D.1 Yeast Growth

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Abstract

The goal of this project is to estimate the population growth overtime of the yeast species *Saccharomyces cerevisiae* using both unrestricted and logistic growth models. We look at the growth of the yeast species over different time frames with different starting population and find that the Unrestricted Growth model is better if we know that the yeast will have the resource and space needed to keep growing undeterred. However, the Logistic Growth Model is much more realistic when we know that there are limitations to the population growth

Introduction

The yeast species *Saccharomyces cerevisiae* is one of the best studied model organism. Under optimal growth conditions, it can have a doubling time as fast as 90 minutes. In our study we explore this doubling behavior to find out how large the yeast population will grow to after 3 days, how much space the yeast population will occupy and how long it will take for the yeast to reach the carrying capacity of its environment. To visualize the population growth trend we represent the doubling growth behavior using two different mathematical models. Unrestricted(exponential) growth model:

$$\frac{dN}{dt} = R_0 N(t) \to N(t) = N_0 e^{R_0 t} = N(t) = N_0 2^{\frac{t}{T_2}}$$

and with the Logistic equation:

$$\frac{dN}{dt} = R_0 N(t) \left(1 - \frac{N(t)}{K}\right) \to N(t) = \frac{K N_0 e^{R_0 t}}{K - N_0 + N_0 e^{R_0 t}}$$

$$
\begin{aligned}
T_2 &= \text{Doubling time, in this case 90 minutes} \\
N_0 &= \text{Starting population} \\
t &= \text{Time elapsed} \\
K &= \text{Carrying capacity} \\
R_0 &= \text{Per capita growth rate}
\end{aligned}
$$

Results and Discussion

To estimate the yeast population growth over 3 days under ideal conditions, we used the unrestricted (exponential) growth model, assuming no limitations due to resources. The MATLAB code used to model this growth is shown in Figure 1. The results of the simulation are shown in Figure 2, which graphs the yeast population over time. The population started with $N_0 = 1$ yeast cell and increased exponentially, reaching a total population$2.81475 * 10^{14}$ cells after 72 hours (3 days).

```
%unrestricted growth model
n0 = 1;
t2 = 90; %min
t = 0:180:72*60;

unrestricted = n0 * 2.^(t/t2);

plot(t/180, unrestricted, '-o');
grid on
xlabel('Time (hours)');
ylabel('Population');
title('Exponential');
legend('Exponential, N_0 = 10000, t2 = 90(min)', 'Location','northwest')
grid on
```

Figure 1: Unrestricted Growth, $N_0 = 1$

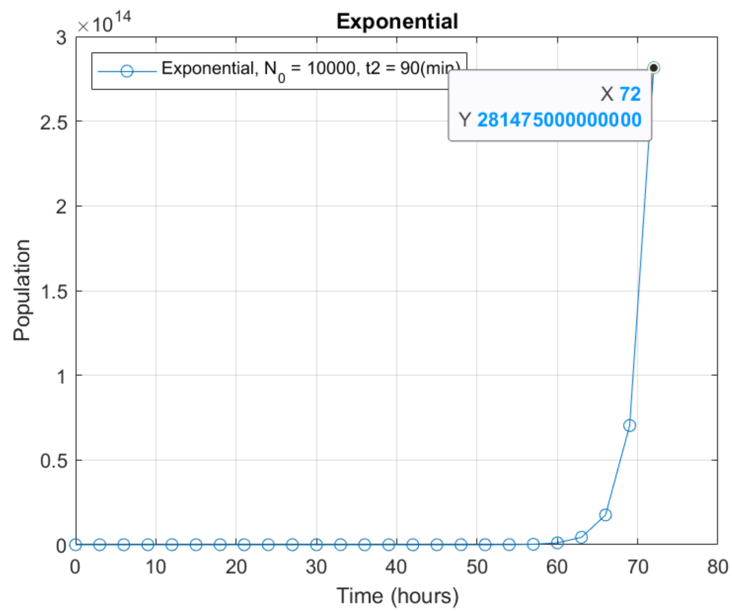

Figure 2: Unrestricted Growth graph, $N_0 = 1$

Next, we aimed to determine how much space the yeast population would occupy. Yeast cells are approximately spherical, with a typical diameter of about 6 µm. The volume of a single yeast cell was calculated using the formula for the volume of a sphere $\frac{4}{3}\pi r^3$, and the conversion $1\text{L} = 10^{-3}\,\text{m}^3$. Given that the diameter is 6 µm, the radius is 3 µm. 1 µm = .000001 m which means 3 µm = .00000 3µm = $3 * 10^{-6}$ m. Plugging this value into the formula for volume of a sphere gives us:

