Expirements Discussed in Section 7

In order to test the architectural components of our AFMP framework, we performed a string of ablation studies. We designed such experiments to break down the model and measure the specific impact of its core features.

We analyzed in two sections. First, we looked at the bottom-up predictive routing engine, which is the base of our architecture. For these initial studies (\ref{sec:sparsity-level} and \ref{sec:sparsity-regu}), we used a version of AFMP where routing was guided only by local similarity and a feed-forward predictive signal. We left out the final top-down refinement pass. This setup allowed us to isolate the effects of learned sparsity and regularization strength on the main routing mechanism. In the second part (\ref{sec:refine-pass}), after finding the best bottom-up configuration, we ran a final ablation to directly measure the benefit of adding the full hierarchical top-down feedback loop.

We performed all ablation experiments on the CIFAR-100 dataset \cite{krizhevsky2009learning}. As a baseline for comparison, we used a "Dense" version of AFMP. This variant uses the full static communication scaffold without any sparse gating, effectively making it a powerful, fully-connected graph neural network.

```
# Author : Zaryab Rahman
# Data : 20/8/25
```

0. Imports and Configuration

This cell imports necessary libraries and defines a `Config` class. This class centralizes all hyperparameters for the model, training, and data, making it easy to modify and track experimental settings.

```
import torch
import torch.nn as nn
import torch.nn.functional as F
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader
import numpy as np
from scipy.spatial import Delaunay
import time
import matplotlib.pyplot as plt


class Config:
    model_type = 'sparse'  # 'sparse' or 'dense'
    sparsity = 0.3
    lambda_sparse = 1e-4

    num_layers = 4
    d_model = 128
    d_position = 2
    patch_size = 8
```

```
        batch_size = 128
        epochs = 100
        lr = 1e-3
        weight_decay = 1e-4

        num_classes = 100
        img_size = 32

    config = Config()
```

1. Static Communication Scaffold

The `StaticGraphBuilder` class is responsible for creating the sparse, static communication graph based on positional priors, as described in **Section 3.5.1** of the paper. This pre-computation is a key step to ensure guaranteed O(N) complexity. It uses Delaunay triangulation for grid-like data (images).

```
class StaticGraphBuilder:
    def __init__(self, mode='delaunay', window_size=8):
        self.mode = mode
        self.window_size = window_size
        self.cache = {}

    def build(self, positions):
        N = positions.shape[0]
        hash_key = f"{N}_{positions.mean().item():.4f}"

        if hash_key in self.cache:
            return self.cache[hash_key]

        if self.mode == 'delaunay' and positions.shape[1] > 1:
            tri = Delaunay(positions.cpu().numpy())
            edges = set()
            for simplex in tri.simplices:
                for i in range(3):
                    for j in range(i+1, 3):
                        a, b = sorted([simplex[i], simplex[j]])
                        edges.add((a, b))
            edge_index = torch.tensor(list(edges)).long().t().contiguous()

        elif self.mode == 'window' or positions.shape[1] == 1:
            edge_index = []
            for i in range(N):
                start = max(0, i - self.window_size)
                end = min(N, i + self.window_size + 1)
                for j in range(start, end):
                    if i != j:
                        edge_index.append([i, j])
            edge_index = torch.tensor(edge_index).long().t().contiguous()
```

```
            row, col = edge_index
            edge_attr = torch.norm(positions[row] - positions[col], dim=1)

            self.cache[hash_key] = (edge_index, edge_attr)
            return edge_index, edge_attr
```

2. Core AFMP Components

This section contains the core PyTorch modules that define the AFMP architecture.

Predictive Router

This module implements the error-driven sparse routing mechanism, which is the central innovation of the AFMP framework (detailed in **Section 3.4**). It computes gates for communication between capsules based on a multi-stage process:

1. **Top-Down Prediction & Error Signal:** Calculates the "surprise" based on predictions from a higher layer.
2. **Routing Score:** Computes a score using a combination of a low-rank bilinear term and contextual features.
3. **Differentiable Sparsification:** Selects the top connections.
4. **Final Gate Computation:** Generates the final modulatory gates.

```
class PredictiveRouter(nn.Module):
    def __init__(self, d_model, k_dim=64, sparsity=0.3, is_sparse=True):
        super().__init__()
        self.d_model = d_model
        self.k_dim = k_dim
        self.sparsity = sparsity
        self.is_sparse = is_sparse
        self.W_pred = nn.Linear(d_model, d_model)
        self.W_q = nn.Linear(d_model, k_dim)
        self.W_k = nn.Linear(d_model, k_dim)
        self.U = nn.Parameter(torch.randn(k_dim, k_dim) * 0.02)
        self.v = nn.Sequential(
            nn.Linear(2 * d_model + 2, 64),
            nn.ReLU(),
            nn.Linear(64, 1)
        )
        self.W_gate = nn.Linear(1, d_model)

    # now expects a 2D tensor H of shape (B*N, D)
    def forward(self, H, edge_index, edge_attr, top_down=None):
        row, col = edge_index
        # H is now 2D, so N is its first dimension
        N, E = H.shape[0], edge_index.shape[1]

        # top-down is also a 2D tensor now
        if top_down is not None:
            pred_j = self.W_pred(top_down[row])
            errors = torch.norm(H[col] - pred_j, dim=1, keepdim=True)
        else:
            errors = torch.zeros(E, 1, device=H.device)
```

```
        Q = self.W_q(H[row])
        K = self.W_k(H[col])
        bilinear = (Q @ self.U) * K
        bilinear = bilinear.sum(dim=1, keepdim=True)



        feat_linear = torch.cat([
            H[row],
            H[col],
            errors,
            edge_attr.unsqueeze(1)
        ], dim=1)


        linear_term = self.v(feat_linear)
        scores = bilinear + linear_term

        # sparsification logic needs to know the original number of nodes per graph
        num_nodes_per_graph = H.shape[0] // config.batch_size
        if self.is_sparse and self.sparsity > 0:
            k = int(self.sparsity * num_nodes_per_graph)
            if k > 0:
                node_scores = torch.full_like(scores, -float('inf'))
                num_edges_to_keep = k * config.batch_size
                if scores.numel() > num_edges_to_keep:
                    threshold = torch.topk(scores.view(-1), num_edges_to_keep).values[-1]
                    mask = (scores >= threshold).float()
                    scores = scores * mask

        gates = torch.sigmoid(self.W_gate(scores))
        return gates
```

Perceptual Capsule Unit (PCU)

