D.4 Making predictions with a simple SIR model

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Abstract

This report investigates the outbreak of epidemics using the SIR (Susceptible-Infectious-Recovered) model, focusing on a baseline epidemic in a city of 10 million people, modeled roughly after the COVID-19 pandemic. We explore whether the healthcare systems in various countries would be prepared for such an epidemic, how different factors effect the spread and duration of the epidemic and how early intervention strategies impact the severity of an epidemic. Our simulation results indicate that early intervention strategies such as mass vaccination and isolation protocols can effectively reduce the impact of an epidemic and potentially even stop an epidemic from taking place. It also highlights the importance of using multiple strategies alongside one another to combat epidemics.

Introduction

Viral respiratory infections are a particularly common class of infectious disease that jump between species relatively often, leading to epidemics in populations with minimal immunity to the novel form. COVID-19 was the most recent of these. With this class of infectious agent, individuals generally develop some degree of immunity after an infection and thus it is common to model these using the SIR model described by:

$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

with a differential equation model written as:

$$\begin{aligned}
\hat{S}_{i+1} &= \hat{S}_i - \beta \hat{S}_i \hat{I}_i, \\
\hat{I}_{i+1} &= \hat{I}_i + \beta \hat{S}_i \hat{I}_i - \gamma \hat{I}_i, \\
\hat{R}_{i+1} &= \hat{R}_i + \gamma \hat{I}_i.
\end{aligned}$$

Here $\hat{S}$, $\hat{I}$ and $\hat{R}$ represent fractional population. $\hat{S}$ is the fractional susceptible population. $\hat{I}$ is the fractional Infective population. $\hat{R}$ is the fractional recovered population.The epidemiological literature commonly reports fractional populations in units of cases per 100,000. β is a parameter that describes the likelihood of new infections, and γ is the rate of recovery. We can express β and γ in terms of the basic reproductive factor, r_0, and the average length of infectivity, τ, as:

$$\gamma = \frac{1}{\tau} \quad \text{and} \quad \beta = \frac{r_0}{\tau}.$$

In our simulation, we model the dynamics of COVID-19 in a city of 10 million people, where 4 initially infected individuals arrive with no prior immunity. We will refer to this as the baseline model. We model using SIR, with parameters consistent with early estimates of COVID-19: basic reproductive factor of about 3.0, with an average length of infectivity of about 10 days.

Results and Discussion

The code for performing our simulation is provided below in Figure 1.

```
N = 10000000;
I0 = 4 / N;
R0 = 0;
S0 = 1 - (I0 + R0);

tau = 10; % in days
r0 = 3;
beta = r0 / tau;
gamma = 1 / tau;

total_time = 160; % Total simulation time in days

S = zeros(1, total_time);
I = zeros(1, total_time);
R = zeros(1, total_time);
S(1) = S0;
I(1) = I0;
R(1) = R0;

for i = 1:total_time
    S(i + 1) = S(i) - beta * S(i) * I(i);
    I(i + 1) = I(i) + (beta * S(i) * I(i) - gamma * I(i));
    R(i + 1) = R(i) + gamma * I(i);

end

% Plotting results
plot(0:1:total_time, S, '-g');
hold on;
plot(0:1:total_time, I, '-r');
plot(0:1:total_time, R, '-b');
xlabel('Days');
ylabel('Fractional Population');
title('COVID-19 SIR Model Simulation');
legend('Susceptible', 'Infected', 'Recovered');
grid on;
hold off;
```

Figure 1: Matlab code for our simulation

We begin our simulation by running the baseline scenario where our population has no prior immunity, roughly modeled with reproductive factor and infectivity length similar to COVID-19. The results of the scenario are shown in Figure 2.