$$V = \frac{4}{3}\pi r^3 \;\; = \frac{4}{3}\pi(3 * 10^{-6})^3 \;\; = 1.13097 * 10^{-16}\,\text{m}^3$$

Thus a singular yeast cell has a volume of about $1.13097 * 10^{-16}\,\text{m}^3$. Using the conversion factor $1\text{L} = 10^{-3}\,\text{m}^3$ we find that 1 yeast cell occupies:

$$V_{\text{cell}} = 1.13097 \times 10^{-16}\,\text{m}^3 \times \frac{1\,\text{L}}{10^{-3}\,\text{m}^3} = 1.13097 \times 10^{-13}\,\text{L}$$

Given that the total yeast population after 3 days is $2.81475 * 10^{14}$cells, the total volume occupied by the yeast population is:

$$V_{\text{total}} = (1.13097 \times 10^{-13}\,\text{L}) \times (2.81475 \times 10^{14}) = 31.83\,\text{L}$$

Now lets say if an experiment biologist puts in 10,000 yeast cells into a 2L flask with 1L of growth medium. Since 1L is already occupied by the growth medium, the yeast has 1L of volume to grow in. Given that 2.81475×10^{14} cells occupy 31.83L of space we can calculate the amount of cells that occupy 1L of space:

$$\frac{2.81475 \times 10^{14}\,\text{cells}}{31.83\,\text{L}} = 8.84307 \times 10^{12}\,\text{cells/L}$$

There are 8.84307×10^{12} cells per Liter. We can use our code in Figure 1 and set $N_0 = 10{,}000$ to see how long it takes to for the yeast population to reach 8.84307×10^{12} cells. In Figure 3 we see the after 44.5 hours passes, the population grows to 8.52229×10^{12} cells which is close to the a 8.84307×10^{12} cells needed to occupy 1L of space.

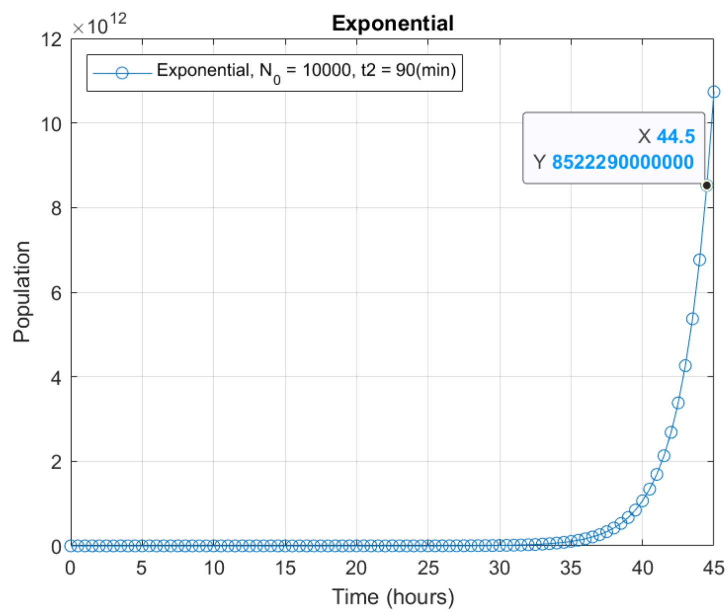

Figure 3: Unrestricted growth graph, $N_0 = 10{,}000$

So far we have used the unrestricted growth model to make our calculations. However this model neglected competition for resources and environmental limitations. To improve our calculations we can use the logistical growth model which sets a carrying capacity, K. Given that for yeast, the maximal density achievable in liquid culture is about 2×10^8 cells per mL, we can calculate the maximal cells per liter to be:

$$V_{\text{cell}} = 2 \times 10^8 \,\text{cells/mL} \times \frac{10^3 \,\text{mL}}{1\,\text{L}} = 2 \times 10^{11} \,\text{cells/L}$$

When environmental factors was not a concern we calculate that 8.84307×10^{12} cells can occupy 1L. However with environmental limitations and competition we find that a maximum of 2×10^{11} cells can occupy 1L. This means about $\frac{2 \times 10^{11}\,\text{cells/L}}{8.84307 \times 10^{12}\,\text{cells/L}} \times 100 = 2.262\,\%$ of 1L is actually occupied by cells.