The PCU is the main computational unit of the AFMP, analogous to a cortical column (see **Section 3.3**). It updates its state by integrating its own recurrent state with the gated messages it receives from its neighbors.

```
class PCU(nn.Module):
    def __init__(self, d_model):
        super().__init__()
        self.W_self = nn.Linear(d_model, d_model)
        self.W_msg = nn.Linear(d_model, d_model)
        self.activation = nn.GELU()

    def forward(self, H, gates, edge_index):
        row, col = edge_index
        self_part = self.W_self(H)
        messages = self.W_msg(H[col])
```

```
        modulated = messages * gates

        # index_add_ works correctly on the flattened 2D tensor
        agg = torch.zeros_like(H)
        agg = agg.index_add_(0, row, modulated)
        return self.activation(self_part + agg)
```

```
class TopDownAggregator(nn.Module):
    def __init__(self, d_model):
        super().__init__()
        self.W_t = nn.Linear(d_model, d_model)

    def forward(self, H_lower, H_higher, hierarchy):
        parent_signals = hierarchy @ H_higher
        modulation = self.W_t(parent_signals)
        return H_lower + modulation
```

AFMP Layer and Full Model

These classes assemble the router and PCU into a full `AFMPLayer`. The `AFMP` class then stacks these layers to create the final model. Note the `use_top_down` flag in the `AFMP` model's constructor, which is crucial for running the ablation study in **Section 7.3**.

```
class AFMPLayer(nn.Module):
    def __init__(self, d_model, d_position, sparsity=0.3, is_sparse=True):
        super().__init__()
        self.router = PredictiveRouter(d_model, sparsity=sparsity, is_sparse=is_sparse)
        self.pcu = PCU(d_model)

    def forward(self, H, edge_index, edge_attr, top_down=None):
        # h arrives as (B, N, D)
        batch_size, num_nodes, d_model = H.shape

        # flatten for graph operations
        H_flat = H.view(-1, d_model) #  (b*n, d)

        top_down_flat = None
        if top_down is not None:
            top_down_flat = top_down.view(-1, d_model)

        gates = self.router(H_flat, edge_index, edge_attr, top_down_flat)

        H_updated_flat = self.pcu(H_flat, gates, edge_index)

        H_out = H_updated_flat.view(batch_size, num_nodes, d_model)

        return H_out, gates
```

```
class SparsityRegularizer(nn.Module):
    def __init__(self, lambda_sparse=0.01):
```

```
        super().__init__()
        self.lambda_sparse = lambda_sparse

    def forward(self, gates):
        return self.lambda_sparse * torch.mean(torch.abs(gates))
```

```
class AFMP(nn.Module):
    def __init__(self, num_layers, d_model, d_position, num_classes,
                 img_size=32, patch_size=8, sparsity=0.3, is_sparse=True,
                 use_top_down=True): # new control flag
        super().__init__()
        self.patch_size = patch_size
        self.img_size = img_size
        self.use_top_down = use_top_down # store the flag

        self.num_patches = (img_size // patch_size) ** 2
        self.embed = nn.Linear(3 * patch_size * patch_size, d_model)
        self.position_embed = nn.Linear(2, d_model)

        grid = torch.arange(0, img_size, patch_size) + patch_size / 2
        grid_x, grid_y = torch.meshgrid(grid, grid, indexing='ij')
        static_positions = torch.stack([grid_x, grid_y], dim=-1).reshape(-1, 2)

        graph_builder = StaticGraphBuilder()
        edge_index, edge_attr = graph_builder.build(static_positions)

        self.register_buffer('static_positions', static_positions)
        self.register_buffer('edge_index', edge_index)
        self.register_buffer('edge_attr', edge_attr)

        self.layers = nn.ModuleList([
            AFMPLayer(d_model, d_position, sparsity, is_sparse)
            for _ in range(num_layers)
        ])

        if self.use_top_down:
            self.top_down_aggregators = nn.ModuleList([
                TopDownAggregator(d_model) for _ in range(num_layers-1)
            ])

        self.readout = nn.Sequential(
            nn.AdaptiveAvgPool1d(1),
            nn.Flatten(),
            nn.Linear(d_model, num_classes)
        )

        self.reg = SparsityRegularizer(
            lambda_sparse=config.lambda_sparse if is_sparse else 0.0
        )

    def forward(self, images, return_internals=False):
        batch_size = images.size(0)
```

```
        p = self.patch_size

        patches = images.unfold(2, p, p).unfold(3, p, p)
        patches = patches.permute(0, 2, 3, 1, 4, 5)
        patches = patches.contiguous().view(batch_size, -1, 3 * p * p)

        pos_emb = self.position_embed(self.static_positions.expand(batch_size, -1, -1))
        H = self.embed(patches) + pos_emb

        states = []
        all_gates = []
        current = H
        for i, layer in enumerate(self.layers):
            top_down_signal_for_routing = states[i-1] if i > 0 else None
            current, gates = layer(current, self.edge_index, self.edge_attr, top_down=top_down_signal_for_routing)
            states.append(current)
            all_gates.append(gates)

        if self.use_top_down:
            for i in range(len(self.layers)-2, -1, -1):
                hierarchy = self.create_hierarchy_map(states[i].shape[1], states[i+1].shape[1])
                hierarchy = hierarchy.to(images.device)
                states[i] = self.top_down_aggregators[i](states[i], states[i+1], hierarchy)

        logits = self.readout(states[-1].permute(0, 2, 1)).squeeze(-1)

        sparsity_loss = self.reg(torch.cat(all_gates)) if all_gates else torch.tensor(0.0, device=images.device)

        if return_internals:
            return logits, sparsity_loss, all_gates, states
        else:
            return logits, sparsity_loss

    def create_hierarchy_map(self, n_low, n_high):

        if n_low == 0 or n_high == 0: return torch.zeros(n_low, n_high)
        if n_low % n_high != 0:
            ratio = n_low // n_high
        else:
            ratio = n_low // n_high
        if ratio == 0: return torch.zeros(n_low, n_high)

        map_matrix = torch.zeros(n_low, n_high)
        for i in range(n_high):
            start_idx = i * ratio
            end_idx = (i + 1) * ratio if i < n_high - 1 else n_low
            map_matrix[start_idx:end_idx, i] = 1.0
        return map_matrix


import os
```

3. Visualization and Analysis

To enhance the interpretability of the model, this `VisualizationHook` generates plots during training. \