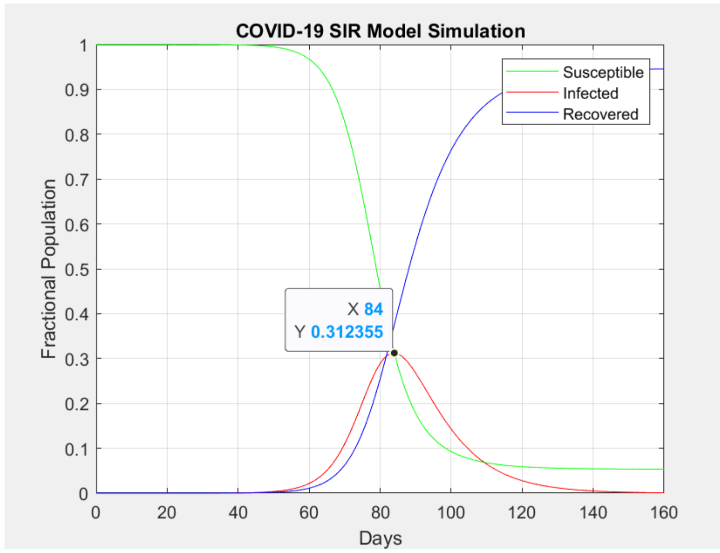

Figure 2: Baseline scenario, $r0 = 3$, $\tau = 10$, R0 $= 0$

As shown in Figure 2, the highest number of actively infected individual occurs on day 85. There are 31,235.5 cases per 100,000 people on that day. After this point, the number of actively infected individuals gradually decrease each day.

The highest number of daily occurrence happens at day 76 as Shown in Figure 3. There are 2,221 new daily cases per 100,000 people at this time. After this point, the number of newly infected individuals starts to decrease each day.

```
highest_slope = 0;
highest_increase_day = 0;

for i = 1:total_time
    S(i + 1) = S(i) - beta * S(i) * I(i);
    I(i + 1) = I(i) + (beta * S(i) * I(i) - gamma * I(i));
    R(i + 1) = R(i) + gamma * I(i);

    if (I(i + 1) - I(i)) > highest_slope
        highest_slope = (I(i + 1) - I(i));
        highest_increase_day = (i + 1);
    end
end
```

Figure 3: Additional find highest number of daily occurrences

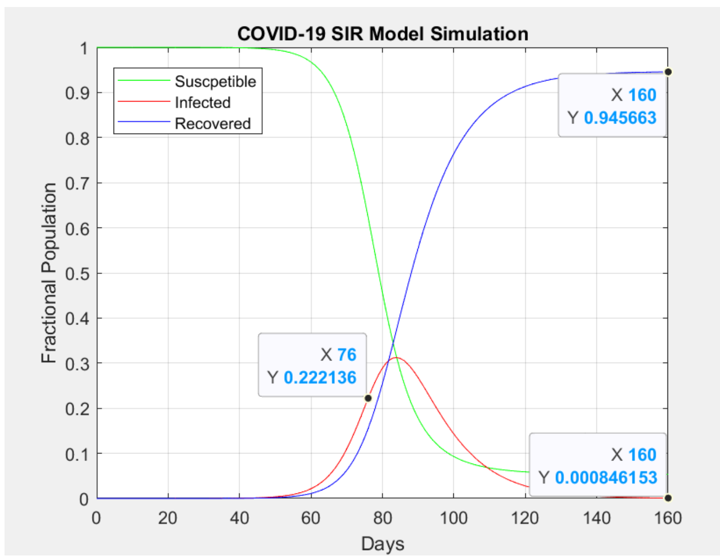

Figure 4: Baseline scenario, $r0 = 3$, $\tau = 10$, R0 = 0

By the end of the epidemic on day 160, we find that 94.65% of the population were infected in total. We find this number by adding up the total recovered and infected individuals at day 160. Almost the entire population was infected by the end of the epidemic.

We want our healthcare system to be prepared for such an epidemic. Assuming that 10% of all actively infected individuals would require hospital beds then at the height of the epidemic we would need .1*31,235.5 = 3,124 beds for every 100,000 people. In addition about .1 * 2,221 = 222 hospital admissions would be expected at peak growth per 100,000 people. With such a high infectivity rate, the human cost would be great regardless of how small the mortality rate is. Assuming our mortality rate is 1.5% which is consistent with early data of COVID, about 0.9465 * 0.015 * 10,000,000 = 141,975 people would die by the end of the epidemic in our model city. 6. The results of our simulation shows that no developed country is prepared to handle such a state. Countries such as the US, Canada and the UK have around 270 hospital beds per 100,000. Japan and South Korea have about 1270 beds per 100,000. This is significantly lower than the 3,124 hospital beds per 100,000 people predicted to be required to manage the epidemic. The situation is even more dire for developing countries as they have as few as 50 beds per 100,000. In the real world, as COVID spread it overwhelmed every countries healthcare system as the number of hospital beds were far less than the number of people who needed hospital care and our simulation reflects that behavior.

