A&O 180 Final Project Documentation
Simulation of Kelvin-Helmholtz Instability

Name: Adrian Lam Ling Ho

Main Goal: Simulate a Kelvin-Helmholtz instability problem in a 2D incompressible flow

Mathematical Problem:

- Equations:

Quasi-geostrophic (QG) Barotropic Vorticity Equations with advection term in 2d Jacobian Form.

$$\frac{\partial \zeta_g}{\partial t} = -J(\psi, \zeta) - \beta \frac{\partial \psi}{\partial x} + v\,\nabla^2 \zeta_g$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \zeta_g(x, y) \qquad J(a,b) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$$

with **v = 3.6E-3, 3.6E-4, 3.6E-5** m^2/s and $\beta = 0$

- Domain: $0 \le x \le L_x$ and $0 \le y \le L_y$ with $L_x = L_y = 1$m
- Boundary Conditions: Periodic in x for all variables (ψ, ζ) and walls in y ($\psi = 0$, $\zeta = 0$)
- Initial Conditions: (Horizontal velocity & corresponding stream-function)

$$\boldsymbol{u}_0(x,y) = \begin{bmatrix} u_\infty \tanh\left(\frac{2y-1}{\delta_0}\right) \\ 0 \end{bmatrix} + c_n \begin{bmatrix} \partial_y \psi(x,y) \\ -\partial_x \psi(x,y) \end{bmatrix}$$

$$\psi(x,y) = u_\infty \exp\left(-\frac{(y-0.5)^2}{\delta_0^2}\right)\left[\cos(8\pi x) + \cos(20\pi x)\right]$$

with $\delta_0 = 1/28$ (initial vorticity thickness), $u_\infty = 1$ (reference velocity), $c_n = 10^{-3}$ (scaling/noise factor)

- Reynolds number: $\mathbf{Re = \delta_0 u_\infty / v = 1/(28v)}$
  - Re = 100,1000,10000
- Total Integration Time: 14.8s; Time Step = 3.6E-5s

Numerical Discretization:

- **Spatial Discretization**: second-order centered differences for all spatial derivatives
- **Jacobian Discretization**: Arawaka Discretization for Jacobian term
- **Time Discretization:**
  1. 3rd Order Adam-Bashford Scheme **(AB3)** for advection
     (Used Euler Forward with Δt for Diffusion, as well as the first 2 advection steps)
  2. 4th Order Runge Kutta Scheme **(RK4)** for advection
     (Used Euler Forward with Δt for Diffusion)

- <u>**Poisson Solver:**</u> SOR iterative solver with estimated **α = 1.9622** & **tolerance = 10E-8**
  Stream-function solved from previous time step becomes initial guess for solver

- <u>**Number of Grid Points:**</u>
  Main Run: 201x201; **High Resolution: 401x401**

<u>**Other Experimental Investigations:**</u>

- **RUN A:**
  - AB3; Comparison between Re = 100,1000,10000; Grid Resolution: 201x201
- **RUN B:**
  - RK4; Comparison between Re = 100,1000,10000; Grid Resolution: 201x201
- **RUN C:**
  - RK4; Re = 10000; Comparison between 201x201 vs 401x401;

- **RUN D:** Random Perturbation
  - Used randomized perturbation in the initial condition
  - Process: Use randomly generated wavenumber and inverse Fourier transform to obtain randomized waves in sines and cosines
  - Investigate growth of the most amplified mode in the stability
- **RUN D:** No perturbation
  - Removal of initial perturbation.

<u>**References:**</u>

<u>Schroeder P.W., John V., Lederer P.L., Lehrenfeld C., Lube G., Schöberl J.On reference solutions and the sensitivity of the 2d kelvin–helmholtz instability problem (2018)</u>