- The learned communication graph over an input image.
- The effective sparsity achieved in each layer.
- The distribution of gate activation values.

```
class VisualizationHook:
    def __init__(self, model, dataloader, device, vis_epoch_interval=10):
        self.model = model
        self.dataloader = dataloader
        self.device = device
        self.vis_epoch_interval = vis_epoch_interval

        self.vis_dir = f'visualizations_{config.model_type}'
        os.makedirs(self.vis_dir, exist_ok=True)

        self.sample_batch = next(iter(self.dataloader))

    def run(self, epoch):
        """main function to generate and save all visualizations for a given epoch."""
        if epoch % self.vis_epoch_interval != 0 and epoch != config.epochs -1:
            return

        print(f"\nGenerating visualizations for Epoch {epoch+1}....")
        self.model.eval() #
        with torch.no_grad():
            image, _ = self.sample_batch
            image = image[0:1].to(self.device)

            gates_per_layer, H_per_layer = self.get_internals(image)

            middle_layer_idx = config.num_layers // 2
            self.plot_communication_graph(
                image.squeeze(0),
                gates_per_layer[middle_layer_idx],
                epoch
            )

            self.plot_sparsity_stats(gates_per_layer, epoch)

            self.plot_gate_distribution(gates_per_layer[middle_layer_idx], epoch, middle_layer_idx)

        print("Visualizations saved...")

    def get_internals(self, image_batch):
        """modified forward pass to capture intermediate gates and states."""
        logits, sparsity_loss, all_gates, all_states = self.model(image_batch, return_internals=True)
        return all_gates, all_states
    def plot_communication_graph(self, image_tensor, gates, epoch):
        """
```

```
        alll communication pathways on top of the image,
        with line width and transparency determined by gate strength.
        """
        inv_normalize = transforms.Normalize(
            mean=[-0.5071/0.2675, -0.4867/0.2565, -0.4408/0.2761],
            std=[1/0.2675, 1/0.2565, 1/0.2761]
        )
        img = inv_normalize(image_tensor).cpu().permute(1, 2, 0)

        edge_index = self.model.edge_index.cpu()
        positions = self.model.static_positions.cpu()
        gate_strengths = gates.mean(dim=1).cpu().numpy()

        plt.figure(figsize=(8, 8))
        plt.imshow(img, interpolation='nearest') # Use 'nearest' for crisp pixels

        for i in range(edge_index.shape[1]):
            start_node, end_node = edge_index[:, i]
            p_start = positions[start_node]
            p_end = positions[end_node]
            strength = gate_strengths[i]

            alpha = max(0.01, strength * 0.6)
            lw = strength * 3.0

            if alpha > 0.05:
                plt.plot([p_start[1], p_end[1]], [p_start[0], p_end[0]],
                         color='cyan',
                         alpha=alpha,
                         lw=lw)

        plt.title(f'Learned Communication Graph (Epoch {epoch+1})', fontsize=16)
        plt.axis('off')
        plt.savefig(f'{self.vis_dir}/comm_graph_epoch_{epoch+1}.png', bbox_inches='tight', dpi=150)
        plt.close()

    def plot_sparsity_stats(self, gates_per_layer, epoch):
        """
        plots a bar chart of sparsity per layer using a more
        meaningful threshold of 0.5 to define a "pruned" connection.
        """
        sparsities = []
        layer_indices = range(len(gates_per_layer))

        for gates in gates_per_layer:
            threshold = 0.5
            sparsity = (gates.mean(dim=1) < threshold).float().mean().item()
            sparsities.append(sparsity * 100)

        plt.figure(figsize=(8, 5))
        plt.bar(layer_indices, sparsities, color='dodgerblue')
        plt.title(f'Connection Sparsity per Layer (Epoch {epoch+1})', fontsize=14)
        plt.xlabel('Layer Index')
        plt.ylabel('Sparsity (% of connections pruned)')
```

```
        plt.xticks(layer_indices)
        plt.ylim(0, 100)
        plt.grid(axis='y', linestyle='--', alpha=0.7)
        plt.savefig(f'{self.vis_dir}/sparsity_stats_epoch_{epoch+1}.png')
        plt.close()


    def plot_gate_distribution(self, gates, epoch, layer_idx):
        """plots a histogram of gate activation values."""
        gate_values = gates.mean(dim=1).view(-1).cpu().numpy()

        plt.figure(figsize=(8, 5))
        plt.hist(gate_values, bins=50, color='mediumseagreen')
        plt.title(f'Gate Activation Distribution - Layer {layer_idx} (Epoch {epoch+1})')
        plt.xlabel('Mean Gate Value')
        plt.ylabel('Frequency')
        plt.yscale('log')
        plt.savefig(f'{self.vis_dir}/gate_dist_epoch_{epoch+1}.png')
        plt.close()
```

4. Data Loading and Training Loop

These are standard helper functions for:

- Loading and transforming the CIFAR-100 dataset.
- Executing a single training epoch, including task loss and sparsity regularization.
- Evaluating the model on the validation set.````

Before the `run_experiment()` and `main()` functions:

5. Main Experiment: Ablation Study (Figure 6)

This is the main driver cell for the notebook. The `run_experiment` function encapsulates a full training run for a given model configuration.

The `main` function is configured to perform the **Hierarchical Top-Down Refinement Pass Ablation Study** from **Section 7.3**. It runs two experiments and saves their results to JSON files:

