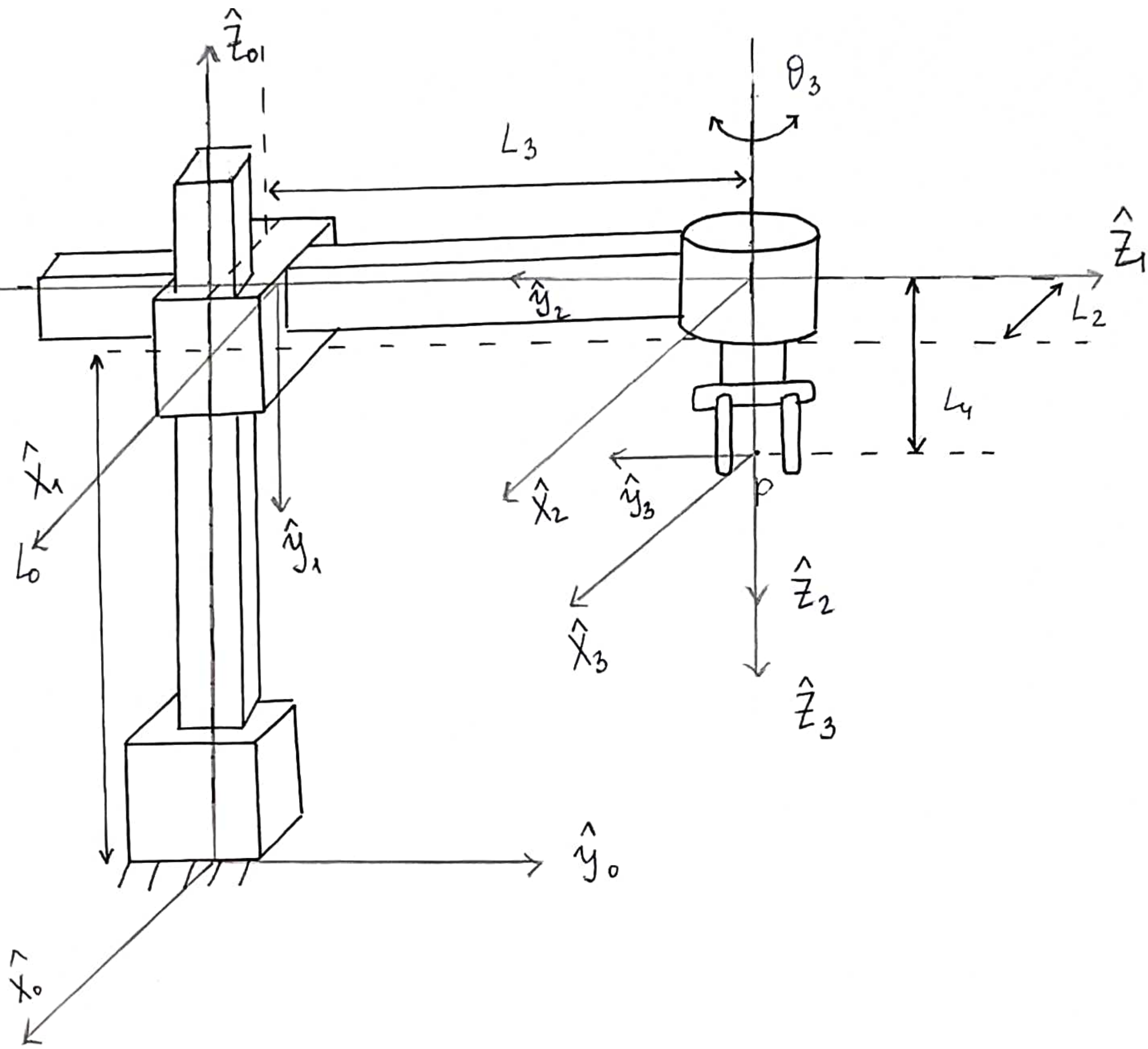
4.1



joint i	α_{i-1}	a_{i-1}	d_i	θ_i
1	$\alpha_0 = 0°$	$a_0 = 0$	$d_1 = d_1$	$\theta_1 = 0$
2	$\alpha_1 = +90°$	$a_1 = L_2$	$d_2 = L_3 + d_2$	$\theta_2 = 0$
3	$\alpha_2 = -90°$	$a_2 = 0$	$d_3 = 0$	$\theta_3 = \theta_3$