Scenarios can widely vary depending on the factors at play leading up to the epidemic. We can adapt our simulation to visualize what happens with varying factors. To view a scenario where the our infectious agent has a very high rate of spread we can increase the reproductive rate of the infectious agent. Figure 5 shows a scenario where the reproductive rate doubled to 6:

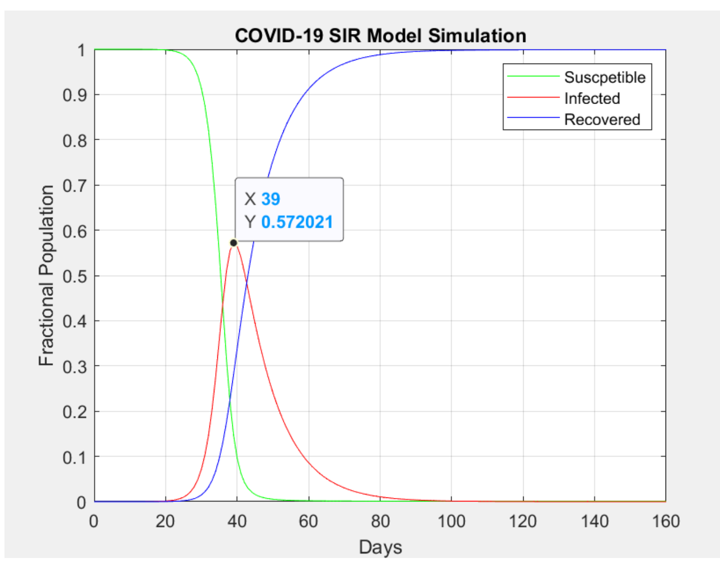

Figure 5: Enter Caption

In such a scenario, the doubling of the reproductive rate results in the epidemic spreading much faster, with a much higher peak number of infections and daily new cases. However, early interventions such as isolating parts of the population or vaccinating to induce prior immunity would have reduced the susceptible population and increased the fraction of individuals in the removed category. These measures would have substantially slowed the spread of the epidemic.

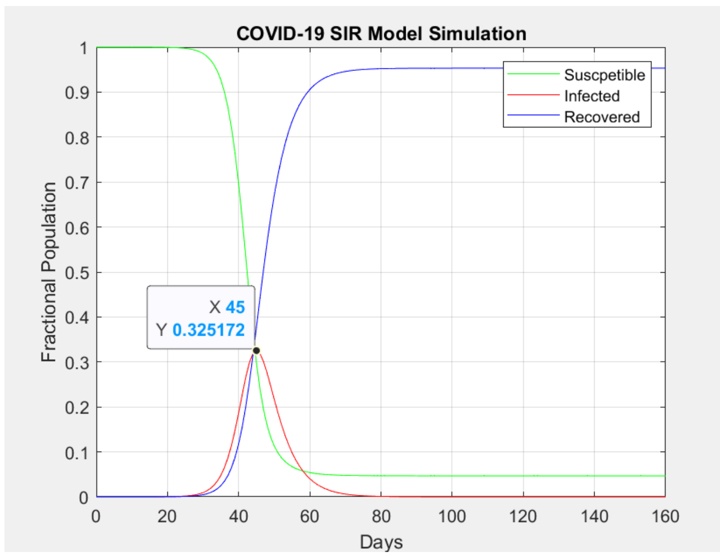

Figure 6: Enter Caption

Figure 6 illustrates a scenario where the infectivity period is reduced to 5 days. We see that the epidemic lasts for a shorter period of time. However the peak number of cases still remains very high, slightly above our base case scenario. This again highlights the need for rapid early intervention, as we have a much shorter time to respond before we reach the height of the epidemic.