Now lets say we have the same experiment biologist putting in 10,000 yeast cells into a 2L flask with 1L of growth medium. Given that we have a density value for the yeast population, this time we can use the logistical growth model to get a more accurate estimate for population growth over time. We can set the carrying capacity, K, in the logistic model to 2×10^{11} cells since we only have 1L of space for the yeast to grow in. We also need to calculate $R_0 = \ln(2)/T_2 = \ln(2)/90 = 0.0077$. Figure 4 shows the MATLAB code for the Logistic growth model and Figure 5 shows the graph produced by the code.

```
% Logistic equation
K = 2*10^11; % Define carrying capacity
n0 = 10000; % Initial population
r0 = 0.0077; % Growth rate
t = 0:90:100*60; % Time in minutes

logistic = n0*exp(r0*t).*(K./(K-n0+n0*exp(r0*t)));

plot(t/60, logistic, '-o', 'DisplayName', 'Logistic');
xlabel('Time (1.5 hour increment)');
ylabel('Population');
title('Logistic');
legend('Logistic, N_0 = 10000, R_0 = 0.0077, K = 2*10^{11}', 'Location', 'northeast')
axis([0,100,10000,2.3e11])
grid on
```

Figure 4: Logistic Growth, $N_0 = 10.000$

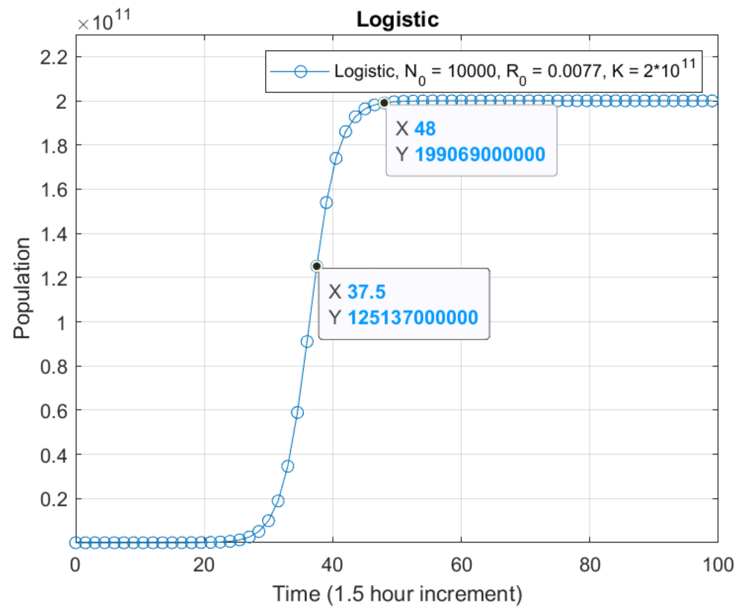

Figure 5: Logistic Growth graph, $N_0 = 10{,}000$

When we compare Figure 3 to Figure 5, we see that both graph reach their maximum capacity at similar times. Figure 3 reaches their max capacity around 44.5 hours while Figure 5 starts reaching its max capacity around 48 hours. However we see that the growth in Figure 5 starts to significantly slow down and does not go above 2×10^{11} cells. This is due to the fact that the logistic equation has a max carrying capacity, K, to account for environmental limitations and competition.

Conclusion

Using the Unrestricted Growth Model, the population will keep increasing till infinity. This approach is fine if we know that the yeast will not run out of resources to help it grow. However, if we know that the yeast will have limitations such as space and resource constraints then it is much more realistic to use a Logistic Growth Model as it takes those factors into consideration and slows down growth as the population size keeps increasing till the yeast reach the maximum carrying capacity of its environment.

Given these results we can also try to estimate if it