MRs Q factor and 3-dB bandwidth

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An ideal cavity would confine light indefinitely (that is, without loss) and would have resonant frequencies at precise values. Deviation from this ideal condition is described by the cavity *Q* factor (which is proportional to the confinement time in units of the optical period). Signals that exit the drop port of a microresonator (on resonance) experience insertion loss dominated by a component that is proportional to 1/*Q*.

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*k2* is defined as the power coupling coefficient between the bus waveguide and the microring resonator.

[16] All losses other than the bus-ring coupling, including the bending loss and radiation loss due to sidewall roughness, is lumped into a parameter *kp2*, which is the fraction of propagation power loss per round-trip in the microring resonator. The propagation loss decreases as the waveguide width increases in silicon waveguides, as the guided light is more confined in the silicon core and scatters less at the rough surfaces of the waveguides. The microring waveguide is approximately of single mode (the lowest TE) at ~ 1.55 μm for waveguide’s width up to 600 nm, and other modes have higher propagation losses in the strongly bended microring waveguides. With a recently reported method, propagation losses in fabricated microring resonators with *R=2.5* μm and different *Wring* were characterized and listed in the following table:



The propagation loss was reduced down to ~ 2 dB/cm in 10 μm-radius microrings, corresponding to intrinsic quality factors up to ~ 400,000.

We define the minimum power transmission in the through-port as γt, the drop -3dB bandwidth as , and the response period of the resonator as *FSR* (free spectral range). The waveguide power coupling coefficient is calculated to be *k2=* π*×*(*)×[1-(*γt*)1/2]/FSR*, and the propagation power loss coefficient is determined to be *kp2= 2*π*×*(*)×(*γt*)1/2/FSR.* To be compared with the losses in straight waveguides, which is often quoted in dB/cm, the propagation loss in a microring resonator can be expressed as *-10×log10(1-* *kp2)/(2*π*R)* (dB/cm), where 2πR is the perimeter of the microring resonator.

The total quality factor is defined as

= = [2]

And the response period of the resonator is

[21]

where λ0 is the resonance wavelength, f0 is the resonance frequency

Δλ is the 3-dB bandwidth in nm, Δf is the 3-dB bandwidth in Hz

neff is the effective index of the microring resonator, and L=2πR

ngroup is the group index of the cavity mode

(The effective index of the TE ground mode in a 500x220 nm2 Si/SiO2 waveguide is 2.44 at 1550 nm, while the group index is 4.2 [20]. For a MR with 3um radius, it can be calculated that FSR=30.35nm. For a MR with 1.5um radius, the FSR is measured to be 62.5nm [22])

And the drop -3dB bandwidth is

And the intrinsic quality factor is defined as

= = *Q/ (γt) 1/2* [2]

For a symmetric coupled sing-ring add-drop filter, the drop-port maximum power transmission is

[17]

[14]

And the theoretical drop loss (in dB) can be expressed by

As the intrinsic losses always exist in any real devices, i.e., kp2 > 0, the drop loss always > 0dB.

If k2 >> kp2 holds, drop loss ≈ 0dB.

It can also be expressed as

where λ0 is the resonance wavelength

Q is the total quality factor

ngroup is the group effective index of the the microring resonator, and L=2πR

*k2* is the fraction of power coupling between the waveguide and the microring resonator

## Drop-port Power transfer of sing-ring add-drop filter

The single-pole filter has a Lorentzian power transfer function [17] with a bandwidth defined by 2δ=Δλ=λ0/Q.



**[17] Mani Hossein-Zadeh etc., Importance of Intrinsic-Q in Microring-Based Optical Filters and Dispersion Compensation Devices, 2007**

Microring resonators with *Q* between approximately 1500 and 100,000 have been demonstrated

using various technologies, including polysilicon ridge waveguides, various silicon-on-insulator (SOI) structures, and even exotic material systems [8].

Table 1 (all are experimentally measured)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | [1] | [2] | [4] | [3] | [5] | [7] | [8] |
| Resonance wavelength  (nm) | 1574 | 1525 | 1552 | 1555 | 1547 | 1565 | 1564 |
| MR radius | 6µm | 5µm | 4.5µm | 17µm | 12µm | N/A | N/A |
| 3-dB bandwidth | 0.04nm  4.8GHz | 0.11nm  14.1GHz | 0.6nm  74.7GHz | 0.04nm  4.8GHz | 0.22nm  26.7GHz | 0.095nm  9~11.6GHz | 0.078nm  9.6GHz |
| Q factor | 39,350 | 14,000 | 2,587 | 40,000 | 7,250 | 16,500 | 20,000 |
| Drop-loss | N/A | <1dB | 0.5dB | 12.7dB | N/A | N/A | N/A |
| FSR | N/A | 16 nm | N/A | 5.5nm | N/A | N/A | N/A |

Power penalty for high-data-rate NRZ signal attenuation

When the FWHM (full-width at half-maximum) of the resonator (in GHz) is similar to the incident optical signal data rate (in Gbps), even the first-order sidebands are attenuated by an order of magnitude more than the carrier frequency [8]. This attenuation affects higher frequency components more, resulting in distortion of NRZ data that leads to transition edge lengthening. These waveform distortions results in increased bit error rate (BER), which can be quantified by the receiver power penalty, defined as the additional power margin required to restore received signal quality and overcome the degradation introduced by a given device [7]. As the ratio of the optical signal data rate (in Gbps) to the resonator FWHM (in GHz) increases, the degradation becomes more severe, causing higher power penalties [6].

Table 2

|  |  |  |
| --- | --- | --- |
|  | [7] | [8] |
| FWHM | 0.095 nm  11.6GHz | 0.078 nm  9.6GHz |
| Passing signal | 10Gbps NRZ | 10Gbps NRZ |
| Power penalty | 1.4dB at a BER of 10-9 for a 231-1 pseudo random binary sequence (measured) | 0.8dB at a BER of 10-9 for a 231-1 pseudo random binary sequence (measured) |

Fig.1 (reference [9])

For this analysis, frequency is given as a unitless parameter: the ratio of the full-width at half-maximum (FWHM) Γ of microring resonator-based filters to the data rate *F* of the optical signal is used.



Fig.6 (reference [6])



Microresonator switching time

Table 3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **[11]** | **[12] microdisk** | **[12] microdisk** | **[13]** |
| **MR radius** | ~5µm | 5µm | 5µm | 6µm |
| **Q factor** | 20,000 | 16,900 | 10,300 | 10,000 |
| **Modulation bandwidth** | 4Gbps | 510MHz | 410MHz | N/A |
| **Driving voltage** | 6.5V  peak-to-peak | 0.85V forward bias  -6V reverse bias | 0.85V forward bias  -6V reverse bias | <1 V forward bias |
| **Rise time** | 40ps (measured) | 417ps  (measured) | 850ps  (measured) | Sub-ns  (not measured) |
| **Fall time** | 60ps (measured) | 1.1ns  (measured) | 2.1ns  (measured) | Sub-ns  (not measured) |
| **DC switching power** | N/A | 20µw | 20µw  (diode current is 24μA) | 20µw (diode current is on the order of 10) |
| **Carrier injection** |  | 6.8x1016 cm-3  (corresponding to **Δn=-3x10-4, Δα=1cm-1**) |  | 1x1017 cm-3 |
| **Observed wavelength blue-shift** |  | 0.06nm |  | 0.15nm |
| **Observed Δn** |  | -1.35x10-4, the accompanied thermo-optic effect raise the refractive index by 2x10-4 |  |  |

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