1. **Control:** The full AFMP model with the top-down pass enabled.
2. **Ablated:** The AFMP model without the top-down pass.

```
def prepare_cifar100(batch_size=128):
    transform_train = transforms.Compose([
        transforms.RandomCrop(32, padding=4),
        transforms.RandomHorizontalFlip(),
        transforms.ToTensor(),
        transforms.Normalize((0.5071, 0.4867, 0.4408), (0.2675, 0.2565, 0.2761))
    ])

    transform_test = transforms.Compose([
```

```
        transforms.ToTensor(),
        transforms.Normalize((0.5071, 0.4867, 0.4408), (0.2675, 0.2565, 0.2761))
    ])

    train_set = torchvision.datasets.CIFAR100(
        root='./data', train=True, download=True, transform=transform_train)
    test_set = torchvision.datasets.CIFAR100(
        root='./data', train=False, download=True, transform=transform_test)

    train_loader = DataLoader(train_set, batch_size=batch_size, shuffle=True, num_workers=4)
    test_loader = DataLoader(test_set, batch_size=batch_size, shuffle=False, num_workers=4)

    return train_loader, test_loader
```

```
def train_epoch(model, train_loader, optimizer, device, epoch):
    model.train()
    total_loss, total_correct, total_samples = 0, 0, 0
    sparsity_loss_sum = 0

    for batch_idx, (inputs, targets) in enumerate(train_loader):
        inputs, targets = inputs.to(device), targets.to(device)
        batch_size = inputs.size(0)

        optimizer.zero_grad()

        logits, sparsity_loss = model(inputs)
        task_loss = F.cross_entropy(logits, targets)
        loss = task_loss + sparsity_loss

        # backward pass
        loss.backward()
        optimizer.step()

        _, predicted = logits.max(1)
        total_correct += predicted.eq(targets).sum().item()
        total_samples += batch_size
        total_loss += loss.item()
        sparsity_loss_sum += sparsity_loss.item()

        if batch_idx % 100 == 0:
            print(f'Epoch: {epoch} | Batch: {batch_idx}/{len(train_loader)} '
                  f'| Loss: {loss.item():.4f} | Acc: {100.*total_correct/total_samples:.2f}%')

    avg_loss = total_loss / len(train_loader)
    avg_acc = 100. * total_correct / total_samples
    avg_sparsity_loss = sparsity_loss_sum / len(train_loader)

    return avg_loss, avg_acc, avg_sparsity_loss

def validate(model, test_loader, device):
    model.eval()
    total_correct, total_samples = 0, 0
```

```
    with torch.no_grad():
        for inputs, targets in test_loader:
            inputs, targets = inputs.to(device), targets.to(device)
            batch_size = inputs.size(0)

            logits, _ = model(inputs)

            _, predicted = logits.max(1)
            total_correct += predicted.eq(targets).sum().item()
            total_samples += batch_size

    accuracy = 100. * total_correct / total_samples
    return accuracy


def get_peak_memory(model, device):
    """Measure peak GPU memory usage during a forward pass"""
    torch.cuda.empty_cache()
    torch.cuda.reset_peak_memory_stats(device)

    # create dummy input
    dummy_input = torch.randn(2, 3, config.img_size, config.img_size).to(device)

    # forward pass
    model(dummy_input)

    # get peak memory in MB
    peak_mem_mb = torch.cuda.max_memory_allocated(device) / (1024 ** 2)
    return peak_mem_mb



def save_results(model_type, model, history, best_acc, peak_mem):
    """saves the model state, training history, and final metrics."""
    print(f"\nSaving results for {model_type.upper()} model....")


    torch.save(model.state_dict(), f'afmp_{model_type}_final.pth')
    print(f"Final model state saved to afmp_{model_type}_final.pth")

    results = {
        'model_type': model_type,
        'best_val_accuracy': best_acc,
        'peak_memory_mb': peak_mem,
        'training_history': history
    }

    results_filename = f'results_{model_type}.json'
    with open(results_filename, 'w') as f:
        json.dump(results, f, indent=4)
    print(f"Training history and metrics saved to {results_filename}")
    print("--------------------------------------------------")
```

```
def run_experiment(model_name, use_top_down):
    """
    runs a single, complete training and validation experiment.

    Args:
        model_name (str): A descriptive name for saving files (e.g., 'sparse_with_top_down').
        use_top_down (bool): Flag to control the use of the top-down refinement pass.
    """
    config.model_type = 'sparse'
    config.sparsity = 0.3
    config.lambda_sparse = 1e-4

    device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
    print(f"Using device: {device}")

    model = AFMP(
        num_layers=config.num_layers,
        d_model=config.d_model,
        d_position=config.d_position,
        num_classes=config.num_classes,
        img_size=config.img_size,
        patch_size=config.patch_size,
        sparsity=config.sparsity,
        is_sparse=(config.model_type == 'sparse'),
        use_top_down=use_top_down
    ).to(device)

    num_params = sum(p.numel() for p in model.parameters())
    print(f"Model: {model_name.upper()} | Parameters: {num_params/1e6:.2f}M")

    optimizer = torch.optim.AdamW(
        model.parameters(),
        lr=config.lr,
        weight_decay=config.weight_decay
    )

    scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(
        optimizer, 'max', patience=5, factor=0.2, verbose=True
    )

    train_loader, test_loader = prepare_cifar100(config.batch_size)
    visualizer = VisualizationHook(model, test_loader, device, vis_epoch_interval=10)

    history = {
        'train_loss': [], 'train_acc': [], 'val_acc': [],
        'sparsity_loss': [], 'time': []
    }

    best_acc = 0
    start_time = time.time()

    for epoch in range(config.epochs):
        epoch_start = time.time()
        train_loss, train_acc, sparsity_loss = train_epoch(
```

```
            model, train_loader, optimizer, device, epoch)

        val_acc = validate(model, test_loader, device)
        scheduler.step(val_acc)
        visualizer.run(epoch)

        epoch_time = time.time() - epoch_start
        history['train_loss'].append(train_loss)
        history['train_acc'].append(train_acc)
        history['val_acc'].append(val_acc)
        history['sparsity_loss'].append(sparsity_loss)
        history['time'].append(epoch_time)

        print(f"Epoch {epoch+1}/{config.epochs} | "
              f"Time: {epoch_time:.1f}s | "
              f"Train Loss: {train_loss:.4f} | "
              f"Train Acc: {train_acc:.2f}% | "
              f"Val Acc: {val_acc:.2f}% | "
              f"Sparsity Loss: {sparsity_loss:.6f}")

        if val_acc > best_acc:
            best_acc = val_acc
            torch.save(model.state_dict(), f'afmp_{model_name}_best.pth')

    total_time = time.time() - start_time
    print(f"Training completed in {total_time/60:.1f} minutes")
    print(f"Best validation accuracy for {model_name.upper()}: {best_acc:.2f}%")

    peak_mem = get_peak_memory(model, device)

    return model, history, best_acc, peak_mem

import json
```

```
def main():
    '''
    # run 1
    # this is the full, powerful model with the updated logic.
    print("="*50)
    print(f"RUNNING HIERARCHY STUDY: CONTROL (WITH TOP-DOWN) sparsity : {config.sparsity}, Lambda sparse : {config.lambda_sparse}")
    print("="*50)
```

```
control_name = 'sparse_with_top_down'
control_model, control_history, control_acc, control_mem = run_experiment(
    model_name=control_name,
    use_top_down=True
)
# Save results immediately after completion
save_results(control_name, control_model, control_history, control_acc, control_mem)'''


# run 2
# this model uses the identical powerful bottom-up pass but skips the final refinement.
print("\n" + "="*50)
print("RUNNING HIERARCHY STUDY: ABLATION (NO TOP-DOWN)")
print("="*50)

ablation_name = 'sparse_no_top_down'
ablation_model, ablation_history, ablation_acc, ablation_mem = run_experiment(
    model_name=ablation_name,
    use_top_down=False
)
save_results(ablation_name, ablation_model, ablation_history, ablation_acc, ablation_mem)




print("\n" + "="*50)
print("HIERARCHY ABLATION STUDY COMPLETE")
print("="*50)
print("Final Comparison:")
print(f"{'Model Variant':<30} {'Val Accuracy':<15}")
print("-"*45)
print(f"{'AFMP with Top-Down':<30} {control_acc:.2f}%")
print(f"{'AFMP without Top-Down':<30} {ablation_acc:.2f}%")


'''

print(f"{'Model Variant':<20} {'Val Accuracy':<15} {'Peak Memory (MB)':<20}")
print("-"*65)
print(f"{'AFMP-Dense':<20} {dense_acc:.2f}% {dense_mem:.2f}")'''
print(f"{'AFMP-Tiny':<20} {sparse_acc:.2f}% {sparse_mem:.2f}")
'''

plt.figure(figsize=(12, 8))

plt.subplot(2, 2, 1)
plt.plot(dense_history['val_acc'], label=f'Dense (Best: {dense_acc:.2f}%)')
plt.plot(sparse_history['val_acc'], label=f'Sparse (Best: {sparse_acc:.2f}%)')
plt.title('Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.legend()
plt.grid(True)
```

```
    plt.subplot(2, 2, 2)
    plt.plot(dense_history['train_loss'], label='Dense')
    plt.plot(sparse_history['train_loss'], label='Sparse')
    plt.title('Training Loss')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.legend()
    plt.grid(True)

    plt.subplot(2, 2, 3)
    plt.plot(sparse_history['sparsity_loss'])
    plt.title('Sparsity Loss (Sparse Model)')
    plt.xlabel('Epoch')
    plt.ylabel('Loss Value')
    plt.grid(True)

    plt.subplot(2, 2, 4)
    avg_dense_time = sum(dense_history['time']) / len(dense_history['time'])
    avg_sparse_time = sum(sparse_history['time']) / len(sparse_history['time'])
    plt.bar(['Dense', 'Sparse'], [avg_dense_time, avg_sparse_time])
    plt.title('Average Per-Epoch Training Time')
    plt.ylabel('Seconds')

    plt.tight_layout()
    plt.savefig('cifar100_results_with_scheduler.png')
    plt.show()'''

if __name__ == '__main__':
    main()
```

```
def check_params():
    config.num_layers = 5
    config.d_model = 512
    config.patch_size = 16

    device = torch.device('cpu')

    model = AFMP(
        num_layers=config.num_layers,
        d_model=config.d_model,
        d_position=config.d_position, # d_position can stay small
        num_classes=config.num_classes, # For CIFAR-100
        img_size=192, # Use a sample large image size
        patch_size=config.patch_size,
        sparsity=0.3,
        is_sparse=True,
        use_top_down=True
    ).to(device)

    num_params = sum(p.numel() for p in model.parameters() if p.requires_grad)
    print(f"AFMP-Small Configuration:")
    print(f"  d_model: {config.d_model}")
    print(f"  num_layers: {config.num_layers}")
```

```
    print(f"  patch_size: {config.patch_size}")
    print(f"----------------------------------")
    print(f"Total Trainable Parameters: {num_params / 1e6:.2f}M")
    print(f"Target (DeiT-Tiny): ~5.7M")

check_params()
```

```
AFMP-Small Configuration:
  d_model: 512
  num_layers: 5
  patch_size: 16
----------------------------------
Total Trainable Parameters: 6.12M
Target (DeiT-Tiny): ~5.7M
```

```
import timm

deit_tiny_model = timm.create_model(
    'deit_tiny_patch16_224',
    pretrained=False,
    num_classes=100
)

num_params_deit = sum(p.numel() for p in deit_tiny_model.parameters() if p.requires_grad)
print(f"DeiT-Tiny Trainable Parameters: {num_params_deit / 1e6:.2f}M")
```

```
DeiT-Tiny Trainable Parameters: 5.54M
```