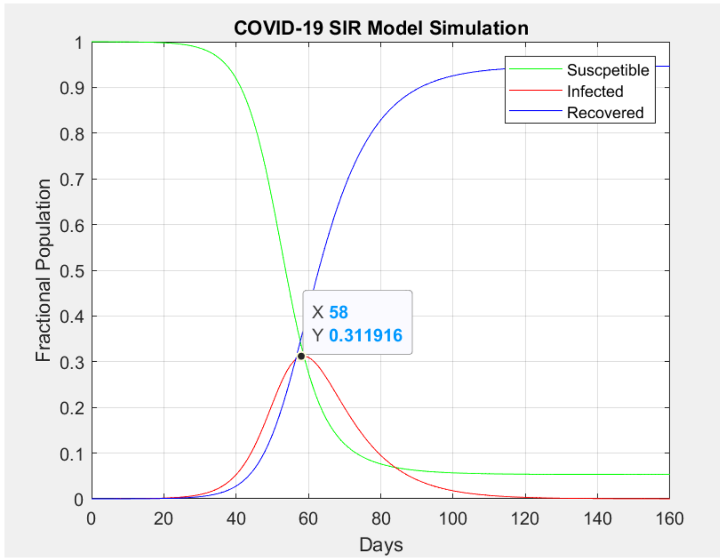

Figure 7: Enter Caption

Figure 7 illustrates a scenario when we have an initial infected population of 400 people. This simulates a case with a large starting outbreak. In this scenario we find that the epidemic progresses much rapidly than in our baseline case with the the height of the epidemic occurring much earlier. This again highlights the need for rapid early intervention.

Often responses to managing a potential epidemic effectively reduce the basic reproductive number. We simulate this in our model by trying various reproductive rates lower than 3.

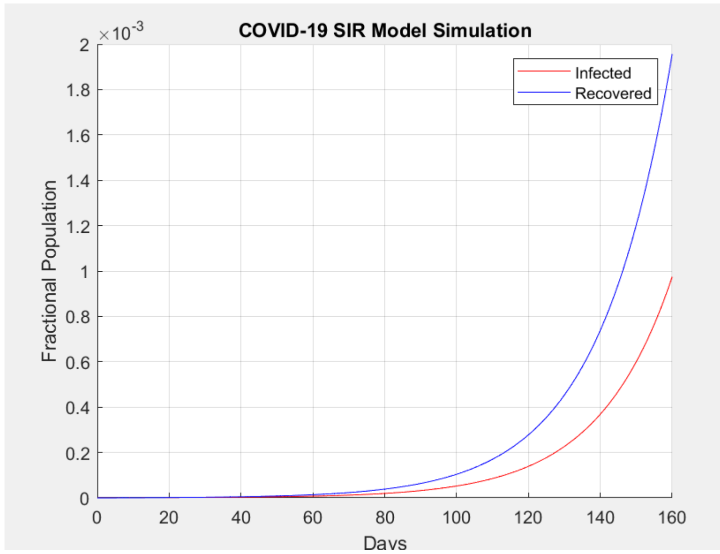

Figure 8: Enter Caption

Figure 8 illustrates a scenario where the reproductive rate is 1.5. We find that the infected rate is lower than the recovered rate which shows that the infection spread is under control.

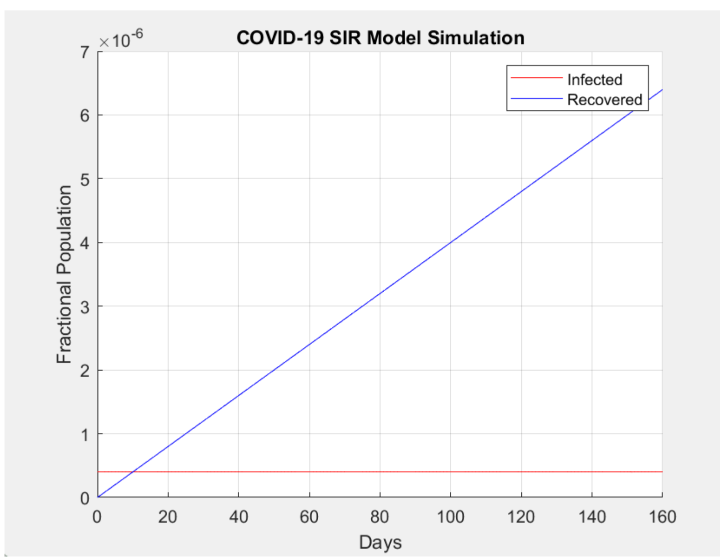

Figure 9: Enter Caption

Figure 9 illustrates a scenario where the reproductive rate is 1.0. We find that the number of infected stays constant while the number of recovered people linearly increases which is expected with a constant infected population.