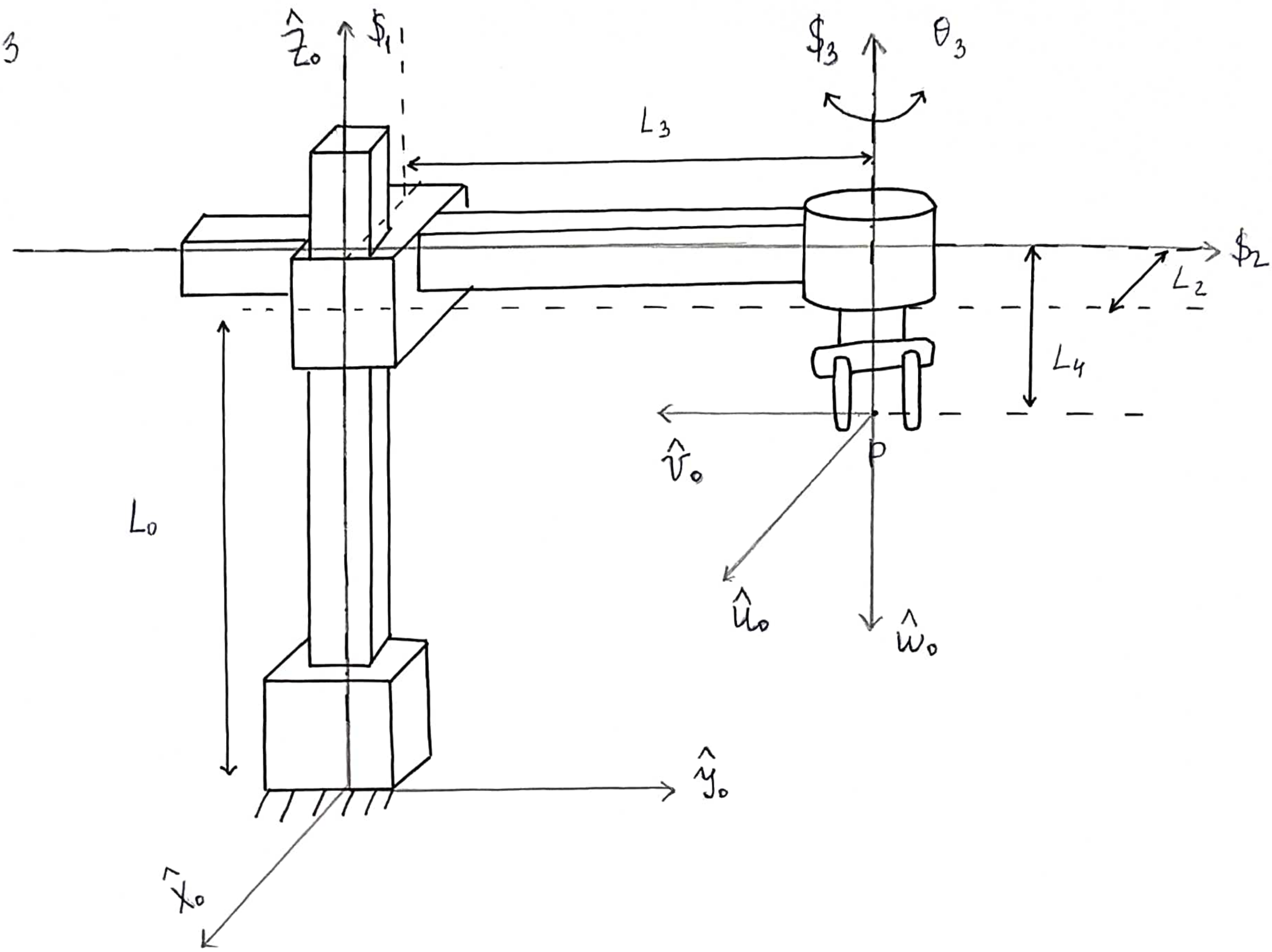
The manipulator has cylindrical workspace

4.2



joint i	α_i	a_i	d_i	θ_i
1	$\alpha_1 = -90^\circ$	$a_1 = L_2$	$d_1 = L_0 + d_1$	$\theta_1 = 0$
2	$\alpha_2 = -90^\circ$	$a_2 = 0$	$d_2 = L_3 + d_2$	$\theta_2 = 0$
3	$\alpha_3 = 0^\circ$	$a_3 = 0$	$d_3 = L_4$	$\theta_3 = \theta_3$

4.3



joint i	S_i	S_{0i}
1	$(0, 0, 1)$	$(0, 0, 0)$
2	$(0, 1, 0)$	$(-L_2, 0, L_0)$
3	$(0, 0, 1)$	$(-L_2, L_3, L_0)$

$$\hat{u}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \hat{v}_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad \hat{w}_0 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad P_0 = \begin{bmatrix} -L_2 \\ L_3 \\ L_0 - L_4 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2(c\theta_3 - 1) + L_3 s\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2 s\theta_3 - L_3(c\theta_3 - 1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_h = A_1 A_2 A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2(c\theta_3 - 1) + L_3 s\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2 s\theta_3 - L_3(c\theta_3 - 1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2(c\theta_3 - 1) + L_3 s\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2 s\theta_3 - L_3(c\theta_3 - 1) + t_2 \\ 0 & 0 & 1 & t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics:

$$P = A_h \begin{bmatrix} u_{x_0} & v_{x_0} & w_{x_0} & P_{ox} \\ u_{y_0} & v_{y_0} & w_{y_0} & P_{oy} \\ u_{z_0} & v_{z_0} & w_{z_0} & P_{oz} \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2(c\theta_3 - 1) + L_3 s\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2\, s\theta_3 - L_3(c\theta_3 - 1) + t_2 \\ 0 & 0 & 1 & t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -L_2 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & -1 & l_0 - L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_3 & s\theta_3 & 0 & -\cancel{L_2 c\theta_3} - \cancel{L_3 s\theta_3} + \cancel{L_2(c\theta_3} - 1) + \cancel{L_3 s\theta_3} \\ s\theta_3 & -c\theta_3 & 0 & -\cancel{L_2 s\theta_3} + \cancel{L_3 c\theta_3} + \cancel{L_2 s\theta_3} - \cancel{L_3(c\theta_3} - 1) + t_2 \\ 0 & 0 & -1 & l_0 - L_4 + t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_3 & s\theta_3 & 0 & -L_2 \\ s\theta_3 & -c\theta_3 & 0 & + L_3 + t_2 \\ 0 & 0 & -1 & l_0 - L_4 + t_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.4

Target pose:

$$\begin{bmatrix} c\varphi & s\varphi & 0 & x \\ s\varphi & -c\varphi & 0 & y \\ 0 & 0 & -1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y = L_3 + t_2 \\ z = l_0 - l_4 + t_1 \\ c\varphi = c\theta_3 \\ s\varphi = s\theta_3 \end{cases} \Rightarrow \begin{cases} t_2 = y - L_3 \\ t_1 = z - l_0 + l_4 \\ \theta_3 = \mathrm{Atan2}(s\varphi, c\varphi) \end{cases}$$