Double-click (or enter) to edit

```
import torch
import torch.nn as nn
import torchvision.transforms as transforms
import time
import matplotlib.pyplot as plt
import numpy as np
import timm
import os
```

```
Start coding or generate with AI.
```

```
def get_afmp_small(num_classes=100, img_size=224):
    """builds and returns the scaled-up AFMP-Small model."""
    model = AFMP(
        num_layers=5, d_model=512, d_position=2,
        num_classes=num_classes, img_size=img_size, patch_size=16,
        sparsity=0.3, is_sparse=True, use_top_down=True
    )
    return model
```

```
def get_deit_tiny(num_classes=100, img_size=224):
    """
    builds and returns the DeiT-Tiny model from timm, configured for a specific image size.
    """
    model = timm.create_model(
        'deit_tiny_patch16_224',
        pretrained=False,
        num_classes=num_classes,
        img_size=img_size
    )
    return model


def measure_performance(model, device, input_tensor):
    """
    measures peak memory and time for a single forward/backward pass.
    returns (peak_memory_mb, time_seconds), or (None, None) on OOM error.
    """
    model.to(device)

    num_classes = model.num_classes if hasattr(model, 'num_classes') else 100
    target = torch.randint(0, num_classes, (input_tensor.size(0),), device=device)

    torch.cuda.synchronize()
    torch.cuda.reset_peak_memory_stats(device)

    try:
        for _ in range(5):
            output = model(input_tensor)
            if isinstance(output, tuple): # Handle models returning multiple values
                output = output[0]
            loss = nn.CrossEntropyLoss()(output, target)
            loss.backward()
            model.zero_grad()

        torch.cuda.synchronize()
        start_time = time.time()

        for _ in range(10): # average over 10 passes for stable time measurement
            output = model(input_tensor)
            if isinstance(output, tuple):
                output = output[0]
            loss = nn.CrossEntropyLoss()(output, target)
            loss.backward()
            model.zero_grad() # clear gradients after each pass

        torch.cuda.synchronize()
        end_time = time.time()

        peak_memory_mb = torch.cuda.max_memory_allocated(device) / (1024**2)
        avg_time_sec = (end_time - start_time) / 10

        return peak_memory_mb, avg_time_sec
```