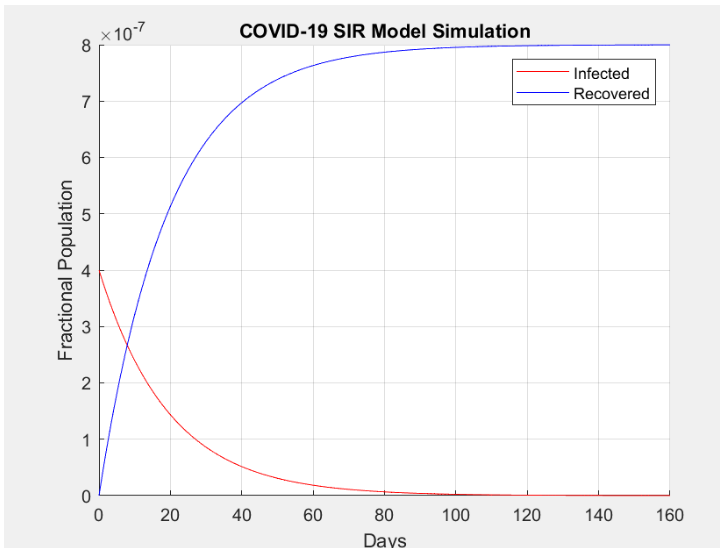

Figure 10: Enter Caption

Figure 10 illustrates a scenario where the reproductive rate is .5.We find that the number of individuals in the infected population continuously decreases towards 0. As such the recovered population stays constant since there are no new infected individuals. In all 3 cases the recovered population growth outpaces the infected population growth indicating that effective response is being provided to slow down and eventually stop the spread of COVID.

To illustrate the effectiveness of early intervention in stopping epidemics, We can simulate a population with pre-existing immunity to COVID before the spread even starts. We can put a portion of the population into the recovered/removed category. We can do this since people with pre-existing immunities are unlikely to infected by COVID again which makes them effectively part of the recovered population. So we set our R0 value in our code equal to the fraction of the population who are already immune. In such cases the initial susceptible population is the total population subtracted by the sum of the initial infected and recovered population. Figure 11 represents a scenario where 25% of the population has pre-existing immunity.

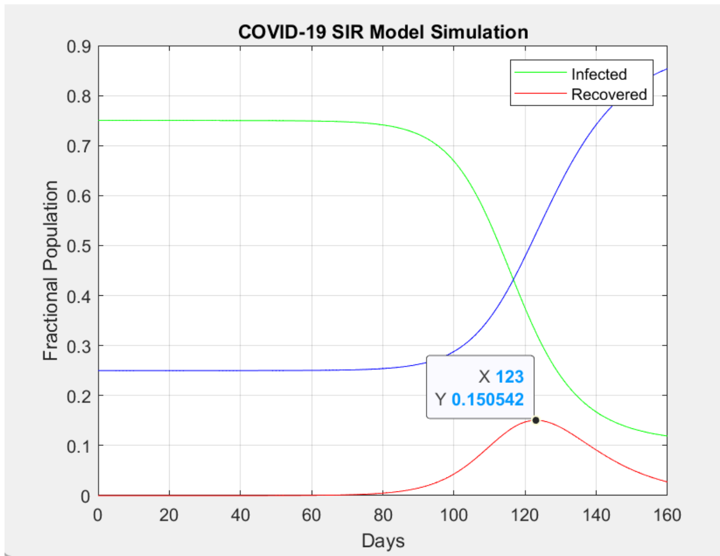

Figure 11: Enter Caption

We find that with a population with prior immunity, the peak of the epidemic takes much longer to reach at 123 days. In addition, the peak is much lower in magnitude than when the population did not have any prior immunity. Figure 12 represents a scenario where 50% of the population has pre-existing immunity.