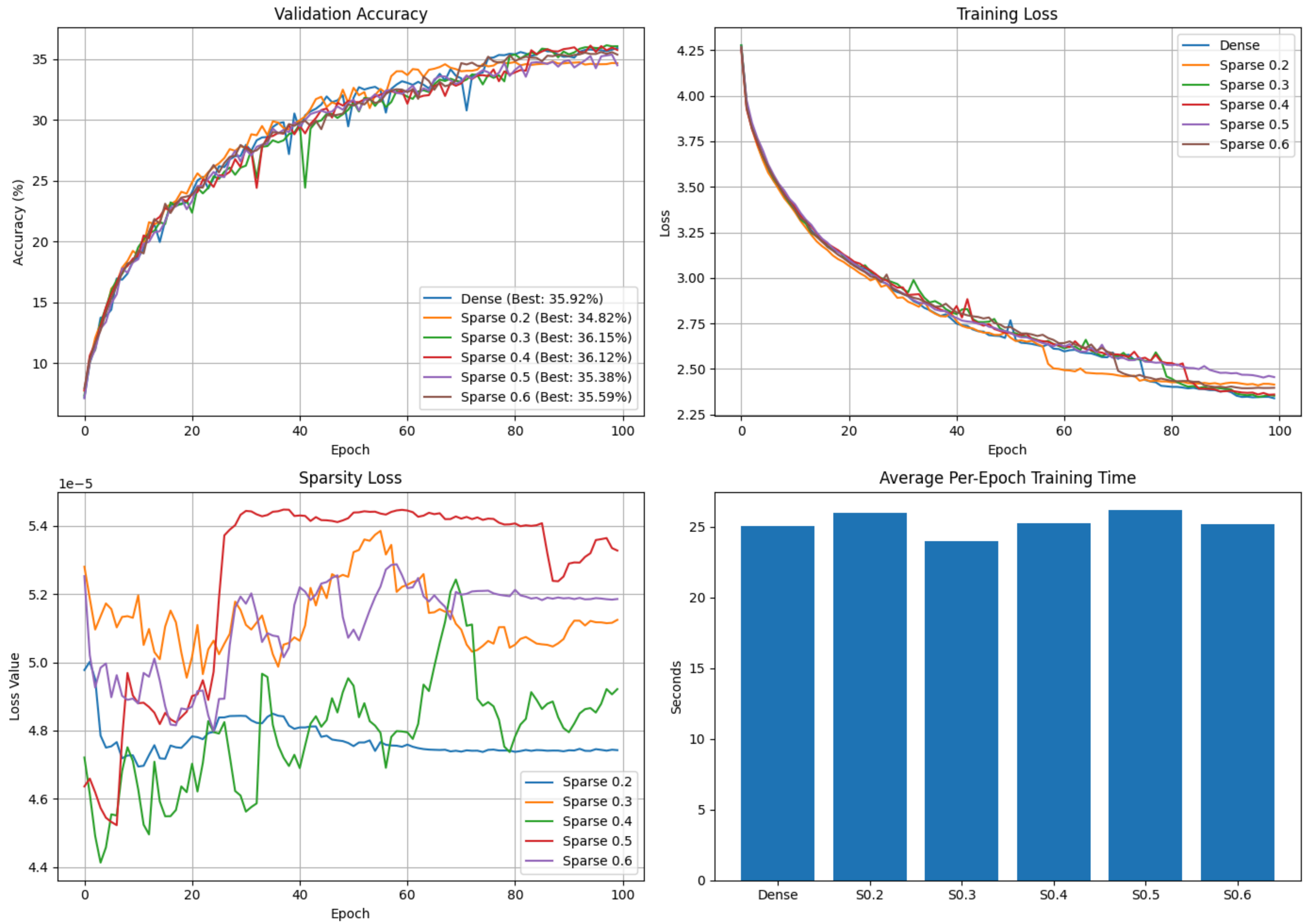
```
    except torch.cuda.OutOfMemoryError:
        print(f"--- OOM ERROR! Model could not run on this input size. ---")
        return None, None
    finally:
        # clean up memory
        del model, input_tensor, target
        torch.cuda.empty_cache()
```

```
def run_scalability_experiment():
    """Main function to run the scalability test and generate plots."""

    demice = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
    if not torch.cuda.is_available():
        print("WARNING: CUDA not available...")
        return

    resolutions = [64, 96, 128, 160, 192, 224]
    patch_size = 16
    batch_size = 64

    results = {
        'resolutions': resolutions,
        'num_patches': [(res // patch_size)**2 for res in resolutions],
        'afmp': {'memory': [], 'time': []},
        'deit': {'memory': [], 'time': []}
    }

    afmp_check = get_afmp_small()
    deit_check = get_deit_tiny()
    print(f"AFMP-Small Parameters: {sum(p.numel() for p in afmp_check.parameters())/1e6:.2f}M")
    print(f"DeiT-Tiny Parameters: {sum(p.numel() for p in deit_check.parameters())/1e6:.2f}M")
    del afmp_check, deit_check

    for res in resolutions:
        num_patches = (res // patch_size)**2
        print(f"\nTesting Resolution: {res}x{res} (N = {num_patches} patches)...")

        input_tensor = torch.randn(batch_size, 3, res, res, device=device)

        print("Testing AFMP-Small...")
        afmp_model = get_afmp_small(img_size=res)
        mem, t = measure_performance(afmp_model, device, input_tensor.clone())
        results['afmp']['memory'].append(mem)
        results['afmp']['time'].append(t)

        print("Testing DeiT-Tiny...")
        deit_model = get_deit_tiny(img_size=res)
        mem, t = measure_performance(deit_model, device, input_tensor.clone())
        results['deit']['memory'].append(mem)
        results['deit']['time'].append(t)
```

```
        plot_results(results)


    def plot_results(results):
        """Generates and saves the final scalability plots."""

        num_patches = results['num_patches']

        plt.style.use('seaborn-v0_8-whitegrid')
        fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(18, 7))

        afmp_mem = [m for m in results['afmp']['memory'] if m is not None]
        deit_mem = [m for m in results['deit']['memory'] if m is not None]

        ax1.plot(num_patches[:len(afmp_mem)], afmp_mem, 'o-', label='AFMP-Small (Ours)', color='blue', lw=2, markersize=8)
        ax1.plot(num_patches[:len(deit_mem)], deit_mem, 's--', label='DeiT-Tiny (Baseline)', color='red', lw=2, markersize=8)

        if len(deit_mem) < len(num_patches):
            oom_patch_idx = len(deit_mem)
            oom_patch_num = num_patches[oom_patch_idx]
            ax1.scatter(oom_patch_num, deit_mem[-1] * 1.05, marker='x', color='red', s=200, zorder=10, label='DeiT OOM Error')
            ax1.text(oom_patch_num, deit_mem[-1] * 0.9, 'OOM Error', color='red', ha='center', va='top', fontsize=12, weight='bold')

        ax1.set_title('Memory Scalability: O(N) vs O(N²)', fontsize=18, weight='bold')
        ax1.set_xlabel('Number of Patches (N)', fontsize=14)
        ax1.set_ylabel('Peak GPU Memory (MB)', fontsize=14)
        ax1.legend(fontsize=12)
        ax1.tick_params(axis='both', which='major', labelsize=12)

        afmp_time = [t * 1000 for t in results['afmp']['time'] if t is not None] # Convert to ms
        deit_time = [t * 1000 for t in results['deit']['time'] if t is not None] # Convert to ms

        ax2.plot(num_patches[:len(afmp_time)], afmp_time, 'o-', label='AFMP-Small (Ours)', color='blue', lw=2, markersize=8)
        ax2.plot(num_patches[:len(deit_time)], deit_time, 's--', label='DeiT-Tiny (Baseline)', color='red', lw=2, markersize=8)

        if len(deit_time) < len(num_patches):
            oom_patch_idx = len(deit_time)
            oom_patch_num = num_patches[oom_patch_idx]
            ax2.scatter(oom_patch_num, deit_time[-1] * 1.05, marker='x', color='red', s=200, zorder=10, label='DeiT OOM Error')

        ax2.set_title('Time Scalability: O(N) vs O(N²)', fontsize=18, weight='bold')
        ax2.set_xlabel('Number of Patches (N)', fontsize=14)
        ax2.set_ylabel('Time per Pass (ms)', fontsize=14)
        ax2.legend(fontsize=12)
        ax2.tick_params(axis='both', which='major', labelsize=12)

        fig.suptitle('AFMP vs. DeiT: Scalability Analysis', fontsize=22, weight='bold')
        plt.tight_layout(rect=[0, 0.03, 1, 0.95])
        plt.savefig('scalability_results.png', dpi=300)
        print("\nScalability plots saved to 'scalability_results.png'")
        plt.show()

    run_scalability_experiment()
```

```
import json
import matplotlib.pyplot as plt

with open('results_dense (1).json', 'r') as f:
    dense_results = json.load(f)
with open('results_sparse_0.2.json', 'r') as f:
    sparse_0_2_results = json.load(f)
with open('results_sparse_0.3.json', 'r') as f:
    sparse_0_3_results = json.load(f)
with open('results_sparse_0.4.json', 'r') as f:
    sparse_0_4_results = json.load(f)
with open('results_sparse_0.5.json', 'r') as f:
    sparse_0_5_results = json.load(f)
with open('results_sparse_0.6.json', 'r') as f:
    sparse_0_6_results = json.load(f)

# Access the training histories
dense_history = dense_results['training_history']
sparse_0_2_history = sparse_0_2_results['training_history']
sparse_0_3_history = sparse_0_3_results['training_history']
sparse_0_4_history = sparse_0_4_results['training_history']
sparse_0_5_history = sparse_0_5_results['training_history']
sparse_0_6_history = sparse_0_6_results['training_history']

dense_acc = max(dense_history['val_acc'])
sparse_0_2_acc = max(sparse_0_2_history['val_acc'])
sparse_0_3_acc = max(sparse_0_3_history['val_acc'])
sparse_0_4_acc = max(sparse_0_4_history['val_acc'])
sparse_0_5_acc = max(sparse_0_5_history['val_acc'])
sparse_0_6_acc = max(sparse_0_6_history['val_acc'])

plt.figure(figsize=(14, 10))

plt.subplot(2, 2, 1)
plt.plot(dense_history['val_acc'], label=f'Dense (Best: {dense_acc:.2f}%)')
plt.plot(sparse_0_2_history['val_acc'], label=f'Sparse 0.2 (Best: {sparse_0_2_acc:.2f}%)')
plt.plot(sparse_0_3_history['val_acc'], label=f'Sparse 0.3 (Best: {sparse_0_3_acc:.2f}%)')
plt.plot(sparse_0_4_history['val_acc'], label=f'Sparse 0.4 (Best: {sparse_0_4_acc:.2f}%)')
plt.plot(sparse_0_5_history['val_acc'], label=f'Sparse 0.5 (Best: {sparse_0_5_acc:.2f}%)')
plt.plot(sparse_0_6_history['val_acc'], label=f'Sparse 0.6 (Best: {sparse_0_6_acc:.2f}%)')
plt.title('Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.legend()
plt.grid(True)

plt.subplot(2, 2, 2)
plt.plot(dense_history['train_loss'], label='Dense')
plt.plot(sparse_0_2_history['train_loss'], label='Sparse 0.2')
plt.plot(sparse_0_3_history['train_loss'], label='Sparse 0.3')
plt.plot(sparse_0_4_history['train_loss'], label='Sparse 0.4')
plt.plot(sparse_0_5_history['train_loss'], label='Sparse 0.5')
plt.plot(sparse_0_6_history['train_loss'], label='Sparse 0.6')
plt.title('Training Loss')
```

```
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.legend()
plt.grid(True)

plt.subplot(2, 2, 3)
plt.plot(sparse_0_2_history['sparsity_loss'], label='Sparse 0.2')
plt.plot(sparse_0_3_history['sparsity_loss'], label='Sparse 0.3')
plt.plot(sparse_0_4_history['sparsity_loss'], label='Sparse 0.4')
plt.plot(sparse_0_5_history['sparsity_loss'], label='Sparse 0.5')
plt.plot(sparse_0_6_history['sparsity_loss'], label='Sparse 0.6')
plt.title('Sparsity Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss Value')
plt.legend()
plt.grid(True)

avg_dense_time = sum(dense_history['time']) / len(dense_history['time'])
avg_sparse_0_2_time = sum(sparse_0_2_history['time']) / len(sparse_0_2_history['time'])
avg_sparse_0_3_time = sum(sparse_0_3_history['time']) / len(sparse_0_3_history['time'])
avg_sparse_0_4_time = sum(sparse_0_4_history['time']) / len(sparse_0_4_history['time'])
avg_sparse_0_5_time = sum(sparse_0_5_history['time']) / len(sparse_0_5_history['time'])
avg_sparse_0_6_time = sum(sparse_0_6_history['time']) / len(sparse_0_6_history['time'])

plt.subplot(2, 2, 4)
plt.bar(['Dense', 'S0.2', 'S0.3', 'S0.4', 'S0.5', 'S0.6'],
        [avg_dense_time, avg_sparse_0_2_time, avg_sparse_0_3_time,
         avg_sparse_0_4_time, avg_sparse_0_5_time, avg_sparse_0_6_time])
plt.title('Average Per-Epoch Training Time')
plt.ylabel('Seconds')

plt.tight_layout()
plt.savefig('comparison_results_full.png')
plt.show()
```



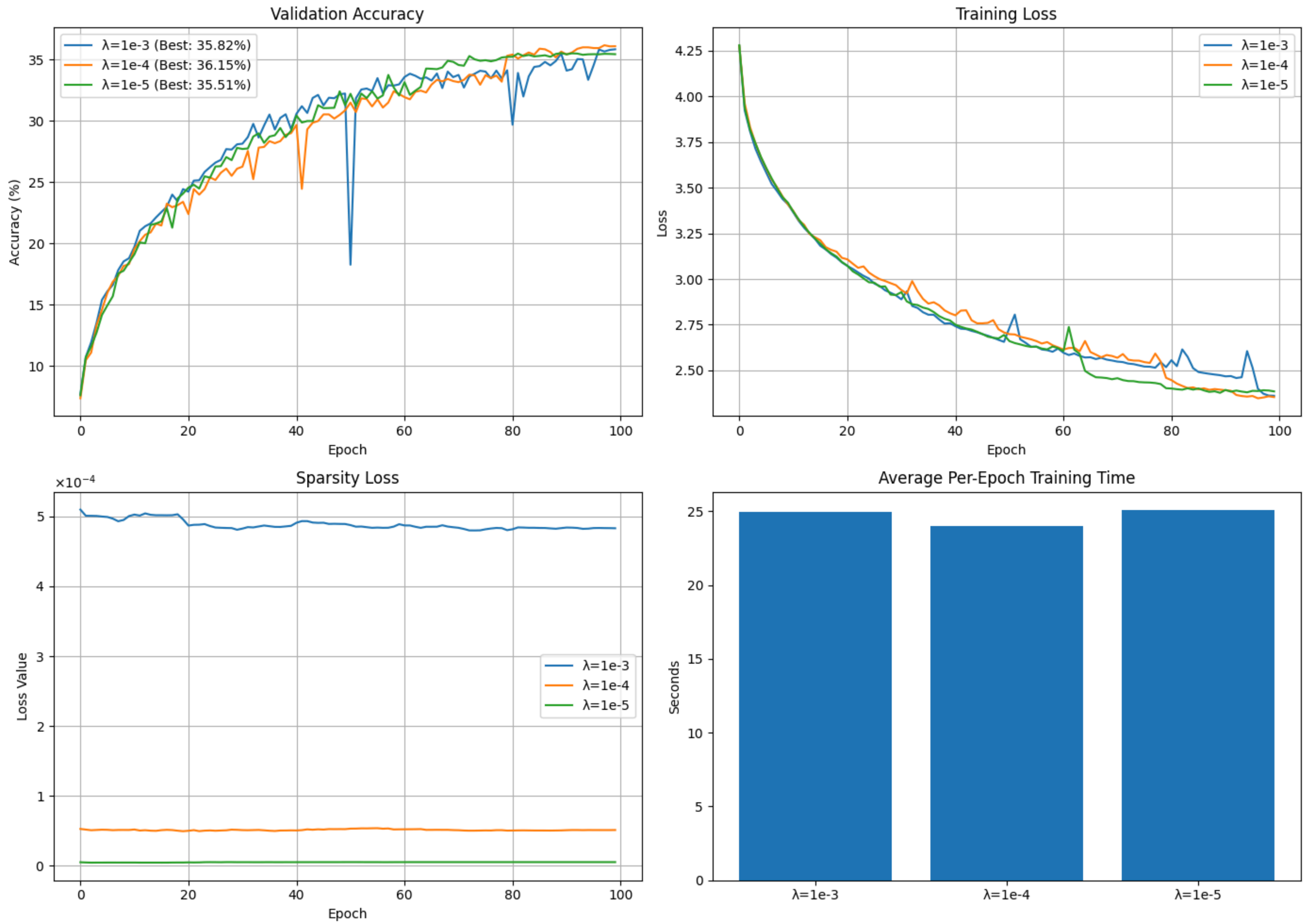

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

```
cp -r /content/visualizations_sparse/  /content/drive/MyDrive/
```