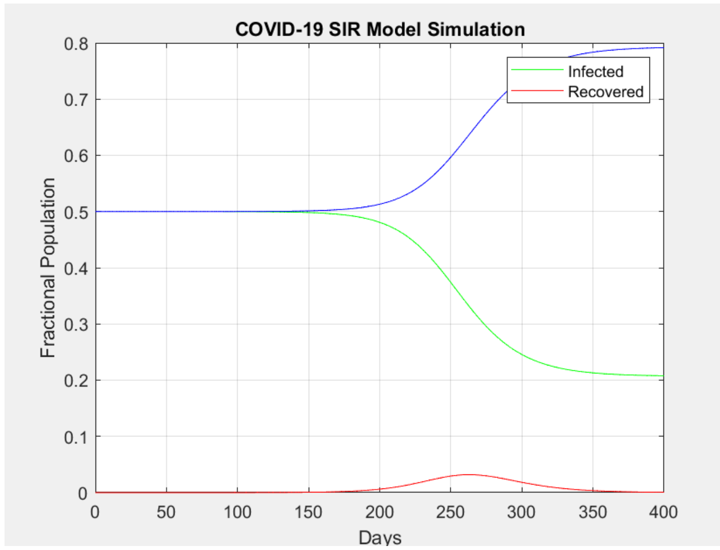

Figure 12: Enter Caption

The peak occurs at around 260 days this time and in addition the peak is significantly lower in magnitude. Figure 13 represents a scenario where 75% of the population has pre-existing immunity.

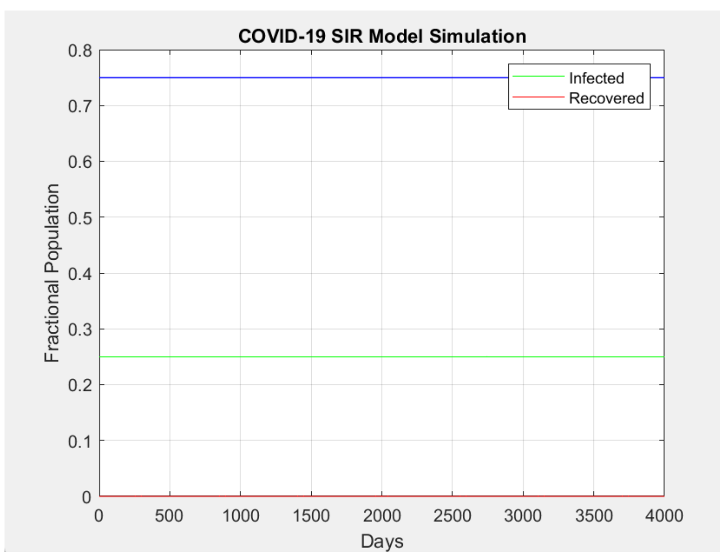

Figure 13: Enter Caption

Even if we increase the length of the simulation to 4000 days we do not find any noticeable peak in infected population. This is an example of herd immunity. The minimum vaccination threshold required to achieve such herd immunity can be derived analytically from our simulation model. To calculate this threshold we use the equation:

$$\text{Fraction immune} = 1 - \frac{1}{r_0}$$

For our model the threshold is: $1 - \frac{1}{3} = .6667 = 66.67\%$. A minimum of 67% of the population need to be vaccinated. This is consistent with our simulation results as seen in our simulations with R0 = .25, .50, .75. Once vaccinated population was set to 75% we see in Figure 13 that the infected population never increases to any noticeable peak. If one strategy is not enough or it is not possible to fully implement one method to mitigate the epidemic then we can combine multiple intervention techniques. For example, if we implement isolation/quarantine protocols to reduce basic reproductive numbers to $r_0 = 1.5$ instead of 3 then we can calculate the vaccination level required for heard immunity to be $1 - \frac{1}{1.5} = .3333 = 33.33\%$. If we implement measures to reduce the spread of the disease such as isolation then we need a much smaller portion of the population to be vaccinated to achieve heard immunity. Combining multiple strategies is especially effective when one strategy is not enough by itself.

Conclusion

Our simulations of the SIR model COVID-19 highlights that without prior immunity, as was the case in the real world, the disease rapidly spreads rapidly and as immunity develops, gradually individuals move to the recovered population. This rapid spread and high infective population rate will overwhelm the healthcare systems of every nation as non of them have enough hospital beds for the number of individuals who will require hospital care during most of the duration of the epidemic. However our simulations also show that early intervention methods such as mass vaccination and social distancing are effective in mitigating the impact of the epidemic which can also prevent our health systems form being overwhelmed.