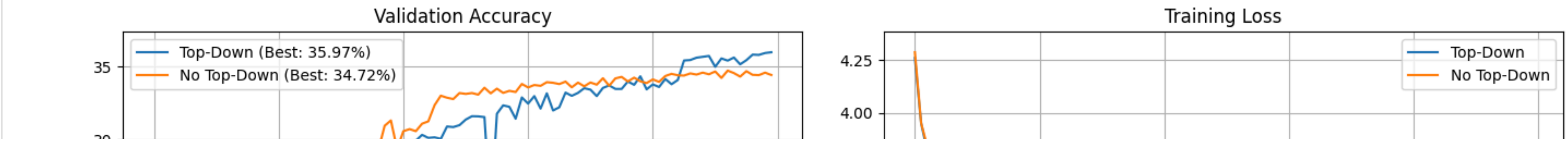
```
import json
import matplotlib.pyplot as plt

with open('/content/results_sparse_0.3_with-lambda_sparse_1e-3.json', 'r') as f:
    results_1e_3 = json.load(f)
with open('/content/results_sparse_0.3_with_lambda_sparse_1e-4.json', 'r') as f:
    results_1e_4 = json.load(f)
with open('/content/results_sparse_0.3_with_lambda_sparse_1e-5.json', 'r') as f:
    results_1e_5 = json.load(f)

history_1e_3 = results_1e_3['training_history']
history_1e_4 = results_1e_4['training_history']
history_1e_5 = results_1e_5['training_history']

acc_1e_3 = max(history_1e_3['val_acc'])
acc_1e_4 = max(history_1e_4['val_acc'])
acc_1e_5 = max(history_1e_5['val_acc'])

plt.figure(figsize=(14, 10))

plt.subplot(2, 2, 1)
plt.plot(history_1e_3['val_acc'], label=f'λ=1e-3 (Best: {acc_1e_3:.2f}%)')
plt.plot(history_1e_4['val_acc'], label=f'λ=1e-4 (Best: {acc_1e_4:.2f}%)')
plt.plot(history_1e_5['val_acc'], label=f'λ=1e-5 (Best: {acc_1e_5:.2f}%)')
plt.title('Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.legend()
plt.grid(True)

# 2. Training Loss
plt.subplot(2, 2, 2)
plt.plot(history_1e_3['train_loss'], label='λ=1e-3')
plt.plot(history_1e_4['train_loss'], label='λ=1e-4')
plt.plot(history_1e_5['train_loss'], label='λ=1e-5')
plt.title('Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.legend()
plt.grid(True)

import matplotlib.ticker as ticker  # Add this import at the top

plt.subplot(2, 2, 3)
plt.plot(history_1e_3['sparsity_loss'], label='λ=1e-3')
plt.plot(history_1e_4['sparsity_loss'], label='λ=1e-4')
plt.plot(history_1e_5['sparsity_loss'], label='λ=1e-5')
```

```
plt.title('Sparsity Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss Value')
plt.yscale('linear')  # keep it linear, just use scientific notation
plt.gca().yaxis.set_major_formatter(ticker.ScalarFormatter(useMathText=True))
plt.gca().yaxis.get_major_formatter().set_powerlimits((-4, 4))  # force scientific notation for small numbers
plt.grid(True)
plt.legend()

avg_time_1e_3 = sum(history_1e_3['time']) / len(history_1e_3['time'])
avg_time_1e_4 = sum(history_1e_4['time']) / len(history_1e_4['time'])
avg_time_1e_5 = sum(history_1e_5['time']) / len(history_1e_5['time'])

plt.subplot(2, 2, 4)
plt.bar(['λ=1e-3', 'λ=1e-4', 'λ=1e-5'],
        [avg_time_1e_3, avg_time_1e_4, avg_time_1e_5])
plt.title('Average Per-Epoch Training Time')
plt.ylabel('Seconds')

plt.tight_layout()
plt.savefig('lambda_sparse_comparison.png')
plt.show()
```




```
import json
import matplotlib.pyplot as plt

with open('/content/results_sparse_with_top_down.json', 'r') as f:
```

```
        topdown_results = json.load(f)

    with open('/content/results_sparse_no_top_down.json', 'r') as f:
        no_topdown_results = json.load(f)

    topdown_history = topdown_results['training_history']
    no_topdown_history = no_topdown_results['training_history']

    topdown_acc = max(topdown_history['val_acc'])
    no_topdown_acc = max(no_topdown_history['val_acc'])

    plt.figure(figsize=(14, 10))

    plt.subplot(2, 2, 1)
    plt.plot(topdown_history['val_acc'], label=f'Top-Down (Best: {topdown_acc:.2f}%)')
    plt.plot(no_topdown_history['val_acc'], label=f'No Top-Down (Best: {no_topdown_acc:.2f}%)')
    plt.title('Validation Accuracy')
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy (%)')
    plt.legend()
    plt.grid(True)

    plt.subplot(2, 2, 2)
    plt.plot(topdown_history['train_loss'], label='Top-Down')
    plt.plot(no_topdown_history['train_loss'], label='No Top-Down')
    plt.title('Training Loss')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.legend()
    plt.grid(True)

    plt.subplot(2, 2, 3)
    plt.plot(topdown_history['sparsity_loss'], label='Top-Down')
    plt.plot(no_topdown_history['sparsity_loss'], label='No Top-Down')
    plt.title('Sparsity Loss')
    plt.xlabel('Epoch')
    plt.ylabel('Loss Value')
    plt.legend()
    plt.grid(True)

    avg_topdown_time = sum(topdown_history['time']) / len(topdown_history['time'])
    avg_no_topdown_time = sum(no_topdown_history['time']) / len(no_topdown_history['time'])

    plt.subplot(2, 2, 4)
    plt.bar(['Top-Down', 'No Top-Down'],
            [avg_topdown_time, avg_no_topdown_time])
    plt.title('Average Per-Epoch Training Time')
    plt.ylabel('Seconds')

    plt.tight_layout()
    plt.savefig('comparison_topdown_vs_no.png')
    plt.show()
```


Validation Accuracy
Top-Down (Best: 35.97%)
No Top-Down (Best: 34.72%)
35
30
Training Loss
Top-Down
No Top-Down
4.25
4.00